A MANUAL

OF

RULES, TABLES, AND DATA

FOR

MECHANICAL ENGINEERS.

A MANUAL

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RULES, TABLES, AND DATA

FOR

MECHANICAL ENGINEERS

BASED ON THE MOST RECENT INVESTIGATIONS:

OF CONSTANT USE
IN CALCULATIONS AND ESTIMATES RELATING TO

STRENGTH OF MATERIALS AND OF ELEMENTARY CONSTRUCTIONS; LABOUR; HEAT AND ITS APPLICATIONS, STEAM AND ITS PROPERTIES, COMBUSTION AND FUELS, STEAM BOILERS, STEAM ENGINES, HOT-AIR ENGINES, GAS-ENGINES; FLOW OF AIR AND OF WATER; AIR MACHINES; HYDRAULIC MACHINES; MILL-GEARING; FRICTION AND THE RESISTANCE OF MACHINERY, &c.; WEIGHTS, MEASURES, AND MONIES, BRITISH AND FOREIGN, WITH THE RECIPROCAL EQUIVALENTS FOR THE CONVERSION OF BRITISH AND FRENCH COMPOUND UNITS OF WEIGHT, PRESSURE, TIME, SPACE, AND MONEY; SPECIFIC GRAVITY AND THE WEIGHT OF BODIES; WEIGHT OF METALS, &c.

WITH

TABLES OF LOGARITHMS, CIRCLES, SQUARES, CUBES, SQUARE-ROOTS, AND CUBE-ROOTS; AND MANY OTHER USEFUL MATHEMATICAL TABLES.

BY

DANIEL KINNEAR CLARK,
MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS;
AUTHOR OF "RAILWAY MACHINERY," "EXHIBITED MACHINERY OF 1862," ETC.

FOURTH EDITION.

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GLASGOW AND EDINBURGH.
1889.

PREFACE.

THIS Work is designed as a book of general reference for Engineers:—to give within a moderate compass the leading rules and data, with numerous tables, of constant use in calculations and estimates relating to Practical Mechanics. The Author has endeavoured to concentrate the results of the latest investigations of others as well as his own, and to present the best information, with perspicuity, conciseness, and scientific accuracy.

Amongst the new and original features of this Work, the following may be named:—

In the section on Weights and Measures, the weight, volume, and relations of water and air as standards of measure, are concisely set forth. The various English measures, abstract and technical, are given in full detail, with tables of various wire-gauges in use: and equivalent values of compound units of weight, power, and measure—as, for example, miles per hour and feet per second. The French Metric Standards are defined, according to the latest determinations, with tables of metric weights and measures, equivalents of British and French weights and measures, and a number of convenient approximate equivalents. There is, in addition, a full table of equivalents of French and English compound units of weight, pressure, time, space, and money—as, for example, pounds per yard and kilogrammes per metre; which will be found of great utility for the reciprocal conversion of English and French units.

The tables of the Weight of bars, tubes, pipes, cylinders, plates, sheets, wires, &c., of iron and other metals, have been calculated expressly for this Work, and they contain several new features designed to add to their usefulness. They are accompanied by a summary of the various units of weight of wrought iron, cast iron, and steel, with plain rules for the weight.

In the section on Heat and its Applications, the received mechanical theory is defined and illustrated by examples. The relations of the pressure, volume, and temperature of air and other gases,

with their specific heat, are investigated in detail. The transmission of heat through plates and pipes, between water and water, steam and air, &c., for purposes of heating or cooling, is verified by many experimental data, which are reduced to units of performance.

The physical properties of steam are deduced from the results of Regnault's experiments, with the aid of the mechanical theory of heat. A very full table of the Properties of Saturated Steam is given. The table is, for the most part, reproduced from the article "Steam," contributed by the Author to the *Encyclopedia Britannica*, 8th edition, and it was the first published table of the same extent, in the English language, based on Regnault's data. An original table of the properties of saturated mixtures of air and aqueous vapour is added.

In the section on Combustion, new and simple formulas and data are given for the quantity of air consumed in combustion, and of the gaseous products of combustion, the heat evolved by combustion, the heating power of combustibles, and the temperature of combustion; with several tables.

On Coal as a Fuel, both English and Foreign, its composition, with the results of many series of experiments on its combustion, are collected and arranged. The quantity of air consumed in its combustion, and of the gaseous products, with the total heat generated, are calculated in detail. Coke, lignite, asphalte, wood, charcoal, peat, and peat-charcoal, are similarly treated; whilst the combustible properties of tan, straw, liquid-fuels, and coal-gas, are shortly treated.

The section on Strength of Materials is wholly new. The great accumulation of experimental data has been explored, and the most important results have been abstracted and tabulated. The results of the experiments of Mr. David Kirkaldy occupy the greater portion of the space, since he has contributed more, probably, than any other experimentalist to our knowledge of the Strength of Materials. The Author has investigated afresh the theory of the transverse strength and deflection of solid beams, and has deduced a new and simple series of formulas from these investigations, the truth of which has been established with remarkable force by the evidence of experiment. These investigations, based on the action of diagonal stress, throw light upon the element called by Mr. W. H. Barlow, "the resistance of flexure:" revealing, in a simple manner, the nature of that hitherto occult entity; and showing that flexure is not the cause, but the effect of the resistance. In addition to formulas

for beams of the ordinary form, special formulas have been deduced for the transverse strength and deflection of railway rails, double-headed or flanged, of iron or steel; in the establishment of which he has availed himself of the important experimental data published by Mr. R. Price Williams, and by Mr. B. Baker. To our knowledge of the strength of timber, Mr. Thomas Laslett has recently made important additions, and the results of his experiments have been somewhat fully abstracted and analyzed. But woods, by their extremely variable nature, are not amenable, like wrought-iron and steel, to the unconditional application of formulas for transverse strength. The Author has, nevertheless, deduced from the evidence, certain formulas for the transverse strength and deflection of woods, with tables of constants, which, if applied with intelligence and a knowledge of the uncertainties, cannot fail to prove of utility.

The Torsional Strength of Solid Bodies has also been investigated afresh, and reduced to new formulas.

In dealing with the Strength of Elementary Constructions, the Author has brought together many important experimental results. In treating of rivet-joints and their employment in steam-boilers, he has, he believes, clearly developed the elements of their strength and their weakness. By a close comparison of the results of tests of cast-iron flanged beams, it is plainly shown that the ultimate strength of a cast-iron beam is scarcely affected by the proportionate size of the upper flange, and that the lower flange and the web are, practically, the only elements which regulate the strength. The tests of solid-rolled and rivetted wrought-iron joists are also analyzed; and for the strength and deflection of these, as for those of cast-iron flanged beams, new and simple rules and formulas are given. A new investigation, with appropriate formulas, is given for the bursting strength of hollow cylinders, of whatever thickness. It is shown that the variation of stress throughout the thickness, follows a diminishing hyperbolic ratio from the inner surface towards the outer surface. The resistance of tubes and cylindrical flues to collapsing pressure is also investigated, and formulas based on the results of experience are given.

On the subject of Mill-gearing, a new and compact table of the pitch, number of teeth, and diameter of toothed wheels is given, with new formulas and tables for the strength and horse-power of the teeth of wheels, and for the weight of toothed wheels. New formulas and tables are given for the driving power of leather

belts, and the weight of cast-iron pulleys. For the strength of Shafting,—cast-iron, wrought-iron, and steel,—a new and complete series of formulas has been constructed, comprising its resistance to transverse deflection and to torsion, with very full tables of the weight, strength, power, and span of shafting.

The Evaporative Performance of Steam-boilers is exhaustively investigated with respect to the proportions of fuel, water, grate-area, and heating-surface, and the relations of these quantities are reduced to simple formulas for different types of boilers. The actual evaporative performances of boilers are abstracted in tabular form.

The Performance of Steam worked expansively, in single and in compound cylinders, is exhaustively analysed by the aid of diagrams; the similarity and the dissimilarity of its action in the Woolf-engine and the Receiver-engine, are investigated; and the principles of calculation to be applied respectively to these, the leading classes of compound engines, are explained. The best working ratios of expansion are deduced from the results of numerous experiments and observations on the performance of steam-engines.

The principles of Air-compressing Machines, and Compressed-air Engines are investigated, and convenient formulas and tables for use are deduced.

The whole of the materials for the preparation of this work have been drawn from the best available sources, foreign as well as English. Vast stores of the results of experience are accumulated in the *Proceedings of the Institution of Civil Engineers*, the *Proceedings of the Institution of Mechanical Engineers*, and other journals. From these and other sources, the Author has drawn much of his material.

D. K. CLARK.

8 Buckingham Street, Adelphi,
LONDON, *20th March*, 1877.

NOTE ON THE FOURTH EDITION.

I have thoroughly revised this book, and, besides correcting it up to date, I have introduced much new matter, which will render this edition even more valuable than the last.

D. K. CLARK.

January, 1889.

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I.

A MANUAL

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GEOMETRICAL PROBLEMS.

PROBLEMS ON STRAIGHT LINES.

PROBLEM I.—*To bisect a straight line, or an arc of a circle*, Fig. 1.—From the ends A, B, as centres, describe arcs intersecting at C and D, and draw C D, which bisects the line, or the arc, at the point E or F.

PROBLEM II.—*To draw a perpendicular to a straight line, or a radial line to a circular arc*, Fig. 1.—Operate

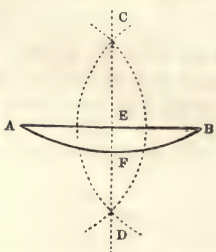


Fig. 1.—Probs. I. and II.

as in the foregoing problem. The line C D is perpendicular to A B: the line C D is also radial to the arc A B.

PROBLEM III.—*To draw a perpendicular to a straight line, from a given point in that line*, Fig. 2.—With any radius, from the given point A, in the line B C, cut the line at B and C; with a longer radius describe arcs from B

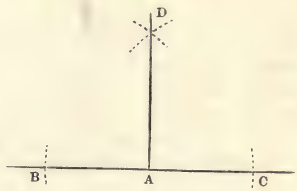


Fig. 2.—Prob. III.

and C, cutting each other at D, and draw the perpendicular D A.

2d Method, Fig. 3.—From any centre F, above B C, describe a circle passing through the given point A,

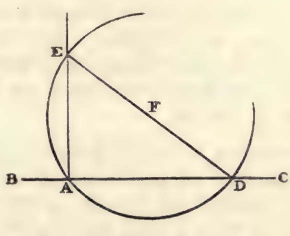


Fig. 3.—Prob. III. 2d method.

and cutting the given line at D; draw D F, and produce it to cut the circle at E; and draw the perpendicular A E.

3d Method, Fig. 4.—From A describe an arc E C, and from E, with the same radius, the arc A C, cutting

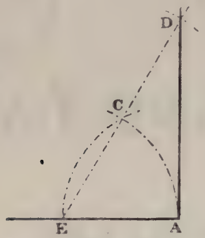


Fig. 4.—Prob. III. 3d method.

the other at C; through C draw a line E C D, and set off C D equal to C E; and through D draw the perpendicular A D.

4th Method, Fig. 5.—From the given point A set off a distance A E

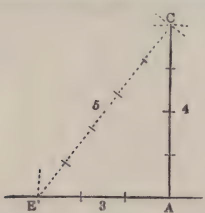


Fig. 5.—Prob. III. 4th method.

equal to three parts, by any scale; and on the centres A and E, with radii of four and five parts respectively, describe arcs intersecting at C. Draw the perpendicular A C.

Note.—This method is most useful on very large scales, where straight edges are inapplicable. Any multiples of the numbers 3, 4, 5 may be taken with the same effect, as 6, 8, 10, or 9, 12, 15.

PROBLEM IV.—*To draw a perpendicular to a straight line from any point without it*, Fig. 6.—From the point A, with a sufficient radius, cut the given line at F and G; and from these points describe arcs cutting at E. Draw the perpendicular A E.

Note.—If there be no room below the line, the intersection may be taken above the line; that is to say, between the line and the given point.

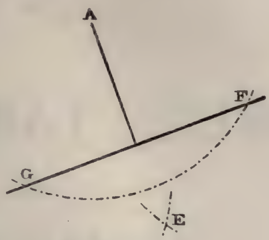


Fig. 6.—Prob. IV.

2d Method, Fig. 7.—From any two points B, C, at some distance apart,

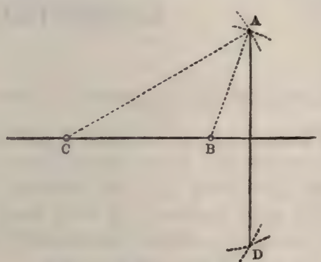


Fig. 7.—Prob. IV. 2d method.

in the given line, and with the radii B A, C A, respectively, describe arcs cutting at A D. Draw the perpendicular A D.

PROBLEM V.—*To draw a straight line parallel to a given line, at a given distance apart*, Fig. 8.—From the cen-

Fig. 8.—Prob. V.

tres A, B, in the given line, with the given distance as radius, describe arcs C, D; and draw the parallel line C D touching the arcs.

PROBLEM VI.—*To draw a parallel through a given point*, Fig. 9.—With a radius equal to the distance of the

given point C from the given line A B, describe the arc D from B, taken

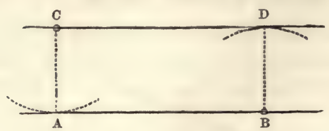


Fig. 9.—Prob. VI.

considerably distant from C. Draw the parallel through C to touch the arc D.

2d Method, Fig. 10.—From A, the

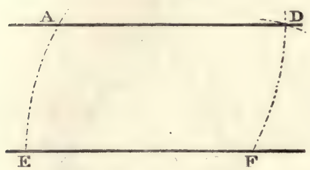


Fig. 10.—Prob. VI. 2d method.

given point, describe the arc F D, cutting the given line at F; from F, with the same radius, describe the arc E A, and set off F D equal to E A. Draw the parallel through the points A, D.

Note, Fig. 11.—When a series of parallels are required perpendicular to a base line A B, they may be drawn, as in Fig. 1, through points in the base line, set off at the required dis-

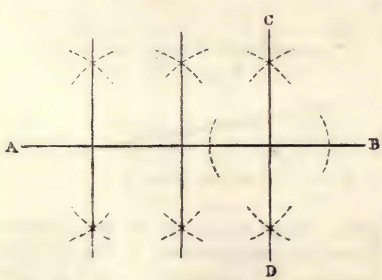


Fig. 11.—Prob. VI.

tances apart. This method is convenient also where a succession of parallels are required to a given line, C D; for the perpendicular A B may be drawn to it, and any number of parallels may be drawn upon the perpendicular.

PROBLEM VII.—*To divide a straight line into a number of equal parts*, Fig. 12.—To divide the line A B into, say, five parts. From A and B draw parallels A C, B D, on opposite sides. Set off any convenient distance four times

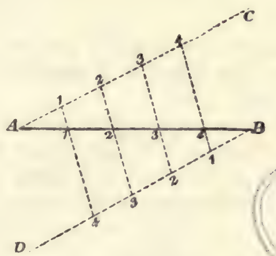


Fig. 12.—Prob. VII.

(one less than the given number) from A on A C, and from B on B D; join the first on A C to the fourth on B D, and so on. The lines so drawn divide A B as required.

2d Method, Fig. 13.—Draw the line A C at an angle from A, set off, say,

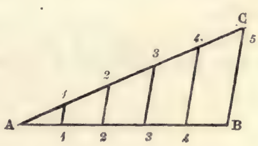


Fig. 13.—Prob. VII. 2d method.

five equal parts; draw B 5, and draw parallels to it from the other points of division in A C. These parallels divide A B as required.

Note.—By a similar process a line may be divided into a number of unequal parts; setting off divisions on A C, proportional by a scale to the required divisions, and drawing parallels cutting A B.

PROBLEM VIII.—*Upon a straight*

line to draw an angle equal to a given angle, Fig. 14.—Let A be the given angle, and FG the line. With any radius, from the points A and F, describe arcs DE, IH, cutting the sides of the angle A, and the line FG. Set

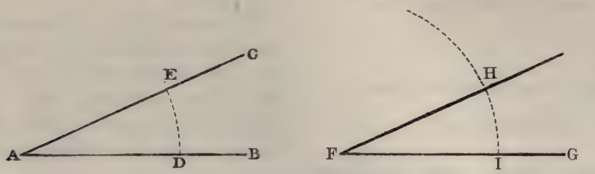


Fig. 14.—Prob. VIII.

off the arc IH equal to DE, and draw FH. The angle F is equal to A, as required.

To draw angles of 60° and 30°, Fig. 15.—From F, with any radius FI, describe an arc IH; and from I, with the same radius, cut the arc at H, and

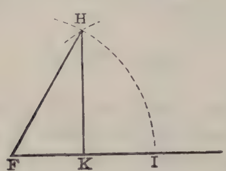


Fig. 15.—Prob. VIII.

draw FH to form the required angle IFH. Draw the perpendicular HK to the base line, to form the angle of 30° FHK.

To draw an angle of 45°, Fig. 16.—Set off the distance FI, draw the

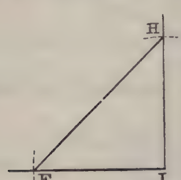


Fig. 16.—Prob. VIII.

perpendicular IH equal to IF, and join HF, to form the angle at F as required. The angle at H is also 45°.

PROBLEM IX.—*To bisect an angle*, Fig. 17.—Let ACB be the angle; on the centre C cut the sides at A, B. On A and B, as centres, describe arcs cutting at D. Draw CD, dividing the angle into two equal parts.

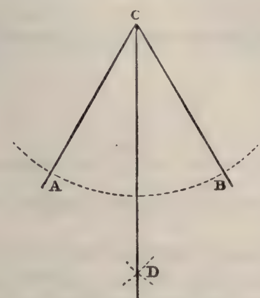


Fig. 17.—Prob. IX.

PROBLEM X.—*To bisect the inclination of two lines, of which the intersection is inaccessible*, Fig. 18.—Upon the

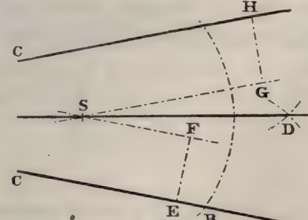


Fig. 18.—Prob. X.

given lines CB, CH, at any points, draw perpendiculars EF, GH, of equal lengths, and through F and G draw parallels to the respective lines, cutting at S; bisect the angle FSG, so formed, by the line SD, which divides equally the inclination of the given lines.

PROBLEMS ON STRAIGHT LINES AND CIRCLES.

PROBLEM XI.—*Through two given points to describe an arc of a circle with a given radius*, Fig. 19.—On the points

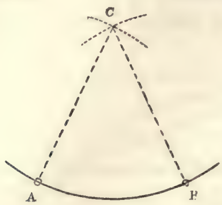


Fig. 19.—Prob. XI.

A and B as centres, with the given radius, describe arcs cutting at C; and from C, with the same radius, describe an arc A B as required.

PROBLEM XII.—*To find the centre of a circle, or of an arc of a circle.*

1st, for a circle, Fig. 20.—Draw the

PROBLEM XIII.—*To describe a circle passing through three given points*, Fig. 21.—Let A, B, C be the given points, and proceed as in last pro-

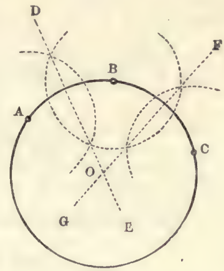


Fig. 21.—Prob. XII. XIII.

blem to find the centre O, from which the circle may be described.

Note.—This problem is variously useful:—in striking out the circular arches of bridges upon centerings, when the span and rise are given; describing shallow pans, or dished

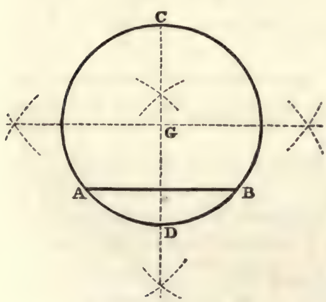


Fig. 20.—Prob. XII.

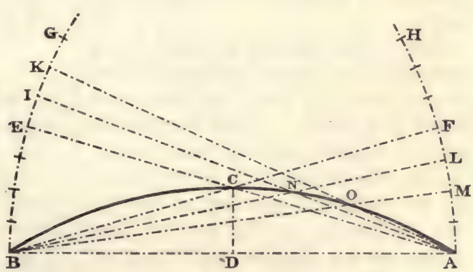


Fig. 22.—Prob. XIV. 1st method.

chord A B, bisect it by the perpendicular C D, bounded both ways by the circle; and bisect C D for the centre G.

2d, for a circle or an arc, Fig. 21.—Select three points, A, B, C, in the circumference, well apart; with the same radius, describe arcs from these three points, cutting each other; and draw the two lines, D E, F G, through their intersections, according to Fig. 1. The point O, where they cut, is the centre of the circle or arc.

covers of vessels; or finding the diameter of a fly-wheel or any other object of large diameter, when only a part of the circumference is accessible.

PROBLEM XIV.—*To describe a circle passing through three given points when the centre is not available.*

1st Method, Fig. 22.—From the extreme points A, B, as centres, describe arcs A H, B G. Through the third point C, draw A E, B F, cutting

the arcs. Divide A F and B E into any number of equal parts, and set off a series of equal parts of the same length on the upper portions of the arcs beyond the points E, F. Draw straight lines, B L, B M, &c., to the divisions in A F; and A I, A K, &c., to the divisions in E G: the successive intersections N, O, &c., of these lines, are points in the circle required, between the given points A and C, which may be filled in accordingly: similarly the remaining part of the curve B C may be described.

2d Method, Fig. 23.—Let A, D, B be the given points. Draw A B, A D, D B,

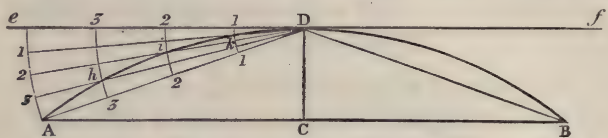


Fig. 23.—Prob. XIV. 2d method.

and *e f* parallel to A B. Divide D A into a number of equal parts at 1, 2, 3, &c., and from D describe arcs through these points to meet *e f*. Divide the arc A *e* into the same number of equal parts, and draw straight lines from D to the points of division. The intersections of these lines successively with the arcs 1, 2, 3, &c., are points in the circle which may be filled in as before.

Note.—The second method is not perfectly exact, but is sufficiently near to exactness for arcs less than one-fourth of a circle. When the middle point is equally distant from the extremes, the vertical C D is the rise of the arc; and this problem is serviceable for setting circular arcs of large radius, as for bridges of very great

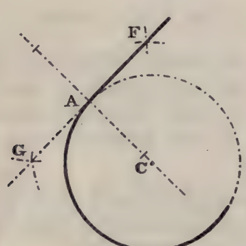


Fig. 24.—Prob. XV.

span, when the centre is unavailable; and for the outlines of bridge-beams, and of beams and connecting-rods of steam-engines, and the like.

PROBLEM XV.—*To draw a tangent to a circle from a given point in the circumference*, Fig. 24.—Through the given point A, draw the radial line

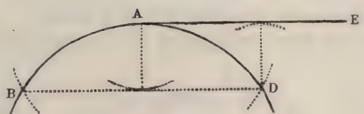


Fig. 25.—Prob. XV. 2d method.

A C, and the perpendicular F G is the tangent.

2d Method, when the centre is not available, Fig. 25.—From A, set off equal segments A B, A D; join B D, and draw A E parallel to it for the tangent.

PROBLEM XVI.—*To draw tangents to a circle from a point without it.*

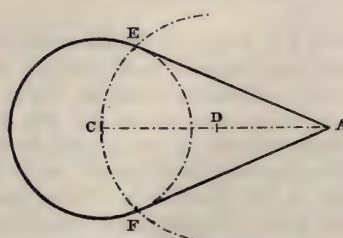


Fig. 26.—Prob. XVI. 1st method.

1*st Method*, Fig. 26.—Draw A C from the given point A to the centre

C; bisect it at D, and from the centre D, describe an arc through C, cutting the circle at E, F. Then A E, A F, are tangents.

2d Method, Fig. 27.—From A, with the radius A C, describe an arc B C D, and from C, with a radius equal to the

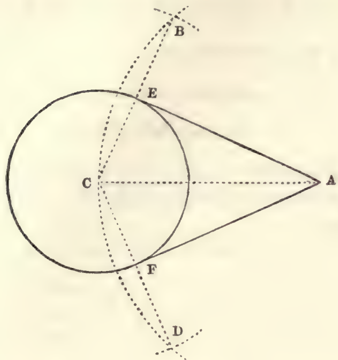


Fig. 27.—Prob. XVI. 2d method.

diameter of the circle, cut the arc at B, D; join B C, C D, cutting the circle at E, F, and draw A E, A F, the tangents.

Note.—When a tangent is already drawn, the exact point of contact may be found by drawing a perpendicular to it from the centre.

PROBLEM XVII.—*Between two inclined lines to draw a series of circles touching these lines and touching each other*, Fig. 28.—Bisect the inclination

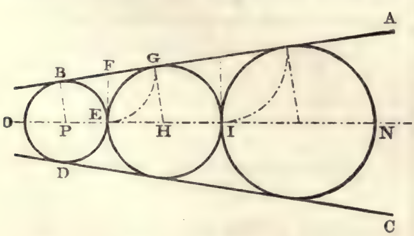


Fig. 28.—Prob. XVII.

of the given lines A B, C D by the line N O. From a point P in this line, draw the perpendicular P B to the line A B, and on P describe the circle B D touching the lines and cutting the centre line at E. From E draw E F perpendicular to the centre line, cutting A B at F, and from F describe an arc E G, cutting A B at G. Draw G H parallel to B P, giving H, the centre of the next circle, to be described with the radius H E, and so on for the next circle I N.

Inversely, the largest circle may be described first, and the smaller ones in succession.

Note.—This problem is of frequent use in scroll work.

PROBLEM XVIII.—*Between two inclined lines to draw a circular segment to fill the angle, and touching the lines*, Fig. 29.—Bisect the inclination

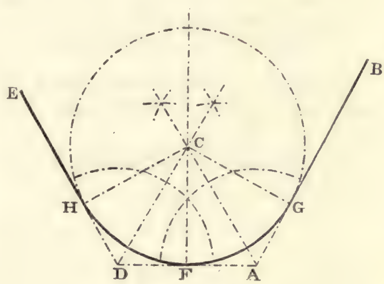


Fig. 29.—Prob. XVIII.

of the lines A B, D E by the line F C, and draw the perpendicular A F D to define the limit within which the circle is to be drawn. Bisect the angles A and D by lines cutting at C, and from C, with radius C F, draw the arc H F G as required.

PROBLEM XIX.—*To describe a circular arc joining two circles, and to touch one of them at a given point*, Fig. 30.—To join the circles A B, F G, by an arc touching one of them at F, draw the radius E F, and produce it both ways; set off F H equal to the radius A C of the other circle, join C H

and bisect it with the perpendicular L I, cutting E F at I. On the centre I,

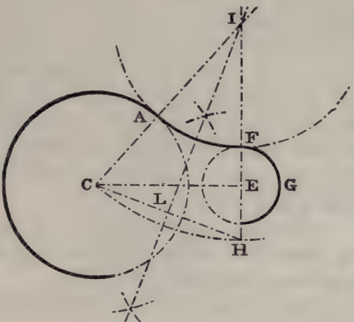


Fig. 30.—Prob. XIX.

with radius I F, describe the arc F A as required.

PROBLEMS ON CIRCLES AND RECTILINEAL FIGURES.

PROBLEM XX.—*To construct a triangle on a given base, the sides being given.*

1st. An equilateral triangle, Fig. 31.

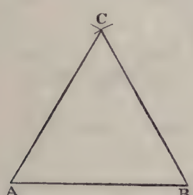


Fig. 31.—Prob. XX.

—On the ends of the given base, A, B, with A B as radius, describe arcs cutting at C, and draw A C, C B.

2d. A triangle of unequal sides, Fig. 32.—On either end of the base A D, with the side B as radius, describe an arc; and with the side C as radius, on the other end of the base as a centre, cut the arc at E. Join A E, D E.

Note.—This construction may be used for finding the position of a point, C or E, at given distances from the ends of a base, not necessarily to form a triangle.

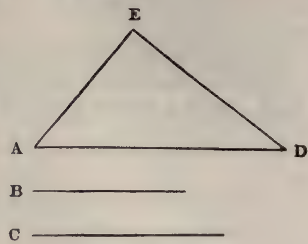


Fig. 32.—Prob. XX.

PROBLEM XXI.—*To construct a square or a rectangle on a given straight line.*

1st. A square, Fig. 33.—On the

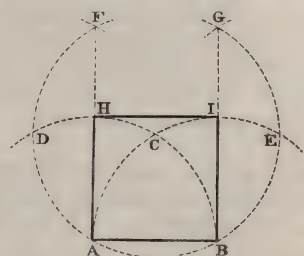


Fig. 33.—Prob. XXI.

ends A, B, as centres, with the line A B as radius, describe arcs cutting at C; on C, describe arcs cutting the others at D E; and on D and E, cut these at F G. Draw A F, B G, and join the intersections H, I.

2d. A rectangle, Fig. 34.—On the base E F, draw the perpendiculars E H,

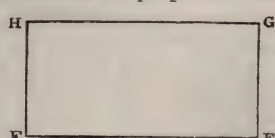


Fig. 34.—Prob. XXI.

F G, equal to the height of the rectangle, and join G H.

When the centre lines, A B, C D, Fig. 35, of a square or a rectangle are given, cutting at E.—Set off E F, E G,

the half lengths of the figure, and E H, E T, the half heights. On the centres H, T, with a radius of half the length,

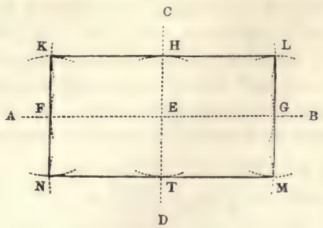


Fig. 35.—Prob. XXI.

describe arcs; and, on the centres F, G, with a radius of half the height, cut these arcs at K, L, M, N. Join these intersections.

PROBLEM XXII.—*To construct a parallelogram, of which the sides and one of the angles are given*, Fig. 36.—

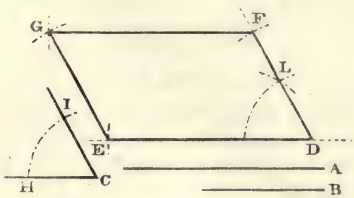


Fig. 36.—Prob. XXII.

Draw the side D E equal to the given length A, and set off the other side D F equal to the other length B, forming the given angle C. From E, with D F as radius, describe an arc, and from F, with the radius D E, cut the arc at G. Draw F G, E G.

Or, the remaining sides may be drawn as parallels to D E, D F.

The formation of the angle D is readily done as indicated, by taking the straight length of the arc H I and C I as radius, and finding the intersection L.

PROBLEM XXIII.—*To describe a circle about a triangle*, Fig. 37.—Bisect two sides A B, A C of the triangle at E, F, and from these points draw perpendiculars cutting at K. On the centre K, with the radius K A, draw the circle A B C.

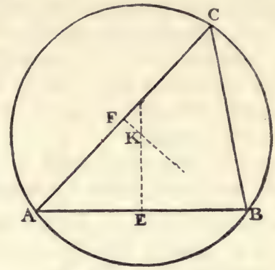


Fig. 37.—Prob. XXIII.

PROBLEM XXIV.—*To inscribe a circle in a triangle*, Fig. 38.—Bisect two of the angles A, C, of the triangle by lines cutting at D; from D draw a perpendicular D E to any side, and with D E as radius describe a circle.

When the triangle is equilateral, the centre of the circle may be found by bisecting two of the sides, and

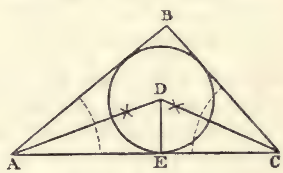


Fig. 38.—Prob. XXIV.

drawing perpendiculars as in the previous problem. Or, draw a perpendicular from one of the angles to the opposite side, and from the side set off one-third of the perpendicular.

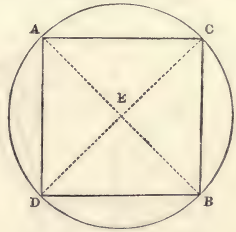


Fig. 39.—Prob. XXV.

PROBLEM XXV.—*To describe a circle about a square, and to inscribe a square in a circle*, Fig. 39.

1st. To describe the circle. Draw the diagonals A B, C D of the square, cutting at E; on the centre E, with the radius E A, describe the circle.

2d. To inscribe the square.—Draw the two diameters A B, C D at right angles, and join the points A, B, C, D to form the square.

Note.—In the same way a circle may be described about a rectangle.

PROBLEM XXVI.—*To inscribe a circle in a square, and to describe a square about a circle*, Fig. 40.

1st. To inscribe the circle.—Draw

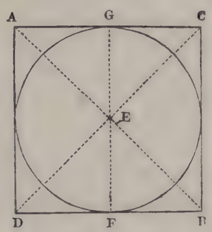


Fig. 40.—Prob. XXVI.

the diagonals A B, C D of the square, cutting at E; draw the perpendicular E F to one side, and with the radius E F describe the circle.

2d. To describe the square.—Draw two diameters A B, C D at right angles, and produce them; bisect the angle D E B at the centre by the diameter F G, and through F and G draw per-

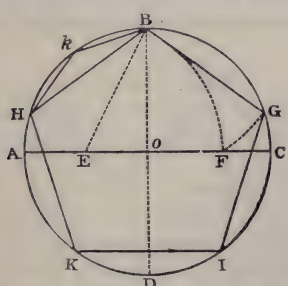


Fig. 41.—Prob. XXVII.

pendiculars A C, B D, and join the points A D and B C, where they cut the diagonals, to complete the square.

PROBLEM XXVII.—*To inscribe a pentagon in a circle*, Fig. 41.—Draw two diameters A C, B D at right angles, cutting at O; bisect A O at E, and from E, with radius E B, cut A C at F; from B, with radius B F, cut the circumference at G, H, and with the same radius step round the circle to I and K; join the points so found to form the pentagon.

PROBLEM XXVIII.—*To construct a hexagon upon a given straight line*, Fig. 42.—From A and B, the ends of the given line, describe arcs cutting at *g*; from *g*, with the radius *g* A, de-

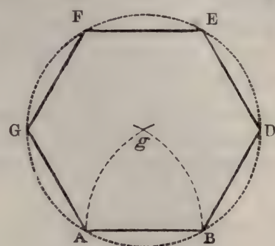


Fig. 42.—Prob. XXVIII.

scribe a circle; with the same radius set off the arcs A G, G F, and B D, D E. Join the points so found to form the hexagon.

PROBLEM XXIX.—*To inscribe a hexagon in a circle*, Fig. 43.—Draw a diameter A C B; from A and B as centres, with the radius of the circle A C, cut the circumference at D, E, F, G; and draw A D, D E, &c. to form the hexagon.

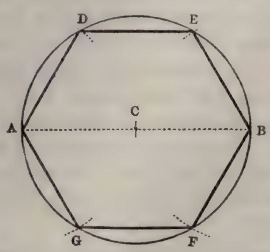


Fig. 43.—Prob. XXIX.

The points D, E, &c., may also be found by stepping the radius six times round the circle.

PROBLEM XXX.—*To describe a hexagon about a circle*, Fig. 44.—Draw a

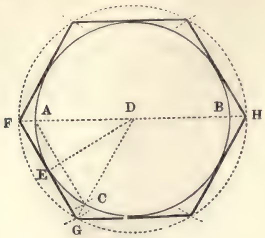


Fig. 44.—Prob. XXX.

diameter A D B, and with the radius A D, on the centre A, cut the circumference at C; join A C, and bisect it with the radius D E; through E draw the parallel F G cutting the diameter at F, and with the radius D F describe the circle F H. Within this circle describe a hexagon by the preceding problem; it touches the given circle.

PROBLEM XXXI.—*To describe an octagon on a given straight line*, Fig. 45.

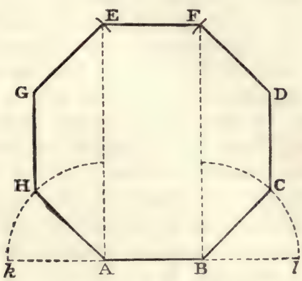


Fig. 45.—Prob. XXXI.

—Produce the given line A B both ways, and draw perpendiculars A E, B F; bisect the external angles A and B, by the lines A H, B C, which make equal to A B. Draw C D and H G parallel to A E, and equal to A B; from the centres G, D, with the radius A B, cut the perpendiculars at E, F, and draw E F to complete the octagon.

PROBLEM XXXII.—*To convert a square into an octagon*, Fig. 46.—Draw the diagonals of the square cutting at *e*; from the corners A, B, C, D, with A *e* as radius, describe arcs cutting the

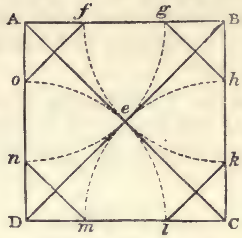


Fig. 46.—Prob. XXXII.

sides at *g*, *h*, &c.; and join the points so found to form the octagon.

PROBLEM XXXIII.—*To inscribe an octagon in a circle*, Fig. 47.—Draw

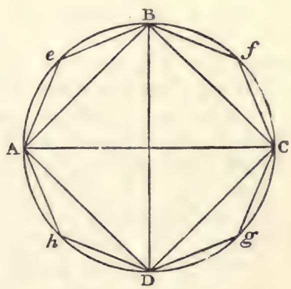


Fig. 47.—Prob. XXXIII.

two diameters A C, B D at right angles; bisect the arcs A B, B C, &c., at *e*, *f*, &c., and join A *e*, *e* B, &c., to form the octagon.

PROBLEM XXXIV.—*To describe an octagon about a circle*, Fig. 48.—

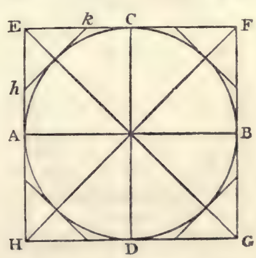


Fig. 48.—Prob. XXXIV.

Describe a square about the given circle A B, draw perpendiculars *h k*,

&c., to the diagonals, touching the circle, to form the octagon.

Or, the points *h*, *k*, &c., may be found by cutting the sides from the corners of the square, as in the second last problem.

PROBLEM XXXV.—*To describe a polygon of any number of sides upon a given straight line*, Fig. 49.—Produce

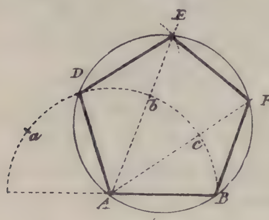


Fig. 49.—Prob. XXXV.

the given line A B, and on A, with the radius A B, describe a semicircle, divide the semi-circumference into as many equal parts as there are to be sides in the polygon; say, in this example, five sides. Draw lines from A through the divisional points D, *b*, and *c*, omitting one point *a*; and on the centres B, D, with the radius A B, cut A *b* at E and A *c* at F. Draw D E, E F, F B to complete the polygon,

PROBLEM XXXVI.—*To inscribe a circle within a polygon*, Figs. 50, 51.—When the polygon has an even number of sides, Fig. 50, bisect two op-

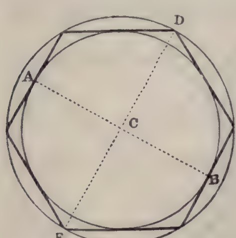


Fig. 50.—Prob. XXXVI. XXXVII.

posite sides at A and B, draw A B, and bisect it at C by a diagonal D E; and with the radius C A describe the circle.

When the number of sides is odd, Fig. 51, bisect two of the sides at A

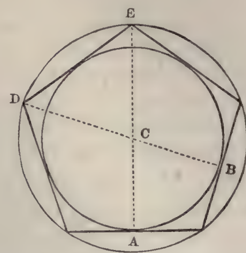


Fig. 51.—Prob. XXXVI. XXXVII.

and B, and draw lines A E, B D to the opposite angles, intersecting at C; from C, with the radius C A, describe the circle.

PROBLEM XXXVII.—*To describe a circle without a polygon*, Figs. 50, 51.—Find the centre C as before, and with the radius C D describe the circle.

The foregoing selection of problems on regular figures are the most useful in mechanical practice on that subject. Several other regular figures may be constructed from them by bisection of the arcs of the circumscribing circles. In this way a decagon, or ten-sided polygon, may be formed from the pentagon, as shown by the bisection of the arc B H at *k* in Fig. 41. Inversely, an equilateral triangle may be inscribed by joining the alternate points of division found for a hexagon.

PROBLEM XXXVIII.—*To inscribe a polygon of any number of sides within a circle*, Fig. 52.—Draw the diameter A B, and through the centre E draw the perpendicular E C, cutting the circle at F. Divide E F into four equal parts, and set off three parts equal to those from F to C. Divide the diameter A B into as many equal parts as the polygon is to have sides; and from C draw C D through the second point of division, cutting the circle at D. Then A D is equal to one

side of the polygon, and by stepping round the circumference with the

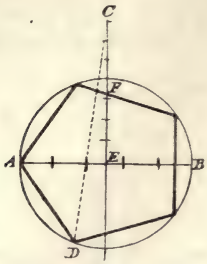


Fig. 52.—Prob. XXXVIII.

length A D, the polygon may be completed.

The constructions for inscribing regular polygons in circles are suitable also for dividing the circumference of a circle into a number of equal parts. To supply a means of dividing the circumference into *any* number of parts, including cases not provided for in the foregoing problems, the annexed table of angles relating to polygons, expressed in degrees, will be found of general utility. In this table the angle at

TABLE OF POLYGONAL ANGLES.

Number of Sides.	Angle at Centre.	Number of Sides.	Angle at Centre.
No.	Degrees.	No.	Degrees.
3	120	12	30
4	90	13	$27\frac{9}{13}$
5	72	14	$25\frac{5}{7}$
6	60	15	24
7	$51\frac{3}{7}$	16	$22\frac{1}{2}$
8	45	17	$21\frac{3}{17}$
9	40	18	20
10	36	19	19
11	$32\frac{8}{11}$	20	18

the centre is found by dividing 360°, the number of degrees in a circle, by the number of sides in the polygon; and by setting off round the centre of the circle a succession of angles by means of the protractor, equal to the angle in the table due to a given number of sides, the radii so drawn will divide the circumference into the same number of parts. The triangles thus formed are termed the elementary triangles of the polygon.

PROBLEM XXXIX.—*To inscribe any regular polygon in a given circle; or to divide the circumference into a given number of equal parts, by means of the angle at the centre.* Fig. 53.—

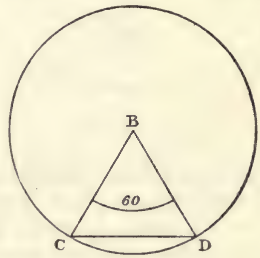


Fig. 53.—Prob. XXXIX.

Suppose the circle is to contain a hexagon, or is to be divided at the circumference into six equal parts. Find the angle at the centre for a hexagon, or 60°; draw any radius B C, and set off, by a protractor or otherwise, the angle at the centre C B D equal to 60°; then the interval C D is one side of the figure, or segment of the circumference; and the remaining points of division may be found either by stepping along the circumference with the distance C D in the dividers, or by setting off the remaining five angles, of 60° each, round the centre.

PROBLEMS ON THE ELLIPSE.

An ellipse is an oval figure, like a circle in perspective. The line A B, Fig. 54, that divides it equally in the direction of its greatest dimension, is the *transverse axis;* and the perpendicular C D, through the centre, is the *conjugate axis.* Two points, F, G, in the transverse axis, are the

foci of the curve, each being called a *focus;* being so placed that the sum of their distances from either end of the conjugate axis, C or D, is equal

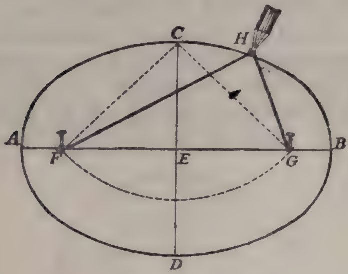


Fig. 54.—Prob. XL.

to the transverse axis. In general, the sum of their distances from any other point in the curve is equal to the transverse axis. A line drawn at right angles to either axis, and terminated by the curve, is a *double ordinate*, and each half of it is an *ordinate.* The segments of an axis between an ordinate and its vertices are called *abscisses.* The double ordinate drawn through a focus is called the *parameter* of the axis.

The squares of any two ordinates to the transverse axis, are to each other as the rectangles of their respective abscisses.

PROBLEM XL.—*To describe an ellipse when the length and breadth are given*, Fig. 54.—On the centre C, with A E as radius, cut the axis A B at F and G, the foci; fix a couple of pins into the axis at F and G, and loop on a thread or cord upon them equal in length to the axis A B, so as when stretched to reach to the extremity C of the conjugate axis, as shown in dotlining. Place a pencil or drawpoint inside the cord, as at H, and guiding the pencil in this way, keeping the cord equally in tension, carry the pencil round the pins F, G, and so describe the ellipse.

Note.—This method is employed in setting off elliptical garden-plots, walks, &c.

2d Method, Fig. 55.—Along the straight edge of a slip of stiff paper, mark off a distance *a c* equal to A C,

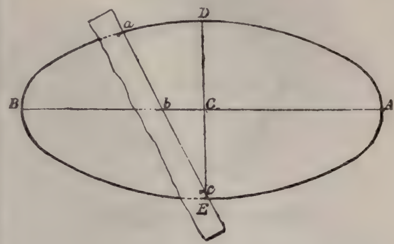


Fig. 55.—Prob. XL. 2d method.

half the transverse axis; and from the same point a distance *a b* equal to C D, half the conjugate axis. Place the slip so as to bring the point *b* on the line A B of the transverse axis, and the point *c* on the line D E; and set off on the drawing the position of the point *a.* Shifting the slip, so that the point *b* travels on the transverse axis, and the point *c* on the conjugate axis, any number of points in the curve may be found, through which the curve may be traced.

3d Method, Fig. 56.—The action of the preceding method may be embodied so as to afford the means of

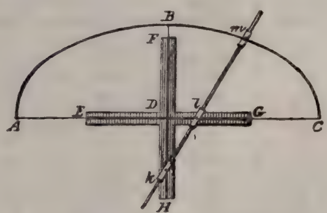


Fig. 56.—Prob. XL. 3d method.

describing a large curve continuously, by means of a bar *m k*, with steel points *m*, *l*, *k*, rivetted into brass slides adjusted to the length of the semiaxes, and fixed with setscrews. A rectangular cross E G, with guiding slots, is placed coinciding with the two

axes of the ellipse, A C and B H; by sliding the points k, l, in the slots, and carrying round the point m, the curve may be continuously described. A pen or pencil may be fixed at m.

4th Method, Fig. 57.—Bisect the transverse axis at C, and through C

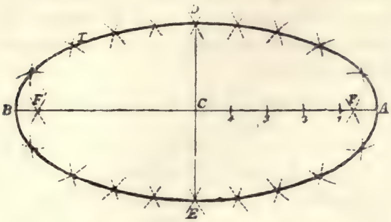


Fig. 57.—Prob. XL. 4th method.

draw the perpendicular D E, making C D and C E each equal to half the conjugate axis. From D or E, with the radius A C, cut the transverse axis at F, F′, for the foci. Divide A C into a number of parts at the points 1, 2, 3, &c. With the radius A 1, on F and F′ as centres, describe arcs; and with the radius B 1, on the same centres, cut these arcs as shown. Repeat the operation for the other divisions of the transverse axis. The series of intersections thus made are points in the curve, through which the curve may be traced.

5th Method, Fig. 58.—On the two

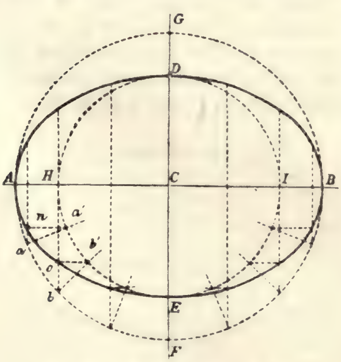


Fig. 58.—Prob. XL. 5th method.

axes A B, D E as diameters, on centre C, describe circles; from a number of points, a, b, &c., in the circumference A F B, draw radii cutting the inner circle at a', b', &c. From a, b, &c., draw perpendiculars to A B; and from a', b', &c., draw parallels to A B, cutting the respective perpendiculars at n, o, &c. The intersections are points in the curve, through which the curve may be traced.

6th Method, Fig. 59.—When the transverse and conjugate diameters

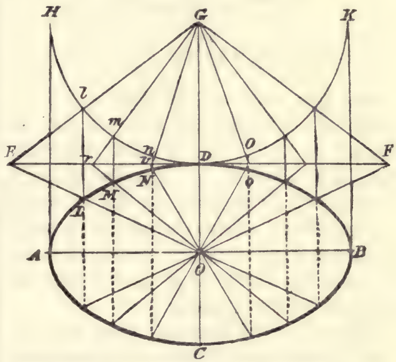


Fig. 59.—Prob. XL. 6th method.

are given, A B, C D, draw the tangent E F parallel to A B. Produce C D, and on the centre G, with the radius of half A B, describe a semicircle H D K; from the centre G draw any number of straight lines to the points E, r, &c., in the line E F, cutting the circumference at l, m, n, &c.; from the centre O of the ellipse draw straight lines to the points E, r, &c., and from the points l, m, n, &c., draw parallels to G C, cutting the lines O E, O r, &c., at L, M, N, &c. These are points in the circumference of the ellipse, and the curve may be traced through them. Points in the other half of the ellipse are formed by extending the intersecting lines as indicated in the figure.

PROBLEM XLI.—*To describe an ellipse approximately by means of circular arcs.—First,* with arcs of two radii, Fig. 60.—Find the difference

of the two axes, and set it off from the centre O to *a* and *c*, on O A and O C;

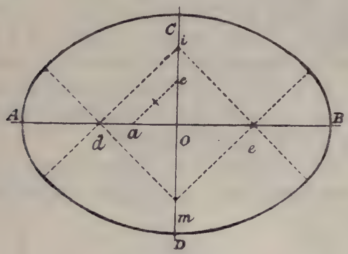


Fig. 60.—Prob. XLI.

draw *a c*, and set off half *a c* to *d*; draw *d i* parallel to *a c*, set off O *e* equal to O *d*, join *e i*, and draw the parallels *e m*, *d m*. From *m*, with radius *m* C, describe an arc through C; and from *i* describe an arc through D; from *d* and *e* describe arcs through A and B. The four arcs form the ellipse approximately.

Note.—This method does not apply satisfactorily when the conjugate axis is less than two-thirds of the transverse axis.

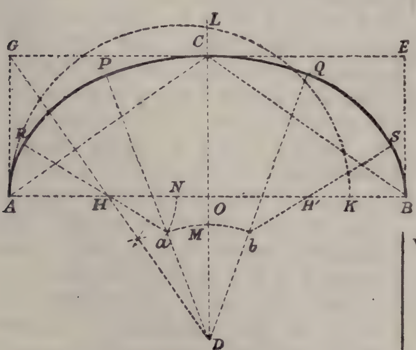


Fig. 61.—Prob. XLI. 2d method.

Second, with arcs of three radii, Fig. 61.—On the transverse axis A B draw the rectangle B G, on the height O C; to the diagonal A C draw the perpendicular G H D; set off O K equal to O C, and describe a semicircle on A K, and produce O C to L; set off O M equal to C L, and on D describe an arc with radius D M; on A, with radius O L, cut this arc at *a*. Thus the five centres D, *a*, *b*, H, H′ are found, from which the arcs are described to form the ellipse.

Note.—This process works well for nearly all proportions of ellipses. It is employed in striking out vaults and stone bridges.

PROBLEM XLII.—*To draw a tangent to an ellipse through a given point in the curve*, Fig. 62.—From the given point T draw straight lines to the foci F, F′; produce F T beyond the curve to *c*, and bisect the exterior angle *c* T F, by the line T *d*, which is the tangent.

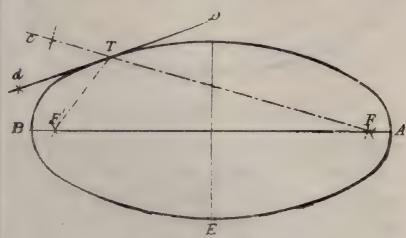


Fig. 62.—Prob. XLII.

PROBLEM XLIII.—*To draw a tangent to an ellipse from a given point without the curve*, Fig. 63.—From the given point T, with a radius to the nearest focus F, describe an arc on the other focus F′, with a radius equal to the transverse axis, cut the arc at K L, and draw K F′, L F′, cutting the curve at M, N. The lines T M, T N are tangents.

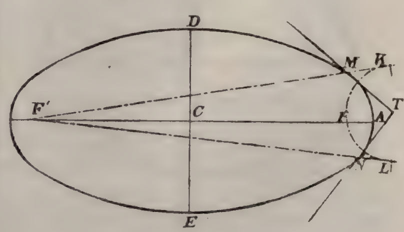


Fig. 63.—Prob. XLIII.

PROBLEMS ON THE PARABOLA.

A parabola, D A C, Fig. 64, is a curve such that every point in the curve is equally distant from the *directrix* K L and the focus F. The focus lies in the *axis* A B drawn from the *vertex* or head of the curve A, so as to divide the figure into two equal parts. The vertex A is equidistant from the directrix and the focus, or A e = A F. Any line parallel to the axis is a diameter. A straight line, as E G or D C, drawn across the figure at right angles to the axis is a double ordinate, and either half of it is an ordinate. The ordinate to the axis E F G, drawn through the focus, is called the *parameter* of the axis. A segment of the axis, reckoned from the vertex, is an *absciss* of the axis; and it is an absciss of the ordinate drawn from the base of the absciss. Thus, A B is an absciss of the ordinate B C.

Abscisses of a parabola are as the squares of their ordinates.

PROBLEM XLIV.—*To describe a parabola when an absciss and its ordinate are given;* that is to say, when the height and breadth are given, Fig. 64.—Bisect the given ordinate

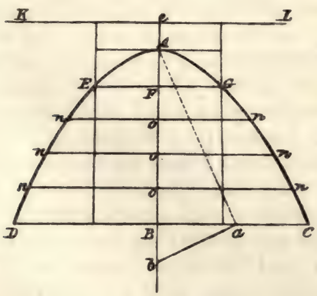


Fig. 64.—Prob. XLIV.

B C at a; draw A a, and then $a\,b$ perpendicular to it, meeting the axis at b. Set off A e, A F, each equal to B b; and draw K e L perpendicular to the axis. Then K L is the directrix and F is the focus. Through F and any number of points, o, o, &c., in the axis, draw double ordinates, $n\ o\ n$, &c.; and on the centre F, with the radii F e, o e, &c., cut the respective ordinates at E, G, n, n, &c. The curve may be traced through these points as shown.

2d Method; by means of a square and a cord, Fig. 65.—Place a straight-

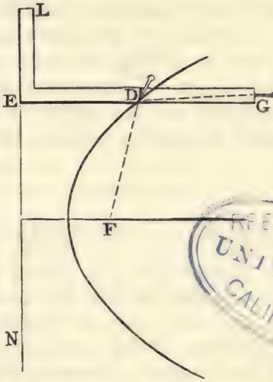


Fig. 65.—Prob. XLIV. 2d method.

edge to the directrix E N, and apply to it a square L E G. Fasten to the end G, one end of a thread or cord equal in length to the edge E G, and attach the other end to the focus F; slide the square along the straight-edge, holding the cord taut against the edge of the square by a draw-point or pencil D, by which the curve is described.

3d Method; when the height and the base are given, Fig. 66.—Let A B be

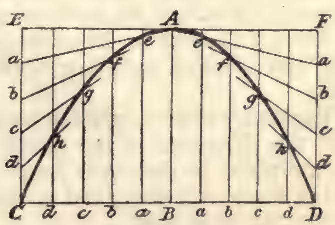


Fig. 66.—Prob. XLIV. 3d method.

the given axis, and C D a double ordinate or base; to describe a parabola

of which the vertex is at A. Through A draw E F parallel to C D, and through C and D draw C E and D F parallel to the axis. Divide B C and B D into any number of equal parts, say five, at *a*, *b*, &c., and divide C E and D F into the same number of parts. Through the points *a*, *b*, *c*, *d* in the base C D, on each side of the axis, draw perpendiculars, and through *a*, *b*, *c*, *d*, in C E and D F, draw lines to the vertex A, cutting the perpendiculars at *e*, *f*, *g*, *h*. These are points in the parabola, and the curve C A D may be traced as shown, passing through them.

PROBLEMS ON THE HYPERBOLA.

The *vertices* A, B, Fig. 67, of opposite hyperbolas, are the heads of the curves, and are points in their centre or axial lines. The *transverse axis* A B is the distance between the vertices, of which the centre C is the *centre.* The *conjugate* axis G H is a straight line drawn through the centre at right angles to the transverse axis. An ordinate F K is a straight line drawn from any point of the curve perpendicular to the axis. The segments of the transverse axis A F, B F, between an ordinate F K and the vertices of the curves, are *abscisses.* The *parameter* is the double ordinate drawn through the focus. The *asymptotes* are two straight lines, S S, R R, drawn from the centre through the ends of a tangent E D at the vertex, equal and parallel to the conjugate axis, and bisected by the transverse axis.

The nature of the hyperbola is such that the difference of the distances of any point in the curve from the foci is always the same, and is equal to the transverse axis.

In a hyperbola the squares of any two ordinates to the transverse axes are to each other as the rectangles of their abscisses.

PROBLEM XLV.—*To describe a hyperbola, the transverse and conjugate axes being given*, Fig. 67.—Draw A B

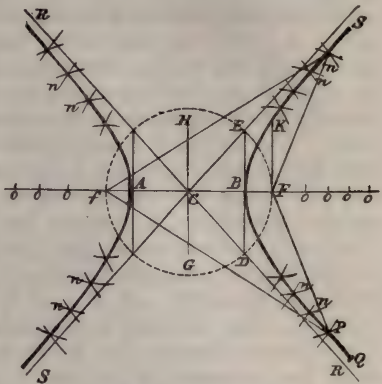


Fig. 67.—Prob. XLV.

equal to the transverse axis, and D E perpendicular to it and equal to the conjugate G H. On C, with the radius C E, describe a circle cutting A B produced, at F *f*; these points are the foci. In A B produced take any number of points *o*, *o*, &c., with the radii A *o*, B *o*, and on centres F, *f* describe arcs cutting each other at *n*, *n*, &c. These are points in the curve, through which it may be traced.

2d Method, Fig. 67.—The curve may be drawn thus:—Let the ends of two threads *f* P Q, F P Q, be fastened at the points *f*, F, and be made to pass through a small bead or pin P, and knotted together at Q. Take hold of Q, and draw the threads tight; move the bead along the threads, and the point P will describe the curve. If the end of the long thread be fixed at F, and the short thread at *f*, the opposite curve may be described in the same manner.

Or, the line *f* Q may be replaced by a straight-edge turning on a pin at *f*, and the cord F Q joined to it at Q. The curve may then be described by means of a point or pencil in the same manner as for the parabola, Fig. 65.

3d Method; when the breadth C D,

height A B, *and transverse axis* A A′ *of the curve are given*, Fig. 68.—Divide

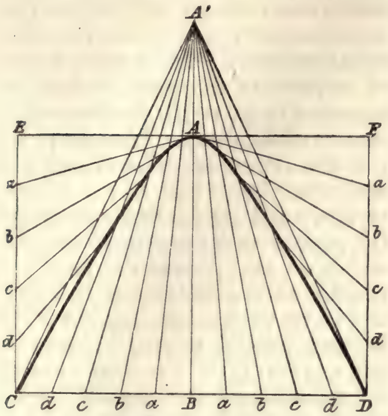


Fig. 68.—Prob. XLV. 3d method.

the base or double ordinate C D into a number of equal parts on each side of the axis at *a, b,* &c.; and divide the parallels C E, D F, into the same number of equal parts at *a, b,* &c. From the points *a, b,* &c., in the base, draw lines to A′, and from the points *a, b,* &c., in the verticals, draw lines to A, cutting the respective lines from the base. Trace the curve through the intersections thus obtained.

THE CYCLOID AND EPICYCLOID.

PROBLEM XLVI.—*To describe a cycloid*, Fig. 69.—When a wheel or a circle D G C rolls along a straight line

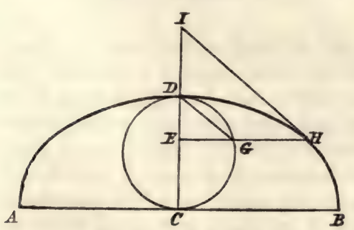


Fig. 69.—Prob. XLVI.

A B, Fig. 69, beginning at A and ending at B, where it has just completed one revolution, it measures off a straight line A B exactly equal to the circumference of the circle D G C, which is called the *generating circle*, and a point or pencil fixed at the point D in the circumference traces out a curvilinear path A D B, called a *cycloid.* A B is the *base* and C D is the *axis* of the cycloid.

Place the generating circle in the middle of the cycloid, as in the figure, draw a line E H parallel to the base, cutting the circle at G; and the tangent H I to the curve at the point H. Then the following are some of the properties of the cycloid:—

The horizontal line H G = arc of the circle G D.

The half-base A C = the half-circumference C G D.

The arc of the cycloid D H = twice the chord D G.

The half-arc of the cycloid D A = twice the diameter of the circle D C.

Or, the whole arc of the cycloid A D B = four times the axis C D.

The area of the cycloid A D B A = three times the area of the generating circle D C.

The tangent H I is parallel to the chord G D.

PROBLEM XLVII.—*To describe an exterior epicycloid*, Fig. 70.—The *epicycloid* differs from the cycloid in this,

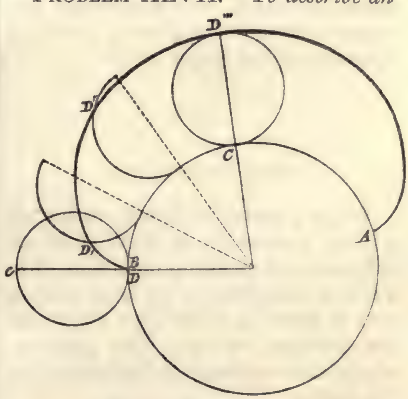


Fig. 70.—Prob. XLVII.

that it is generated by a point D in one circle D C rolling upon the circumference of another circle A C B, instead of on a flat surface or line; the former being the *generating circle*, and the latter the *fundamental circle.* The generating circle is shown in four positions, in which the generating point is successively marked D, D′, D″, D‴. A D‴ B is the epicycloid.

PROBLEM XLVIII.—*To describe*

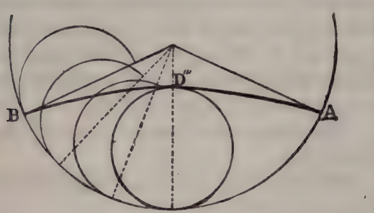


Fig. 71.—Prob. XLVIII.

an interior epicycloid, Fig. 71.—If the generating circle be rolled on the inside of the fundamental circle, as in Fig. 71, it forms an *interior epicycloid, or hypocycloid*, A D‴ B, which becomes in this case nearly a straight line. The other points of reference in the figure correspond to those in Fig. 70. When the diameter of the generating circle is equal to half that of the fundamental circle, the epicycloid becomes a straight line, being in fact a diameter of the larger circle.

THE CATENARY.

When a perfectly flexible string, or a chain consisting of short links, is suspended from two points M, N, Fig. 72, it is stretched by its own weight, and it forms a curve line known as the catenary, M C N. The point C, where the catenary is horizontal, is the *vertex.*

PROBLEM XLIX.—*To describe a catenary*, Fig. 72.—Draw the vertical C G equal to the length of the arc of the chain, M C, on one side of the vertex, and divide it into a great number of equal parts, at (1), (2), (3), &c. Draw the horizontal line C H equal to the length of so much of the rope or chain as measures by its weight the horizontal tension of the chain. From the point C as the vertex, set off C (1) on the horizontal line equal to C 1 on the vertical; and (1) (2) from the point (1), parallel to H 1 and equal to C (1); and again (2) (3) from the point (2) parallel to H 2 and equal to C (1); and so on till the last segment (6) M is drawn parallel to H G. The polygon C (1) (2) (3) . . . M, thus formed, is approximately the catenary curve, which may be traced through the middle points of the sides of the polygon. A similar process being performed for the other side of the curve, the catenary is completed.

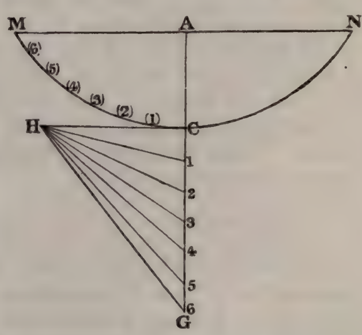


Fig. 72.—Prob. XLIX.

2d Method.—Suspend a finely linked chain against a vertical wall. The curve may be traced from it, on the wall, answering the conditions of given length and height, or of given width or length of arc. A cord having numerous equal weights suspended from it at short and equal distances may be used.

CIRCLES.

The circumference of a circle is commonly signified in mathematical discussions by the symbol π, which indicates the length of the circumference when the diameter = 1.

The area of a circle is as the square of the diameter, or the square of the circumference.

The ratio of the diameter to the circumference is as 1 to 3·141593—
commonly abbreviated,............................as 1 to 3·1416
approximately,............................as 1 to $3\frac{1}{7}$
or as 7 to 22

When the diameter = 1, the area is equal to..................·785398 +
or, commonly abbreviated,..............................·7854
approximately,..............................$\frac{4}{5}$ths.

When the circumference = 1, the area is equal to............·079577 +
or, abbreviated,..............................·0796
approximately,..............................$\frac{2}{25}$ths, or ·08.

In these ratios, the diameter and the circumference are taken lineally, and the area superficially. So that if the diameter = 1 foot, the circumference is equal to 3·1416 feet, and the area is equal to ·7854 square foot.

Note.—If the first three odd figures, 1, 3, 5, be each put down twice, the first three of these will be to the last three, that is 113 is to 355, as the diameter to the circumference.

PLANE TRIGONOMETRY.

The circumference of a circle is supposed to be divided into 360 degrees or divisions, and as the total angularity about the centre is equal to four right angles, each right angle contains 90 degrees, or 90°, and half a right angle

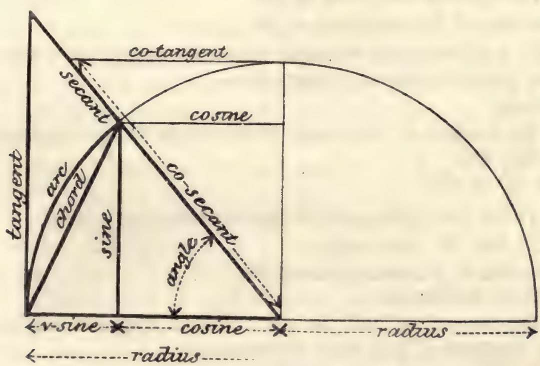


Fig. 73.—Definitions in Plane Trigonometry.

contains 45°. Each degree is divided into 60 minutes, or 60′; and, for the sake of still further minuteness of measurement, each minute is divided into 60 seconds, or 60″. In a whole circle there are, therefore, 360 × 60 × 60 =

1,296,000 seconds. The annexed diagram, Fig. 73, exemplifies the relative positions of the sine, cosine, versed sine, tangent, co-tangent, secant, and co-secant of an angle. It may be stated, generally, that the correlated quantities, namely, the cosine, co-tangent, and co-secant of an angle, are the sine, tangent, and secant, respectively, of the complement of the given angle, the complement being the difference between the given angle and a right angle. The supplement of an angle is the amount by which it is less than two right angles.

When the sines and cosines of angles have been calculated (by means of formulas which it is not necessary here to particularize), the tangents, co-tangents, secants, and co-secants are deduced from them according to the following relations:—

$$\text{tan.} = \frac{\text{rad.} \times \text{sin.}}{\text{cos.}}; \quad \text{cotan.} = \frac{\text{rad.}^2}{\text{tan.}}; \quad \text{sec.} = \frac{\text{rad.}^2}{\text{cos.}}; \quad \text{cosec.} = \frac{\text{rad.}^2}{\text{sin.}}.$$

For these the values will be amplified in tabular form.

A triangle consists of three sides and three angles. When any three of these are given, including a side, the other three may be found by calculation:—

CASE 1.—*When a side and its opposite angle are two of the given parts.*

RULE 1. *To find a side*, work the following proportion:—
as the sine of the angle opposite the given side
is to the sine of the angle opposite the required side,
so is the given side
to the required side.

RULE 2. *To find an angle:—*
as the side opposite to the given angle
is to the side opposite to the required angle,
so is the sine of the given angle
to the sine of the required angle.

RULE 3. *In a right-angled triangle, when the angles and one side next the right angle are given, to find the other side:—*
as radius
is to the tangent of the angle adjacent to the given side,
so is this side
to the other side.

CASE 2.—*When two sides and the included angle are given.*

RULE 4. *To find the other side:—*
as the sum of the two given sides
is to their difference,
so is the tangent of half the sum of their opposite angles
to the tangent of half their difference—
add this half difference to the half sum, to find the greater angle; and subtract the half difference from the half sum, to find the less angle. The other side may then be found by Rule 1.

RULE 5. *When the sides of a right-angled triangle are given, to find the angles:—*

as one side
is to the other side,
so is the radius
to the tangent of the angle adjacent to the first side.

CASE 3.—*When the three sides are given.*

RULE 6. *To find an angle.* Subtract the sum of the logarithms of the sides which contain the required angle, from 20; to the remainder add the logarithm of half the sum of the three sides, and that of the difference between this half sum and the side opposite to the required angle. Half the sum of these three logarithms will be the logarithmic cosine of half the required angle. The other angles may be found by Rule 1.

RULE 7. Subtract the sum of the logarithms of the two sides which contain the required angle, from 20, and to the remainder add the logarithms of the differences between these two sides and half the sum of the three sides. Half the result will be the logarithmic sine of half the required angle.

Note.—In all ordinary cases either of these rules gives sufficiently accurate results. It is recommended that Rule 6 should be used when the required angle exceeds 90°; and Rule 7 when it is less than 90°.

MENSURATION OF SURFACES.

To find the area of a parallelogram. Multiply the length by the height, or perpendicular breadth.

Or, multiply the product of two contiguous sides by the natural sine of the included angle.

To find the area of a triangle. Multiply the base by the perpendicular height, and take half the product.

Or, multiply half the product of two contiguous sides by the natural sine of the included angle.

To find the area of a trapezoid. Multiply half the sum of the parallel sides by the perpendicular distance between them.

To find the area of a quadrilateral inscribed in a circle. From half the sum of the four sides subtract each side severally; multiply the four remainders together; the square root of the product is the area.

To find the area of any quadrilateral figure. Divide the quadrilateral into two triangles; the sum of the areas of the triangles is the area.

Or, multiply half the product of the two diagonals by the natural sine of the angle at their intersection.

Note.—As the diagonals of a square and a rhombus intersect at right angles (the natural sine of which is 1), half the product of their diagonals is the area.

To find the area of any polygon. Divide the polygon into triangles and trapezoids by drawing diagonals; find the areas of these as above shown, for the area.

To find the area of a regular polygon. Multiply half the perimeter of the polygon by the perpendicular drawn from the centre to one of the sides.

Note.—To find the perpendicular when the side is given—

as radius
to tangent of half-angle at perimeter (see table No. 1),
so is half length of side
to perpendicular.

Or, multiply the square of a side of any regular polygon by the corresponding area in the following table:—

TABLE No. 1.—ANGLES AND AREAS OF REGULAR POLYGONS.

NAME.	Number of Sides.	One half Angle at the Perimeter.	Area. (Side=1)	Perpendicular. (Side=1)
Equilateral triangle,......	3	30°	0·4330	0·2887
Square,.....................	4	45°	1·0000	0·5000
Pentagon,.................	5	54°	1·7205	0·6882
Hexagon,..................	6	60°	2·5981	0·8660
Heptagon,.................	7	$64°\frac{2}{7}$	3·6339	1·0383
Octagon, ...	8	$67°\frac{1}{2}$	4·8284	1·2071
Nonagon,..................	9	70°	6·1818	1·3737
Decagon,..................	10	72°	7·6942	1·5388
Undecagon,..............	11	$73°\frac{7}{11}$	9·3656	1·7028
Dodecagon,..............	12	75°	11·1962	1·8660

To find the circumference of a circle. Multiply the diameter by 3·1416.

Or, multiply the area by 12·5664; the square root of the product is the circumference.

To find the diameter of a circle. Divide the circumference by 3·1416.

Or, multiply the circumference by ·3183.

Or, divide the area by ·7854; the square root of the quotient is the diameter.

To find the area of a circle. Multiply the square of the diameter by ·7854.

Or, multiply the circumference by one-fourth of the diameter.

Or, multiply the square of the circumference by ·07958.

To find the length of an arc of a circle. Multiply the number of degrees in the arc by the radius, and by ·01745.

Or, the length may be found nearly, by subtracting the chord of the whole arc from eight times the chord of half the arc, and taking one-third of the remainder.

To find the area of a sector of a circle. Multiply half the length of the arc of the sector by the radius.

Or, multiply the number of degrees in the arc by the square of the radius, and by ·008727.

To find the area of a segment of a circle. Find the area of the sector which has the same arc as the segment; also the area of the triangle formed by the radial sides of the sector and the chord of the arc; the difference or the sum of these areas will be the area of the segment, according as it is less or greater than a semicircle.

To find the area of a ring included between the circumferences of two con-

centric circles. Multiply the sum of the diameters by their difference, and by ·7854.

To find the area of a cycloid. Multiply the area of the generating circle by 3.

To find the length of an arc of a parabola, cut off by a double ordinate to the axis. To the square of the ordinate add four-fifths of the square of the absciss; twice the square root of the sum is the length nearly.

Note.—This rule is an approximation which applies to those cases only in which the absciss does not exceed half the ordinate.

To find the area of a parabola. Multiply the base by the height; two-thirds of the product is the area.

To find the circumference of an ellipse. Multiply the square root of half the sum of the squares of the two axes by 3·1416.

To find the area of an ellipse. Multiply the product of the two axes by ·7854.

Note.—The area of an ellipse is equal to the area of a circle of which the diameter is a mean proportional between the two axes.

To find the area of an elliptic segment, the base of which is parallel to either axis of the ellipse. Divide the height of the segment by the axis of which it is a part, and find the area of a circular segment, by table No. VII., of which the height is equal to this quotient; multiply the area thus found by the two axes of the ellipse successively; the product is the area.

To find the length of an arc of a hyperbola, beginning at the vertex. To 19 times the transverse axis add 21 times the parameter to this axis, and multiply the sum by the quotient of the absciss divided by the transverse. 2d. To 9 times the transverse add 21 times the parameter, and multiply the sum by the quotient of the absciss divided by the transverse. 3d. To each of these products add 15 times the parameter, and then

as the latter sum
is to the former sum,
so is the ordinate
to the length of the arc, nearly.

To find the area of a hyperbola. To the product of the transverse and absciss add five-sevenths of the square of the absciss, and multiply the square root of the sum by 21; to this product add 4 times the square root of the product of the transverse and absciss; multiply the sum by 4 times the product of the conjugate and absciss, and divide by 75 times the transverse. The quotient is the area nearly.

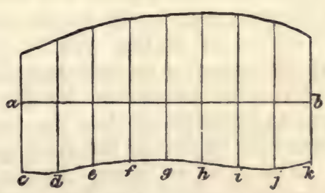


Fig. 74.—For Area of Curvilinear Figure.

To find the area of any curvilinear figure, bounded at the ends by parallel straight lines, Fig. 74. Divide the length of the figure *a b* into any even number of equal parts, and draw ordinates *c*, *d*, *e*, &c., through the points of division, to touch the boundary lines. Add together the first and last ordinates (*c* and *k*), and call the sum A; add together the even ordinates (that is, *d*, *f*, *h*, *j*), and call the sum B; add together the odd ordinates, except the first and last (*e*, *g*, *i*), and call the sum C. Let D be the common distance of the ordinates, then

$$\frac{(\text{A} + 4\,\text{B} + 2\,\text{C})}{3} \times \text{D} = \text{area of figure.}$$

This is known as Simpson's Rule.

2d Method, Fig. 74.—Having divided the figure into an even or an odd number of equal parts, add together the first and last ordinates, making the sum A; and add together all the intermediate ordinates, making the sum B. Let L = the length of the figure, and n = the number of divisions, then

$$\frac{\text{A} + 2\,\text{B}}{2\,n} \times \text{L} = \text{area of figure.}$$

That is to say, twice the sum of the intermediate ordinates, plus the first and last ordinates, divided by twice the number of divisions, and multiplied by the length, is equal to the area of the figure.

This method is that commonly used; it is sufficiently near to exactness for most purposes.

3d Method, Fig. 74.—Having divided the figure as above, measure by a scale the mean depth of each division, at the middle of the division; add together the depths of all the divisions, and divide the sum by the number of divisions, for the average depth; multiply the average depth by the length, which gives the area.

For the sake of obtaining a more nearly exact result, the figure may be divided into two half-parts, *c*, *k*, Fig. 75, one at each end, and a number of whole equal parts, *d*, *e*, *f*, *g*, *h*, *i*, *j*, intermediately. Then the ordinates separating these parts, excluding the extreme ordinates, may be measured

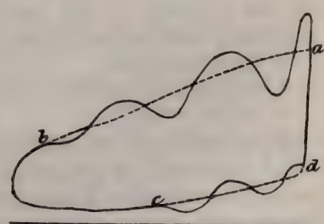


Fig. 75. For Area of Curvilineal Figures. Fig. 76.

direct, and the sum of the measurements divided by the number of them, and multiplied by the length, for the area.

Note.—In dealing with figures of excessively irregular outline, as in Fig. 76, representing an indicator-diagram from a steam-engine, mean lines, *a b*, *c d*, may be substituted for the actual lines, being so traced as to intersect the undulations, so that the total area of the spaces cut off may be compensated by that of the extra spaces inclosed.

Note 2.—The figures have been supposed to be bounded at the ends by parallel planes. But they may be terminated by curves or angles, as in Fig. 76, at *b*, when the extreme ordinates become nothing.

MENSURATION OF SOLIDS.

To find the surface of a prism or a cylinder. The perimeter of the end multiplied by the height gives the upright surface; add twice the area of an end.

To find the cubic contents of a prism or a cylinder. Multiply the area of the base by the height.

To find the surface of a pyramid or a cone. Multiply the perimeter of the base by half the slant height, and add the area of the base.

To find the cubic contents of a pyramid or a cone. Multiply the area of the base by one-third of the perpendicular height.

To find the surface of a frustum of a pyramid or a cone.—Multiply the sum of the perimeters of the ends by half the slant height, and add the areas of the ends.

To find the cubic contents of a frustum of a pyramid, or a cone.—Add together the areas of the two ends, and the mean proportional between them (that is, the square root of their product), and multiply the sum by one-third of the perpendicular height.

Or, when the ends are circles, add together the square of each diameter, and the product of the diameters, and multiply the sum by ·7854, and by one-third of the height.

To find the cubic contents of a wedge.—To twice the length of the base add the length of the edge; multiply the sum by the breadth of the base, and by one-sixth of the height.

To find the cubic contents of a prismoid (*a solid of which the two ends are unequal but parallel plane figures of the same number of sides*).—To the sum of the areas of the two ends, add four times the area of a section parallel to and equally distant from both ends; and multiply the sum by one-sixth of the length.

Note.—This rule gives the true content of all frustums, and of all solids of which the parallel sections are similar figures; and is a good approximation for other kinds of areas and solidities.

To find the surface of a sphere.—Multiply the square of the diameter by 3·1416.

Note.—The surface of a sphere is equal to 4 times the area of one of its great circles.

2. The surface of a sphere is equal to the convex surface of its circumscribing cylinder.

3. The surfaces of spheres are to one another as the squares of their diameters.

To find the curve surface of any segment or zone of a sphere.—Multiply the diameter of the sphere by the height of the zone or segment, and by 3·1416.

Note.—The curve surfaces of segments or zones of the same sphere are to one another as their heights.

To find the cubic contents of a sphere.—Multiply the cube of the diameter by ·5236.

Or, multiply the surface by one-sixth of the diameter.

Note.—The contents of a sphere are two-thirds of the contents of its circumscribing cylinder.

2. The contents of spheres are to one another as the cubes of their diameters.

To find the cubic contents of a segment of a sphere.—From 3 times the diameter of the sphere subtract twice the height of the segment; multiply the difference by the square of the height, and by ·5236.

Or, to 3 times the square of the radius of the base of the segment, add the square of its height; and multiply the sum by the height, and by ·5236.

To find the cubic contents of a frustum or zone of a sphere.—To the sum of the squares of the radii of the ends add ⅓ of the square of the height; multiply the sum by the height, and by 1·5708.

To find the cubic contents of a spheroid.—Multiply the square of the revolving axis by the fixed axis and by ·5236.

Note.—The contents of a spheroid are two-thirds of the contents of its circumscribing cylinder.

2. If the fixed and revolving axes of an oblate spheroid be equal to the revolving and fixed axes of an oblong spheroid respectively, the contents of the oblate are to those of the oblong spheroid as the greater to the less axis.

To find the cubic contents of a segment of a spheroid.—1st. When the base is parallel to the revolving axis. Multiply the difference between thrice the fixed axis and double the height of the segment, by the square of the height, and the product by ·5236. Then,

as the square of the fixed axis
is to the square of the revolving axis,
so is the last product
to the content of the segment.

2d. When the base is perpendicular to the revolving axis. Multiply the difference between thrice the revolving axis and double the height of the segment, by the square of the height, and the product by ·5236. Then,

as the revolving axis
is to the fixed axis,
so is the last product
to the content of the segment.

To find the solidity of the middle frustum of a spheroid.—1st. When the ends are circular, or parallel to the revolving axis. To twice the square of the middle diameter, add the square of the diameter of one end; multiply the sum by the length of the frustum, and the product by ·2618 for the content.

2d. When the ends are elliptical, or perpendicular to the revolving axis. To twice the product of the transverse and conjugate diameters of the middle section, add the product of the transverse and conjugate diameters of one end; multiply the sum by the length of the frustum, and by ·2618 for the content.

To find the cubic contents of a parabolic conoid.—Multiply the area of the base by half the height.

Or, multiply the square of the diameter of the base by the height, and by ·3927.

To find the cubic contents of a frustum of a parabolic conoid.—Multiply half the sum of the areas of the two ends by the height of the frustum.

Or, multiply the sum of the squares of the diameters of the two ends by the height, and by ·3927.

To find the cubic contents of a parabolic spindle.—Multiply the square of the middle diameter by the length, and by ·41888.

To find the cubic contents of the middle frustum of a parabolic spindle.—Add together 8 times the square of the largest diameter, 3 times the square of the diameter at the ends, and 4 times the product of the diameters; multiply the sum by the length of the frustum, and by ·05236.

To find the surface and the cubic contents of any of the five regular solids, Figs.

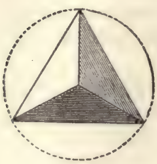


Fig. 77.

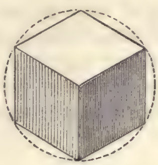


Fig. 78.

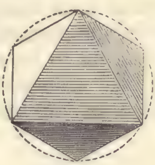


Fig. 79.

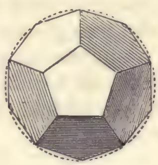


Fig. 80.

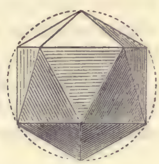


Fig. 81.

77, 78, 79, 80, 81.—For the surface, multiply the tabular area below, by the square of the edge of the solid.

For the contents, multiply the tabular contents below, by the cube of the given edge.

Note.—A regular solid is bounded by similar and regular plane figures. There are five regular solids, shown by Figs. 77 to 81, namely:—

The *tetrahedron*, bounded by four equilateral triangles.

The *hexahedron*, or cube, bounded by six squares.

The *octahedron*, bounded by eight equilateral triangles.

The *dodecahedron*, bounded by twelve pentagons.

The *icosahedron*, bounded by twenty equilateral triangles.

Regular solids may be circumscribed by spheres; and spheres may be inscribed in regular solids.

SURFACES AND CUBIC CONTENTS OF REGULAR SOLIDS.

Number of sides.	Name.	AREA. Edge = 1.	CONTENTS. Edge = 1.
4	Tetrahedron.............	1·7320	0·1178
6	Hexahedron.............	6·0000	1·0000
8	Octahedron..............	3·4641	0·4714
12	Dodecahedron..........	20·6458	7·6631
20	Icosahedron............	8·6603	2·1817

To find the cubic contents of an irregular solid.—Suppose it divided into parts, resembling prisms or other bodies measurable by preceding rules; find the content of each part; the sum of the contents is the cubic contents of the solid.

Note.—The content of a small part is found nearly by multiplying half the sum of the areas of each end by the perpendicular distance between them.

Or, the contents of small irregular solids may sometimes be found by immersing them under water in a prismatic or cylindrical vessel, and observing the amount by which the level of the water descends when the solid is withdrawn. The sectional area of the vessel being multiplied by the descent of the level, gives the cubic contents.

Or, when the solid is very large, and a great degree of accuracy is not requisite, measure its length, breadth, and depth in several different places, and take the mean of the measurement for each dimension, and multiply the three means together.

Or, when the surface of the solid is very extensive, it is better to divide it into triangles, to find the area of each triangle, and to multiply it by the mean depth of the triangle for the contents of each triangular portion; the contents of the triangular sections are to be added together.

The mean depth of a triangular section is obtained by measuring the depth at each angle, adding together the three measurements, and taking one-third of the sum.

MENSURATION OF HEIGHTS AND DISTANCES.

To find the height of an accessible object.—Measure the distance from the base of the object to any convenient station on the same horizontal plane; and at this station take the angle of altitude. Then

as radius
to tangent of the angle of altitude,
so is the horizontal distance
to the height of the object above the horizontal plane passing through the eye of the observer. Add the height of the eye, and the sum is the height of the object.

Note.—The station should be chosen so that the angle of altitude should be as near to 45° as practicable; because the nearer to 45°, the less is the error in altitude arising from error of observation.

When the angle of elevation is 45°, the height above the plane of the eye is equal to the distance. When it is 26° 34′, the height is half the distance.

To find approximately the height of an accessible object.—There are four methods based on the principle of similar triangles.

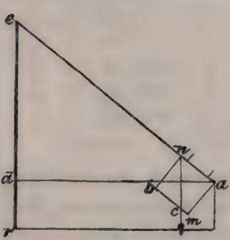


Fig. 82.—Mensuration of a Height.

1st. By a *geometrical square*, Fig. 82.—This is a square, *a b*, with two sights on one of its sides, *a n*, a plumb-line hung from one extremity, *n*, of that side, and each of the two sides opposite to that extremity, *m b*, *m a*, divided into 100 equal parts; the division beginning at the remote ends, so that the 100th divisions meet at the corner *m*. Let *r e* be the object, and the sights be directed to the summit *e*, at the known distance *a d*. When the plummet cuts the side *b m* at, say, *c*, then by similar triangles, *n b* : *b c* : : *a d* : *d e*. Or, if the plumb-line cuts the side *a m*, then the part of *a m* cut off is to *a n* : : *a d* : *d e*. Adding to *d e* the height of the eye *r d*, the sum is the height of the object, *r e*.

2d. By *shadows*, Fig. 83.—When the sun shines, fix a pole *b c* in the ground, vertically, and measure its shadow *a b*. Measure also the shadow *d e*

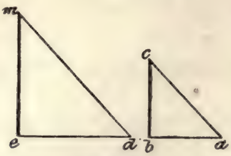
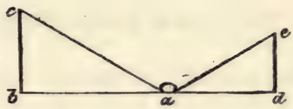


Fig. 83. Mensuration of a Height. Fig. 84.



of the object *e m*; then, by similar triangles, *a b* : *b c* : : *d e* : *e m*, the height of the object.

3d. By *reflection*, Fig. 84.—Place a basin of water, or any horizontal reflecting surface, at *a*, level with the base of the object *d e*, and retire from it till the eye at *c* sees the top of the object *e*, in the centre of the basin at *a*. Then, by similar triangles, *a b* : *b c* : : *a d* : *d e*.

4th. By *two poles*, Fig. 85.—Fix two poles *a m*, *c n*, of unequal lengths, parallel to the object *e r*, so that the eye of the observer at *a*, the top of the shorter pole, may see *c*, the top of the longer pole, in a line with *e*, the summit of the object *r e*. By similar triangles, *a b* : *b c* : : *a d* : *d e*; and adding *r d*, the height of the eye, to *d e*, the sum *r e* is the height of the object.

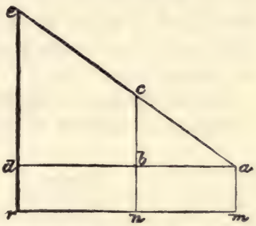


Fig. 85. Mensuration of a Height.

To find the distance of the visible horizon.—To half the logarithm of the height of the eye, add 3·8105; the sum is the logarithm of the distance in feet, nearly.

To find the distance of an object by the motion of sound.—Multiply the number of seconds that elapse between the flash or other sign of the generation of the sound and the arrival of the sound to the ear, by 1120. The product is the distance in feet.

Note.—When a sound generated near the ear returns as an echo, half the interval of time is to be taken, to find the distance of the reflecting surface.

MATHEMATICAL TABLES.

TABLE No. I.—OF LOGARITHMS OF NUMBERS FROM 1 TO 10,000.

Logarithms consist of integers and decimals; but, for the sake of compactness, the integers have been omitted in the table, except in the short preliminary section containing the complete logarithms of numbers from 1 to 100. The table No. I. contains the decimal parts, to six places, of the logarithms of numbers from 1 to 10,000. The integer, or index, or characteristic of a logarithm, standing on the left-hand side of the decimal point, is a number less by 1 than the number of figures or places in the integer of the number. If a number contains both integers and decimals, the index is regulated according to the integers. If it contain only decimals, the index is equal to the number of cyphers next the decimal point, plus 1; moreover, the index is negative, and is so distinguished by the sign minus, –, written over it.

For example, to illustrate the adjustment of the integer of the logarithm to the composition of the number:—

Number.	Logarithm.
4743	3.676053
474·3	2.676053
47·43	1.676053
4·743	0.676053
·4743	$\bar{1}$.676053
·04743	$\bar{2}$.676053
·004743	$\bar{3}$.676053

Still more for the sake of compactness, the first two figures of the logarithms are given only at the beginning of each line of logarithms, to save repetition, only the remaining four decimal places being given for each logarithm. In seeking for a logarithm, the eye readily takes in the prefixed two digits at the commencement of each line.

Rules.—To find the logarithm of a number containing one or two digits, look for the number in the preliminary tablet in one of the columns marked No., and find the logarithm next it. Or, look in the body of the table for the given number in the columns marked N, with one or two cyphers following it; the decimal part of the logarithm is in the column next to it. For example, the decimal part of the logarithm of 3 is found, in the column next to the number 300, to be .477121, and as there is but one digit, the logarithm is completed with a cypher, thus, 0.477121. The same logarithm stands for 30, except that, when completed, it becomes 1.477121. Again, take the number 37; look for 370 in column N, and the decimal part of the logarithm is found, in the column next it, to be .568202, which, being completed, becomes 1.568202. If the number be .37, the logarithm becomes $\bar{1}$.568202.

To find the logarithm of a number consisting of three digits, look for the

number in column N, and find the logarithm in the column next it, as already exemplified, for which the index is to be settled and prefixed as before.

If the number consist of four digits, look for the first three in column N, and the fourth in the horizontal line at the head or at the foot of the table. The decimal part of the logarithm is found opposite the three first digits and under or over the fourth. Take the number 5432; opposite 543 in column N, and in the column headed 2, is the logarithm .734960, to which 3 is to be prefixed, making 3.734960. If the number be 5.432, the complete logarithm is 0.734960.

If the number consist of five or more digits, find the logarithm for the first four as above; multiply the difference, in column D, by the remaining digits, and divide by 10 if there be only one digit more, by 100 if there be two more, and so on; add the quotient to the logarithm for the first four. The sum is the decimal part of the required logarithm, to which the index is to be prefixed. For example, take 3.1416. The logarithm of 3141 is .497068, decimal part; and the difference, $138 \times 6 \div 10 = 83$, is to be added, thus—

$$\begin{array}{r} 0.497068 \\ 83 \\ \hline \end{array}$$

making the complete logarithm,..................0.497151

To find the number corresponding to a given logarithm, look for the logarithm without the index. If it be found exactly or within two or three units of the right-hand digit, then the first three figures of the indicated number will be found in the number column, in a line with the logarithm, and the fourth figure at the top or the foot of the column containing the logarithm. Annex the fourth figure to the first three, and place the decimal point in its proper position, on the principles already explained.

If the given logarithm differs by more than two or three units from the nearest in the table, find the number for the next less tabulated logarithm, which will give the four first digits of the required number. To find the fifth and sixth digits, subtract the tabulated logarithm from the given logarithm, add two cyphers, and divide by the difference found in column D opposite the logarithm. Annex the quotient to the four digits already found, and place the decimal point. For example, to find the number represented by the logarithm 2.564732:—

2.564732 given logarithm.
Log..................367.0 = ...2.564666 nearest less.

56	D 118)6600 (56 nearly.
	590
367.056	
	700
	708

Showing that the required number is 367.056.

To multiply together two or more numbers, add together the logarithms

of the numbers, and the sum is the logarithm of the product. Thus, to multiply 365 by 3.146:—

$$
\begin{array}{lll}
\text{Log } 365 \ldots\ldots\ldots\ldots\ldots & = 2.562293 & \\
\text{Log } 3.146 \ldots\ldots\ldots\ldots & = 0.497759 & \\
\hline
 & \ 3.060052 & \\
\text{Log } 1148. \ldots\ldots\ldots\ldots\ldots & \ 3.059942 & \\
\hline
\end{array}
$$

$$
\begin{array}{ll}
29 & \text{D } 380)11000\ (29 \text{ nearly.} \\
\hline
1148.29 & \phantom{\text{D } 380)1}760 \\
 & \phantom{\text{D } 380)}3400 \\
 & \phantom{\text{D } 380)}3420
\end{array}
$$

Showing that the product is..........1148.29.

To divide one number by another, subtract the logarithm of the divisor from that of the dividend, and the remainder is the logarithm of the quotient.

To find any power of a given number, multiply the logarithm of the number by the exponent of the power. The product is the logarithm of the power.

To find any root of a given number, divide the logarithm of the number by the index of the root. The quotient is the logarithm of the root.

To find the reciprocal of a number, subtract the decimal part of the logarithm of the number from 0.000000; add 1 to the index of the logarithm, and change the sign of the index. This completes the logarithm of the reciprocal. For example, to find the reciprocal of 230:—

$$
\begin{array}{ll}
 & 0.000000 \\
\text{Log } 230 = \ldots\ldots\ldots\ldots & 2.361728 \\
\hline
 & \bar{3}.638272 = \log\ 0.004348 \text{ (reciprocal).}
\end{array}
$$

Inversely, to find the reciprocal of the decimal .004348:—

$$
\begin{array}{ll}
 & 0.000000 \\
\text{Log } .004348 = \ldots\ldots & \bar{3}.638272 \\
\hline
 & 2.361728 = \log\ 230 \text{ (reciprocal).}
\end{array}
$$

Note.—It will be found in practice, for the most part, unnecessary to note the indices of logarithms, as the decimal parts are in most cases sufficiently indicative of the numbers without the indices. The exact calculation of differences may also in most cases be dispensed with—rough mental approximations being sufficiently near for the purpose—particularly when the numbers contain decimals. The indices are, however, indispensable in the calculation of the roots of numbers.

Table No. II.—Of Hyperbolic Logarithms of Numbers.

In this table, the numbers range from 1.01 to 30, advancing by .01, up to the whole number 10; and thence by larger intervals up to 30. The hyperbolic logarithms of numbers, or Neperian logarithms, as they are sometimes called, are calculated by multiplying the common logarithms of the given numbers, in table No. I., by the constant multiplier, 2.302585. The hyperbolic logarithms of numbers intermediate between those which are given in the table, may be readily obtained by interpolating proportional differences.

Table No. III.—Of Circumferences, Circular Areas, Squares and Cubes; and of Square Roots and Cube Roots.

It has been shown how to calculate the powers and roots of numbers by means of logarithms. The table No. III. will be useful for reference. It contains the powers and roots of numbers consecutively from 1 to 1000. The circumferences and areas of circles, due to the numbers contained in the first columns, considered as diameters, are also given.

By a suitable adjustment of decimal points the circumferences, areas, squares and cubes, may be determined from the contents of the table for diameters ten or a hundred times as much as, or less than, the values given in the first column.

For example, if the number 378 in the first column, page 73, be taken as 37.8, the corresponding circumference, area, square and cube are as follows:

	Original.		Decimalized.
Number	378	......	37.8
Circumference	1,187.52	......	118.752
Circular area	112,221.09	......	1122.2109
Square	142,884	......	1,428.84
Cube	54,010,152	......	54,010.152

Table No. IV.—Of Circumferences and Areas of Circles with Sides of Equal Squares.

The table No. IV. gives the circumferences and areas of circles from $\frac{1}{16}$ inch to 120 inches in diameter, advancing by sixteenths of an inch up to 6 inches diameter; thence by eighths of an inch to 50 inches diameter; thence by quarters of an inch to 100 inches diameter; and thence by half-inches to 120 inches diameter.

Whilst the diameters are here expressed as inches, they may be taken as feet, or as measures of any other denomination.

The column of *sides of equal squares*, contains the sides of squares having the same area as the circles in the same lines of the table respectively.

Tables Nos. V. and VI.—Of Lengths of Circular Arcs.

The lengths of circular arcs are given proportionally to that of the radius, and to that of the chord, in the tables Nos. V. and VI. In the first of these tables, the radius is taken = 1, and the number of degrees in the arc are given in the first column. The length of the arc as compared with the radius is given decimally in the second column.

In the second table, the chord is taken = 1, and the rise or height of the arc, expressed decimally as compared with the chord, is given in the first column. The length of the arc relatively to the chord is given in the second column.

To use the first table, No. V., find the proportional length of the arc corresponding to the degrees in the arc, and multiply it by the actual length of the radius; the product is the actual length of the arc.

To use the second table, No. VI., divide the height of the arc by the chord for the proportional height of the arc, which find in the first column of the table; the proportional length of the arc corresponding to it being multiplied by the actual length of the chord, gives the actual length of the arc.

Note.—The length of an arc of a circle may be found nearly thus:—Subtract the chord of the whole arc from 8 times the chord of half the arc. A third of the remainder is the length nearly.

Table No. VII.—Of Areas of Circular Segments.

The areas of circular segments are given in Table No. VII., in proportional superficial measure, the diameter of the circle of which the segment forms a portion being = 1. The height of the segment, expressed decimally in proportion to the diameter, is given in the first column, and the relative area in the second column.

To use the table, divide the height by the diameter, find the quotient in the table, and multiply the corresponding area by the square of the actual length of the diameter; the product will be the actual area.

Table No. VIII.—Sines, Cosines, Tangents, Cotangents, Secants, and Cosecants of Angles from 0° to 90°.

This table, No. VIII., is constructed for angles of from 0° to 90°, advancing by 10′, or one-sixth of a degree. The length of the radius is equal to 1, and forms the basis for the relative lengths given in the table, and which are given to six places of decimals. Each entry in the table has a duplicate significance, being the sine, tangent, or secant of one angle, and at the same time the cosine, cotangent, or cosecant of its complement. For this reason, and for the sake of compactness, the headings of the columns are reversed at the foot; so that the upper headings are correct for the angles named in the left hand margin of the table, and the lower headings for those named in the right hand margin.

To find the sine, or other element, to odd minutes, divide the difference between the sines, &c., of the two angles greater and less than the given angle, in the same proportion that the given angle divides the difference of the two angles, and add one of the parts to the sine next it.

By an inverse process the angle may be found for any given sine, &c., not found in the table.

Table No. IX.—Of Logarithmic Sines, Cosines, Tangents, and Cotangents of Angles from 0° to 90°.

This table, No. IX., is constructed similarly to the table of natural sines, &c., preceding. To avoid the use of logarithms with negative indices, the radius is assumed, instead of being equal to 1, to be equal to 10^{10}, or

10,000,000,000; consequently the logarithm of the radius = 10 log 10 = 10. Whence, if, to log sine of any angle, when calculated for a radius = 1, there be added 10, the sum will be the log sine of that angle for a radius = 10^{10}.

For example, to find the logarithmic sine of the angle 15° 50′.

$$\begin{array}{rl} \text{Nat. sine } 15^\circ\ 50' = {\cdot}272840;\ \text{its log} = & \bar{1}{\cdot}435908 \\ \text{add} = & 10 \\ \hline \text{Logarithmic sine of } 15^\circ\ 50' = & 9{\cdot}435908 \end{array}$$

When the logarithmic sines and cosines have been found in this manner, the logarithmic tangents, cotangents, secants, and cosecants are found from those by addition or subtraction, according to the correlations of the trigonometrical elements already given, and here repeated in logarithmic form:—

Log tan.................. = 10 + log sin. − log. cosin.
Log cotan................ = 20 − log tan.
Log sec.................. = 20 − log cosin.
Log cosec................ = 20 − log sin.

To find the logarithmic sine, tangent, &c., of any angle.—When the number of degrees is less than 45°, find the degrees and minutes in the left hand column headed *angle*, and under the heading *sine*, or *tangent*, &c., as required, the logarithm is found in a line with the angle.

When the number of degrees is above 45°, and less than 90°, find the degrees and minutes in the right hand column headed *angle*, and in the same line, above the title at the foot of the page, *sine* or *tangent*, &c., find the logarithm in a line with the angle.

When the number of degrees is between 90° and 180°, take their supplement to 180°; when between 180° and 270°, diminish them by 180°; and when between 270° and 360°, take their complement to 360°, and find the logarithm of the remainder as before.

If the exact number of minutes is not found in the table, the logarithm of the nearest tabular angle is to be taken and increased or diminished as the case may be, by the due proportion of the difference of the logarithms of the angles greater and less than the given angle.

Table No. X.—Rhumbs, or Points of the Compass.

The Mariner's Compass is a circular card suspended horizontally, having a thin bar of steel magnetized,—the *needle*,—for one of its diameters; the circumference of the card being divided into 32 equal parts, or *points*, and each point subdivided into quarters. A point of the compass is, therefore, equal to (360° ÷ 32 =) 11° 15′.

Table No. XI.—Of Reciprocals of Numbers.

The table No. XI. contains the reciprocals of numbers from 1 to 1000. It has already been shown how to find the reciprocal of a number by means of logarithms.

TABLE No. I.—LOGARITHMS OF NUMBERS
FROM 1 TO 10,000.

No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0.000000	26	1.414973	**51**	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
6	0.778151	**31**	1.491362	56	1.748188	**81**	1.908485
7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
10	1.000000	35	1.544068	60	1.778151	85	1.929419
11	1.041393	36	1.556303	**61**	1.785330	86	1.934498
12	1.079181	37	1.568202	62	1.792392	87	1.939519
13	1.113943	38	1.579784	63	1.799341	88	1.944483
14	1.146128	39	1.591065	64	1.806180	89	1.949390
15	1.176091	40	1.602060	65	1.812913	90	1.954243
16	1.204120	**41**	1.612784	66	1.819544	**91**	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
21	1.322219	46	1.662758	**71**	1.851258	96	1.982271
22	1.342423	47	1.672098	72	1.857332	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
24	1.380211	49	1.690196	74	1.869232	99	1.995635
25	1.397940	50	1.698970	75	1.875061	100	2.000000

N	0	1	2	3	4	5	6	7	8	9	D
100	00– 0000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	00– 4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	00– 8600	9026	9451	9876	......	......	......	......	......	......	425
102	01–	......	......	......	0300	0724	1147	1570	1993	2415	424
103	01– 2837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
104	01– 7033	7451	7868	8284	8700	9116	9532	9947	......	......	417
104	02–	......	......	......	......	......	......	......	0361	0775	416
105	02– 1189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	02– 5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	02– 9384	9789	......	......	......	......	......	......	......	......	405
107	03–	......	0195	0600	1004	1408	1812	2216	2619	3021	404
108	03– 3424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	03– 7426	7825	8223	8620	9017	9414	9811	......	......	......	398
109	04–	......	......	......	......	......	......	0207	0602	0998	397
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
110	04– 1393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	04– 5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	04– 9218	9606	9993	......	......	......	......	......	......	......	388
112	05–	......	......	0380	0766	1153	1538	1924	2309	2694	386
113	05– 3078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
114	05– 6905	7286	7666	8046	8426	8805	9185	9563	9942	......	383
114	06–	......	......	......	......	......	......	......	......	0320	379
115	06– 0698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	06– 4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
117	06– 8186	8557	8927	9298	9668	......	......	......	......	......	380
117	07–	......	......	......	......	0038	0407	0776	1145	1514	370
118	07– 1882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	07– 5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	07– 9181	9543	9904	......	......	......	......	......	......	......	362
120	08–	......	......	0266	0626	0987	1347	1707	2067	2426	360
121	08– 2785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	08– 6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	08– 9905	......	......	......	......	......	......	......	......	......	355
123	09–	0258	0611	0963	1315	1667	2018	2370	2721	3071	353
124	09– 3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	09– 6910	7257	7604	7951	8298	8644	8990	9335	9681	......	348
125	10–	......	......	......	......	......	......	......	......	0026	346
126	10– 0371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	10– 3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
128	10– 7210	7549	7888	8227	8565	8903	9241	9579	9916	......	338
128	11–	......	......	......	......	......	......	......	......	0253	337
129	11– 0590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	11– 3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	11– 7271	7603	7934	8265	8595	8926	9256	9586	9915	......	331
131	12–	......	......	......	......	......	......	......	......	0245	330
132	12– 0574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	12– 3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	12– 7105	7429	7753	8076	8399	8722	9045	9368	9690	......	323
134	13–	......	......	......	......	......	......	......	......	0012	323
135	13– 0334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	13– 3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	13– 6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	13– 9879	......	......	......	......	......	......	......	......	......	315
138	14–	0194	0508	0822	1136	1450	1763	2076	2389	2702	314
139	14– 3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	14– 6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	14– 9219	9527	9835	......	......	......	......	......	......	......	308
141	15–	......	......	0142	0449	0756	1063	1370	1676	1982	307
142	15– 2288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	15– 5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	15– 8362	8664	8965	9266	9567	9868	......	......	......	......	302
144	16–	......	......	......	......	......	0168	0469	0769	1068	301
145	16– 1368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	16– 4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	16– 7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
148	17– 0262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	17– 3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	17– 6091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	17– 8977	9264	9552	9839	……	……	……	……	……	……	287
151	18– ……	……	……	……	0126	0413	0699	0986	1272	1558	287
152	18– 1844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	18– 4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	18– 7521	7803	8084	8366	8647	8928	9209	9490	9771	……	281
154	19– ……	……	……	……	……	……	……	……	……	0051	281
155	19– 0332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	19– 3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	19– 5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	19– 8657	8932	9206	9481	9755	……	……	……	……	……	275
158	20– ……	……	……	……	……	0029	0303	0577	0850	1124	274
159	20– 1397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	20– 4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	20– 6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	20– 9515	9783	……	……	……	……	……	……	……	……	268
162	21– ……	……	0051	0319	0586	0853	1121	1388	1654	1921	267
163	21– 2188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	21– 4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	21– 7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	22– 0108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	22– 2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	22– 5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	22– 7887	8144	8400	8657	8913	9170	9426	9682	9938	……	257
169	23– ……	……	……	……	……	……	……	……	……	0193	256
170	23– 0449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
171	23– 2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	23– 5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	23– 8046	8297	8548	8799	9049	9299	9550	9800	……	……	251
173	24– ……	……	……	……	……	……	……	……	0050	0300	250
174	24– 0549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	24– 3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	24– 5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	24– 7973	8219	8464	8709	8954	9198	9443	9687	9932	……	246
177	25– ……	……	……	……	……	……	……	……	……	0176	245
178	25– 0420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	25– 2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	25– 5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	25– 7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26– 0071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	26– 2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	26– 4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	26– 7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	26– 9513	9746	9980	……	……	……	……	……	……	……	234
186	27– ……	……	……	0213	0446	0679	0912	1144	1377	1609	233
187	27– 1842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	27– 4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
189	27– 6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	27– 8754	8982	9211	9439	9667	9895	......	......	......	......	228
190	28–	......	......	......	......	......	0123	0351	0578	0806	228
191	28– 1033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	28– 3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	28– 5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	28– 7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	29– 0035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	29– 2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	29– 4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	29– 6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	29– 8853	9071	9289	9507	9725	9943	......	......	......	......	218
199	30–	......	......	......	......	......	0161	0378	0595	0813	218
200	30– 1030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	30– 3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	30– 5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	30– 7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	30– 9630	9843	......	......	......	......	......	......	......	......	213
204	31–	......	0056	0268	0481	0693	0906	1118	1330	1542	212
205	31– 1754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	31– 3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	31– 5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	31– 8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	32– 0146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	32– 2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	32– 4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	32– 6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	32– 8380	8583	8787	8991	9194	9398	9601	9805	......	......	204
213	33–	......	......	......	......	......	......	......	0008	0211	203
214	33– 0414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	33– 2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	33– 4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	33– 6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	33– 8456	8656	8855	9054	9253	9451	9650	9849	......	......	200
218	34–	......	......	......	......	......	......	......	0047	0246	199
219	34– 0444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	34– 2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	34– 4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	34– 6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	34– 8305	8500	8694	8889	9083	9278	9472	9666	9860	......	194
223	35–	......	......	......	......	......	......	......	......	0054	194
224	35– 0248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	35– 2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	35– 4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	35– 6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	35– 7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	35– 9835	......	......	......	......	......	......	......	......	......	189
229	36–	0025	0215	0404	0593	0783	0972	1161	1350	1539	189
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
230	36– 1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	36– 3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	36– 5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	36– 7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	36– 9216	9401	9587	9772	9958	......	......	......	......	......	186
234	37–	......	......	......	......	0143	0328	0513	0698	0883	185
235	37– 1068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	37– 2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	37– 4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	37– 6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	37– 8398	8580	8761	8943	9124	9306	9487	9668	9849	......	182
239	38–	......	......	......	......	......	......	......	......	0030	181
240	38– 0211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	38– 2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	38– 3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	38– 5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	38– 7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	38– 9166	9343	9520	9698	9875	......	......	......	......	......	177
245	39–	......	......	......	......	0051	0228	0405	0582	0759	177
246	39– 0935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	39– 2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	39– 4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	39– 6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	39– 7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	39– 9674	9847	......	......	......	......	......	......	......	......	173
251	40–	......	0020	0192	0365	0538	0711	0883	1056	1228	173
252	40– 1401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	40– 3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	40– 4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	40– 6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	40– 8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	40– 9933	......	......	......	......	......	......	......	......	......	169
257	41–	0102	0271	0440	0609	0777	0946	1114	1283	1451	169
258	41– 1620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	41– 3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	41– 4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	41– 6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	41– 8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	41– 9956	......	......	......	......	......	......	......	......	......	165
263	42–	0121	0286	0451	0616	0781	0945	1110	1275	1439	165
264	42– 1604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
265	42– 3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	42– 4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	42– 6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	42– 8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	42– 9752	9914	......	......	......	......	......	......	......	......	162
269	43–	......	0075	0236	0398	0559	0720	0881	1042	1203	161
270	43– 1364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	43– 2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
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272	43– 4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	43– 6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
274	43– 7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	43– 9333	9491	9648	9806	9964	......	......	......	......	......	158
275	44–	......	......	......	......	0122	0279	0437	0594	0752	158
276	44– 0909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	44– 2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
278	44– 4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
279	44– 5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	44– 7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
281	44– 8706	8861	9015	9170	9324	9478	9633	9787	9941	......	154
281	45–	......	......	......	......	......	......	......	......	0095	154
282	45– 0249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
283	45– 1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
284	45– 3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	45– 4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
286	45– 6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
287	45– 7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
288	45– 9392	9543	9694	9845	9995	......	......	......	......	......	151
288	46–	......	......	......	......	0146	0296	0447	0597	0748	151
289	46– 0898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	46– 2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	46– 3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
292	46– 5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	46– 6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
294	46– 8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
295	46– 9822	9969	......	......	......	......	......	......	......	......	147
295	47–	......	0116	0263	0410	0557	0704	0851	0998	1145	147
296	47– 1292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	47– 2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	47– 4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	47– 5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	47– 7121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
301	47– 8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
302	48– 0007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
303	48– 1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	48– 2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
305	48– 4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	48– 5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
307	48– 7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	48– 8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	48– 9958	......	......	......	......	......	......	......	......	......	140
309	49–	0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	49– 1362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	49– 2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	49– 4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	49– 5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	49– 6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
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315	49– 8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
316	49– 9687	9824	9962	......	......	......	......	......	......	......	137
316	50–	......	......	0099	0236	0374	0511	0648	0785	0922	137
317	50– 1059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
318	50– 2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
319	50– 3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	50– 5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	50– 6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
322	50– 7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
323	50– 9203	9337	9471	9606	9740	9874	......	......	......	......	134
323	51–	......	......	......	......	......	0009	0143	0277	0411	134
324	51– 0545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
325	51– 1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	51– 3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
327	51– 4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	51– 5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	51– 7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	51– 8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	51– 9828	9959	......	......	......	......	......	......	......	......	131
331	52–	......	0090	0221	0353	0484	0615	0745	0876	1007	131
332	52– 1138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
333	52– 2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	52– 3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	52– 5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	52– 6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	52– 7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
338	52– 8917	9045	9174	9302	9430	9559	9687	9815	9943	......	128
338	53–	......	......	......	......	......	......	......	......	0072	128
339	53– 0200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128
340	53– 1479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
341	53– 2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	53– 4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
343	53– 5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
344	53– 6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	53– 7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	53– 9076	9202	9327	9452	9578	9703	9829	9954	......	......	126
346	54–	......	......	......	......	......	......	......	0079	0204	125
347	54– 0329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
348	54– 1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	54– 2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	54– 4068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
351	54– 5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	54– 6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	54– 7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	54– 9003	9126	9249	9371	9494	9616	9739	9861	9984	......	123
354	55–	......	......	......	......	......	......	......	......	0106	123
355	55– 0228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
356	55– 1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	55– 2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
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358	55– 3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	55– 5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	55– 6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	55– 7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	55– 8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	55– 9907	......	......	......	......	......	......	......	......	......	120
363	56–	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
364	56– 1101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	56– 2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	56– 3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	56– 4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	56– 5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	56– 7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	56– 8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	56– 9374	9491	9608	9725	9842	9959	......	......	......	......	117
371	57–	......	......	......	......	......	0076	0193	0309	0426	117
372	57– 0543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	57– 1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	57– 2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	57– 4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	57– 5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	57– 6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	57– 7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	57– 8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	57– 9784	9898	......	......	......	......	......	......	......	......	114
380	58–	......	0012	0126	0241	0355	0469	0583	0697	0811	114
381	58– 0925	1039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	58– 2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	58– 3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	58– 4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	58– 5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	58– 6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	58– 7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	58– 8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	58– 9950	......	......	......	......	......	......	......	......	......	112
389	59–	0061	0173	0284	0396	0507	0619	0730	0842	0953	112
390	59– 1065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	59– 2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	59– 3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	59– 4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	59– 5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	59– 6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	59– 7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	59– 8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	59– 9883	9992	......	......	......	......	......	......	......	......	109
398	60–	......	0101	0210	0319	0428	0537	0646	0755	0864	109
399	60– 0973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
400	60– 2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
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401	60– 3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
402	60– 4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	60– 5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	60– 6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	60– 7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	60– 8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	60– 9594	9701	9808	9914	......	......	......	......	......	......	107
407	61–	......	......	......	0021	0128	0234	0341	0447	0554	107
408	61– 0660	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	61– 1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	61– 2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
411	61– 3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	61– 4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	61– 5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	61– 7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	61– 8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	61– 9093	9198	9302	9406	9511	9615	9719	9824	9928	......	105
416	62–	......	......	......	......	......	......	......	......	0032	104
417	62– 0136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
418	62– 1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	62– 2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	62– 3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	62– 4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	62– 5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	62– 6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	62– 7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
425	62– 8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	62– 9410	9512	9613	9715	9817	9919	......	......	......	......	102
426	63–	......	......	......	......	......	0021	0123	0224	0326	102
427	63– 0428	0530	0631	0733	0835	0936	1038	1139	1241	1342	102
428	63– 1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	63– 2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	63– 3468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
431	63– 4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
432	63– 5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	63– 6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
434	63– 7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
435	63– 8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	100
436	63– 9486	9586	9686	9785	9885	9984	......	......	......	......	100
436	64–	......	......	......	......	......	0084	0183	0283	0382	99
437	64– 0481	0581	0680	0779	0879	0978	1077	1177	1276	1375	99
438	64– 1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
439	64– 2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	64– 3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	99
441	64– 4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	64– 5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	64– 6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	64– 7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
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445	64– 8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	64– 9335	9432	9530	9627	9724	9821	9919	......	......	......	97
446	65–	......	......	......	......	......	......	0016	0113	0210	97
447	65– 0308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
448	65– 1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	65– 2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	65– 3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96
451	65– 4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	65– 5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	65– 6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	65– 7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	65– 8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	65– 8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
457	65– 9916	......	......	......	......	......	......	......	......	......	95
457	66–	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
458	66– 0865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
459	66– 1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
460	66– 2758	2852	2947	3041	3135	3230	3324	3418	3512	3607	94
461	66– 3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	66– 4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	66– 5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	66– 6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	66– 7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	66– 8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	66– 9317	9410	9503	9596	9689	9782	9875	9967	......	......	93
467	67–	......	......	......	......	......	......	......	0060	0153	93
468	67– 0246	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
469	67– 1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	67– 2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	67– 3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	67– 3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	67– 4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	67– 5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	67– 6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	67– 7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	67– 8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	67– 9428	9519	9610	9700	9791	9882	9973	......	......	......	91
478	68–	......	......	......	......	......	......	0063	0154	0245	91
479	68– 0336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
480	68– 1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	68– 2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	68– 3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	68– 3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	68– 4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	68– 5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	68– 6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	68– 7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	68– 8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	68– 9309	9398	9486	9575	9664	9753	9841	9930	......	......	89
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489	69–	......	......	......	......	......	......	......	0019	0107	89
490	69– 0196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	69– 1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	69– 1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	69– 2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	69– 3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	69– 4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	69– 5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	69– 6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	69– 7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	69– 8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	69– 8970	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
501	69– 9838	9924	......	......	......	......	......	......	......	......	87
501	70–	......	0011	0098	0184	0271	0358	0444	0531	0617	87
502	70– 0704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	70– 1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	70– 2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	70– 3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	70– 4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	70– 5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	70– 5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	70– 6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	70– 7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	70– 8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	70– 9270	9355	9440	9524	9609	9694	9779	9863	9948	......	85
512	71–	......	......	......	......	......	......	......	......	0033	85
513	71– 0117	0202	0287	0371	0456	0540	0625	0710	0794	0879	85
514	71– 0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	71– 1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	71– 2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	71– 3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
518	71– 4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	71– 5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
520	71– 6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	83
521	71– 6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	71– 7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	71– 8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	71– 9331	9414	9497	9580	9663	9745	9828	9911	9994	......	83
524	72–	......	......	......	......	......	......	......	......	0077	83
525	72– 0159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	72– 0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	72– 1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	72– 2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	72– 3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	72– 4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	72– 5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	72– 5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	72– 6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
N	0	1	2	3	4	5	6	7	8	9	D

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534	72– 7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	72– 8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	72– 9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	72– 9974	......	......	......	......	......	......	......	......	......	81
537	73–	0055	0136	0217	0298	0378	0459	0540	0621	0702	81
538	73– 0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	73– 1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	73– 2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
541	73– 3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	73– 3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
543	73– 4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
544	73– 5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	73– 6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	73– 7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	73– 7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	73– 8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	73– 9572	9651	9731	9810	9889	9968	......	......	......	......	79
549	74–	......	......	......	......	......	0047	0126	0205	0284	79
550	74– 0363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
551	74– 1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	74– 1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	74– 2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	74– 3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	74– 4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	74– 5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	74– 5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	74– 6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	74– 7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	74– 8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	74– 8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	74– 9736	9814	9891	9968	......	......	......	......	......	......	77
562	75–	......	......	......	0045	0123	0200	0277	0354	0431	77
563	75– 0508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
564	75– 1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	75– 2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	75– 2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	75– 3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	75– 4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	75– 5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	75– 5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	75– 6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	75– 7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	75– 8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	75– 8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	75– 9668	9743	9819	9894	9970	......	......	......	......	......	76
575	76–	......	......	......	......	0045	0121	0196	0272	0347	75
576	76– 0422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
577	76– 1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
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578	76– 1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
579	76– 2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
580	76– 3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	75
581	76– 4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
582	76– 4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	76– 5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	76– 6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	76– 7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	76– 7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	76– 8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	76– 9377	9451	9525	9599	9673	9746	9820	9894	9968	......	74
588	77–	......	......	......	......	......	......	......	......	0042	74
589	77– 0115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	77– 0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	77– 1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	77– 2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	77– 3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	77– 3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	77– 4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	77– 5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	77– 5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	77– 6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	77– 7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	77– 8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	77– 8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	77– 9596	9669	9741	9813	9885	9957	......	......	......	......	72
602	78–	......	......	......	......	......	0029	0101	0173	0245	72
603	78– 0317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	78– 1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	78– 1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	78– 2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	78– 3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	78– 3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	78– 4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	78– 5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	78– 6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	78– 6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	78– 7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	78– 8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	78– 8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	78– 9581	9651	9722	9792	9863	9933	......	......	......	......	70
616	79–	......	......	......	......	......	0004	0074	0144	0215	70
617	79– 0285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	79– 0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
619	79– 1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	79– 2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	79– 3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	79– 3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
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623	79– 4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	79– 5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	79– 5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	79– 6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	79– 7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	79– 7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	79– 8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	79– 9341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	80– 0029	0098	0167	0236	0305	0373	0442	0511	0580	0648	69
632	80– 0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	80– 1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	80– 2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	80– 2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	80– 3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	80– 4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	80– 4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	80– 5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	80– 6180	6248	6316	6384	6451	6519	6587	6655	6723	6790	68
641	80– 6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	80– 7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	80– 8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	80– 8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	80– 9560	9627	9694	9762	9829	9896	9964	……	……	……	67
645	81– ……	……	……	……	……	……	……	0031	0098	0165	67
646	81– 0233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	81– 0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	81– 1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	81– 2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	81– 2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	81– 3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	81– 4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	81– 4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	81– 5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	81– 6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	81– 6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	81– 7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	81– 8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	81– 8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	81– 9544	9610	9676	9741	9807	9873	9939	……	……	……	66
660	82– ……	……	……	……	……	……	……	0004	0070	0136	66
661	82– 0201	0267	0333	0399	0464	0530	0595	0661	0727	0792	66
662	82– 0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	82– 1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	82– 2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	82– 2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	82– 3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	82– 4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	82– 4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
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669	82– 5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	82– 6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
671	82– 6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	82– 7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	82– 8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	82– 8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	82– 9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	82– 9947	......	......	......	......	......	......	......	......	......	64
676	83–	0011	0075	0139	0204	0268	0332	0396	0460	0525	64
677	83– 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	83– 1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	83– 1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	83– 2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	64
681	83– 3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	83– 3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	83– 4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	83– 5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	83– 5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	83– 6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	83– 6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	83– 7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	83– 8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	83– 8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	83– 9478	9541	9604	9667	9729	9792	9855	9918	9981	......	63
691	84–	......	......	......	......	......	......	......	......	0043	63
692	84– 0106	0169	0232	0294	0357	0420	0482	0545	0608	0671	63
693	84– 0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	84– 1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	84– 1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	84– 2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	84– 3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	84– 3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	84– 4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
700	84– 5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
701	84– 5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	84– 6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	84– 6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	84– 7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	84– 8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	84– 8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	84– 9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85– 0033	0095	0156	0217	0279	0340	0401	0462	0524	0585	61
709	85– 0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	85– 1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	85– 1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	85– 2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	85– 3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	85– 3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
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715	85– 4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	85– 4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	85– 5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	85– 6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	85– 6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	85– 7332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	85– 7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	85– 8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	85– 9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	85– 9739	9799	9859	9918	9978	......	......	......	......	......	60
724	86–	......	......	......	......	0038	0098	0158	0218	0278	60
725	86– 0338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	86– 0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	86– 1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	86– 2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	86– 2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	86– 3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	59
731	86– 3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	86– 4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	86– 5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	86– 5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	86– 6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	86– 6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	86– 7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	86– 8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	86– 8644	8703	8762	8821	8870	8938	8997	9056	9114	9173	59
740	86– 9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	59
741	86– 9818	9877	9935	9994	......	......	......	......	......	......	59
741	87–	......	......	......	0053	0111	0170	0228	0287	0345	59
742	87– 0404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	87– 0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	87– 1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	87– 2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	87– 2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	87– 3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	87– 3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	87– 4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	87– 5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	87– 5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	87– 6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	87– 6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	87– 7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	87– 7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	87– 8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	87– 9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	87– 9669	9726	9784	9841	9898	9956	......	......	......	......	57
758	88–	......	......	......	......	......	0013	0070	0127	0185	57
759	88– 0242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
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760	88– 0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	57
761	88– 1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	88– 1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	88– 2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	88– 3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	88– 3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	88– 4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	88– 4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	88– 5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	88– 5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	88– 6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	56
771	88– 7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	88– 7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	88– 8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	88– 8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	88– 9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	88– 9862	9918	9974	......	......	......	......	......	......	......	56
776	89–	......	......	0030	0086	0141	0197	0253	0309	0365	56
777	89– 0421	0477	0533	0589	0645	0700	0756	0812	0868	0924	56
778	89– 0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	89– 1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	89– 2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
781	89– 2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	89– 3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	89– 3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	89– 4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	89– 4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	89– 5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	89– 5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	89– 6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	89– 7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	89– 7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	89– 8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	89– 8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	89– 9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	89– 9821	9875	9930	9985	......	......	......	......	......	......	55
794	90–	......	......	......	0039	0094	0149	0203	0258	0312	55
795	90– 0367	0422	0476	0531	0586	0640	0695	0749	0804	0859	55
796	90– 0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	90– 1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	90– 2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	90– 2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	90– 3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54
801	90– 3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	90– 4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	90– 4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	90– 5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	90– 5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
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806	90– 6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	90– 6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	90– 7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	90– 7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	90– 8485	8539	8592	8646	8699	8753	8807	8860	8914	8967	54
811	90– 9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	90– 9556	9610	9663	9716	9770	9823	9877	9930	9984	......	54
812	91–	......	......	......	......	......	......	......	......	0037	53
813	91– 0091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	91– 0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	91– 1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	91– 1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	91– 2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	91– 2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	91– 3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	91– 3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	53
821	91– 4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	91– 4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	91– 5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	91– 5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	91– 6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	91– 6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	91– 7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	91– 8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	91– 8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	91– 9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	91– 9601	9653	9706	9758	9810	9862	9914	9967	......	......	52
831	92–	......	......	......	......	......	......	......	0019	0071	52
832	92– 0123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	92– 0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	92– 1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	92– 1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	92– 2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	92– 2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	92– 3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	92– 3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	92– 4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	92– 4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	92– 5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	92– 5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	92– 6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	92– 6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	92– 7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	92– 7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	92– 8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	92– 8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	92– 9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	92– 9930	9981	......	......	......	......	......	......	......	......	51
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
851	93-	......	0032	0083	0134	0185	0236	0287	0338	0389	51
852	93- 0440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	93- 0949	1000	1051	1102	1153	1203	1254	1305	1356	1407	51
854	93- 1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	93- 1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	93- 2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	93- 2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	93- 3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	93- 3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	93- 4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	93- 5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	93- 5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	93- 6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	93- 6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	93- 7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	93- 7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	93- 8019	8069	8119	8169	8219	8269	8319	8370	8420	8470	50
868	93- 8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	93- 9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	93- 9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	94- 0018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	94- 0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	94- 1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	94- 1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	94- 2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	94- 2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	94- 3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	94- 3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	94- 3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	94- 4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	94- 4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	94- 5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	94- 5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	94- 6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	94- 6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	94- 7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	94- 7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	94- 8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	94- 8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	94- 9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	94- 9878	9926	9975	......	......	......	......	......	......	......	49
891	95-	......	......	0024	0073	0121	0170	0219	0267	0316	49
892	95- 0365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	95- 0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	95- 1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	95- 1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	95- 2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	95- 2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
898	95- 3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	95- 3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	95- 4243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
901	95- 4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	95- 5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	95- 5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	95- 6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	95- 6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	95- 7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	95- 7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	95- 8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	95- 8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	95- 9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	95- 9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	95- 9995	......	......	......	......	......	......	......	......	......	48
912	96-	0042	0090	0138	0185	0233	0280	0328	0376	0423	48
913	96- 0471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	96- 0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	96- 1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	96- 1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	96- 2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	96- 2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	96- 3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	96- 3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	96- 4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	96- 4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	96- 5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	96- 5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	96- 6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	96- 6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	96- 7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	96- 7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	96- 8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	96- 8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	96- 8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	96- 9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	96- 9882	9928	9975	......	......	......	......	......	......	......	47
933	97-	......	......	0021	0068	0114	0161	0207	0254	0300	47
934	97- 0347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	97- 0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	97- 1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	97- 1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	97- 2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	97- 2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
940	97- 3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	97- 3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	97- 4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	97- 4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
944	97– 4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	97– 5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	97– 5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	97– 6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	97– 6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	97– 7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	97– 7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	97– 8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	97– 8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	97– 9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	97– 9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	98– 0003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	98– 0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	98– 0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	98– 1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	98– 1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	98– 2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	98– 2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	98– 3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	98– 3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	98– 4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	98– 4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	98– 4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	98– 5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	98– 5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	98– 6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	98– 6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	98– 7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	98– 7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	98– 8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	98– 8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	98– 9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	98– 9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	98– 9895	9939	9983	......	......	......	......	......	......	......	44
977	99–	......	......	0028	0072	0117	0161	0206	0250	0294	44
978	99– 0339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
979	99– 0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	99– 1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	99– 1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	99– 2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	99– 2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	99– 2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	99– 3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	99– 3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	99– 4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	99– 4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	99– 5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
990	99– 5635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	99– 6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	99– 6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	99– 6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	99– 7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	99– 7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	99– 8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	99– 8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	99– 9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	99– 9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N	0	1	2	3	4	5	6	7	8	9	D

TABLE No. II.—HYPERBOLIC LOGARITHMS OF NUMBERS FROM 1.01 TO 30.

Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
1.01	.0099	1.36	.3075	1.71	.5365	2.06	.7227
1.02	.0198	1.37	.3148	1.72	.5423	2.07	.7275
1.03	.0296	1.38	.3221	1.73	.5481	2.08	.7324
1.04	.0392	1.39	.3293	1.74	.5539	2.09	.7372
1.05	.0488	1.40	.3365	1.75	.5596	2.10	.7419
1.06	.0583	1.41	.3436	1.76	.5653	2.11	.7467
1.07	.0677	1.42	.3507	1.77	.5710	2.12	.7514
1.08	.0770	1.43	.3577	1.78	.5766	2.13	.7561
1.09	.0862	1.44	.3646	1.79	.5822	2.14	.7608
1.10	.0953	1.45	.3716	1.80	.5878	2.15	.7655
1.11	.1044	1.46	.3784	1.81	.5933	2.16	.7701
1.12	.1133	1.47	.3853	1.82	.5988	2.17	.7747
1.13	.1222	1.48	.3920	1.83	.6043	2.18	.7793
1.14	.1310	1.49	.3988	1.84	.6098	2.19	.7839
1.15	.1398	1.50	.4055	1.85	.6152	2.20	.7885
1.16	.1484	1.51	.4121	1.86	.6206	2.21	.7930
1.17	.1570	1.52	.4187	1.87	.6259	2.22	.7975
1.18	.1655	1.53	.4253	1.88	.6313	2.23	.8020
1.19	.1740	1.54	.4318	1.89	.6366	2.24	.8065
1.20	.1823	1.55	.4383	1.90	.6419	2.25	.8109
1.21	.1906	1.56	.4447	1.91	.6471	2.26	.8154
1.22	.1988	1.57	.4511	1.92	.6523	2.27	.8198
1.23	.2070	1.58	.4574	1.93	.6575	2.28	.8242
1.24	.2151	1.59	.4637	1.94	.6627	2.29	.8286
1.25	.2231	1.60	.4700	1.95	.6678	2.30	.8329
1.26	.2311	1.61	.4762	1.96	.6729	2.31	.8372
1.27	.2390	1.62	.4824	1.97	.6780	2.32	.8416
1.28	.2469	1.63	.4886	1.98	.6831	2.33	.8458
1.29	.2546	1.64	.4947	1.99	.6881	2.34	.8502
1.30	.2624	1.65	.5008	2.00	.6931	2.35	.8544
1.31	.2700	1.66	.5068	2.01	.6981	2.36	.8587
1.32	.2776	1.67	.5128	2.02	.7031	2.37	.8629
1.33	.2852	1.68	.5188	2.03	.7080	2.38	.8671
1.34	.2927	1.69	.5247	2.04	.7129	2.39	.8713
1.35	.3001	1.70	.5306	2.05	.7178	2.40	.8755

Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
2.41	.8796	2.81	1.0332	3.21	1.1663	3.61	1.2837
2.42	.8838	2.82	1.0367	3.22	1.1694	3.62	1.2865
2.43	.8879	2.83	1.0403	3.23	1.1725	3.63	1.2892
2.44	.8920	2.84	1.0438	3.24	1.1756	3.64	1.2920
2.45	.8961	2.85	1.0473	3.25	1.1787	3.65	1.2947
2.46	.9002	2.86	1.0508	3.26	1.1817	3.66	1.2975
2.47	.9042	2.87	1.0543	3.27	1.1848	3.67	1.3002
2.48	.9083	2.88	1.0578	3.28	1.1878	3.68	1.3029
2.49	.9123	2.89	1.0613	3.29	1.1909	3.69	1.3056
2.50	.9163	2.90	1.0647	3.30	1.1939	3.70	1.3083
2.51	.9203	2.91	1.0682	3.31	1.1969	3.71	1.3110
2.52	.9243	2.92	1.0716	3.32	1.1999	3.72	1.3137
2.53	.9282	2.93	1.0750	3.33	1.2030	3.73	1.3164
2.54	.9322	2.94	1.0784	3.34	1.2060	3.74	1.3191
2.55	.9361	2.95	1.0818	3.35	1.2090	3.75	1.3218
2.56	.9400	2.96	1.0852	3.36	1.2119	3.76	1.3244
2.57	.9439	2.97	1.0886	3.37	1.2149	3.77	1.3271
2.58	.9478	2.98	1.0919	3.38	1.2179	3.78	1.3297
2.59	.9517	2.99	1.0953	3.39	1.2208	3.79	1.3324
2.60	.9555	3.00	1.0986	3.40	1.2238	3.80	1.3350
2.61	.9594	3.01	1.1019	3.41	1.2267	3.81	1.3376
2.62	.9632	3.02	1.1053	3.42	1.2296	3.82	1.3403
2.63	.9670	3.03	1.1086	3.43	1.2326	3.83	1.3429
2.64	.9708	3.04	1.1119	3.44	1.2355	3.84	1.3455
2.65	.9746	3.05	1.1151	3.45	1.2384	3.85	1.3481
2.66	.9783	3.06	1.1184	3.46	1.2413	3.86	1.3507
2.67	.9821	3.07	1.1217	3.47	1.2442	3.87	1.3533
2.68	.9858	3.08	1.1249	3.48	1.2470	3.88	1.3558
2.69	.9895	3.09	1.1282	3.49	1.2499	3.89	1.3584
2.70	.9933	3.10	1.1314	3.50	1.2528	3.90	1.3610
2.71	.9969	3.11	1.1346	3.51	1.2556	3.91	1.3635
2.72	1.0006	3.12	1.1378	3.52	1.2585	3.92	1.3661
2.73	1.0043	3.13	1.1410	3.53	1.2613	3.93	1.3686
2.74	1.0080	3.14	1.1442	3.54	1.2641	3.94	1.3712
2.75	1.0116	3.15	1.1474	3.55	1.2669	3.95	1.3737
2.76	1.0152	3.16	1.1506	3.56	1.2698	3.96	1.3762
2.77	1.0188	3.17	1.1537	3.57	1.2726	3.97	1.3788
2.78	1.0225	3.18	1.1569	3.58	1.2754	3.98	1.3813
2.79	1.0260	3.19	1.1600	3.59	1.2782	3.99	1.3838
2.80	1.0296	3.20	1.1632	3.60	1.2809	4.00	1.3863

Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
4.01	1.3888	4.41	1.4839	4.81	1.5707	5.21	1.6506
4.02	1.3913	4.42	1.4861	4.82	1.5728	5.22	1.6525
4.03	1.3938	4.43	1.4884	4.83	1.5748	5.23	1.6514
4.04	1.3962	4.44	1.4907	4.84	1.5769	5.24	1.6563
4.05	1.3987	4.45	1.4929	4.85	1.5790	5.25	1.6582
4.06	1.4012	4.46	1.4951	4.86	1.5810	5.26	1.6601
4.07	1.4036	4.47	1.4974	4.87	1.5831	5.27	1.6620
4.08	1.4061	4.48	1.4996	4.88	1.5851	5.28	1.6639
4.09	1.4085	4.49	1.5019	4.89	1.5872	5.29	1.6658
4.10	1.4110	4.50	1.5041	4.90	1.5892	5.30	1.6677
4.11	1.4134	4.51	1.5063	4.91	1.5913	5.31	1.6696
4.12	1.4159	4.52	1.5085	4.92	1.5933	5.32	1.6715
4.13	1.4183	4.53	1.5107	4.93	1.5953	5.33	1.6734
4.14	1.4207	4.54	1.5129	4.94	1.5974	5.34	1.6752
4.15	1.4231	4.55	1.5151	4.95	1.5994	5.35	1.6771
4.16	1.4255	4.56	1.5173	4.96	1.6014	5.36	1.6790
4.17	1.4279	4.57	1.5195	4.97	1.6034	5.37	1.6808
4.18	1.4303	4.58	1.5217	4.98	1.6054	5.38	1.6827
4.19	1.4327	4.59	1.5239	4.99	1.6074	5.39	1.6845
4.20	1.4351	4.60	1.5261	5.00	1.6094	5.40	1.6864
4.21	1.4375	4.61	1.5282	5.01	1.6114	5.41	1.6882
4.22	1.4398	4.62	1.5304	5.02	1.6134	5.42	1.6901
4.23	1.4422	4.63	1.5326	5.03	1.6154	5.43	1.6919
4.24	1.4446	4.64	1.5347	5.04	1.6174	5.44	1.6938
4.25	1.4469	4.65	1.5369	5.05	1.6194	5.45	1.6956
4.26	1.4493	4.66	1.5390	5.06	1.6214	5.46	1.6974
4.27	1.4516	4.67	1.5412	5.07	1.6233	5.47	1.6993
4.28	1.4540	4.68	1.5433	5.08	1.6253	5.48	1.7011
4.29	1.4563	4.69	1.5454	5.09	1.6273	5.49	1.7029
4.30	1.4586	4.70	1.5476	5.10	1.6292	5.50	1.7047
4.31	1.4609	4.71	1.5497	5.11	1.6312	5.51	1.7066
4.32	1.4633	4.72	1.5518	5.12	1.6332	5.52	1.7084
4.33	1.4656	4.73	1.5539	5.13	1.6351	5.53	1.7102
4.34	1.4679	4.74	1.5560	5.14	1.6371	5.54	1.7120
4.35	1.4702	4.75	1.5581	5.15	1.6390	5.55	1.7138
4.36	1.4725	4.76	1.5602	5.16	1.6409	5.56	1.7156
4.37	1.4748	4.77	1.5623	5.17	1.6429	5.57	1.7174
4.38	1.4770	4.78	1.5644	5.18	1.6448	5.58	1.7192
4.39	1.4793	4.79	1.5665	5.19	1.6467	5.59	1.7210
4.40	1.4816	4.80	1.5686	5.20	1.6487	5.60	1.7228

Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
5.61	1.7246	6.01	1.7934	6.41	1.8579	6.81	1.9184
5.62	1.7263	6.02	1.7951	6.42	1.8594	6.82	1.9199
5.63	1.7281	6.03	1.7967	6.43	1.8610	6.83	1.9213
5.64	1.7299	6.04	1.7984	6.44	1.8625	6.84	1.9228
5.65	1.7317	6.05	1.8001	6.45	1.8641	6.85	1.9242
5.66	1.7334	6.06	1.8017	6.46	1.8656	6.86	1.9257
5.67	1.7352	6.07	1.8034	6.47	1.8672	6.87	1.9272
5.68	1.7370	6.08	1.8050	6.48	1.8687	6.88	1.9286
5.69	1.7387	6.09	1.8066	6.49	1.8703	6.89	1.9301
5.70	1.7405	6.10	1.8083	6.50	1.8718	6.90	1.9315
5.71	1.7422	6.11	1.8099	6.51	1.8733	6.91	1.9330
5.72	1.7440	6.12	1.8116	6.52	1.8749	6.92	1.9344
5.73	1.7457	6.13	1.8132	6.53	1.8764	6.93	1.9359
5.74	1.7475	6.14	1.8148	6.54	1.8779	6.94	1.9373
5.75	1.7492	6.15	1.8165	6.55	1.8795	6.95	1.9387
5.76	1.7509	6.16	1.8181	6.56	1.8810	6.96	1.9402
5.77	1.7527	6.17	1.8197	6.57	1.8825	6.97	1.9416
5.78	1.7544	6.18	1.8213	6.58	1.8840	6.98	1.9430
5.79	1.7561	6.19	1.8229	6.59	1.8856	6.99	1.9445
5.80	1.7579	6.20	1.8245	6.60	1.8871	7.00	1.9459
5.81	1.7596	6.21	1.8262	6.61	1.8886	7.01	1.9473
5.82	1.7613	6.22	1.8278	6.62	1.8901	7.02	1.9488
5.83	1.7630	6.23	1.8294	6.63	1.8916	7.03	1.9502
5.84	1.7647	6.24	1.8310	6.64	1.8931	7.04	1.9516
5.85	1.7664	6.25	1.8326	6.65	1.8946	7.05	1.9530
5.86	1.7681	6.26	1.8342	6.66	1.8961	7.06	1.9544
5.87	1.7699	6.27	1.8358	6.67	1.8976	7.07	1.9559
5.88	1.7716	6.28	1.8374	6.68	1.8991	7.08	1.9573
5.89	1.7733	6.29	1.8390	6.69	1.9006	7.09	1.9587
5.90	1.7750	6.30	1.8405	6.70	1.9021	7.10	1.9601
5.91	1.7766	6.31	1.8421	6.71	1.9036	7.11	1.9615
5.92	1.7783	6.32	1.8437	6.72	1.9051	7.12	1.9629
5.93	1.7800	6.33	1.8453	6.73	1.9066	7.13	1.9643
5.94	1.7817	6.34	1.8469	6.74	1.9081	7.14	1.9657
5.95	1.7834	6.35	1.8485	6.75	1.9095	7.15	1.9671
5.96	1.7851	6.36	1.8500	6.76	1.9110	7.16	1.9685
5.97	1.7867	6.37	1.8516	6.77	1.9125	7.17	1.9699
5.98	1.7884	6.38	1.8532	6.78	1.9140	7.18	1.9713
5.99	1.7901	6.39	1.8547	6.79	1.9155	7.19	1.9727
6.00	1.7918	6.40	1.8563	6.80	1.9169	7.20	1.9741

Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
7.21	1.9755	7.61	2.0295	8.01	2.0807	8.41	2.1294
7.22	1.9769	7.62	2.0308	8.02	2.0819	8.42	2.1306
7.23	1.9782	7.63	2.0321	8.03	2.0832	8.43	2.1318
7.24	1.9796	7.64	2.0334	8.04	2.0844	8.44	2.1330
7.25	1.9810	7.65	2.0347	8.05	2.0857	8.45	2.1342
7.26	1.9824	7.66	2.0360	8.06	2.0869	8.46	2.1353
7.27	1.9838	7.67	2.0373	8.07	2.0882	8.47	2.1365
7.28	1.9851	7.68	2.0386	8.08	2.0894	8.48	2.1377
7.29	1.9865	7.69	2.0399	8.09	2.0906	8.49	2.1389
7.30	1.9879	7.70	2.0412	8.10	2.0919	8.50	2.1401
7.31	1.9892	7.71	2.0425	8.11	2.0931	8.51	2.1412
7.32	1.9906	7.72	2.0438	8.12	2.0943	8.52	2.1424
7.33	1.9920	7.73	2.0451	8.13	2.0956	8.53	2.1436
7.34	1.9933	7.74	2.0464	8.14	2.0968	8.54	2.1448
7.35	1.9947	7.75	2.0477	8.15	2.0980	8.55	2.1459
7.36	1.9961	7.76	2.0490	8.16	2.0992	8.56	2.1471
7.37	1.9974	7.77	2.0503	8.17	2.1005	8.57	2.1483
7.38	1.9988	7.78	2.0516	8.18	2.1017	8.58	2.1494
7.39	2.0001	7.79	2.0528	8.19	2.1029	8.59	2.1506
7.40	2.0015	7.80	2.0541	8.20	2.1041	8.60	2.1518
7.41	2.0028	7.81	2.0554	8.21	2.1054	8.61	2.1529
7.42	2.0042	7.82	2.0567	8.22	2.1066	8.62	2.1541
7.43	2.0055	7.83	2.0580	8.23	2.1078	8.63	2.1552
7.44	2.0069	7.84	2.0592	8.24	2.1090	8.64	2.1564
7.45	2.0082	7.85	2.0605	8.25	2.1102	8.65	2.1576
7.46	2.0096	7.86	2.0618	8.26	2.1114	8.66	2.1587
7.47	2.0109	7.87	2.0631	8.27	2.1126	8.67	2.1599
7.48	2.0122	7.88	2.0643	8.28	2.1138	8.68	2.1610
7.49	2.0136	7.89	2.0656	8.29	2.1150	8.69	2.1622
7.50	2.0149	7.90	2.0669	8.30	2.1163	8.70	2.1633
7.51	2.0162	7.91	2.0681	8.31	2.1175	8.71	2.1645
7.52	2.0176	7.92	2.0694	8.32	2.1187	8.72	2.1656
7.53	2.0189	7.93	2.0707	8.33	2.1199	8.73	2.1668
7.54	2.0202	7.94	2.0719	8.34	2.1211	8.74	2.1679
7.55	2.0215	7.95	2.0732	8.35	2.1223	8.75	2.1691
7.56	2.0229	7.96	2.0744	8.36	2.1235	8.76	2.1702
7.57	2.0242	7.97	2.0757	8.37	2.1247	8.77	2.1713
7.58	2.0255	7.98	2.0769	8.38	2.1258	8.78	2.1725
7.59	2.0268	7.99	2.0782	8.39	2.1270	8.79	2.1736
7.60	2.0281	8.00	2.0794	8.40	2.1282	8.80	2.1748

Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
8.81	2.1759	9.11	2.2094	9.41	2.2418	9.71	2.2732
8.82	2.1770	9.12	2.2105	9.42	2.2428	9.72	2.2742
8.83	2.1782	9.13	2.2116	9.43	2.2439	9.73	2.2752
8.84	2.1793	9.14	2.2127	9.44	2.2450	9.74	2.2762
8.85	2.1804	9.15	2.2138	9.45	2.2460	9.75	2.2773
8.86	2.1815	9.16	2.2148	9.46	2.2471	9.76	2.2783
8.87	2.1827	9.17	2.2159	9.47	2.2481	9.77	2.2793
8.88	2.1838	9.18	2.2170	9.48	2.2492	9.78	2.2803
8.89	2.1849	9.19	2.2181	9.49	2.2502	9.79	2.2814
8.90	2.1861	9.20	2.2192	9.50	2.2513	9.80	2.2824
8.91	2.1872	9.21	2.2203	9.51	2.2523	9.81	2.2834
8.92	2.1883	9.22	2.2214	9.52	2.2534	9.82	2.2844
8.93	2.1894	9.23	2.2225	9.53	2.2544	9.83	2.2854
8.94	2.1905	9.24	2.2235	9.54	2.2555	9.84	2.2865
8.95	2.1917	9.25	2.2246	9.55	2.2565	9.85	2.2875
8.96	2.1928	9.26	2.2257	9.56	2.2576	9.86	2.2885
8.97	2.1939	9.27	2.2268	9.57	2.2586	9.87	2.2895
8.98	2.1950	9.28	2.2279	9.58	2.2597	9.88	2.2905
8.99	2.1961	9.29	2.2289	9.59	2.2607	9.89	2.2915
9.00	2.1972	9.30	2.2300	9.60	2.2618	9.90	2.2925
9.01	2.1983	9.31	2.2311	9.61	2.2628	9.91	2.2935
9.02	2.1994	9.32	2.2322	9.62	2.2638	9.92	2.2946
9.03	2.2006	9.33	2.2332	9.63	2.2649	9.93	2.2956
9.04	2.2017	9.34	2.2343	9.64	2.2659	9.94	2.2966
9.05	2.2028	9.35	2.2354	9.65	2.2670	9.95	2.2976
9.06	2.2039	9.36	2.2364	9.66	2.2680	9.96	2.2986
9.07	2.2050	9.37	2.2375	9.67	2.2690	9.97	2.2996
9.08	2.2061	9.38	2.2386	9.68	2.2701	9.98	2.3006
9.09	2.2072	9.39	2.2396	9.69	2.2711	9.99	2.3016
9.10	2.2083	9.40	2.2407	9.70	2.2721	10.00	2.3026
10.25	2.3279	12.75	2.5455	15.50	2.7408	21.0	3.0445
10.50	2.3513	13.00	2.5649	16.0	2.7726	22.0	3.0911
10.75	2.3749	13.25	2.5840	16.5	2.8034	23.0	3.1355
11.00	2.3979	13.50	2.6027	17.0	2.8332	24.0	3.1781
11.25	2.4201	13.75	2.6211	17.5	2.8621	25.0	3.2189
11.50	2.4430	14.00	2.6391	18.0	2.8904	26.0	3.2581
11.75	2.4636	14.25	2.6567	18.5	2.9173	27.0	3.2958
12.00	2.4849	14.50	2.6740	19.0	2.9444	28.0	3.3322
12.25	2.5052	14.75	2.6913	19.5	2.9703	29.0	3.3673
12.50	2.5262	15.00	2.7081	20.0	2.9957	30.0	3.4012

TABLE No. III.—NUMBERS, OR DIAMETERS OF CIRCLES, CIRCUMFERENCES, AREAS, SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS.

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
1	3.1416	0.7854	1	1	1.000	1.000
2	6.28	3.14	4	8	1.414	1.259
3	9.42	7.07	9	27	1.732	1.442
4	12.57	12.57	16	64	2.000	1.587
5	15.71	19.63	25	125	2.236	1.709
6	18.85	28.27	36	216	2.449	1.817
7	21.99	38.48	49	343	2.645	1.912
8	25.13	50.27	64	512	2.828	2.000
9	28.27	63.62	81	729	3.000	2.080
10	31.42	78.54	100	1,000	3.162	2.154
11	34.56	95.03	121	1,331	3.316	2.223
12	37.70	113.10	144	1,728	3.464	2.289
13	40.84	132.73	169	2,197	3.605	2.351
14	43.98	153.94	196	2,744	3.741	2.410
15	47.12	176.71	225	3,375	3.872	2.466
16	50.26	201.06	256	4,096	4.000	2.519
17	53.41	226.98	289	4,913	4.123	2.571
18	56.55	254.47	324	5,832	4.242	2.620
19	59.69	283.53	361	6,859	4.358	2.668
20	62.83	314.16	400	8,000	4.472	2.714
21	65.97	346.36	441	9,261	4.582	2.758
22	69.11	380.13	484	10,648	4.690	2.802
23	72.26	415.48	529	12,167	4.795	2.843
24	75.40	452.39	576	13,824	4.898	2.884
25	78.54	490.87	625	15,625	5.000	2.924
26	81.68	530.93	676	17,576	5.099	2.962
27	84.82	572.56	729	19,683	5.196	3.000
28	87.96	615.75	784	21,952	5.291	3.036
29	91.11	660.52	841	24,389	5.385	3.072
30	94.25	706.86	900	27,000	5.477	3.107
31	97.39	754.77	961	29,791	5.567	3.141
32	100.53	804.25	1,024	32,768	5.656	3.174
33	103.67	855.30	1,089	35,937	5.744	3.207
34	106.81	907.92	1,156	39,304	5.830	3.239
35	109.96	962.11	1,225	42,875	5.916	3.271
36	113.10	1017.88	1,296	46,656	6.000	3.301
37	116.24	1075.21	1,369	50,653	6.082	3.332
38	119.38	1134.11	1,444	54,872	6.164	3.361
39	122.52	1194.59	1,521	59,319	6.244	3.391
40	125.66	1256.64	1,600	64,000	6.324	3.419
41	128.80	1320.25	1,681	68,921	6.403	3.448
42	131.95	1385.44	1,764	74,088	6.480	3.476

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
43	135.09	1452.20	1,849	79,507	6.557	3.503
44	138.23	1520.53	1,936	85,184	6.633	3.530
45	141.37	1590.43	2,025	91,125	6.708	3.556
46	144.51	1661.90	2,116	97,336	6.782	3.583
47	147.65	1734.94	2,209	103,823	6.855	3.608
48	150.80	1809.56	2,304	110,592	6.928	3.634
49	153.94	1885.74	2,401	117,649	7.000	3.659
50	157.08	1963.50	2,500	125,000	7.071	3.684
51	160.22	2042.82	2,601	132,651	7.141	3.708
52	163.36	2123.72	2,704	140,608	7.211	3.732
53	166.50	2206.18	2,809	148,877	7.280	3.756
54	169.65	2290.22	2,916	157,464	7.348	3.779
55	172.79	2375.83	3,025	166,375	7.416	3.802
56	175.93	2463.01	3,136	175,616	7.483	3.825
57	179.07	2551.76	3,249	185,193	7.549	3.848
58	182.21	2642.08	3,364	195,112	7.615	3.870
59	185.35	2733.97	3,481	205,379	7.681	3.892
60	188.50	2827.43	3,600	216,000	7.745	3.914
61	191.64	2922.47	3,721	226,981	7.810	3.936
62	194.78	3019.07	3,844	238,328	7.874	3.957
63	197.92	3117.25	3,969	250,047	7.937	3.979
64	201.06	3216.99	4,096	262,144	8.000	4.000
65	204.20	3318.31	4,225	274,625	8.062	4.020
66	207.34	3421.19	4,356	287,496	8.124	4.041
67	210.49	3525.65	4,489	300,763	8.185	4.061
68	213.63	3631.68	4,624	314,432	8.246	4.081
69	216.77	3739.28	4,761	328,509	8.306	4.101
70	219.91	3848.45	4,900	343,000	8.366	4.121
71	223.05	3959.19	5,041	357,911	8.426	4.140
72	226.19	4071.50	5,184	373,248	8.485	4.160
73	229.34	4185.39	5,329	389,017	8.544	4.179
74	232.48	4300.84	5,476	405,224	8.602	4.198
75	235.62	4417.86	5,625	421,875	8.660	4.217
76	238.76	4536.46	5,776	438,976	8.717	4.235
77	241.90	4656.63	5,929	456,533	8.744	4.254
78	245.04	4778.36	6,084	474,552	8.831	4.272
79	248.19	4901.67	6,241	493,039	8.888	4.290
80	251.33	5026.55	6,400	512,000	8.944	4.308
81	254.47	5153.00	6,561	531,441	9.000	4.326
82	257.61	5281.02	6,724	551,368	9.055	4.344
83	260.75	5410.61	6,889	571,787	9.110	4.362
84	263.89	5541.77	7,056	592,704	9.165	4.379
85	267.03	5674.50	7,225	614,125	9.219	4.396
86	270.18	5808.80	7,396	636,056	9.273	4.414
87	273.32	5944.68	7,569	658,503	9.327	4.431
88	276.46	6082.12	7,744	681,472	9.380	4.447
89	279.60	6221.14	7,921	704,969	9.433	4.461
90	282.74	6361.73	8,100	729,000	9.486	4.481

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
91	285.88	6503.88	8,281	753,571	9.539	4.497
92	289.03	6647.61	8,464	778,688	9.591	4.514
93	292.17	6792.91	8,649	804,357	9.643	4.530
94	295.31	6939.78	8,836	830,584	9.695	4.546
95	298.45	7088.22	9,025	857,375	9.746	4.562
96	301.59	7238.23	9,216	884,736	9.797	4.578
97	304.73	7389.81	9,409	912,673	9.848	4.594
98	307.88	7542.96	9,604	941,192	9.899	4.610
99	311.02	7697.69	9,801	970,299	9.949	4.626
100	314.16	7853.98	10,000	1,000,000	10.000	4.641
101	317.30	8011.85	10,201	1,030,301	10.049	4.657
102	320.41	8171.28	10,404	1,061,208	10.099	4.672
103	323.58	8332.29	10,609	1,092,727	10.148	4.687
104	326.73	8494.87	10,816	1,124,864	10.198	4.702
105	329.87	8659.01	11,025	1,157,625	10.246	4.717
106	333.01	8824.73	11,236	1,191,016	10.295	4.732
107	336.15	8992.02	11,449	1,225,043	10.344	4.747
108	339.29	9160.88	11,664	1,259,712	10.392	4.762
109	342.43	9331.32	11,881	1,295,029	10.440	4.776
110	345.57	9503.32	12,100	1,331,000	10.488	4.791
111	348.72	9676.89	12,321	1,367,631	10.535	4.805
112	351.86	9852.03	12,544	1,404,928	10.583	4.820
113	355.00	10028.75	12,769	1,442,897	10.630	4.834
114	358.14	10207.03	12,996	1,481,544	10.677	4.848
115	361.28	10386.89	13,225	1,520,875	10.723	4.862
116	364.42	10568.32	13,456	1,560,896	10.770	4.876
117	367.57	10751.32	13,689	1,601,613	10.816	4.890
118	370.71	10935.88	13,924	1,643,032	10.862	4.904
119	373.85	11122.02	14,161	1,685,159	10.908	4.918
120	376.99	11309.73	14,400	1,728,000	10.954	4.932
121	380.13	11499.01	14,641	1,771,561	11.000	4.946
122	383.27	11689.87	14,884	1,815,848	11.045	4.959
123	386.42	11882.29	15,129	1,860,867	11.090	4.973
124	389.56	12076.28	15,376	1,906,624	11.135	4.986
125	392.70	12271.85	15,625	1,953,125	11.180	5.000
126	395.84	12468.98	15,876	2,000,376	11.224	5.013
127	398.98	12667.69	16,129	2,048,383	11.269	5.026
128	402.12	12867.96	16,384	2,097,152	11.313	5.039
129	405.26	13069.81	16,641	2,146,689	11.357	5.052
130	408.41	13273.23	16,900	2,197,000	11.401	5.065
131	411.55	13478.22	17,161	2,248,091	11.445	5.078
132	414.69	13684.78	17,424	2,299,968	11.489	5.091
133	417.83	13892.91	17,689	2,352,637	11.532	5.104
134	420.97	14102.61	17,956	2,406,104	11.575	5.117
135	424.11	14313.88	18,225	2,460,375	11.618	5.129
136	427.26	14526.72	18,496	2,515,456	11.661	5.142
137	430.40	14741.14	18,769	2,571,353	11.704	5.155
138	433.54	14957.12	19,044	2,620,872	11.747	5.167

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
139	436.68	15174.68	19,321	2,685,619	11.789	5.180
140	439.82	...15393.80	...19,600	... 2,744,000	11.832	5.192
141	442.96	15614.50	19,881	2,803,221	11.874	5.204
142	446.11	...15836.77	...20,164	... 2,863,288	11.916	5.217
143	449.25	16060.61	20,449	2,924,207	11.958	5.229
144	452.39	...16286.02	...20,736	... 2,985,984	12.000	5.241
145	455.53	16513.00	21,025	3,048,625	12.041	5.253
146	458.67	...16741.55	...21,316	... 3,112,136	12.083	5.265
147	461.81	16971.67	21,609	3,176,523	12.124	5.277
148	464.96	...17203.36	...21,904	... 3,241,792	12.165	5.289
149	468.10	17436.62	22,201	3,307,949	12.206	5.301
150	471.24	...17671.46	...22,500	... 3,375,000	12.247	5.313
151	474.38	17907.86	22,801	3,442,951	12.288	5.325
152	477.52	...18145.84	...23,104	... 3,511,808	12.328	5.336
153	480.66	18385.39	23,409	3,581,577	12.369	5.348
154	483.80	...18626.50	...23,716	... 3,652,264	12.409	5.360
155	486.95	18869.19	24,025	3,723,875	12.449	5.371
156	490.09	...19113.45	...24,336	... 3,796,416	12.489	5.383
157	493.23	19359.28	24,649	3,869,893	12.529	5.394
158	496.37	...19606.68	...24,964	... 3,944,312	12.569	5.406
159	499.51	19855.65	25,281	4,019,679	12.609	5.417
160	502.65	...20106.19	...25,600	... 4,096,000	12.649	5.428
161	505.80	20358.34	25,921	4,173,281	12.688	5.440
162	508.94	...20611.99	...26,244	... 4,251,528	12.727	5.451
163	512.08	20867.24	26,569	4,330,747	12.767	5.462
164	515.22	...21124.07	...26,896	... 4,410,944	12.806	5.473
165	518.36	21382.46	27,225	4,492,125	12.845	5.484
166	521.50	...21642.43	...27,556	... 4,574,296	12.884	5.495
167	524.65	21903.97	27,889	4,657,463	12.922	5.506
168	527.79	...22167.08	...28,224	... 4,741,632	12.961	5.517
169	530.93	22431.76	28,561	4,826,809	13.000	5.528
170	534.07	...22698.01	...28,900	... 4,913,000	13.038	5.539
171	537.21	22965.83	29,241	5,000,211	13.076	5.550
172	540.35	...23235.22	...29,584	... 5,088,448	13.114	5.561
173	543.50	23506.18	29,929	5,177,717	13.152	5.572
174	546.64	...23778.71	...30,276	... 5,268,024	13.190	5.582
175	549.78	24052.82	30,625	5,359,375	13.228	5.593
176	552.92	...24328.49	...30,976	... 5,451,776	13.266	5.604
177	556.06	24605.79	31,329	5,545,233	13.304	5.614
178	559.20	...24884.56	...31,684	... 5,639,752	13.341	5.625
179	562.34	25164.94	32,041	5,735,339	13.379	5.635
180	565.49	...25446.90	...32,400	... 5,832,000	13.416	5.646
181	568.63	25730.43	32,761	5,929,741	13.453	5.656
182	571.77	...26015.53	...33,124	... 6,028,568	13.490	5.667
183	574.91	26302.20	33,489	6,128,487	13.527	5.677
184	578.05	...26590.44	...33,856	... 6,229,504	13.564	5.687
185	581.19	26880.25	34,225	6,331,625	13.601	5.698
186	584.34	...27171.63	...34,596	... 6,434,856	13.638	5.708

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
187	587.48	27464.59	34,969	6,539,203	13.674	5.718
188	590.62	...27759.11	...35,344	... 6,644,672	13.711	5.728
189	593.76	28055.21	35,721	6,751,269	13.747	5.738
190	596.90	...28352.87	...36,100	... 6,859,000	13.784	5.748
191	600.04	28652.11	36,481	6,967,871	13.820	5.758
192	603.19	...28952.92	...36,864	... 7,077,888	13.856	5.768
193	606.33	29255.30	37,249	7,189,057	13.892	5.778
194	609.47	...29559.26	...37,636	... 7,301,384	13.928	5.788
195	612.61	29864.77	38,025	7,414,875	13.964	5.798
196	615.75	...30171.86	...38,416	... 7,529,536	14.000	5.808
197	618.89	30480.52	38,809	7,645,373	14.035	5.818
198	622.03	...30790.75	...39,204	... 7,762,392	14.071	5.828
199	625.18	31102.55	39,601	7,880,599	14.106	5.838
200	628.32	...31415.93	...40,000	... 8,000,000	14.142	5.848
201	631.46	31730.87	40,401	8,120,601	14.177	5.857
202	634.60	...32047.39	...40,804	... 8,242,408	14.212	5.867
203	637.74	32365.47	41,209	8,365,427	14.247	5.877
204	640.88	...32685.13	...41,616	... 8,489,664	14.282	5.886
205	644.03	33006.36	42,025	8,615,125	14.317	5.896
206	647.17	...33329.16	...42,436	... 8,741,816	14.352	5.905
207	650.31	33653.53	42,849	8,869,743	14.387	5.915
208	653.45	...33979.47	...43,264	... 8,998,912	14.422	5.924
209	656.59	34306.98	43,681	9,123,329	14.456	5.934
210	659.73	...34636.06	...44,100	... 9,261,000	14.491	5.943
211	662.88	34966.71	44,521	9,393,931	14.525	5.953
212	666.02	...35298.94	...44,944	... 9,528,128	14.560	5.962
213	669.16	35632.73	45,369	9,663,597	14.594	5.972
214	672.30	...35968.09	...45,796	... 9,800,344	14.628	5.981
215	675.44	36305.03	46,225	9,938,375	14.662	5.990
216	678.58	...36643.61	...46,656	...10,077,696	14.696	6.000
217	681.73	36983.61	47,089	10,218,313	14.730	6.009
218	684.87	...37325.26	...47,524	...10,360,232	14.764	6.018
219	688.01	37668.48	47,961	10,503,459	14.798	6.027
220	691.15	...38013.27	...48,400	...10,648,000	14.832	6.036
221	694.29	38359.63	48,841	10,793,861	14.866	6.045
222	697.43	...38707.56	...49,284	...10,941,048	14.899	6.055
223	700.57	39057.07	49,729	11,089,567	14.933	6.064
224	703.72	...39408.14	...50,176	...11,239,424	14.966	6.073
225	706.86	39760.78	50,625	11,390,625	15.000	6.082
226	710.00	...40115.00	...51,076	...11,543,176	15.033	6.091
227	713.14	40470.78	51,529	11,697,083	15.066	6.100
228	716.28	...40828.14	...51,984	...11,852,352	15.099	6.109
229	719.42	41187.07	52,441	12,008,989	15.132	6.118
230	722.57	...41547.56	...52,900	...12,167,000	15.165	6.126
231	725.71	41909.63	53,361	12,326,391	15.198	6.135
232	728.85	...42273.27	...53,824	...12,487,168	15.231	6.144
233	731.99	42638.48	54,289	12,649,337	15.264	6.153
234	735.13	...43005.26	...54,756	...12,812,904	15.297	6.162

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
235	738.27	43373.61	55,225	12,977,875	15.329	6.171
236	741.42	43743.54	55,696	13,144,256	15.362	6.179
237	744.56	44115.03	56,169	13,312,053	15.394	6.188
238	747.70	44488.09	56,644	13,481,272	15.427	6.197
239	750.84	44862.73	57,121	13,651,919	15.459	6.205
240	753.98	45238.93	57,600	13,824,000	15.491	6.214
241	757.12	45616.71	58,081	13,997,521	15.524	6.223
242	760.26	45996.06	58,564	14,172,488	15.556	6.231
243	763.41	46376.98	59,049	14,348,907	15.588	6.240
244	766.55	46759.47	59,536	14,526,784	15.620	6.248
245	769.69	47143.52	60,025	14,706,125	15.652	6.257
246	772.83	47529.16	60,516	14,886,936	15.684	6.265
247	775.97	47916.36	61,009	15,069,223	15.716	6.274
248	779.11	48305.13	61,504	15,252,992	15.748	6.282
249	782.26	48695.47	62,001	15,438,249	15.779	6.291
250	785.40	49087.39	62,500	15,625,000	15.811	6.299
251	788.54	49480.87	63,001	15,813,251	15.842	6.307
252	791.68	49875.92	63,504	16,003,008	15.874	6.316
253	794.82	50272.55	64,009	16,194,277	15.905	6.324
254	797.96	50670.75	64,516	16,387,064	15.937	6.333
255	801.11	51070.52	65,025	16,581,375	15.968	6.341
256	804.25	51471.86	65,536	16,777,216	16.000	6.349
257	807.39	51874.76	66,049	16,974,593	16.031	6.357
258	810.53	52279.24	66,564	17,173,512	16.062	6.366
259	813.67	52685.29	67,081	17,373,979	16.093	6.374
260	816.81	53092.96	67,600	17,576,000	16.124	6.382
261	819.96	53502.11	68,121	17,779,581	16.155	6.390
262	823.10	53912.87	68,644	17,984,728	16.186	6.398
263	826.24	54325.21	69,169	18,191,447	16.217	6.406
264	829.38	54739.11	69,696	18,399,744	16.248	6.415
265	832.52	55154.59	70,225	18,609,625	16.278	6.423
266	835.66	55571.63	70,756	18,821,096	16.309	6.431
267	838.80	55990.25	71,289	19,034,163	16.340	6.439
268	841.95	56410.44	71,824	19,248,832	16.370	6.447
269	845.09	56832.20	72,361	19,465,109	16.401	6.455
270	848.23	57255.53	72,900	19,683,000	16.431	6.463
271	851.37	57680.43	73,441	19,902,511	16.462	6.471
272	854.51	58106.90	73,984	20,123,648	16.492	6.479
273	857.65	58534.94	74,529	20,346,417	16.522	6.487
274	860.80	58964.55	75,076	20,570,824	16.552	6.495
275	863.94	59395.74	75,625	20,796,875	16.583	6.502
276	867.08	59828.49	76,176	21,024,576	16.613	6.510
277	870.22	60262.82	76,729	21,253,933	16.643	6.518
278	873.36	60698.72	77,284	21,484,952	16.673	6.526
279	876.50	61136.18	77,841	21,717,639	16.703	6.534
280	879.65	61575.22	78,400	21,952,000	16.733	6.542
281	882.79	62015.82	78,961	22,188,041	16.763	6.549
282	885.93	62458.00	79,524	22,425,768	16.792	6.557

Number, or Diameter.	Circum-ference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
283	889.07	62901.75	80,089	22,665,187	16.822	6.565
284	892.21	...63347.07	...80,656	...22,906,304	16.852	6.573
285	895.35	63793.97	81,225	23,149,125	16.881	6.580
286	898.49	...64242.43	...81,796	...23,393,656	16.911	6.588
287	901.64	64692.46	82,369	23,639,903	16.941	6.596
288	904.78	...65144.07	...82,944	...23,887,872	16.970	6.603
289	907.92	65597.24	83,521	24,137,569	17.000	6.611
290	911.06	...66051.99	...84,100	...24,389,000	17.029	6.619
291	914.20	66508.30	84,681	24,642,171	17.059	6.627
292	917.34	...66966.19	...85,264	...24,897,088	17.088	6.634
293	920.49	67425.65	85,849	25,153,757	17.117	6.642
294	923.63	...67886.68	...86,436	...25,412,184	17.146	6.649
295	926.77	68349.28	87,025	25,672,375	17.176	6.657
296	929.91	...68813.45	...87,616	...25,934,336	17.205	6.664
297	933.05	69279.19	88,209	26,198,073	17.234	6.672
298	936.19	...69746.50	...88,804	...26,463,592	17.263	6.679
299	939.34	70215.38	89,401	26,730,899	17.292	6.687
300	942.48	...70685.83	...90,000	...27,000,000	17.320	6.694
301	945.62	71157.86	90,601	27,270,901	17.349	6.702
302	948.76	...71631.45	...91,204	...27,543,608	17.378	6.709
303	951.90	72106.62	91,809	27,818,127	17.407	6.717
304	955.04	...72583.36	...92,416	...28,094,464	17.436	6.724
305	958.19	73061.66	93,025	28,372,625	17.464	6.731
306	961.33	...73541.54	...93,636	...28,652,616	17.493	6.739
307	964.47	74022.99	94,249	28,934,443	17.521	6.746
308	967.61	...74506.01	...94,864	...29,218,112	17.549	6.753
309	970.75	74990.60	95,481	29,503,629	17.578	6.761
310	973.89	...75476.76	...96,100	...29,791,000	17.607	6.768
311	977.03	75964.50	96,721	30,080,231	17.635	6.775
312	980.18	...76453.80	...97,344	...30,371,328	17.663	6.782
313	983.32	76944.67	97,969	30,664,297	17.692	6.789
314	986.46	...77437.12	...98,596	...30,959,144	17.720	6.797
315	989.60	77931.13	99,225	31,255,875	17.748	6.804
316	992.74	...78426.72	...99,856	...31,554,496	17.776	6.811
317	995.88	78923.88	100,489	31,855,013	17.804	6.818
318	999.03	...79422.60	101,124	...32,157,432	17.832	6.826
319	1002.17	79922.90	101,761	32,461,759	17.860	6.833
320	1005.31	...80424.77	102,400	...32,768,000	17.888	6.839
321	1008.45	80928.21	103,041	33,076,161	17.916	6.847
322	1011.59	...81433.22	103,684	...33,386,248	17.944	6.854
323	1014.73	81939.80	104,329	33,698,267	17.972	6.861
324	1017.88	...82447.96	104,976	...34,012,224	18.000	6.868
325	1021.02	82957.68	105,625	34,328,125	18.028	6.875
326	1024.16	...83468.98	106,276	...34,645,976	18.055	6.882
327	1027.30	83981.84	106,929	34,965,783	18.083	6.889
328	1030.44	...84496.28	107,584	...35,287,552	18.111	6.896
329	1033.58	85012.28	108,241	35,611,289	18.138	6.903
330	1036.73	...85529.86	108,900	...35,937,000	18.166	6.910

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
331	1039.87	86049.01	109,561	36,264,691	18.193	6.917
332	1043.01	...86569.73	110,224	...36,594,368	18.221	6.924
333	1046.15	87092.02	110,889	36,926,037	18.248	6.931
334	1049.29	...87615.88	111,556	...37,259,704	18.276	6.938
335	1052.43	88141.31	112,225	37,595,375	18.303	6.945
336	1055.57	...88668.31	112,896	...37,933,056	18.330	6.952
337	1058.72	89196.88	113,569	38,272,753	18.357	6.959
338	1061.86	...89727.03	114,244	...38,614,472	18.385	6.966
339	1065.00	90258.74	114,921	38,958,219	18.412	6.973
340	1068.14	...90792.03	115,600	...39,304,000	18.439	6.979
341	1071.28	91326.88	116,281	39,651,821	18.466	6.986
342	1074.42	...91863.31	116,964	...40,001,688	18.493	6.993
343	1077.57	92401.31	117,649	40,353,607	18.520	7.000
344	1080.71	...92940.88	118,336	...40,707,584	18.547	7.007
345	1083.85	93482.02	119,025	41,063,625	18.574	7.014
346	1086.99	...94024.73	119,716	...41,421,736	18.601	7.020
347	1090.13	94569.01	120,409	41,781,923	18.628	7.027
348	1093.27	...95114.86	121,104	...42,144,192	18.655	7.034
349	1096.42	95662.28	121,801	42,508,549	18.681	7.040
350	1099.56	...96211.28	122,500	...42,875,000	18.708	7.047
351	1102.70	96761.84	123,201	43,243,551	18.735	7.054
352	1105.84	...97314.76	123,904	...43,614,208	18.762	7.061
353	1108.98	97867.68	124,609	43,986,977	18.788	7.067
354	1112.12	...98422.96	125,316	...44,361,864	18.815	7.074
355	1115.26	98979.80	126,025	44,738,875	18.842	7.081
356	1118.41	...99538.22	126,736	...45,118,016	18.868	7.087
357	1121.55	100098.21	127,449	45,499,293	18.894	7.094
358	1124.69	100659.27	128,164	...45,882,712	18.921	7.101
359	1127.83	101222.90	128,881	46,268,279	18.947	7.107
360	1130.97	101787.60	129,600	...46,656,000	18.974	7.114
361	1134.11	102353.87	130,321	47,045,881	19.000	7.120
362	1137.26	102921.72	131,044	...47,437,928	19.026	7.127
363	1140.40	103491.13	131,769	47,832,147	19.052	7.133
364	1143.54	104062.12	132,496	...48,228,544	19.079	7.140
365	1146.68	104634.67	133,225	48,627,125	19.105	7.146
366	1149.82	105208.80	133,956	...49,027,896	19.131	7.153
367	1152.96	105784.49	134,689	49,430,863	19.157	7.159
368	1156.11	106361.76	135,424	...49,836,032	19.183	7.166
369	1159.25	106940.60	136,161	50,243,409	19.209	7.172
370	1162.39	107521.01	136,900	...50,653,000	19.235	7.179
371	1165.53	108102.99	137,641	51,064,811	19.261	7.185
372	1168.67	108686.54	138,384	...51,478,848	19.287	7.192
373	1171.81	109271.66	139,129	51,895,117	19.313	7.198
374	1174.96	109858.35	139,876	...52,313,624	19.339	7.205
375	1178.10	110446.62	140,625	52,734,375	19.365	7.211
376	1181.24	111036.45	141,376	...53,157,376	19.391	7.218
377	1184.38	111627.86	142,129	53,582,633	19.416	7.224
378	1187.52	112220.83	142,884	...54,010,152	19.442	7.230

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
379	1190.66	112815.38	143,641	54,439,939	19.468	7.237
380	1193.80	113411.49	144,400	...54,872,000	19.493	7.243
381	1196.95	114009.18	145,161	55,306,341	19.519	7.249
382	1200.09	114608.44	145,924	...55,742,968	19.545	7.256
383	1203.23	115209.27	146,689	56,181,887	19.570	7.262
384	1206.37	115811.67	147,456	...56,623,104	19.596	7.268
385	1209.51	116415.64	148,225	57,066,625	19.621	7.275
386	1212.65	117021.18	148,996	...57,512,456	19.647	7.281
387	1215.80	117628.30	149,769	57,960,603	19.672	7.287
388	1218.94	118236.98	150,544	...58,411,072	19.698	7.294
389	1222.08	118847.24	151,321	58,863,869	19.723	7.299
390	1225.22	119459.06	152,100	...59,319,000	19.748	7.306
391	1228.36	120072.46	152,881	59,776,471	19.774	7.312
392	1231.50	120687.42	153,664	...60,236,288	19.799	7.319
393	1234.65	121303.96	154,449	60,698,457	19.824	7.325
394	1237.79	121922.07	155,236	...61,162,984	19.849	7.331
395	1240.93	122541.75	156,025	61,629,875	19.875	7.337
396	1244.07	123163.00	156,816	...62,099,136	19.899	7.343
397	1247.21	123785.82	157,609	62,570,773	19.925	7.349
398	1250.35	124410.21	158,404	...63,044,792	19.949	7.356
399	1253.49	125036.17	159,201	63,521,199	19.975	7.362
400	1256.64	125663.71	160,000	...64,000,000	20.000	7.368
401	1259.78	126292.81	160,801	64,481,201	20.025	7.374
402	1262.92	126923.48	161,604	...64,964,808	20.049	7.380
403	1266.06	127553.73	162,409	65,450,827	20.075	7.386
404	1269.20	128189.55	163,216	...65,939,264	20.099	7.392
405	1272.34	128824.93	164,025	66,430,125	20.125	7.399
406	1275.49	129461.89	164,836	...66,923,416	20.149	7.405
407	1278.63	130100.42	165,649	67,419,143	20.174	7.411
408	1281.77	130740.52	166,464	...67,911,312	20.199	7.417
409	1284.91	131382.19	167,281	68,417,929	20.224	7.422
410	1288.05	132025.43	168,100	...68,921,000	20.248	7.429
411	1291.19	132670.24	168,921	69,426,531	20.273	7.434
412	1294.34	133316.63	169,744	...69,934,528	20.298	7.441
413	1297.48	133964.58	170,569	70,444,997	20.322	7.447
414	1300.62	134614.10	171,396	...70,957,944	20.347	7.453
415	1303.76	135265.20	172,225	71,473,375	20.371	7.459
416	1306.90	135917.86	173,056	...71,991,296	20.396	7.465
417	1310.04	136572.10	173,889	72,511,713	20.421	7.471
418	1313.19	137227.91	174,724	...73,034,632	20.445	7.477
419	1316.33	137885.29	175,561	73,560,059	20.469	7.483
420	1319.47	138544.24	176,400	...74,088,000	20.494	7.489
421	1322.61	139204.70	177,241	74,618,461	20.518	7.495
422	1325.75	139866.85	178,084	...75,151,448	20.543	7.501
423	1328.89	140530.51	178,929	75,686,967	20.567	7.507
424	1332.03	141195.74	179,776	...76,225,024	20.591	7.513
425	1335.18	141862.54	180,625	76,765,625	20.615	7.518
426	1338.32	142530.92	181,476	...77,308,776	20.639	7.524

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427	1341.46	143200.86	182,329	77,854,483	20.664	7.530
428	1344.60	143872.38	183,184	...78,402,752	20.688	7.536
429	1347.74	144545.46	184,041	78,953,589	20.712	7.542
430	1550.88	145220.12	184,900	...79,507,000	20.736	7.548
431	1354.03	145896.35	185,761	80,062,991	20.760	7.554
432	1357.17	146574.15	186,624	...80,621,568	20.785	7.559
433	1360.31	147253.52	187,489	81,182,737	20.809	7.565
434	1363.45	147934.46	188,356	...81,746,504	20.833	7.571
435	1366.59	148616.97	189,225	82,312,875	20.857	7.577
436	1369.73	149301.05	190,096	...82,881,856	20.881	7.583
437	1372.88	149986.70	190,969	83,453,453	20.904	7.588
438	1376.02	150673.93	191,844	...84,027,672	20.928	7.594
439	1379.16	151362.72	192,721	84,604,519	20.952	7.600
440	1382.30	152053.08	193,600	...85,184,000	20.976	7.606
441	1385.44	152745.02	194,481	85,766,121	21.000	7.612
442	1388.58	153438.53	195,364	...86,350,388	21.024	7.617
443	1391.73	154133.60	196,249	86,938,307	21.047	7.623
444	1394.87	154830.25	197,136	...87,528,384	21.071	7.629
445	1398.01	155528.47	198,025	88,121,125	21.095	7.635
446	1401.15	156228.26	198,916	...88,716,536	21.119	7.640
447	1404.29	156929.62	199,809	89,314,623	21.142	7.646
448	1407.43	157632.55	200,704	...89,915,392	21.166	7.652
449	1410.57	158337.06	201,601	90,518,849	21.189	7.657
450	1413.72	159043.13	202,500	...91,125,000	21.213	7.663
451	1416.86	159750.77	203,401	91,733,851	21.237	7.669
452	1420.00	160459.99	204,304	...92,345,408	21.260	7.674
453	1423.14	161170.77	205,209	92,959,677	21.284	7.680
454	1426.28	161883.13	206,106	...93,576,664	21.307	7.686
455	1429.42	162597.06	207,025	94,196,375	21.331	7.691
456	1432.57	163312.55	207,936	...94,818,816	21.354	7.697
457	1435.71	164029.62	208,849	95,443,993	21.377	7.703
458	1438.85	164748.26	209,764	...96,071,912	21.401	7.708
459	1441.99	165468.47	210,681	96,702,579	21.424	7.714
460	1445.13	166190.25	211,600	...97,336,000	21.447	7.719
461	1448.27	166913.60	212,521	97,972,181	21.471	7.725
462	1451.42	167638.53	213,444	...98,611,128	21.494	7.731
463	1454.56	168365.02	214,369	99,252,847	21.517	7.736
464	1457.70	169093.08	215,296	...99,897,345	21.541	7.742
465	1460.84	169822.72	216,225	100,544,625	21.564	7.747
466	1463.98	170553.92	217,156	101,194,696	21.587	7.753
467	1467.12	171286.70	218,089	101,847,563	21.610	7.758
468	1470.26	172021.05	219,024	102,503,232	21.633	7.764
469	1473.41	172756.97	219,961	103,161,709	21.656	7.769
470	1476.55	173494.45	220,900	103,823,000	21.679	7.775
471	1479.69	174233.51	221,841	104,487,111	21.702	7.780
472	1482.83	174974.14	222,784	105,154,048	21.725	7.786
473	1485.97	175716.35	223,729	105,823,817	21.749	7.791
474	1489.11	176460.12	224,676	106,496,424	21.771	7.797

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475	1492.26	177205.46	225,625	107,171,875	21.794	7.802
476	1495.40	177952.37	226,576	107,850,176	21.817	7.808
477	1498.54	178700.86	227,529	108,531,333	21.840	7.813
478	1501.68	179450.91	228,484	109,215,352	21.863	7.819
479	1504.82	180202.54	229,441	109,902,239	21.886	7.824
480	1507.96	180955.74	230,400	110,592,000	21.909	7.830
481	1511.11	181710.50	231,361	111,284,641	21.932	7.835
482	1514.25	182466.84	232,324	111,980,168	21.954	7.840
483	1517.39	183224.75	233,289	112,678,587	21.977	7.846
484	1520.53	183984.23	234,256	113,379,904	22.000	7.851
485	1523.67	184745.28	235,225	114,084,125	22.023	7.857
486	1526.81	185507.90	236,196	114,791,256	22.045	7.862
487	1529.96	186272.10	237,169	115,501,303	22.069	7.868
488	1533.10	187037.86	238,144	116,214,272	22.091	7.873
489	1536.24	187805.19	239,121	116,936,169	22.113	7.878
490	1539.38	188574.10	240,100	117,649,000	22.136	7.884
491	1542.52	189344.57	241,081	118,370,771	22.158	7.889
492	1545.66	190116.62	242,064	119,095,488	22.181	7.894
493	1548.80	190890.24	243,049	119,823,157	22.204	7.899
494	1551.95	191665.43	244,036	120,553,784	22.226	7.905
495	1555.09	192442.19	245,025	121,287,375	22.248	7.910
496	1558.23	193220.51	246,016	122,023,936	22.271	7.915
497	1561.37	194000.42	247,009	122,763,473	22.293	7.921
498	1564.51	194781.89	248,004	123,505,992	22.316	7.926
499	1567.65	195564.93	249,001	124,251,499	22.338	7.932
500	1570.80	196349.54	250,000	125,000,000	22.361	7.937
501	1573.94	197135.72	251,001	125,751,501	22.383	7.942
502	1577.08	197923.48	252,004	126,506,008	22.405	7.947
503	1580.22	198712.80	253,009	127,263,527	22.428	7.953
504	1583.36	199503.70	254,016	128,024,864	22.449	7.958
505	1586.50	200296.17	255,025	128,787,625	22.472	7.963
506	1589.65	201090.20	256,036	129,554,216	22.494	7.969
507	1592.79	201885.81	257,049	130,323,843	22.517	7.974
508	1595.93	202682.99	258,064	131,096,512	22.539	7.979
509	1599.07	203481.74	259,081	131,872,229	22.561	7.984
510	1602.21	204282.06	260,100	132,651,000	22.583	7.989
511	1605.35	205083.95	261,121	133,432,831	22.605	7.995
512	1608.49	205887.42	262,144	134,217,728	22.627	8.000
513	1611.64	206692.45	263,169	135,005,697	22.649	8.005
514	1614.78	207499.05	264,196	135,796,744	22.671	8.010
515	1617.92	208307.23	265,225	136,590,875	22.694	8.016
516	1621.06	209116.97	266,256	137,388,096	22.716	8.021
517	1624.20	209928.29	267,289	138,188,413	22.738	8.026
518	1627.34	210741.18	268,324	138,991,832	22.759	8.031
519	1630.49	211555.63	269,361	139,798,359	22.782	8.036
520	1633.63	212371.66	270,400	140,608,000	22.803	8.041
521	1636.77	213189.26	271,441	141,420,761	22.825	8.047
522	1639.91	214008.43	272,484	142,236,648	22.847	8.052

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
523	1643.05	214829.17	273,529	143,055,667	22.869	8.057
524	1646.19	215651.49	274,576	143,877,824	22.891	8.062
525	1649.34	216475.37	275,625	144,703,125	22.913	8.067
526	1652.48	217300.82	276,676	145,531,576	22.935	8.072
527	1655.62	218127.85	277,729	146,363,183	22.956	8.077
528	1658.76	218956.44	278,784	147,197,952	22.978	8.082
529	1661.90	219786.61	279,841	148,035,889	23.000	8.087
530	1665.04	220618.32	280,900	148,877,000	23.022	8.093
531	1668.19	221451.65	281,961	149,721,291	23.043	8.098
532	1671.33	222286.53	283,024	150,568,768	23.065	8.103
533	1674.47	223122.98	284,089	151,419,437	23.087	8.108
534	1677.61	223961.00	285,156	152,273,304	23.108	8.113
535	1680.75	224800.59	286,225	153,130,375	23.130	8.118
536	1683.89	225641.75	287,296	153,990,656	23.152	8.123
537	1687.04	226484.48	288,369	154,854,153	23.173	8.128
538	1690.18	227328.77	289,444	155,720,872	23.195	8.133
539	1693.32	228174.66	290,521	156,590,819	23.216	8.138
540	1696.46	229022.10	291,600	157,464,000	23.238	8.143
541	1699.60	229871.12	292,681	158,340,421	23.259	8.148
542	1702.74	230721.71	293,764	159,220,088	23.281	8.153
543	1705.88	231573.86	294,849	160,103,007	23.302	8.158
544	1709.03	232427.59	295,936	160,989,184	23.324	8.163
545	1712.17	233282.89	297,025	161,878,625	23.345	8.168
546	1715.31	234139.76	298,116	162,771,336	23.367	8.173
547	1718.45	234998.20	299,209	163,667,323	23.388	8.178
548	1721.59	235858.21	300,304	164,566,592	23.409	8.183
549	1724.73	236719.79	301,401	165,469,149	23.431	8.188
550	1727.88	237582.94	302,500	166,375,000	23.452	8.193
551	1731.02	238447.67	303,601	167,284,151	23.473	8.198
552	1734.16	239313.96	304,704	168,196,608	23.495	8.203
553	1737.30	240181.83	305,809	169,112,377	23.516	8.208
554	1740.44	241051.26	306,916	170,031,464	23.537	8.213
555	1743.58	241922.27	308,025	170,953,875	23.558	8.218
556	1746.73	242794.85	309,136	171,879,616	23.579	8.223
557	1749.87	243668.99	310,249	172,808,693	23.601	8.228
558	1753.00	244544.61	311,364	173,741,112	23.622	8.233
559	1756.15	245422.00	312,481	174,676,879	23.643	8.238
560	1759.29	246300.86	313,600	175,616,000	23.664	8.242
561	1762.43	247181.30	314,721	176,558,481	23.685	8.247
562	1765.57	248062.30	315,844	177,504,328	23.706	8.252
563	1768.72	248946.87	316,969	178,453,547	23.728	8.257
564	1771.86	249832.01	318,096	179,406,144	23.749	8.262
565	1775.00	250718.73	319,225	180,362,125	23.769	8.267
566	1778.14	251607.01	320,356	181,321,496	23.791	8.272
567	1781.28	252496.87	321,489	182,284,263	23.812	8.277
568	1784.42	253388.30	322,624	183,250,432	23.833	8.282
569	1787.57	254281.30	323,761	184,220,009	23.854	8.286
570	1790.71	255175.86	324,900	185,193,000	23.875	8.291

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
571	1793.85	256072.00	326,041	186,169,411	23.896	8.296
572	1796.99	256969.71	327,184	187,149,248	23.916	8.301
573	1800.13	257868.99	328,329	188,132,517	23.937	8.306
574	1803.27	258769.85	329,476	189,119,224	23.958	8.311
575	1806.42	259672.27	330,625	190,109,375	23.979	8.315
576	1809.56	260576.26	331,776	191,102,976	24.000	8.320
577	1812.70	261481.83	332,929	192,100,033	24.021	8.325
578	1815.84	262388.96	334,084	193,100,552	24.042	8.330
579	1818.98	263297.67	335,241	194,104,539	24.062	8.335
580	1822.12	264207.94	336,400	195,112,000	24.083	8.339
581	1825.26	265119.79	337,561	196,122,941	24.104	8.344
582	1828.41	266033.21	338,724	197,137,368	24.125	8.349
583	1831.55	266948.20	339,889	198,155,287	24.145	8.354
584	1834.69	267864.76	341,056	199,176,704	24.166	8.359
585	1837.83	268782.80	342,225	200,201,625	24.187	8.363
586	1840.97	269702.59	343,396	201,230,056	24.207	8.368
587	1844.11	270623.86	344,569	202,262,003	24.228	8.373
588	1847.26	271546.70	345,744	203,297,472	24.249	8.378
589	1850.40	272471.12	346,921	204,336,469	24.269	8.382
590	1853.54	273397.10	348,100	205,379,000	24.289	8.387
591	1856.68	274324.66	349,281	206,425,071	24.310	8.392
592	1859.82	275253.78	350,464	207,474,688	24.331	8.397
593	1862.96	276184.48	351,649	208,527,857	24.351	8.401
594	1866.11	277116.75	352,836	209,584,584	24.372	8.406
595	1869.25	278050.59	354,025	210,644,875	24.393	8.411
596	1872.39	278985.99	355,216	211,708,736	24.413	8.415
597	1875.53	279922.97	356,409	212,776,173	24.433	8.420
598	1878.67	280861.53	357,604	213,847,192	24.454	8.425
599	1881.81	281801.65	358,801	214,921,799	24.474	8.429
600	1884.96	282743.34	360,000	216,000,000	24.495	8.434
601	1888.10	283686.60	361,201	217,081,801	24.515	8.439
602	1891.24	284631.44	362,404	218,167,208	24.536	8.444
603	1894.38	285577.84	363,609	219,256,227	24.556	8.448
604	1897.52	286525.82	364,816	220,348,864	24.576	8.453
605	1900.66	287475.36	366,025	221,445,125	24.597	8.458
606	1903.80	288426.48	367,236	222,545,016	24.617	8.462
607	1906.95	289379.17	368,449	223,648,543	24.637	8.467
608	1910.09	290333.43	369,664	224,755,712	24.658	8.472
609	1913.23	291289.26	370,881	225,866,529	24.678	8.476
610	1916.37	292246.66	372,100	226,981,000	24.698	8.481
611	1919.51	293205.63	373,321	228,099,131	24.718	8.485
612	1922.65	294166.17	374,544	229,220,928	24.739	8.490
613	1925.80	295128.28	375,769	230,346,397	24.758	8.495
614	1928.94	296091.97	376,996	231,475,544	24.779	8.499
615	1932.08	297057.22	378,225	232,608,375	24.799	8.504
616	1935.22	298024.05	379,456	233,744,896	24.819	8.509
617	1938.36	298992.44	380,689	234,885,113	24.839	8.513
618	1941.50	299962.41	381,924	236,029,032	24.859	8.518

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
619	1944.65	300933.95	383,161	237,176,659	24.879	8.522
620	1947.79	301907.05	384,400	238,628,000	24.899	8.527
621	1950.93	302881.73	385,641	239,483,061	24.919	8.532
622	1954.07	303857.98	386,884	240,641,848	24.939	8.536
623	1957.21	304835.80	388,129	241,804,367	24.959	8.541
624	1960.35	305815.20	389,376	242,970,624	24.980	8.545
625	1963.50	306796.16	390,625	244,140,625	25.000	8.549
626	1966.64	307778.69	391,876	245,314,376	25.019	8.554
627	1969.78	308762.79	393,129	246,491,883	25.040	8.559
628	1972.92	309748.47	394,384	247,673,152	25.059	8.563
629	1976.06	310735.71	395,641	248,858,189	25.079	8.568
630	1979.20	311724.53	396,900	250,047,000	25.099	8.573
631	1982.34	312714.92	398,161	251,239,591	25.119	8.577
632	1985.49	313706.88	399,424	252,435,968	25.139	8.582
633	1988.63	314700.40	400,689	253,636,137	25.159	8.586
634	1991.77	315695.50	401,956	254,840,104	25.179	8.591
635	1994.91	316692.17	403,225	256,047,875	25.199	8.595
636	1998.05	317690.42	404,496	257,259,456	25.219	8.599
637	2001.19	318690.23	405,769	258,474,853	25.239	8.604
638	2004.34	319691.61	407,044	259,694,072	25.259	8.609
639	2007.48	320694.56	408,321	260,917,119	25.278	8.613
640	2010.62	321699.09	409,600	262,144,000	25.298	8.618
641	2013.76	322705.18	410,881	263,374,721	25.318	8.622
642	2016.90	323712.85	412,164	264,609,288	25.338	8.627
643	2020.04	324722.09	413,449	265,847,707	25.357	8.631
644	2023.19	325732.89	414,736	267,089,984	25.377	8.636
645	2026.33	326745.27	416,025	268,836,125	25.397	8.640
646	2029.47	327759.22	417,316	269,586,136	25.416	8.644
647	2032.61	328774.74	418,609	270,840,023	25.436	8.649
648	2035.75	329791.83	419,904	272,097,792	25.456	8.653
649	2038.89	330810.49	421,201	273,359,449	25.475	8.658
650	2042.04	331830.72	422,500	274,625,000	25.495	8.662
651	2045.18	332852.53	423,801	275,894,451	25.515	8.667
652	2048.32	333875.90	425,104	277,167,808	25.534	8.671
653	2051.46	334900.85	426,409	278,445,077	25.554	8.676
654	2054.60	335927.36	427,716	279,726,264	25.573	8.680
655	2057.74	336955.45	429,025	281,011,375	25.593	8.684
656	2060.88	337985.10	430,336	282,800,416	25.612	8.689
657	2064.03	339016.33	431,649	283,593,393	25.632	8.693
658	2067.17	340049.13	432,964	284,890,312	25.651	8.698
659	2070.31	341083.50	434,281	286,191,179	25.671	8.702
660	2073.45	342119.44	435,600	287,496,000	25.690	8.706
661	2076.59	343156.95	436,921	288,804,781	25.710	8.711
662	2079.73	344196.03	438,244	290,117,528	25.720	8.715
663	2082.88	345236.69	439,569	291,434,247	25.749	8.719
664	2086.02	346278.91	440,896	292,754,944	25.768	8.724
665	2089.16	347322.70	442,225	294,079,625	25.787	8.728
666	2092.30	348368.07	443,556	295,408,296	25.807	8.733

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
667	2095.44	349415.00	444,889	296,740,963	25.826	8.737
668	2098.58	350463.51	446,224	298,077,632	25.846	8.742
669	2101.73	351513.59	447,561	299,418,309	25.865	8.746
670	2104.87	352565.24	448,900	300,763,000	25.884	8.750
671	2108.01	353618.43	450,241	302,111,711	25.904	8.753
672	2111.15	354673.24	451,584	303,464,448	25.923	8.759
673	2114.29	355729.60	452,929	304,821,217	25.942	8.763
674	2117.43	356787.54	454,276	306,182,024	25.961	8.768
675	2120.58	357847.04	455,625	307,546,875	25.981	8.772
676	2123.72	358908.11	456,976	308,915,776	26.000	8.776
677	2126.86	359970.75	458,329	310,288,733	26.019	8.781
678	2130.00	361034.97	459,684	311,665,752	26.038	8.785
679	2133.14	362100.75	461,041	313,046,839	26.058	8.789
680	2136.28	363168.11	462,400	314,432,000	26.077	8.794
681	2139.42	364237.04	463,761	315,821,241	26.096	8.798
682	2142.57	365307.54	465,124	317,214,568	26.115	8.802
683	2145.71	366379.60	466,489	318,611,987	26.134	8.807
684	2148.85	367453.24	467,856	320,013,504	26.153	8.811
685	2151.99	368528.45	469,225	321,419,125	26.172	8.815
686	2155.13	369605.23	470,596	322,828,856	26.192	8.819
687	2158.27	370683.59	471,969	324,242,703	26.211	8.824
688	2161.42	371763.51	473,344	325,660,672	26.229	8.828
689	2164.56	372845.00	474,721	327,082,769	26.249	8.832
690	2167.70	373928.07	476,100	328,509,000	26.268	8.836
691	2170.84	375012.70	477,481	329,939,371	26.287	8.841
692	2173.98	376098.91	478,864	331,373,888	26.306	8.845
693	2177.12	377186.68	480,249	332,812,557	26.325	8.849
694	2180.27	378276.03	481,636	334,255,384	26.344	8.853
695	2183.41	379366.95	483,025	335,702,375	26.363	8.858
696	2186.55	380459.44	484,416	337,153,536	26.382	8.862
697	2189.69	381553.50	485,809	338,608,873	26.401	8.866
698	2192.83	382649.43	487,204	340,068,392	26.419	8.870
699	2195.97	383746.33	488,601	341,532,099	26.439	8.875
700	2199.12	384845.10	490,000	343,000,000	26.457	8.879
701	2202.26	385945.44	491,401	344,472,101	26.476	8.883
702	2205.40	387047.36	492,804	345,948,088	26.495	8.887
703	2208.54	388150.84	494,209	347,428,927	26.514	8.892
704	2211.68	389255.90	495,616	348,913,664	26.533	8.896
705	2214.82	390362.52	497,025	350,402,625	26.552	8.900
706	2217.96	391470.32	498,436	351,895,816	26.571	8.904
707	2221.11	392580.49	499,849	353,393,243	26.589	8.908
708	2224.25	393691.83	501,264	354,894,912	26.608	8.913
709	2227.39	394804.74	502,681	356,400,829	26.627	8.917
710	2230.53	395919.21	504,100	357,911,000	26.644	8.921
711	2233.67	397035.27	505,521	359,425,431	26.664	8.925
712	2236.81	398152.89	506,944	360,944,128	26.683	8.929
713	2239.96	399272.08	508,369	362,467,097	26.702	8.934
714	2243.10	400392.84	509,796	363,994,344	26.721	8.938

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
715	2246.24	401515.18	511,225	365,525,875	26.739	8.942
716	2249.38	402639.08	512,656	367,061,696	26.758	8.946
717	2252.52	403764.56	514,089	368,601,813	26.777	8.950
718	2255.66	404891.60	515,524	370,146,232	26.795	8.954
719	2258.81	406020.22	516,961	371,694,959	26.814	8.959
720	2261.95	407150.41	518,400	373,248,000	26.833	8.963
721	2265.09	408282.17	519,841	374,805,361	26.851	8.967
722	2268.23	409415.50	521,284	376,367,048	26.870	8.971
723	2271.37	410550.40	522,729	377,933,067	26.889	8.975
724	2274.51	411686.87	524,176	379,503,424	26.907	8.979
725	2277.66	412824.91	525,625	381,078,125	26.926	8.983
726	2280.80	413964.52	527,076	382,657,176	26.944	8.988
727	2283.94	415105.71	528,529	384,240,583	26.963	8.992
728	2287.08	416248.46	529,984	385,828,352	26.991	8.996
729	2290.22	417392.79	531,441	387,420,489	27.000	9.000
730	2293.36	418538.68	532,900	389,017,000	27.018	9.004
731	2296.50	419686.15	534,361	390,617,891	27.037	9.008
732	2299.65	420835.19	535,824	392,223,168	27.055	9.012
733	2302.79	421985.79	537,289	393,832,837	27.074	9.016
734	2305.93	423137.97	538,756	395,446,904	27.092	9.020
735	2309.07	424291.72	540,225	397,065,375	27.111	9.023
736	2312.21	425447.04	541,696	398,688,256	27.129	9.029
737	2315.35	426603.93	543,169	400,315,553	27.148	9.033
738	2318.50	427762.40	544,644	401,947,272	27.166	9.037
739	2321.64	428922.43	546,121	403,583,419	27.184	9.041
740	2324.78	430084.03	547,600	405,224,000	27.203	9.045
741	2327.92	431247.21	549,081	406,869,021	27.221	9.049
742	2331.06	432411.95	550,564	408,518,488	27.239	9.053
743	2334.20	433578.27	552,049	410,172,407	27.258	9.057
744	2337.35	434746.16	553,536	411,830,784	27.276	9.061
745	2340.49	435915.62	555,025	413,493,625	27.295	9.065
746	2343.63	437086.64	556,516	415,160,936	27.313	9.069
747	2346.77	438259.24	558,009	416,832,723	27.331	9.073
748	2349.91	439433.41	559,504	418,508,992	27.349	9.077
749	2353.05	440609.16	561,001	420,189,749	27.368	9.081
750	2356.20	441786.47	562,500	421,875,000	27.386	9.086
751	2359.34	442965.35	564,001	423,564,751	27.404	9.089
752	2362.48	444145.80	565,504	424,525,900	27.423	9.094
753	2365.62	445327.83	567,009	426,957,777	27.441	9.098
754	2368.76	446511.42	568,516	428,661,064	27.459	9.102
755	2371.90	447696.59	570,025	430,368,875	27.477	9.106
756	2375.04	448883.32	571,536	432,081,216	27.495	9.109
757	2378.19	450071.63	573,049	433,798,093	27.514	9.114
758	2381.33	451261.51	574,564	435,519,512	27.532	9.118
759	2384.47	452452.96	576,081	437,245,479	27.549	9.122
760	2387.61	453645.98	577,600	438,976,000	27.568	9.126
761	2390.75	454840.57	579,121	440,711,081	27.586	9.129
762	2393.89	456036.73	580,644	442,450,728	27.604	9.134

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
763	2397.04	457234.46	582,169	444,194,947	27.622	9.138
764	2400.18	458433.77	583,696	445,943,744	27.640	9.142
765	2403.32	459634.64	585,225	447,697,125	27.659	9.146
766	2406.46	460837.08	586,756	449,455,096	27.677	9.149
767	2409.60	462041.10	588,289	451,217,663	27.695	9.154
768	2412.74	463246.69	589,824	452,984,832	27.713	9.158
769	2415.89	464453.84	591,361	454,756,609	27.731	9.162
770	2419.03	465662.57	592,900	456,533,000	27.749	9.166
771	2422.17	466872.87	594,441	458,314,011	27.767	9.169
772	2425.31	468084.74	595,984	460,099,648	27.785	9.173
773	2428.45	469298.18	597,529	461,889,917	27.803	9.177
774	2431.59	470513.19	599,076	463,684,824	27.821	9.181
775	2434.73	471729.77	600,625	465,484,375	27.839	9.185
776	2437.88	472947.92	602,176	467,288,576	27.857	9.189
777	2441.02	474167.65	603,729	469,097,433	27.875	9.193
778	2444.16	475388.94	605,284	470,910,952	27.893	9.197
779	2447.30	476611.81	606,841	472,729,139	27.910	9.201
780	2450.44	477836.24	608,400	474,552,000	27.928	9.205
781	2453.58	479062.25	609,961	476,379,541	27.946	9.209
782	2456.73	480289.83	611,524	478,211,768	27.964	9.213
783	2459.87	481518.97	613,089	480,048,687	27.982	9.217
784	2463.01	482749.69	614,656	481,890,304	28.000	9.221
785	2466.15	483981.98	616,225	483,736,025	28.017	9.225
786	2469.29	485215.84	617,796	485,587,656	28.036	9.229
787	2472.43	486451.28	619,369	487,443,403	28.053	9.233
788	2475.58	487688.28	620,944	489,303,872	28.071	9.237
789	2478.72	488926.85	622,521	491,169,069	28.089	9.240
790	2481.86	490166.99	624,100	493,039,000	28.107	9.244
791	2485.00	491408.71	625,681	494,913,671	28.125	9.248
792	2488.14	492651.99	627,264	496,793,088	28.142	9.252
793	2491.28	493896.85	628,849	498,677,257	28.160	9.256
794	2494.43	495143.28	630,436	500,566,184	28.178	9.260
795	2497.57	496391.27	632,025	502,459,875	28.196	9.264
796	2500.71	497640.84	633,616	504,358,336	28.213	9.268
797	2503.85	498891.98	635,209	506,261,573	28.231	9.271
798	2506.99	500144.69	636,804	508,169,592	28.249	9.275
799	2510.13	501398.97	638,401	510,082,399	28.266	9.279
800	2513.27	502654.82	640,000	512,000,000	28.284	9.283
801	2516.42	503912.25	641,601	513,922,401	28.302	9.287
802	2519.56	505171.24	643,204	515,849,608	28.319	9.291
803	2522.70	506431.80	644,809	517,781,627	28.337	9.295
804	2525.84	507693.94	646,416	519,718,464	28.355	9.299
805	2528.98	508957.65	648,025	521,660,125	28.372	9.302
806	2532.12	510222.92	649,636	523,606,616	28.390	9.306
807	2535.27	511489.77	651,249	525,557,943	28.408	9.310
808	2538.41	512758.19	652,864	527,514,112	28.425	9.314
809	2541.55	514028.19	654,481	529,474,129	28.443	9.318
810	2544.09	515299.74	656,100	531,441,000	28.460	9.321

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
811	2547.83	516572.86	657,721	533,411,731	28.478	9.325
812	2550.97	517847.57	659,344	535,387,328	28.496	9.329
813	2554.12	519123.84	660,969	537,366,797	28.513	9.333
814	2557.26	520401.68	662,596	539,353,144	28.531	9.337
815	2560.40	521681.10	664,225	541,343,375	28.548	9.341
816	2563.54	522962.08	665,856	543,338,496	28.566	9.345
817	2566.68	524244.63	667,489	545,338,513	28.583	9.348
818	2569.82	525528.76	669,124	547,343,432	28.601	9.352
819	2572.96	526814.46	670,761	549,353,259	28.618	9.356
820	2576.11	528101.73	672,400	551,368,000	28.636	9.360
821	2579.25	529390.56	674,041	553,387,661	28.653	9.364
822	2582.39	530680.97	675,684	555,412,248	28.670	9.367
823	2585.53	531972.95	677,329	557,441,767	28.688	9.371
824	2588.67	533266.50	678,976	559,476,224	28.705	9.375
825	2591.81	534561.63	680,625	561,515,625	28.723	9.379
826	2594.96	535858.32	682,276	563,559,976	28.740	9.383
827	2598.10	537156.58	683,929	565,609,283	28.758	9.386
828	2601.24	538456.41	685,584	567,663,552	28.775	9.390
829	2604.38	539757.82	687,241	569,722,789	28.792	9.394
830	2607.52	541060.79	688,900	571,787,000	28.810	9.398
831	2610.66	542365.34	690,561	573,856,191	28.827	9.401
832	2613.81	543671.46	692,224	575,930,368	28.844	9.405
833	2616.95	544979.15	693,889	578,009,537	28.862	9.409
834	2620.09	546288.40	695,556	580,093,704	28.879	9.413
835	2623.23	547599.23	697,225	582,182,875	28.896	9.417
836	2626.37	548911.63	698,896	584,277,056	28.914	9.420
837	2629.51	550225.61	700,569	586,376,253	28.931	9.424
838	2632.64	551541.15	702,244	588,480,472	28.948	9.428
839	2635.80	552858.26	703,921	590,589,719	28.965	9.432
840	2638.94	554176.94	705,600	592,704,000	28.983	9.435
841	2642.08	555497.20	707,281	594,823,321	29.000	9.439
842	2645.22	556819.02	708,964	596,947,688	29.017	9.443
843	2648.36	558142.42	710,649	599,077,107	29.034	9.447
844	2651.50	559467.39	712,336	601,211,584	29.052	9.450
845	2654.65	560793.92	714,025	603,351,125	29.069	9.454
846	2657.79	562122.03	715,716	605,495,736	29.086	9.458
847	2660.93	563451.71	717,409	607,645,423	29.103	9.461
848	2664.07	564782.96	719,104	609,800,192	29.120	9.465
849	2667.21	566115.78	720,801	611,960,049	29.138	9.469
850	2670.35	567450.17	722,500	614,125,000	29.155	9.473
851	2673.50	568786.14	724,201	616,295,051	29.172	9.476
852	2676.64	570123.67	725,904	618,470,208	29.189	9.480
853	2679.78	571462.77	727,609	620,650,477	29.206	9.483
854	2682.92	572803.45	729,316	622,835,864	29.223	9.487
855	2686.06	574145.69	731,025	625,026,375	29.240	9.491
856	2689.20	575489.51	732,736	627,222,016	29.257	9.495
857	2692.35	576834.90	734,449	629,422,793	29.274	9.499
858	2695.49	578181.85	736,164	631,628,712	29.292	9.502

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
859	2698.63	579530.38	737,881	633,839,779	29.309	9.506
860	2701.77	580880.48	739,600	636,056,000	29.326	9.509
861	2704.91	582232.15	741,321	638,277,381	29.343	9.513
862	2708.05	583585.39	743,044	640,503,928	29.360	9.517
863	2711.19	584940.21	744,769	642,735,647	29.377	9.520
864	2714.34	586296.59	746,496	644,972,544	29.394	9.524
865	2717.45	587654.54	748,225	647,214,625	29.411	9.528
866	2720.62	589014.07	749,956	649,461,896	29.428	9.532
867	2723.76	590375.16	751,689	651,714,363	29.445	9.535
868	2726.90	591737.83	753,424	653,972,032	29.462	9.539
869	2730.04	593102.06	755,161	656,234,909	29.479	9.543
870	2733.19	594467.87	756,900	658,503,000	29.496	9.546
871	2736.33	595835.25	758,641	660,776,311	29.513	9.550
872	2739.47	597204.20	760,384	663,054,848	29.529	9.554
873	2742.61	598574.72	762,129	665,338,617	29.546	9.557
874	2745.75	599946.81	763,876	667,627,624	29.563	9.561
875	2748.89	601320.47	765,625	669,921,875	29.580	9.565
876	2752.04	602695.70	767,376	672,221,376	29.597	9.568
877	2755.18	604072.50	769,129	674,526,133	29.614	9.572
878	2758.32	605450.88	770,884	676,836,152	29.631	9.575
879	2761.46	606830.82	772,641	679,151,439	29.648	9.579
880	2764.60	608212.34	774,400	681,472,000	29.665	9.583
881	2767.74	609595.42	776,161	683,797,841	29.682	9.586
882	2770.89	610980.08	777,924	686,128,968	29.698	9.590
883	2774.03	612366.31	779,689	688,465,387	29.715	9.594
884	2777.17	613754.11	781,456	690,807,104	29.732	9.597
885	2780.31	615143.48	783,225	693,154,125	29.749	9.601
886	2783.45	616534.42	784,996	695,506,456	29.766	9.604
887	2786.59	617926.93	786,769	697,864,103	29.782	9.608
888	2789.73	619321.01	788,544	700,227,072	29.799	9.612
889	2792.88	620716.66	790,321	702,595,369	29.816	9.615
890	2796.02	622113.89	792,100	704,969,000	29.833	9.619
891	2799.16	623512.68	793,881	707,347,971	29.850	9.623
892	2802.30	624913.04	795,664	709,732,288	29.866	9.626
893	2805.44	626314.98	797,449	712,121,957	29.883	9.630
894	2808.58	627718.49	799,236	714,516,984	29.900	9.633
895	2811.73	629123.56	801,025	716,917,375	29.916	9.637
896	2814.87	630530.21	802,816	719,323,136	29.933	9.640
897	2818.01	631938.43	804,609	721,734,273	29.950	9.644
898	2821.15	633348.22	806,404	724,150,792	29.967	9.648
899	2824.29	634759.58	808,201	726,572,699	29.983	9.651
900	2827.43	636172.51	810,000	729,000,000	30.000	9.655
901	2830.58	637587.01	811,804	731,432,701	30.017	9.658
902	2833.72	639003.09	813,604	733,870,808	30.033	9.662
903	2836.86	640420.73	815,409	736,314,327	30.050	9.666
904	2840.00	641839.95	817,216	738,763,264	30.066	9.669
905	2843.14	643260.73	819,025	741,217,625	30.083	9.673
906	2846.28	644683.09	820,836	743,677,416	30.100	9.676

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
907	2849.43	646107.01	822,649	746,142,643	30.116	9.680
908	2852.57	647532.51	824,464	748,613,312	30.133	9.683
909	2855.71	648959.58	826,281	751,089,429	30.150	9.687
910	2858.85	650388.21	828,100	753,571,000	30.163	9.690
911	2861.99	651818.43	829,921	756,058,031	30.183	9.694
912	2865.13	653250.21	831,744	758,550,528	30.199	9.698
913	2868.27	654683.56	833,569	761,048,497	30.216	9.701
914	2871.42	656118.48	835,396	763,551,944	30.232	9.705
915	2874.56	657554.98	837,225	766,060,875	30.249	9.708
916	2877.70	658993.04	839,056	768,575,296	30.265	9.712
917	2880.84	660432.68	840,889	771,095,213	30.282	9.715
918	2883.98	661873.88	842,724	773,620,632	30.298	9.718
919	2887.12	663316.66	844,561	776,151,559	30.315	9.722
920	2890.27	664761.01	846,400	778,688,000	30.331	9.726
921	2893.41	666206.92	848,241	781,229,961	30.348	9.729
922	2896.55	667654.41	850,084	783,777,448	30.364	9.733
923	2899.69	669103.47	851,929	786,330,467	30.381	9.736
924	2902.83	670554.10	853,776	788,889,024	30.397	9.740
925	2905.97	672006.30	855,625	791,453,125	30.414	9.743
926	2909.12	673460.08	857,476	794,022,776	30.430	9.747
927	2912.26	674915.42	859,329	796,597,983	30.447	9.750
928	2915.40	676372.33	861,184	799,178,752	30.463	9.754
929	2918.54	677830.82	863,041	801,765,089	30.479	9.757
930	2921.68	679290.87	864,900	804,357,000	30.496	9.761
931	2924.82	680752.50	866,761	806,954,491	30.512	9.764
932	2927.96	682215.69	868,624	809,557,568	30.529	9.768
933	2931.11	683680.46	870,489	812,166,237	30.545	9.771
934	2934.25	685146.80	872,356	814,780,504	30.561	9.775
935	2937.39	686614.71	874,225	817,400,375	30.578	9.778
936	2940.53	688084.19	876,096	820,025,856	30.594	9.783
937	2943.67	689555.24	877,969	822,656,953	30.610	9.785
938	2946.81	691027.86	879,844	825,293,672	30.627	9.789
939	2949.96	692502.05	881,721	827,936,019	30.643	9.792
940	2953.10	693977.82	883,600	830,584,000	30.659	9.796
941	2956.24	695455.15	885,481	833,237,621	30.676	9.799
942	2959.38	696934.06	887,364	835,896,888	30.692	9.803
943	2962.52	698414.53	889,249	838,561,807	30.708	9.806
944	2965.66	699896.58	891,136	841,232,384	30.724	9.810
945	2968.81	701380.28	893,025	843,908,625	30.741	9.813
946	2971.95	702865.38	894,916	846,590,536	30.757	9.817
947	2975.09	704352.14	896,809	849,278,123	30.773	9.820
948	2978.23	705840.47	898,704	851,971,392	30.790	9.823
949	2981.37	707330.37	900,601	854,670,349	30.806	9.827
950	2984.51	708821.84	902,500	857,375,000	30.822	9.830
951	2987.66	710314.88	904,401	860,085,351	30.838	9.834
952	2990.80	711809.58	906,304	862,801,408	30.854	9.837
953	2993.94	713305.68	908,209	865,523,177	30.871	9.841
954	2997.08	714803.48	910,116	868,250,664	30.887	9.844

Number, or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
955	3000.22	716302.76	912,025	870,983,875	30.903	9.848
956	3003.36	717803.66	913,936	873,722,816	30.919	9.851
957	3006.50	719306.12	915,849	876,467,493	30.935	9.854
958	3009.65	720810.16	917,764	879,217,912	30.951	9.858
959	3012.79	722315.77	919,681	881,974,079	30.968	9.861
960	3015.93	723822.95	921,600	884,736,000	30.984	9.865
961	3019.07	725331.70	923,521	887,503,681	31.000	9.868
962	3022.21	726842.02	925,444	890,277,128	31.016	9.872
963	3025.35	728353.91	927,369	893,056,347	31.032	9.875
964	3028.50	729867.37	929,296	895,841,344	31.048	9.878
965	3031.64	731382.40	931,225	898,632,125	31.064	9.881
966	3034.78	732899.01	933,156	901,428,696	31.080	9.885
967	3037.92	734417.18	935,089	904,231,063	31.097	9.889
968	3041.06	735936.93	937,024	907,039,232	31.113	9.892
969	3044.20	737458.25	938,961	909,853,209	31.129	9.895
970	3047.35	738981.13	940,900	912,673,000	31.145	9.899
971	3050.49	740505.59	942,841	915,498,611	31.161	9.902
972	3053.63	742031.62	944,784	918,330,048	31.177	9.906
973	3056.77	743559.22	946,729	921,167,317	31.193	9.909
974	3059.91	745088.39	948,676	924,010,424	31.209	9.912
975	3063.05	746619.13	950,625	926,859,375	31.225	9.916
976	3066.19	748151.44	952,576	929,714,176	31.241	9.919
977	3069.34	749685.32	954,529	932,574,833	31.257	9.923
978	3072.48	751220.78	956,484	935,441,352	31.273	9.926
979	3075.62	752757.80	958,441	938,313,739	31.289	9.929
980	3078.76	754296.40	960,400	941,192,000	31.305	9.933
981	3081.90	755836.56	962,361	944,076,141	31.321	9.936
982	3085.04	757378.30	964,324	946,966,168	31.337	9.940
983	3088.19	758921.61	966,289	949,862,087	31.353	9.943
984	3091.33	760466.48	968,256	952,763,904	31.369	9.946
985	3094.47	762012.93	970,225	955,671,625	31.385	9.950
986	3097.61	763560.95	972,196	958,585,256	31.401	9.953
987	3100.75	765110.54	974,169	961,504,803	31.416	9.956
988	3103.89	766661.71	976,144	964,430,272	31.432	9.960
989	3107.04	768214.44	978,121	967,361,669	31.448	9.963
990	3110.18	769768.74	980,100	970,299,000	31.464	9.966
991	3113.32	771324.61	982,081	973,242,271	31.480	9.970
992	3116.46	772882.06	984,064	976,191,488	31.496	9.973
993	3119.60	774441.07	986,049	979,146,657	31.512	9.977
994	3122.74	776001.66	988,036	982,107,784	31.528	9.980
995	3125.89	777563.82	990,025	985,074,875	31.544	9.983
996	3129.03	779127.54	992,016	988,047,936	31.559	9.987
997	3132.17	780692.84	994,009	991,026,973	31.575	9.990
998	3135.31	782259.71	996,004	994,011,992	31.591	9.993
999	3138.45	783828.15	998,001	997,002,999	31.607	9.997
1000	3141.60	785398.16	1,000,000	1,000,000,000	31.623	10.000

TABLE No. IV. CIRCLES:—DIAMETER, CIRCUMFERENCE, AREA, AND SIDE OF EQUAL SQUARE.

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).	Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
				3	9.4248	7.0686	2.6586
1/16	.1963	.00307	.0553	3 1/16	9.6211	7.3662	2.7140
⅛	.3927	.01227	.1107	3⅛	9.8175	7.6699	2.7694
3/16	.5890	.02761	.1661	3 3/16	10.014	7.9798	2.8248
¼	.7854	.04909	.2215	3¼	10.210	8.2957	2.8801
5/16	.9817	.07670	.2770	3 5/16	10.406	8.6180	2.9355
⅜	1.1781	.1104	.3323	3⅜	10.602	8.9462	2.9909
7/16	1.3744	.1503	.3877	3 7/16	10.799	9.2807	3.0463
½	1.5708	.1963	.4431	3½	10.995	9.6211	3.1017
9/16	1.7771	.2485	.4984	3 9/16	11.191	9.9680	3.1571
⅝	1.9635	.3068	.5539	3⅝	11.388	10.320	3.2124
11/16	2.1598	.3712	.6092	3 11/16	11.584	10.679	3.2678
¾	2.3562	.4418	.6646	3¾	11.781	11.044	3.3232
13/16	2.5525	.5185	.7200	3 13/16	11.977	11.416	3.3786
⅞	2.7489	.6013	.7754	3⅞	12.173	11.793	3.4340
15/16	2.9452	.6903	.8308	3 15/16	12.369	12.177	3.4894
1	3.1416	.7854	.8862	4	12.566	12.566	3.5448
1 1/16	3.3379	.8866	.9416	4 1/16	12.762	12.962	3.6002
1⅛	3.5343	.9940	.9969	4⅛	12.959	13.364	3.6555
1 3/16	3.7306	1.1075	1.0524	4 3/16	13.155	13.772	3.7109
1¼	3.9270	1.2271	1.1017	4¼	13.351	14.186	3.7663
1 5/16	4.1233	1.3530	1.1631	4 5/16	13.547	14.606	3.8217
1⅜	4.3197	1.4848	1.2185	4⅜	13.744	15.033	3.8771
1 7/16	4.5160	1.6229	1.2739	4 7/16	13.940	15.465	3.9325
1½	4.7124	1.7671	1.3293	4½	14.137	15.904	3.9880
1 9/16	4.9087	1.9175	1.3847	4 9/16	14.333	16.349	4.0434
1⅝	5.1051	2.0739	1.4401	4⅝	14.529	16.800	4.0987
1 11/16	5.3014	2.2365	1.4955	4 11/16	14.725	17.257	4.1541
1¾	5.4978	2.4052	1.5508	4¾	14.922	17.720	4.2095
1 13/16	5.6941	2.5800	1.6062	4 13/16	15.119	18.190	4.2648
1⅞	5.8905	2.7611	1.6616	4⅞	15.315	18.665	4.3202
1 15/16	6.0868	2.9483	1.7170	4 15/16	15.511	19.147	4.3756
2	6.2832	3.1416	1.7724	5	15.708	19.635	4.4310
2 1/16	6.4795	3.3380	1.8278	5 1/16	15.904	20.129	4.4864
2⅛	6.6759	3.5465	1.8831	5⅛	16.100	20.629	4.5417
2 3/16	6.8722	3.7584	1.9385	5 3/16	16.296	21.135	4.5971
2¼	7.0686	3.9760	1.9939	5¼	16.493	21.647	4.6525
2 5/16	7.2649	4.2000	2.0493	5 5/16	16.689	22.166	4.7079
2⅜	7.4613	4.4302	2.1047	5⅜	16.886	22.690	4.7633
2 7/16	7.6576	4.7066	2.1601	5 7/16	17.082	23.221	4.8187
2½	7.8540	4.9087	2.2155	5½	17.278	23.758	4.8741
2 9/16	8.0503	5.1573	2.2709	5 9/16	17.474	24.301	4.9295
2⅝	8.2467	5.4119	2.3262	5⅝	17.671	24.850	4.9848
2 11/16	8.4430	5.6723	2.3816	5 11/16	17.867	25.406	5.0402
2¾	8.6394	5.9395	2.4370	5¾	18.064	25.967	5.0956
2 13/16	8.8357	6.2126	2.4924	5 13/16	18.261	26.535	5.1510
2⅞	9.0321	6.4918	2.5478	5⅞	18.457	27.108	5.2064
2 15/16	9.2284	6.7772	2.6032	5 15/16	18.653	27.688	5.2618

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).	Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
6	18.849	28.274	5.3172	12	37.699	113.097	10.634
6⅛	19.242	29.464	5.4280	12⅛	38.091	115.466	10.745
6¼	19.635	30.679	5.5388	12¼	38.484	117.859	10.856
6⅜	20.027	31.919	5.6495	12⅜	38.877	120.276	10.966
6½	20.420	33.183	5.7603	12½	39.270	122.718	11.077
6⅝	20.813	34.471	5.8711	12⅝	39.662	125.184	11.188
6¾	21.205	35.784	5.9819	12¾	40.055	127.676	11.299
6⅞	21.598	37.122	6.0927	12⅞	40.448	130.192	11.409
7	21.991	38.484	6.2034	13	40.840	132.732	11.520
7⅛	22.383	39.871	6.3142	13⅛	41.233	135.297	11.631
7¼	22.776	41.282	6.4350	13¼	41.626	137.886	11.742
7⅜	23.169	42.718	6.5358	13⅜	42.018	140.500	11.853
7½	23.562	44.178	6.6465	13½	42.411	143.139	11.963
7⅝	23.954	45.663	6.7573	13⅝	42.804	145.802	12.074
7¾	24.347	47.173	6.8681	13¾	43.197	148.489	12.185
7⅞	24.740	48.707	6.9789	13⅞	43.589	151.201	12.296
8	25.132	50.265	7.0897	14	43.982	153.938	12.406
8⅛	25.515	51.848	7.2005	14⅛	44.375	156.699	12.517
8¼	25.918	53.456	7.3112	14¼	44.767	159.485	12.628
8⅜	26.310	55.088	7.4220	14⅜	45.160	162.295	12.739
8½	26.703	56.745	7.5328	14½	45.553	165.130	12.850
8⅝	27.096	58.426	7.6436	14⅝	45.945	167.989	12.960
8¾	27.489	60.132	7.7544	14¾	46.338	170.873	13.071
8⅞	27.881	61.862	7.8651	14⅞	46.731	173.782	13.182
9	28.274	63.617	7.9760	15	47.124	176.715	13.293
9⅛	28.667	65.396	8.0866	15⅛	47.516	179.672	13.403
9¼	29.059	67.200	8.1974	15¼	47.909	182.654	13.514
9⅜	29.452	69.029	8.3081	15⅜	48.302	185.661	13.625
9½	29.845	70.882	8.4190	15½	48.694	188.692	13.736
9⅝	30.237	72.759	8.5297	15⅝	49.087	191.748	13.847
9¾	30.630	74.662	8.6405	15¾	49.480	194.828	13.957
9⅞	31.023	76.588	8.7513	15⅞	49.872	197.933	14.068
10	31.416	78.540	8.8620	16	50.265	201.062	14.179
10⅛	31.808	80.515	8.9728	16⅛	50.658	204.216	14.290
10¼	32.201	82.516	9.0836	16¼	51.051	207.394	14.400
10⅜	32.594	84.540	9.1943	16⅜	51.443	210.597	14.511
10½	32.986	86.590	9.3051	16½	51.836	213.825	14.622
10⅝	33.379	88.664	9.4159	16⅝	52.229	217.077	14.732
10¾	33.772	90.762	9.5267	16¾	52.621	220.353	14.843
10⅞	34.164	92.885	9.6375	16⅞	53.014	223.654	14.954
11	34.558	95.033	9.7482	17	53.407	226.980	15.065
11⅛	34.950	97.205	9.8590	17⅛	53.799	230.330	15.176
11¼	35.343	99.402	9.9698	17¼	54.192	233.705	15.286
11⅜	35.735	101.623	10.080	17⅜	54.585	237.104	15.397
11½	36.128	103.869	10.191	17½	54.978	240.528	15.508
11⅝	36.521	106.139	10.302	17⅝	55.370	243.977	15.619
11¾	36.913	108.434	10.413	17¾	55.763	247.450	15.730
11⅞	37.306	110.753	10.523	17⅞	56.156	250.947	15.840

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).	Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
18	56.548	254.469	15.951	24	75.398	452.390	21.268
18⅛	56.941	258.016	16.062	24⅛	75.791	457.115	21.379
18¼	57.334	261.587	16.173	24¼	76.183	461.864	21 490
18⅜	57.726	265.182	16.283	24⅜	76.576	466.638	21.601
18½	58.119	268.803	16.394	24½	76.969	471.436	21.712
18⅝	58.512	272.447	16.505	24⅝	77.361	476.259	21.822
18¾	58.905	276.117	16.616	24¾	77.754	481.106	21.933
18⅞	59.297	279.811	16.727	24⅞	78.147	485.978	22.044
19	59.690	283.529	16.837	25	78.540	490.875	22.155
19⅛	60.083	287.272	16.948	25⅛	78.932	495.796	22.265
19¼	60.475	291.039	17.060	25¼	79.325	500.741	22.376
19⅜	60.868	294.831	17.170	25⅜	79.718	505.711	22.487
19½	61.261	298.648	17.280	25½	80.110	510.706	22.598
19⅝	61.653	302.489	17.391	25⅝	80.503	515.725	22.709
19¾	62.046	306.355	17.502	25¾	80.896	520.769	22.819
19⅞	62.439	310.245	17.613	25⅞	81.288	525.837	22.930
20	62.832	314.160	17.724	26	81.681	530.930	23.041
20⅛	63.224	318.099	17.834	26⅛	82.074	536.047	23.152
20¼	63.617	322.063	17.945	26¼	82.467	541.189	23.062
20⅜	64.010	326.051	18.056	26⅜	82.859	546.356	23.373
20½	64.402	330.064	18.167	26½	83.252	551.547	23.484
20⅝	64.795	334.101	18.277	26⅝	83.645	556.762	23.595
20¾	65.188	338.163	18.388	26¾	84.037	562.002	23.708
20⅞	65.580	342.250	18.499	26⅞	84.430	567.267	23.816
21	65.973	346.361	18.610	27	84.823	572.556	23.927
21⅛	66.366	350.497	18.721	27⅛	85.215	577.870	24.038
21¼	66.759	354.657	18.831	27¼	85.608	583.208	24.149
21⅜	67.151	358.841	18.942	27⅜	86.001	588.571	24.259
21½	67.544	363.051	19.053	27½	86.394	593.958	24.370
21⅝	67.937	367.284	19.164	27⅝	86.786	599.370	24.481
21¾	68.329	371.543	19.274	27¾	87.179	604.807	24.592
21⅞	68.722	375.826	19.385	27⅞	87.572	610.268	24.703
22	69.115	380.133	19.496	28	87.964	615.753	24.813
22⅛	69.507	384.465	19.607	28⅛	88.357	621.263	24.924
22¼	69.900	388.822	19.718	28¼	88.750	626.798	25.035
22⅜	70.293	393.203	19.828	28⅜	89.142	632.357	25.146
22½	70.686	397.608	19.939	28½	89.535	637.941	25.256
22⅝	71.078	402.038	20.050	28⅝	89.928	643.594	25.367
22¾	71.471	406.493	20.161	28¾	90.321	649.182	25.478
22⅞	71.864	410.972	20.271	28⅞	90.713	654.839	25.589
23	72.256	415.476	20.382	29	91.106	660.521	25.699
23⅛	72.649	420.004	20.493	29⅛	91.499	666.227	25.810
23¼	73.042	424.557	20.604	29¼	91.891	671.958	25.921
23⅜	73.434	429.135	20.715	29⅜	92.284	677.714	26.032
23½	73.827	433.731	20.825	29½	92.677	683.494	26.143
23⅝	74.220	438.363	20.936	29⅝	93.069	689.298	26.253
23¾	74.613	443.014	21.047	29¾	93.462	695.128	26.364
23⅞	75.005	447.699	21.158	29⅞	93.855	700.981	26.478

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).	Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
30	94.248	706.860	26.586	36	113.097	1017.88	31.903
30⅛	94.640	712.762	26.696	36⅛	113.490	1024.95	32.014
30¼	95.033	718.690	26.807	36¼	113.883	1032.06	32.124
30⅜	95.426	724.641	26.918	36⅜	114.275	1039.19	32.235
30½	95.818	730.618	27.029	36½	114.668	1046.35	32.349
30⅝	96.211	736.619	27.139	36⅝	115.061	1053.52	32.457
30¾	96.604	742.644	27.250	36¾	115.453	1060.73	32.567
30⅞	96.996	748.694	27.361	36⅞	115.846	1067.95	32.678
31	97.389	754.769	27.472	37	116.239	1075.21	32.789
31⅛	97.782	760.868	27.583	37⅛	116.631	1082.48	32.900
31¼	98.175	766.992	27.693	37¼	117.024	1089.79	33.011
31⅜	98.567	773.140	27.804	37⅜	117.417	1097.11	33.021
31½	98.968	779.313	27.915	37½	117.810	1104.46	33.232
31⅝	99.353	785.510	28.026	37⅝	118.202	1111.84	33.343
31¾	99.745	791.732	28.136	37¾	118.595	1119.24	33.454
31⅞	100.138	797.978	28.247	37⅞	118.988	1126.66	33.564
32	100.531	804.249	28.358	38	119.380	1134.11	33.675
32⅛	100.924	810.545	28.469	38⅛	119.773	1141.59	33.786
32¼	101.316	816.865	28.580	38¼	120.166	1149.08	33.897
32⅜	101.709	823.209	28.691	38⅜	120.558	1156.61	34.008
32½	102.102	829.578	28.801	38½	120.951	1164.15	34.118
32⅝	102.494	835.972	28.912	38⅝	121.344	1171.73	34.229
32¾	102.887	842.390	29.023	38¾	121.737	1179.32	34.340
32⅞	103.280	848.833	29.133	38⅞	122.129	1186.94	34.451
33	103.672	855.30	29.244	39	122.522	1194.59	34.561
33⅛	104.055	861.79	29.355	39⅛	122.915	1202.26	34.672
33¼	104.458	868.30	29.466	39¼	123.307	1209.95	34.783
33⅜	104.850	874.84	29.577	39⅜	123.700	1217.67	34.894
33½	105.243	881.41	29.687	39½	124.093	1225.42	35.005
33⅝	105.636	888.00	29.798	39⅝	124.485	1233.18	35.115
33¾	106.029	894.61	29.909	39¾	124.878	1240.98	35.226
33⅞	106.421	901.25	30.020	39⅞	125.271	1248.79	35.337
34	106.814	907.92	30.131	40	125.664	1256.64	35.448
34⅛	107.207	914.61	30.241	40⅛	126.056	1264.50	35.558
34¼	107.599	921.32	30.352	40¼	126.449	1272.39	35.669
34⅜	107.992	928.06	30.463	40⅜	126.842	1280.31	35.780
34½	108.385	934.82	30.574	40½	127.234	1288.25	35.891
34⅝	108.777	941.60	30.684	40⅝	127.627	1296.21	36.002
34¾	109.170	948.41	30.795	40¾	128.020	1304.20	36.112
34⅞	109.563	955.25	30.906	40⅞	128.412	1312.21	36.223
35	109.956	962.11	31.017	41	128.805	1320.25	36.334
35⅛	110.348	968.99	31.128	41⅛	129.198	1328.32	36.445
35¼	110.741	975.90	31.238	41¼	129.591	1336.40	36.555
35⅜	111.134	982.84	31.349	41⅜	129.983	1344.51	36.666
35½	111.526	989.80	31.460	41½	130.376	1352.65	36.777
35⅝	111.919	996.78	31.571	41⅝	130.769	1360.81	36.888
35¾	112.312	1003.78	31.681	41¾	131.161	1369.00	36.999
35⅞	112.704	1010.82	31.792	41⅞	131.554	1377.21	37.109

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
42	131.947	1385.44	37.220
42⅛	132.339	1393.70	37.331
42¼	132.732	1401.98	37.442
42⅜	133.125	1410.29	37.552
42½	133.518	1418.62	37.663
42⅝	133.910	1426.98	37.774
42¾	134.303	1435.36	37.885
42⅞	134.696	1443.77	37.996
43	135.088	1452.20	38.106
43⅛	135.481	1460.65	38.217
43¼	135.874	1469.13	38.328
43⅜	136.266	1477.63	38.439
43½	136.659	1486.17	38.549
43⅝	137.052	1494.72	38.660
43¾	137.445	1503.30	38.771
43⅞	137.837	1511.90	38.882
44	138.230	1520.53	38.993
44⅛	138.623	1529.18	39.103
44¼	139.015	1537.86	39.214
44⅜	139.408	1546.55	39.325
44½	139.801	1555.28	39.436
44⅝	140.193	1564.03	39.546
44¾	140.586	1572.81	39.657
44⅞	140.979	1581.61	39.768
45	141.372	1590.43	39.879
45⅛	141.764	1599.28	39.989
45¼	142.157	1608.15	40.110
45⅜	142.550	1617.04	40.211
45½	142.942	1625.97	40.322
45⅝	143.335	1634.92	40.432
45¾	143.728	1643.89	40.543
45⅞	144.120	1652.88	40.654
46	144.513	1661.90	40.765
46⅛	144.906	1670.95	40.876
46¼	145.299	1680.01	40.986
46⅜	145.691	1689.10	41.097
46½	146.084	1698.23	41.208
46⅝	146.477	1707.37	41.319
46¾	146.869	1716.54	41.429
46⅞	147.262	1725.73	41.540
47	147.655	1734.94	41.651
47⅛	148.047	1744.18	41.762
47¼	148.440	1753.45	41.873
47⅜	148.833	1762.73	41.983
47½	149.226	1772.05	42.094
47⅝	149.618	1781.39	42.205
47¾	150.011	1790.76	42.316
47⅞	150.404	1800.14	42.427

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
48	150.796	1809.56	42.537
48⅛	151.189	1818.99	42.648
48¼	151.582	1828.46	42.759
48⅜	151.974	1837.93	42.870
48½	152.367	1847.45	42.980
48⅝	152.760	1856.99	43.091
48¾	153.153	1866.55	43.202
48⅞	153.545	1876.13	43.313
49	153.938	1885.74	43.423
49⅛	154.331	1895.37	43.534
49¼	154.723	1905.03	43.645
49⅜	155.116	1914.70	43.756
49½	155.509	1924.42	43.867
49⅝	155.901	1934.15	43.977
49¾	156.294	1943.91	44.088
49⅞	156.687	1953.69	44.199
50	157.080	1963.50	44.310
50¼	157.865	1983.18	44.531
50½	158.650	2002.96	44.753
50¾	159.436	2022.84	44.974
51	160.221	2042.82	45.196
51¼	161.007	2062.90	45.417
51½	161.792	2083.07	45.639
51¾	162.577	2103.35	45.861
52	163.363	2123.72	46.082
52¼	164.148	2144.19	46.304
52½	164.934	2164.75	46.525
52¾	165.719	2185.42	46.747
53	166.504	2206.18	46.968
53¼	167.290	2227.05	47.190
53½	168.075	2248.01	47.411
53¾	168.861	2269.06	47.633
54	169.646	2290.22	47.854
54¼	170.431	2311.48	48.076
54½	171.217	2332.83	48.298
54¾	172.002	2354.28	48.519
55	172.788	2375.83	48.741
55¼	173.573	2397.48	48.962
55½	174.358	2419.22	49.184
55¾	175.144	2441.07	49.405
56	175.929	2463.01	49.627
56¼	176.715	2485.05	49.848
56½	177.500	2507.19	50.070
56¾	178.285	2529.42	50.291

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).	Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
57	179.071	2551.76	50.513	68	213.628	3631.68	60.261
57¼	179.856	2574.19	50.735	68¼	214.414	3658.44	60.483
57½	180.642	2596.72	50.956	68½	215.199	3685.29	60.704
57¾	181.427	2619.35	51.178	68¾	215.985	3712.24	60.926
58	182.212	2642.08	51.399	69	216.770	3739.28	61.147
58¼	182.998	2664.91	51.621	69¼	217.555	3766.43	61.369
58½	183.783	2687.83	51.842	69½	218.341	3793.67	61.591
58¾	184.569	2710.85	52.064	69¾	219.126	3821.02	61.812
59	185.354	2733.97	52.285	70	219.912	3848.45	62.034
59¼	186.139	2757.19	52.507	70¼	220.697	3875.99	62.255
59½	186.925	2780.51	52.729	70½	221.482	3903.63	62.477
59¾	187.710	2803.92	52.950	70¾	222.268	3931.36	62.698
60	188.496	2827.43	53.172	71	223.053	3959.19	62.920
60¼	189.281	2851.05	53.393	71¼	223.839	3987.13	63.141
60½	190.066	2874.76	53.615	71½	224.624	4015.16	63.363
60¾	190.852	2898.56	53.836	71¾	225.409	4043.28	63.545
61	191.637	2922.47	54.048	72	226.195	4071.50	63.806
61¼	192.423	2946.47	54.279	72¼	226.980	4099.83	64.028
61½	193.208	2970.57	54.501	72½	227.766	4128.25	64.249
61¾	193.993	2994.77	54.723	72¾	228.551	4156.77	64.471
62	194.779	3019.07	54.944	73	229.336	4185.39	64.692
62¼	195.564	3043.47	55.166	73¼	230.122	4214.11	64.914
62½	196.350	3067.96	55.387	73½	230.907	4242.92	65.135
62¾	197.135	3092.56	55.609	73¾	231.693	4271.83	65.357
63	197.920	3117.25	55.830	74	232.478	4300.84	65.578
63¼	198.706	3142.04	56.052	74¼	233.263	4329.95	65.800
63½	199.491	3166.92	56.273	74½	234.049	4359.16	66.022
63¾	200.277	3191.91	56.495	74¾	234.834	4388.47	66.243
64	201.062	3216.99	56.716	75	235.620	4417.86	66.465
64¼	201.847	3242.17	56.938	75¼	236.405	4447.37	66.686
64½	202.633	3267.46	57.159	75½	237.190	4476.97	66.908
64¾	203.418	3292.83	57.381	75¾	237.976	4506.67	67.129
65	204.204	3318.31	57.603	76	238.761	4536.46	67.351
65¼	204.989	3343.88	57.824	76¼	239.547	4566.36	67.572
65½	205.774	3369.56	58.046	76½	240.332	4596.35	67.794
65¾	206.560	3395.33	58.267	76¾	241.117	4626.44	68.016
66	207.345	3421.19	58.489	77	241.903	4656.63	68.237
66¼	208.131	3447.16	58.710	77¼	242.688	4686.92	68.459
66½	208.916	3473.23	58.932	77½	243.474	4717.30	68.680
66¾	209.701	3499.39	59.154	77¾	244.259	4747.79	68.902
67	210.487	3525.66	59.375	78	245.044	4778.36	69.123
67¼	211.272	3552.01	59.597	78¼	245.830	4809.05	69.345
67½	212.058	3578.47	59.818	78½	246.615	4839.83	69.566
67¾	212.843	3605.03	60.040	78¾	247.401	4870.70	69.788

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).	Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
79	248.186	4901.68	70.009	90	282.744	6361.73	79.758
79¼	248.971	4932.75	70.231	90¼	283.529	6399.12	79.980
79½	249.757	4963.92	70.453	90½	284.314	6432.62	80.201
79¾	250.542	4995.19	70.674	90¾	285.099	6468.16	80.423
80	251.328	5026.55	70.896	91	285.885	6503.88	80.644
80¼	252.113	5058.00	71.118	91¼	286.670	6539.68	80.866
80½	252.898	5089.58	71.339	91½	287.456	6573.56	81.087
80¾	253.683	5121.22	71.561	91¾	288.242	6611.52	81.308
81	254.469	5153.00	71.782	92	289.027	6647.61	81.530
81¼	255.254	5184.84	72.004	92¼	289.812	6683.80	81.752
81½	256.040	5216.82	72.225	92½	290.598	6720.07	81.973
81¾	256.825	5248.84	72.447	92¾	291.383	6756.40	82.195
82	257.611	5281.02	72.668	93	292.168	6792.91	82.416
82¼	258.396	5313.28	72.890	93¼	292.953	6829.48	82.638
82½	259.182	5345.62	73.111	93½	293.739	6866.16	82.859
82¾	259.967	5378.04	73.333	93¾	294.524	6882.92	83.081
83	260.752	5410.61	73.554	94	295.310	6939.78	83.302
83¼	261.537	5443.24	73.776	94¼	296.095	6976.72	83.524
83½	262.323	5476.00	73.997	94½	296.881	7013.81	83.746
83¾	263.108	5508.84	74.219	94¾	297.666	7050.92	83.968
84	263.894	5541.77	74.440	95	298.452	7088.22	84.189
84¼	264.679	5574.80	74.662	95¼	299.237	7125.56	84.411
84½	265.465	5607.95	74.884	95½	300.022	7163.04	84.632
84¾	266.250	5641.16	75.106	95¾	300.807	7200.56	84.854
85	267.035	5674.51	75.327	96	301.593	7238.23	85.077
85¼	267.821	5707.92	75.549	96¼	302.378	7275.96	85.299
85½	268.606	5741.47	75.770	96½	302.164	7313.84	85.520
85¾	269.392	5775.09	75.992	96¾	303.948	7351.72	85.742
86	270.177	5808.80	76.213	97	304.734	7389.81	85.963
86¼	270.962	5842.60	76.435	97¼	305.520	7427.96	86.185
86½	271.748	5876.55	76.656	97½	306.306	7474.20	86.407
86¾	272.533	5910.52	76.878	97¾	307.090	7504.52	86.628
87	273.319	5944.68	77.099	98	307.876	7542.96	86.850
87¼	274.104	5978.88	77.321	98¼	308.662	7581.48	87.072
87½	274.890	6013.21	77.542	98½	309.446	7620.12	87.293
87¾	275.675	6047.60	77.764	98¾	310.232	7658.80	87.515
88	276.460	6082.12	77.985	99	311.018	7697.69	87.736
88¼	277.245	6116.72	78.207	99¼	311.802	7736.60	87.958
88½	278.031	6151.44	78.428	99½	312.588	7775.64	88.180
88¾	278.816	6186.20	78.650	99¾	313.374	7814.76	88.401
89	279.602	6221.14	78.871	100	314.159	7853.98	88.623
89¼	280.387	6256.12	79.093	100½	315.730	7932.72	89.066
89½	281.173	6291.25	79.315	101	317.301	8011.85	89.509
89¾	281.958	6326.44	79.537	101½	318.872	8091.36	89.952

Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).	Diameter.	Circumference.	Area.	Side of Equal Square (Square Root of Area).
102	320.442	8171.28	90.395	112	351.858	9852.03	99.258
102½	322.014	8251.60	90.838	112½	353.430	9940.20	99.701
103	323.584	8332.29	91.282	113	355.000	10028.75	100.144
103½	325.154	8413.40	91.725	113½	356.570	10117.68	100.587
104	326.726	8494.87	92.168	114	358.142	10207.03	101.031
104½	328.296	8576.76	92.611	114½	359.712	10296.76	101.474
105	329.867	8659.01	93.054	115	361.283	10386.89	101.917
105½	331.438	8741.68	93.497	115½	362.854	10477.40	102.360
106	333.009	8824.73	93.940	116	364.425	10568.32	102.803
106½	334.580	8908.20	94.383	116½	365.996	10659.64	103.247
107	336.150	8992.02	94.826	117	367.566	10751.32	103.690
107½	337.722	9076.24	95.269	117½	369.138	10843.40	104.133
108	339.292	9160.88	95.713	118	370.708	10935.88	104.576
108½	340.862	9245.92	96.156	118½	372.278	11028.76	105.019
109	342.434	9331.32	96.599	119	373.849	11122.02	105.463
109½	344.004	9417.12	97.042	119½	375.420	11215.68	105.906
110	345.575	9503.32	97.485	120	376.991	11309.73	106.350
110½	347.146	9589.92	97.928				
111	348.717	9676.89	98.371				
111½	350.288	9764.28	98.815				

TABLE No. V.—LENGTHS OF CIRCULAR ARCS FROM 1° TO 180°. GIVEN, THE DEGREES.

(RADIUS = 1.)

Degrees.	Length.	Degrees.	Length.	Degrees.	Length.	Degrees.	Length.
1	.0175	40	.6981	79	1.3788	117	2.0420
2	.0349	41	.7156			118	2.0595
3	.0524	42	.7330	80	1.3963	119	2.0769
4	.0698	43	.7505	81	1.4137		
5	.0873	44	.7679	82	1.4312	120	2.0944
6	.1047	45	.7854	83	1.4486	121	2.1118
7	.1222	46	.8028	84	1.4661	122	2.1293
8	.1396	47	.8203	85	1.4835	123	2.1468
9	.1571	48	.8377	86	1.5010	124	2.1642
10	.1745	49	.8552	87	1.5184	125	2.1817
11	.1920	50	.8727	88	1.5359	126	2.1991
12	.2094	51	.8901	89	1.5533	127	2.2166
13	.2269	52	.9076	90	1.5708	128	2.2340
14	.2443	53	.9250	91	1.5882	129	2.2515
15	.2618	54	.9425	92	1.6057	130	2.2689
16	.2793	55	.9599	93	1.6232	131	2.2864
17	.2967	56	.9774	94	1.6406	132	2.3038
18	.3142	57	.9948	95	1.6581	133	2.3213
19	.3316	58	1.0123	96	1.6755	134	2.3387
20	.3491	59	1.0297	97	1.6930	135	2.3562
21	.3665	60	1.0472	98	1.7104	136	2.3736
22	.3840	61	1.0647	99	1.7279	137	2.3911
23	.4014	62	1.0821	100	1.7453	138	2.4086
24	.4189	63	1.0996	101	1.7628	139	2.4260
25	.4363	64	1.1170	102	1.7802	140	2.4435
26	.4538	65	1.1345	103	1.7977	141	2.4609
27	.4712	66	1.1519	104	1.8151	142	2.4784
28	.4887	67	1.1694	105	1.8326	143	2.4958
29	.5061	68	1.1868	106	1.8500	144	2.5133
30	.5236	69	1.2043	107	1.8675	145	2.5307
31	.5411	70	1.2217	108	1.8850	146	2.5482
32	.5585	71	1.2392	109	1.9024	147	2.5656
33	.5760	72	1.2566	110	1.9199	148	2.5831
34	.5934	73	1.2741	111	1.9373	149	2.6005
35	.6109	74	1.2915	112	1.9548	150	2.6180
36	.6283	75	1.3090	113	1.9722	151	2.6354
37	.6458	76	1.3265	114	1.9897	152	2.6529
38	.6632	77	1.3439	115	2.0071	153	2.6704
39	.6807	78	1.3614	116	2.0246	154	2.6878

Degrees.	Length.	Degrees.	Length.	Degrees.	Length.	Degrees.	Length.
155	2.7053	161	2.8100	168	2.9321	174	3.0369
156	2.7227	162	2.8274	169	2.9496	175	3.0543
157	2.7402	163	2.8449			176	3.0718
158	2.7576	164	2.8623	170	2.9670	177	3.0892
159	2.7751	165	2.8798	171	2.9845	178	3.1067
		166	2.8972	172	3.0020	179	3.1241
160	2.7925	167	2.9147	173	3.0194	180	3.1416

TABLE No. VI.—LENGTHS OF CIRCULAR ARCS, UP TO A SEMICIRCLE. GIVEN, THE HEIGHT.

(CHORD = 1.)

Height.	Length.	Height.	Length.	Height.	Length.	Height.	Length.
.100	1.02646	.140	1.05147	.180	1.08428	.220	1.12444
.101	1.02698	.141	1.05220	.181	1.08519	.221	1.12554
.102	1.02752	.142	1.05293	.182	1.08611	.222	1.12664
.103	1.02806	.143	1.05367	.183	1.08704	.223	1.12774
.104	1.02860	.144	1.05441	.184	1.08797	.224	1.12885
.105	1.02914	.145	1.05516	.185	1.08890	.225	1.12997
.106	1.02970	.146	1.05591	.186	1.08984	.226	1.13108
.107	1.03026	.147	1.05667	.187	1.09079	.227	1.13219
.108	1.03082	.148	1.05743	.188	1.09174	.228	1.13331
.109	1.03139	.149	1.05819	.189	1.09269	.229	1.13444
.110	1.03196	.150	1.05896	.190	1.09365	.230	1.13557
.111	1.03254	.151	1.05973	.191	1.09461	.231	1.13671
.112	1.03312	.152	1.06051	.192	1.09557	.232	1.13785
.113	1.03371	.153	1.06130	.193	1.09654	.233	1.13900
.114	1.03430	.154	1.06209	.194	1.09752	.234	1.14015
.115	1.03490	.155	1.06288	.195	1.09850	.235	1.14131
.116	1.03551	.156	1.06368	.196	1.09949	.236	1.14247
.117	1.03611	.157	1.06449	.197	1.10048	.237	1.14363
.118	1.03672	.158	1.06530	.198	1.10147	.238	1.14480
.119	1.03734	.159	1.06611	.199	1.10247	.239	1.14597
.120	1.03797	.160	1.06693	.200	1.10347	.240	1.14714
.121	1.03860	.161	1.06775	.201	1.10447	.241	1.14832
.122	1.03923	.162	1.06858	.202	1.10548	.242	1.14951
.123	1.03987	.163	1.06941	.203	1.10650	.243	1.15070
.124	1.04051	.164	1.07025	.204	1.10752	.244	1.15189
.125	1.04116	.165	1.07109	.205	1.10855	.245	1.15308
.126	1.04181	.166	1.07194	.206	1.10958	.246	1.15428
.127	1.04247	.167	1.07279	.207	1.11062	.247	1.15549
.128	1.04313	.168	1.07365	.208	1.11165	.248	1.15670
.129	1.04380	.169	1.07451	.209	1.11269	.249	1.15791
.130	1.04447	.170	1.07537	.210	1.11374	.250	1.15912
.131	1.04515	.171	1.07624	.211	1.11479	.251	1.16034
.132	1.04584	.172	1.07711	.212	1.11584	.252	1.16156
.133	1.04652	.173	1.07799	.213	1.11690	.253	1.16279
.134	1.04722	.174	1.07888	.214	1.11796	.254	1.16402
.135	1.04792	.175	1.07977	.215	1.11904	.255	1.16526
.136	1.04862	.176	1.08066	.216	1.12011	.256	1.16650
.137	1.04932	.177	1.08156	.217	1.12118	.257	1.16774
.138	1.05003	.178	1.08246	.218	1.12225	.258	1.16899
.139	1.05075	.179	1.08337	.219	1.12334	.259	1.17024

Height.	Length.	Height.	Length.	Height.	Length.	Height.	Length.
.260	1.17150	.307	1.23492	.354	1.30634	.401	1.38496
.261	1.17276	.308	1.23636	.355	1.30794	.402	1.38671
.262	1.17403	.309	1.23781	.356	1.30954	.403	1.38846
.263	1.17530	.310	1.23926	.357	1.31115	.404	1.39021
.264	1.17657	.311	1.24070	.358	1.31276	.405	1.39196
.265	1.17784	.312	1.24216	.359	1.31437	.406	1.39372
.266	1.17912	.313	1.24361	.360	1.31599	.407	1.30548
.267	1.18040	.314	1.24507	.361	1.31761	.408	1.39724
.268	1.18169	.315	1.24654	.362	1.31923	.409	1.39900
.269	1.18299	.316	1.24801	.363	1.32086	.410	1.40077
.270	1.18429	.317	1.24948	.364	1.32249	.411	1.40254
.271	1.18559	.318	1.25095	.365	1.32413	.412	1.40432
.272	1.18689	.319	1.25243	.366	1.32577	.413	1.40610
.273	1.18820	.320	1.25391	.367	1.32741	.414	1.40788
.274	1.18951	.321	1.25540	.368	1.32905	.415	1.40966
.275	1.19082	.322	1.25689	.369	1.33069	.416	1.41145
.276	1.19214	.323	1.25838	.370	1.33234	.417	1.41324
.277	1.19346	.324	1.25988	.371	1.33399	.418	1.41503
.278	1.19479	.325	1.26138	.372	1.33564	.419	1.41682
.279	1.19612	.326	1.26288	.373	1.33730	.420	1.41861
.280	1.19746	.327	1.26437	.374	1.33896	.421	1.42041
.281	1.19880	.328	1.26588	.375	1.34063	.422	1.42221
.282	1.20014	.329	1.26740	.376	1.34229	.423	1.42402
.283	1.20149	.330	1.26892	.377	1.34396	.424	1.42583
.284	1.20284	.331	1.27044	.378	1.34563	.425	1.42764
.285	1.20419	.332	1.27196	.379	1.34731	.426	1.42945
.286	1.20555	.333	1.27349	.380	1.34899	.427	1.43127
.287	1.20691	.334	1.27502	.381	1.35068	.428	1.43309
.288	1.20827	.335	1.27656	.382	1.35237	.429	1.43491
.289	1.20964	.336	1.27810	.383	1.35406	.430	1.43673
.290	1.21202	.337	1.27864	.384	1.35575	.431	1.43856
.291	1.21239	.338	1.28118	.385	1.35744	.432	1.44039
.292	1.21377	.339	1.28273	.386	1.35914	.433	1.44222
.293	1.21515	.340	1.28428	.387	1.36084	.434	1.44405
.294	1.21654	.341	1.28583	.388	1.36254	.435	1.44589
.295	1.21794	.342	1.28739	.389	1.36425	.436	1.44773
.296	1.21933	.343	1.28895	.390	1.36596	.437	1.44957
.297	1.22073	.344	1.29052	.391	1.36767	.438	1.45142
.298	1.22213	.345	1.29209	.392	1.36939	.439	1.45327
.299	1.22354	.346	1.29366	.393	1.37111	.440	1.45512
.300	1.22495	.347	1.29523	.394	1.37283	.441	1.45697
.301	1.22636	.348	1.29681	.395	1.37455	.442	1.45883
.302	1.22778	.349	1.29839	.396	1.37628	.443	1.46069
.303	1.22920	.350	1.29997	.397	1.37801	.444	1.46255
.304	1.23063	.351	1.30156	.398	1.37974	.445	1.46441
.305	1.23206	.352	1.30315	.399	1.38148	.446	1.46628
.306	1.23349	.353	1.30474	.400	1.38322	.447	1.46815

Height.	Length.	Height.	Length.	Height.	Length.	Height.	Length.
.448	1.47002	.461	1.49460	.475	1.52152	.489	1.54893
.449	1.47189	.462	1.49651	.476	1.52346	.490	1.55091
.450	1.47377	.463	1.49842	.477	1.52541	.491	1.55289
.451	1.47565	.464	1.50033	.478	1.52736	.492	1.55487
.452	1.47753	.465	1.50224	.479	1.52931	.493	1.55685
.453	1.47942	.466	1.50416	.480	1.53126	.494	1.55854
.454	1.48131	.467	1.50608	.481	1.53322	.495	1.56083
.455	1.48320	.468	1.50800	.482	1.53518	.496	1.56282
.456	1.48509	.469	1.50992	.483	1.53714	.497	1.56481
.457	1.48699	.470	1.51185	.484	1.53910	.498	1.56681
.458	1.48889	.471	1.51378	.485	1.54106	.499	1.56881
.459	1.49079	.472	1.51571	.486	1.54302	.500	1.57080
.460	1.49269	.473	1.51764	.487	1.54499		
		.474	1.51958	.488	1.54696		

TABLE No. VII.—AREAS OF CIRCULAR SEGMENTS, UP TO A SEMICIRCLE.

(DIAMETER OF CIRCLE = 1.)

Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
.001	.00004	.040	.01054	.079	.02889	.118	.05209
.002	.00012	.041	.01093	.080	.02943	.119	.05274
.003	.00022	.042	.01133	.081	.02997	.120	.05338
.004	.00034	.043	.01173	.082	.03053	.121	.05404
.005	.00047	.044	.01214	.083	.03108	.122	.05469
.006	.00062	.045	.01255	.084	.03163	.123	.05535
.007	.00078	.046	.01297	.085	.03219	.124	.05600
.008	.00095	.047	.01340	.086	.03275	.125	.05666
.009	.00114	.048	.01382	.087	.03331	.126	.05733
.010	.00133	.049	.01425	.088	.03385	.127	.05799
.011	.00153	.050	.01468	.089	.03444	.128	.05866
.012	.00175	.051	.01512	.090	.03501	.129	.05933
.013	.00197	.052	.01556	.091	.03538	.130	.06000
.014	.00220	.053	.01601	.092	.03616	.131	.06067
.015	.00244	.054	.01646	.093	.03674	.132	.06135
.016	.00268	.055	.01691	.094	.03732	.133	.06203
.017	.00294	.056	.01737	.095	.03790	.134	.06271
.018	.00320	.057	.01783	.096	.03850	.135	.06339
.019	.00347	.058	.01830	.097	.03909	.136	.06407
.020	.00375	.059	.01877	.098	.03968	.137	.06476
.021	.00403	.060	.01924	.099	.04028	.138	.06545
.022	.00432	.061	.01972	.100	.04087	.139	.06614
.023	.00461	.062	.02020	.101	.04148	.140	.06683
.024	.00492	.063	.02068	.102	.04208	.141	.06753
.025	.00523	.064	.02117	.103	.04269	.142	.06822
.026	.00555	.065	.02166	.104	.04330	.143	.06892
.027	.00587	.066	.02215	.105	.04391	.144	.06963
.028	.00619	.067	.02265	.106	.04452	.145	.07033
.029	.00653	.068	.02315	.107	.04514	.146	.07103
.030	.00687	.069	.02366	.108	.04576	.147	.07174
.031	.00721	.070	.02417	.109	.04638	.148	.07245
.032	.00756	.071	.02468	.110	.04701	.149	.07316
.033	.00792	.072	.02520	.111	.04763	.150	.07387
.034	.00828	.073	.02571	.112	.04826	.151	.07459
.035	.00864	.074	.02624	.113	.04889	.152	.07530
.036	.00901	.075	.02676	.114	.04953	.153	.07603
.037	.00939	.076	.02729	.115	.05016	.154	.07675
.038	.00977	.077	.02782	.116	.05080	.155	.07747
.039	.01015	.078	.02836	.117	.05145	.156	.07819

Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
.157	.07892	.203	.11423	.249	.15268	.295	.19360
.158	.07965	.204	.11504	.250	.15355	.296	.19451
.159	.08038	.205	.11584	.251	.15442	.297	.19543
.160	.08111	.206	.11665	.252	.15528	.298	.19634
.161	.08185	.207	.11746	.253	.15615	.299	.19725
.162	.08258	.208	.11827	.254	.15702	.300	.19817
.163	.08332	.209	.11908	.255	.15789	.301	.19908
.164	.08406	.210	.11990	.256	.15876	.302	.20000
.165	.08480	.211	.12071	.257	.15964	.303	.20092
.166	.08554	.212	.12153	.258	.16051	.304	.20184
.167	.08629	.213	.12235	.259	.16139	.305	.20276
.168	.08704	.214	.12317	.260	.16226	.306	.20368
.169	.08778	.215	.12399	.261	.16314	.307	.20460
.170	.08854	.216	.12481	.262	.16402	.308	.20553
.171	.08929	.217	.12563	.263	.16490	.309	.20645
.172	.09004	.218	.12646	.264	.16578	.310	.20738
.173	.09080	.219	.12729	.265	.16666	.311	.20830
.174	.09155	.220	.12811	.266	.16755	.312	.20923
.175	.09231	.221	.12894	.267	.16843	.313	.21015
.176	.09307	.222	.12977	.268	.16932	.314	.21108
.177	.09383	.223	.13060	.269	.17020	.315	.21201
.178	.09460	.224	.13144	.270	.17109	.316	.21294
.179	.09537	.225	.13227	.271	.17198	.317	.21387
.180	.09613	.226	.13311	.272	.17287	.318	.21480
.181	.09690	.227	.13395	.273	.17376	.319	.21573
.182	.09767	.228	.13478	.274	.17465	.320	.21667
.183	.09845	.229	.13562	.275	.17554	.321	.21760
.184	.09922	.230	.13646	.276	.17644	.322	.21853
.185	.09200	.231	.13731	.277	.17733	.323	.21947
.186	.10077	.232	.13815	.278	.17823	.324	.22040
.187	.10153	.233	.13899	.279	.17912	.325	.22134
.188	.10233	.234	.13984	.280	.18002	.326	.22228
.189	.10317	.235	.14069	.281	.18092	.327	.22322
.190	.10390	.236	.14154	.282	.18182	.328	.22415
.191	.10469	.237	.14239	.283	.18272	.329	.22509
.192	.10547	.238	.14324	.284	.18362	.330	.22603
.193	.10626	.239	.14409	.285	.18452	.331	.22697
.194	.10705	.240	.14494	.286	.18542	.332	.22792
.195	.10784	.241	.14580	.287	.18633	.333	.22886
.196	.10864	.242	.14665	.288	.18723	.334	.22980
.197	.10943	.243	.14752	.289	.18814	.335	.23074
.198	.11023	.244	.14837	.290	.18905	.336	.23169
.199	.11102	.245	.14923	.291	.18996	.337	.23263
.200	.11182	.246	.15009	.292	.19086	.338	.23358
.201	.11262	.247	.15096	.293	.19177	.339	.23453
.202	.11343	.248	.15182	.294	.19268	.340	.23547

Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
.341	.23642	.376	.26998	.411	.30417	.446	.33880
.342	.23737	.377	.27095	.412	.30516	.447	.33980
.343	.23832	.378	.27192	.413	.30614	.448	.34079
.344	.23927	.379	.27289	.414	.30712	.449	.34179
.345	.24025	.380	.27386	.415	.30811	.450	.34278
.346	.24117	.381	.27483	.416	.30910	.451	.34378
.347	.24212	.382	.27580	.417	.31008	.453	.34577
.348	.24307	.383	.27678	.418	.31107	.455	.34776
.349	.24403	.384	.27775	.419	.31205	.457	.34975
.350	.24498	.385	.27872	.420	.31304	.459	.35174
.351	.24593	.386	.27969	.421	.31403	.462	.35174
.352	.24689	.387	.28070	.422	.31502	.464	.35673
.353	.24784	.388	.28164	.423	.31600	.466	.35873
.354	.24880	.389	.28262	.424	.31699	.468	.36072
.355	.24976	.390	.28359	.425	.31798	.470	.36272
.356	.25071	.391	.28457	.426	.31897	.471	.36371
.357	.25167	.392	.28554	.427	.31996	.473	.36571
.358	.25263	.393	.28652	.428	.32095	.475	.36771
.359	.25359	.394	.28750	.429	.32194	.477	.36971
.360	.25455	.395	.28848	.430	.32293	.479	.37170
.361	.25551	.396	.28945	.431	.32392	.482	.37470
.362	.25647	.397	.29043	.432	.32491	.484	.37670
.363	.25743	.398	.29141	.433	.32590	.486	.37870
.364	.25839	.399	.29239	.434	.32689	.488	.38070
.365	.25936	.400	.29337	.435	.32788	.490	.38270
.366	.26032	.401	.29435	.436	.32887	.491	.38370
.367	.26128	.402	.29533	.437	.32987	.492	.38470
.368	.26225	.403	.29631	.438	.33086	.493	.38570
.369	.26321	.404	.29729	.439	.33185	.494	.38670
.370	.26418	.405	.29827	.440	.33284	.495	.38770
.371	.26514	.406	.29926	.441	.33384	.496	.38870
.372	.26611	.407	.30024	.442	.33483	.497	.38970
.373	.26708	.408	.30122	.443	.33582	.498	.39070
.374	.26805	.409	.30220	.444	.33682	.499	.39170
.375	.26901	.410	.30319	.445	.33781	.500	.39270

TABLE No. VIII.—SINES, COSINES, TANGENTS, COTANGENTS, SECANTS, AND COSECANTS OF ANGLES FROM 0° TO 90°.

ADVANCING BY 10′ OR ONE-SIXTH OF A DEGREE. (RADIUS = 1.)

Angle.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	
0° 0′	.000000	Infinite.	.000000	Infinite.	1.00000	1.000000	90° 0′
10	.002909	343.77516	.002909	343.77371	1.00000	.999996	50
20	.005818	171.88831	.005818	171.88540	1.00002	.999983	40
30	.008727	114.59301	.008727	114.58865	1.00004	.999962	30
40	.011635	85.945609	.011636	85.939791	1.00007	.999932	20
50	.014544	68.757360	.014545	68.750087	1.00011	.999894	10
1 0	.017452	57.298688	.017455	57.289962	1.00015	.999848	89 0
10	.020361	49.114062	.020365	49.103881	1.00021	.999793	50
20	.023269	42.975713	.023275	42.964077	1.00027	.999729	40
30	.026177	38.201550	.026186	38.188459	1.00034	.999657	30
40	.029085	34.382316	.029097	34.367771	1.00042	.999577	20
50	.031992	31.257577	.032009	31.241577	1.00051	.999488	10
2 0	.034899	28.653708	.034921	28.636253	1.00061	.999391	88 0
10	.037806	26.450510	.037834	26.431600	1.00072	.999285	50
20	.040713	24.562123	.040747	24.541758	1.00083	.999171	40
30	.043619	22.925586	.043661	22.903766	1.00095	.999048	30
40	.046525	21.493676	.046576	21.470401	1.00108	.998917	20
50	.049431	20.230284	.049491	20.205553	1.00122	.998778	10
3 0	.052336	19.107323	.052408	19.081137	1.00137	.998630	87 0
10	.055241	18.102619	.055325	18.074977	1.00153	.998473	50
20	.058145	17.198434	.058243	17.169337	1.00169	.998308	40
30	.061049	16.380408	.061163	16.349855	1.00187	.998135	30
40	.063952	15.636793	.064083	15.604784	1.00205	.997357	20
50	.066854	14.957882	.067004	14.924417	1.00224	.997763	10
4 0	.069756	14.335587	.069927	14.300666	1.00244	.997564	86 0
10	.072658	13.763115	.072851	13.726738	1.00265	.997357	50
20	.075559	13.234717	.075776	13.196888	1.00287	.997141	40
30	.078459	12.745495	.078702	12.706205	1.00309	.996917	30
40	.081359	12.291252	.081629	12.250505	1.00333	.996685	20
50	.084258	11.868370	.084558	11.826167	1.00357	.996444	10
5 0	.087156	11.473713	.087489	11.430052	1.00382	.996195	85 0
10	.090053	11.104549	.090421	11.059431	1.00408	.995937	50
20	.092950	10.758488	.093354	10.711913	1.00435	.995671	40
30	.095846	10.433431	.096289	10.385397	1.00463	.995396	30
40	.098741	10.127522	.099226	10.078031	1.00491	.995113	20
50	.101635	9.8391227	.102164	9.7881732	1.00521	.994822	10
	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	Angle.

Angle.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	
6° 0′	.104528	9.5667722	.105104	9.5143645	1.00551	.994522	84° 0′
10	.107421	9.3091699	.108046	9.2553035	1.00582	.994214	50
20	.110313	9.0651512	.110990	9.0098261	1.00614	.993897	40
30	.113203	8.8336715	.113936	8.7768874	1.00647	.993572	30
40	.116093	8.6137901	.116883	8.5555468	1.00681	.993238	20
50	.118982	8.4045586	.119833	8.3449558	1.00715	.992896	10
7 0	.121869	8.2055090	.122785	8.1443464	1.00751	.992546	83 0
10	.124756	8.0156450	.125738	7.9530224	1.00787	.992187	50
20	.127642	7.8344335	.128694	7.7703506	1.00825	.991820	40
30	.130526	7.6612976	.131653	7.5957541	1.00863	.991445	30
40	.133410	7.4957100	.134613	7.4287064	1.00902	.991061	20
50	.136292	7.3371909	.137576	7.2687255	1.00942	.990669	10
8 0	.139173	7.1852965	.140541	7.1153697	1.00983	.990268	82 0
10	.142053	7.0396220	.143508	6.9682335	1.01024	.989859	50
20	.144932	6.8997942	.146478	6.8269437	1.01067	.989442	40
30	.147809	6.7654691	.149451	6.6911562	1.01111	.989016	30
40	.150686	6.6363293	.152426	6.5605538	1.01155	.988582	20
50	.153561	6.5120812	.155404	6.4348428	1.01200	.988139	10
9 0	.156434	6.3924532	.158384	6.3137515	1.01247	.987688	81 0
10	.159307	6.2771933	.161368	6.1970279	1.01294	.987229	50
20	.162178	6.1660674	.164354	6.0844381	1.01432	.986762	40
30	.165048	6.0588980	.167343	5.9757644	1.01391	.986286	30
40	.167916	5.9553625	.170334	5.8708042	1.01440	.985801	20
50	.170783	5.8553921	.173329	5.7693688	1.01491	.985309	10
10 0	.173648	5.7587705	.176327	5.6712818	1.01543	.984808	80 0
10	.176512	5.6653331	.179328	5.5763786	1.01595	.984298	50
20	.179375	5.5749258	.182332	5.4845052	1.01649	.983781	40
30	.182236	5.4874043	.185339	5.3955172	1.01703	.983255	30
40	.185095	5.4026333	.188359	5.3092793	1.01758	.982721	20
50	.187953	5.3204860	.191363	5.2256647	1.01815	.982178	10
11 0	.190809	5.2408431	.194380	5.1445540	1.01872	.981627	79 0
10	.193664	5.1635924	.197401	5.0658352	1.01930	.981068	50
20	.196517	5.0886284	.200425	4.9894027	1.01989	.980500	40
30	.199368	5.0158317	.203452	4.9151570	1.02049	.979925	30
40	.202218	4.9451687	.206483	4.8430045	1.02110	.979341	20
50	.205065	4.8764907	.209518	4.7728568	1.02171	.978748	10
12 0	.207912	4.8097343	.212557	4.7046301	1.02234	.978148	78 0
10	.210756	4.7448206	.215599	4.6382457	1.02298	.977539	50
20	.213599	4.6816748	.218645	4.5736287	1.02362	.976921	40
30	.216440	4.6202263	.221695	4.5107085	1.02428	.976296	30
40	.219279	4.5604080	.224748	4.4494181	1.02494	.975662	20
50	.222116	4.5021565	.227806	4.3896940	1.02562	.975020	10
	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	Angle.

Angle.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	
13° 0′	.224951	4.4454115	.230868	4.3314759	1.02630	.974370	77° 0′
10	.227784	4.3901158	.233934	4.2747066	1.02700	.973712	50
20	.230616	4.3362150	.237004	4.2193318	1.02770	.973045	40
30	.233445	4.2836576	.240079	4.1652998	1.02842	.972370	30
40	.236273	4.2323943	.243158	4.1125614	1.02914	.971687	20
50	.239098	4.1823785	.246241	4.0610700	1.02987	.970995	10
14 0	.241922	4.1335655	.249328	4.0107809	1.03061	.970296	76 0
10	.244743	4.0859130	.252420	3.9616518	1.03137	.969588	50
20	.247563	4.0393804	.255517	3.9136420	1.03213	.968872	40
30	.250380	3.9939292	.258618	3.8667131	1.03290	.968148	30
40	.253195	3.9495224	.261723	3.8208281	1.03363	.967415	20
50	.256008	3.9061250	.264834	3.7759519	1.03447	.966675	10
15 0	.258819	3.8637033	.267949	3.7320508	1.03528	.965926	75 0
10	.261628	3.8222251	.271069	3.6890927	1.03609	.965169	50
20	.264434	3.7816596	.274195	3.6470467	1.03691	.964404	40
30	.267238	3.7419775	.277325	3.6058835	1.03774	.963630	30
40	.270040	3.7031506	.280460	3.5655749	1.03858	.962849	20
50	.272840	3.6651518	.283600	3.5260938	1.03944	.962059	10
16 0	.275637	3.6279553	.286745	3.4874144	1.04030	.961262	74 0
10	.278432	3.5915363	.289896	3.4495120	1.04117	.960456	50
20	.281225	3.5558710	.293052	3.4123626	1.04206	.959642	40
30	.284015	3.5209365	.296214	3.3759434	1.04295	.958820	30
40	.286803	3.4867110	.299380	3.3402326	1.04385	.957990	20
50	.289589	3.4531735	.302553	3.3052091	1.04477	.957151	10
17 0	.292372	3.4203036	.305731	3.2708526	1.04569	.956305	73 0
10	.295152	3.3880820	.308914	3.2371438	1.04663	.955450	50
20	.297930	3.3564900	.312104	3.2040638	1.04757	.954588	40
30	.300706	3.3255095	.315299	3.1715948	1.04853	.953717	30
40	.303479	3.2951234	.318500	3.1397194	1.04950	.952838	20
50	.306249	3.2653149	.321707	3.1084210	1.05047	.951951	10
18 0	.309017	3.2360680	.324920	3.0776835	1.05146	.951057	72 0
10	.311782	3.2073673	.328139	3.0474915	1.05246	.950154	50
20	.314545	3.1791978	.331364	3.0178301	1.05347	.949243	40
30	.317305	3.1515453	.334595	2.9886850	1.05449	.948324	30
40	.320062	3.1243959	.337833	2.9600422	1.05552	.947397	20
50	.322816	3.0977363	.341077	2.9318885	1.05657	.946462	10
19 0	.325568	3.0715535	.344328	2.9042109	1.05762	.945519	71 0
10	.328317	3.0458352	.347585	2.8769970	1.05869	.944568	50
20	.331063	3.0205693	.350848	2.8502349	1.05976	.943609	40
30	.333807	2.9957443	.354119	2.8239129	1.06085	.942641	30
40	.336547	2.9713490	.357396	2.7980198	1.06195	.941666	20
50	.339285	2.9473724	.360680	2.7725448	1.06306	.940684	10
	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	Angle.

Angle.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	
20° 0′	.342020	2.9238044	.363970	2.7474774	1.06418	.939693	70° 0′
10	.344752	2.9006346	.367268	2.7228076	1.06531	.938694	50
20	.347481	2.8778532	.370573	2.6985254	1.06645	.937687	40
30	.350207	2.8554510	.373885	2.6746215	1.06761	.936672	30
40	.352931	2.8334185	.377204	2.6510867	1.06878	.935650	20
50	.355651	2.8117471	.380530	2.6279121	1.06995	.934619	10
21 0	.358368	2.7904281	.383864	2.6050891	1.07115	.933580	69 0
10	.361082	2.7694532	.387205	2.5826094	1.07235	.932534	50
20	.363793	2.7488144	.390554	2.5604649	1.07356	.931480	40
30	.366501	2.7285038	.393911	2.5386479	1.07479	.930418	30
40	.369206	2.7085139	.397275	2.5171507	1.07602	.929348	20
50	.371908	2.6888374	.400647	2.4959661	1.07727	.928270	10
22 0	.374607	2.6694672	.404026	2.4750869	1.07853	.927184	68 0
10	.377302	2.6503962	.407414	2.4545061	1.07981	.926090	50
20	.379994	2.6316180	.410810	2.4342172	1.08109	.924989	40
30	.382683	2.6131259	.414214	2.4142136	1.08239	.923880	30
40	.385369	2.5949137	.417626	2.3944889	1.08370	.922762	20
50	.388052	2.5769753	.421046	2.3750372	1.08503	.921638	10
23 0	.390731	2.5593047	.424475	2.3558524	1.08636	.920505	67 0
10	.393407	2.5418961	.427912	2.3369287	1.08771	.919364	50
20	.396080	2.5247440	.431358	2.3182606	1.08907	.918216	40
30	.398749	2.5078428	.434812	2.2998425	1.09044	.917060	30
40	.401415	2.4911874	.438276	2.2816693	1.09183	.915896	20
50	.404078	2.4747726	.441748	2.2637357	1.09323	.914725	10
24 0	.406737	2.4585933	.445229	2.2460368	1.09464	.913545	66 0
10	.409392	2.4426448	.448719	2.2285676	1.09606	.912358	50
20	.412045	2.4269222	.452218	2.2113234	1.09750	.911164	40
30	.414693	2.4114210	.455726	2.1942997	1.09895	.909961	30
40	.417338	2.3961367	.459244	2.1774920	1.10041	.908751	20
50	.419980	2.3810650	.462771	2.1608958	1.10189	.907533	10
25 0	.422618	2.3662016	.466308	2.1445069	1.10338	.906308	65 0
10	.425253	2.3515424	.469854	2.1283213	1.10488	.905075	50
20	.427884	2.3370833	.473410	2.1123348	1.10640	.903834	40
30	.430511	2.3228205	.476976	2.0965436	1.10793	.902585	30
40	.433135	2.3087501	.480551	2.0809438	1.10947	.901329	20
50	.435755	2.2948685	.484137	2.0655318	1.11103	.900065	10
26 0	.438371	2.2811720	.487733	2.0503038	1.11260	.898794	64 0
10	.440984	2.2676571	.491339	2.0352565	1.11419	.897515	50
20	.443593	2.2543204	.494955	2.0203862	1.11579	.896229	40
30	.446198	2.2411585	.498582	2.0056897	1.11740	.894934	30
40	.448799	2.2281681	.502219	1.9911637	1.11903	.893633	20
50	.451397	2.2153460	.505867	1.9768050	1.12067	.892323	10
	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	Angle.

Angle.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	
27° 0′	.453990	2.2026893	.509525	1.9626105	1.12233	.891007	63° 0′
10	.456580	2.1901947	.513195	1.9485772	1.12400	.889682	50
20	.459166	2.1778595	.516876	1.9347020	1.12568	.888350	40
30	.461749	2.1656806	.520567	1.9209821	1.12738	.887011	30
40	.464327	2.1536553	.524270	1.9074147	1.12910	.885664	20
50	.466901	2.1417808	.527984	1.8939971	1.13083	.884309	10
28 0	.469472	2.1300545	.531709	1.8807265	1.13257	.882948	62 0
10	.472038	2.1184737	.535547	1.8676003	1.13433	.881578	50
20	.474600	2.1070359	.539195	1.8546159	1.13610	.880201	40
30	.477159	2.0957385	.542956	1.8417409	1.13789	.878817	30
40	.479713	2.0845792	.546728	1.8290628	1.13970	.877425	20
50	.482263	2.0735556	.550515	1.8164892	1.14152	.876026	10
29 0	.484810	2.0626653	.554309	1.8040478	1.14335	.874620	61 0
10	.487352	2.0519061	.558118	1.7917362	1.14521	.873206	50
20	.489890	2.0412757	.561939	1.7795524	1.14707	.871784	40
30	.492424	2.0307720	.565773	1.7674940	1.14896	.870356	30
40	.494953	2.0203929	.569619	1.7555590	1.15085	.868920	20
50	.497479	2.0101362	.573478	1.7437453	1.15277	.867476	10
30 0	.500000	2.0000000	.577350	1.7320508	1.15470	.866025	60 0
10	.502517	1.9899822	.581235	1.7204736	1.15665	.864567	50
20	.505030	1.9800810	.585134	1.7090116	1.15861	.863102	40
30	.507538	1.9702944	.589045	1.6976631	1.16059	.861629	30
40	.510043	1.9606206	.592970	1.6864261	1.16259	.860149	20
50	.512543	1.9510577	.596908	1.6752988	1.16460	.858662	10
31 0	.515038	1.9416040	.600861	1.6642795	1.16663	.857167	59 0
10	.517529	1.9322578	.604827	1.6533663	1.16868	.855665	50
20	.520016	1.9230173	.608807	1.6425576	1.17075	.854156	40
30	.522499	1.9138809	.612801	1.6318517	1.17283	.852640	30
40	.524977	1.9048469	.616809	1.6212469	1.17493	.851117	20
50	.527450	1.8959138	.620832	1.6107417	1.17704	.849586	10
32 0	.529919	1.8870799	.624869	1.6003345	1.17918	.848048	58 0
10	.532384	1.8783438	.628921	1.5900238	1.18133	.846503	50
20	.534844	1.8697040	.632988	1.5798079	1.18350	.844951	40
30	.537300	1.8611590	.637079	1.5696856	1.18569	.843391	30
40	.539751	1.8527073	.641167	1.5596552	1.18790	.841825	20
50	.542197	1.8443476	.645280	1.5497155	1.19012	.840251	10
33 0	.544639	1.8360785	.649408	1.5398650	1.19236	.838671	57 0
10	.547076	1.8278985	.653531	1.5301025	1.19463	.837083	50
20	.549509	1.8198065	.657710	1.5204261	1.19691	.835488	40
30	.551937	1.8118010	.661886	1.5108352	1.19920	.833886	30
40	.554360	1.8038809	.666077	1.5013282	1.20152	.832277	20
50	.556779	1.7960449	.670285	1.4919039	1.20386	.830661	10
	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	Angle.

Angle.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	
34° 0′	.559193	1.7882916	.674509	1.4825610	1.20622	.829038	56° 0′
10	.561602	1.7806201	.678749	1.4732983	1.20859	.827407	50
20	.564007	1.7730290	.683007	1.4641147	1.21099	.825770	40
30	.566406	1.7655173	.687281	1.4550090	1.21341	.824126	30
40	.568801	1.7580837	.691573	1.4459801	1.21584	.822475	20
50	.571191	1.7507273	.695881	1.4370268	1.21830	.820817	10
35 0	.573576	1.7434468	.700208	1.4281480	1.22077	.819152	55 0
10	.575957	1.7362413	.704552	1.4193427	1.22327	.817480	50
20	.578332	1.7291096	.708913	1.4106098	1.22579	.815801	40
30	.580703	1.7220508	.713293	1.4019483	1.22833	.814116	30
40	.583069	1.7150639	.717691	1.3933571	1.23089	.812423	20
50	.585429	1.7081478	.722108	1.3848355	1.23347	.810723	10
36 0	.587785	1.7013016	.726543	1.3763810	1.23607	.809017	54 0
10	.590136	1.6945244	.730996	1.3679959	1.23869	.807304	50
20	.592482	1.6878151	.735469	1.3596764	1.24134	.805584	40
30	.594823	1.6811730	.739961	1.3514224	1.24400	.803857	30
40	.597159	1.6745970	.744472	1.3432331	1.24669	.802123	20
50	.599489	1.6680864	.749003	1.3351075	1.24940	.800383	10
37 0	.601815	1.6616401	.753554	1.3270448	1.25214	.798636	53 0
10	.604136	1.6552575	.758125	1.3190441	1.25489	.796882	50
20	.606451	1.6489376	.762716	1.3111046	1.25767	.795121	40
30	.608761	1.6426796	.767627	1.3032254	1.26047	.793353	30
40	.611067	1.6364828	.771959	1.2954057	1.26330	.791579	20
50	.613367	1.6303462	.776612	1.2876447	1.26615	.789798	10
38 0	.615661	1.6242692	.781286	1.2799416	1.26902	.788011	52 0
10	.617951	1.6182510	.785981	1.2722957	1.27191	.786217	50
20	.620235	1.6122908	.790698	1.2647062	1.27483	.784416	40
30	.622515	1.6063879	.795436	1.2571723	1.27778	.782608	30
40	.624789	1.6005416	.800196	1.2496933	1.28075	.780794	20
50	.627057	1.5947511	.804080	1.2422685	1.28374	.778973	10
39 0	.629320	1.5890157	.809784	1.2348972	1.28676	.777146	51 0
10	.631578	1.5833318	.814612	1.2275786	1.28980	.775312	50
20	.633831	1.5777077	.819463	1.2203121	1.29287	.773472	40
30	.636078	1.5721337	.824336	1.2130970	1.29597	.771625	30
40	.638320	1.5666121	.829234	1.2059327	1.29909	.769771	20
50	.640557	1.5611424	.834155	1.1988184	1.30223	.767911	10
40 0	.642788	1.5557238	.839100	1.1917536	1.30541	.766044	50 0
10	.645013	1.5503558	.844069	1.1847376	1.30861	.764171	50
20	.647233	1.5450378	.849062	1.1777698	1.31183	.762292	40
30	.649448	1.5397690	.854081	1.1708496	1.31509	.760406	30
40	.651657	1.5345491	.859124	1.1639763	1.31837	.758514	20
50	.653861	1.5293773	.864193	1.1571495	1.32168	.756615	10
	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	Angle.

Angle.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	
41° 0′	.656059	1.5242531	.869287	1.1503684	1.32501	.754710	49° 0′
10	.658252	1.5191759	.874407	1.1436326	1.32838	.752798	50
20	.660439	1.5141452	.879553	1.1369414	1.33177	.750880	40
30	.662620	1.5091605	.884725	1.1302944	1.33519	.748956	30
40	.664796	1.5042211	.889924	1.1236909	1.33864	.747025	20
50	.666966	1.4993267	.895151	1.1171305	1.34212	.745088	10
42 0	.669131	1.4944765	.900404	1.1106125	1.34563	.743145	48 0
10	.671289	1.4896703	.905685	1.1041365	1.34917	.741195	50
20	.673443	1.4849073	.910994	1.0977020	1.35274	.739239	40
30	.675590	1.4801872	.916331	1.0913085	1.35634	.737277	30
40	.677732	1.4755095	.921697	1.0849554	1.35997	.735309	20
50	.679868	1.4708736	.927091	1.0786423	1.36363	.733335	10
43 0	.681998	1.4662792	.932515	1.0723687	1.36733	.731354	47 0
10	.684123	1.4617257	.937968	1.0661341	1.37105	.729367	50
20	.686242	1.4572127	.943451	1.0599381	1.37481	.727374	40
30	.688355	1.4527397	.948965	1.0537801	1.37860	.725374	30
40	.690462	1.4483063	.954508	1.0476598	1.38242	.723369	20
50	.692563	1.4439120	.960083	1.0415767	1.38628	.721357	10
44 0	.694658	1.4395565	.965689	1.0355303	1.39016	.719340	46 0
10	.696748	1.4352393	.971326	1.0295203	1.39409	.717316	50
20	.698832	1.4309602	.976996	1.0235461	1.39804	.715286	40
30	.700909	1.4267182	.982697	1.0176074	1.40203	.713251	30
40	.702981	1.4225134	.988432	1.0117088	1.40606	.711209	20
50	.705047	1.4183454	.994199	1.0058348	1.41012	.709161	10
45 0	.707107	1.4142136	1.000000	1.0000000	1.41421	.707107	45 0
	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	Angle.

TABLE No. IX.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS OF ANGLES FROM 0° TO 90°.

ADVANCING BY 10′, OR ONE-SIXTH OF A DEGREE.

Angle.	Sine.	Tangent.	Cotangent.	Cosine.	
0°	0.000000	0.000000	Infinite.	10.000000	90°
10′	7.463726	7.463727	12.536273	9.999998	50′
20	7.764754	7.764761	12.235239	9.999993	40
30	7.940842	7.940858	12.059142	9.999983	30
40	8.065776	8.065806	11.934194	9.999971	20
50	8.162681	8.162727	11.837273	9.999954	10
1	8.241855	8.241921	11.758079	9.999934	89
10	8.308794	8.308884	11.691116	9.999910	50
20	8.366777	8.366895	11.633105	9.999882	40
30	8.417919	8.418068	11.581932	9.999851	30
40	8.463665	8.463849	11.536151	9.999816	20
50	8.505045	8.505267	11.494733	9.999778	10
2	8.542819	8.543084	11.456916	9.999735	88
10	8.577566	8.577877	11.422123	9.999689	50
20	8.609734	8.610094	11.389906	9.999640	40
30	8.639680	8.640093	11.359907	9.999586	30
40	8.667689	8.668160	11.331840	9.999529	20
50	8.693998	8.694529	11.305471	9.999469	10
3	8.718800	8.719396	11.280604	9.999404	87
10	8.742259	8.742922	11.257078	9.999336	50
20	8.764511	8.765246	11.234754	9.999265	40
30	8.785675	8.786486	11.213514	9.999189	30
40	8.805852	8.806742	11.193258	9.999110	20
50	8.825130	8.826103	11.173897	9.999027	10
4	8.843585	8.844644	11.155356	9.998941	86
10	8.861283	8.862433	11.137567	9.998851	50
20	8.878285	8.879529	11.120471	9.998757	40
30	8.894643	8.895984	11.104016	9.998659	30
40	8.910404	8.911846	11.088154	9.998558	20
50	8.925609	8.927156	11.072844	9.998453	10
5	8.940296	8.941952	11.058048	9.998344	85
10	8.954499	8.956267	11.043733	9.998232	50
20	8.968249	8.970133	11.029867	9.998116	40
30	8.981573	8.983577	11.016423	9.997996	30
40	8.994497	8.996624	11.003376	9.997872	20
50	9.007044	9.009298	10.990702	9.997745	10
	Cosine.	Cotangent.	Tangent.	Sine.	Angle.

Angle.	Sine.	Tangent.	Cotangent.	Cosine.	
6°	9.019235	9.021620	10.978380	9.997614	84°
10′	9.031089	9.033609	10.966391	9.997480	50′
20	9.042625	9.045284	10.954716	9.997341	40
30	9.053859	9.056659	10.943341	9.997199	30
40	9.064806	9.067752	10.932248	9.997053	20
50	9.075480	9.078576	10.921424	9.996904	10
7	9.085894	9.089144	10.910856	9.996751	83
10	9.096062	9.099468	10.900532	9.996594	50
20	9.105992	9.109559	10.890441	9.996433	40
30	9.115698	9.119429	10.880571	9.996269	30
40	9.125187	9.129087	10.870913	9.996100	20
50	9.134470	9.138542	10.861458	9.995928	10
8	9.143555	9.147803	10.852197	9.995753	82
10	9.152451	9.156877	10.843123	9.995573	50
20	9.161164	9.165774	10.834226	9.995390	40
30	9.169702	9.174499	10.825501	9.995203	30
40	9.178072	9.183059	10.816941	9.995013	20
50	9.186280	9.191462	10.808538	9.994818	10
9	9.194332	9.199713	10.800287	9.994620	81
10	9.202234	9.207817	10.792183	9.994418	50
20	9.209992	9.215780	10.784220	9.994212	40
30	9.217609	9.223607	10.776393	9.994003	30
40	9.225092	9.231302	10.768698	9.993789	20
50	9.232444	9.238872	10.761128	9.993572	10
10	9.239670	9.246319	10.753681	9.993351	80
10	9.246775	9.253648	10.746352	9.993127	50
20	9.253761	9.260863	10.739137	9.992898	40
30	9.260633	9.267967	10.732033	9.992666	30
40	9.267395	9.274964	10.725036	9.992430	20
50	9.274049	9.281858	10.718142	9.992190	10
11	9.280599	9.288652	10.711348	9.991947	79
10	9.287048	9.295349	10.704651	9.991699	50
20	9.293399	9.301951	10.698049	9.991448	40
30	9.299655	9.308463	10.691537	9.991193	30
40	9.305819	9.314885	10.685115	9.990934	20
50	9.311893	9.321222	10.678778	9.990671	10
12	9.317879	9.327475	10.672525	9.990404	78
10	9.323780	9.333646	10.666354	9.990134	50
20	9.329599	9.339739	10.660261	9.989860	40
30	9.335337	9.345755	10.654245	9.989582	30
40	9.340996	9.351697	10.648303	9.989300	20
50	9.346779	9.357566	10.642434	9.989014	10
	Cosine.	Cotangent.	Tangent.	Sine.	Angle.

Angle.	Sine.	Tangent.	Cotangent.	Cosine.	
13°	9.352088	9.363364	10.636636	9.988724	77°
10′	9.357524	9.369094	10.630906	9.988430	50′
20	9.362889	9.374756	10.625244	9.988133	40
30	9.368185	9.380354	10.619646	9.987832	30
40	9.373414	9.385888	10.614112	9.987526	20
50	9.378577	9.391360	10.608640	9.987217	10
14	9.383675	9.396771	10.603229	9.986904	76
10	9.388711	9.402124	10.597876	9.986587	50
20	9.393685	9.407419	10.592581	9.986266	40
30	9.398600	9.412658	10.587342	9.985942	30
40	9.403455	9.417842	10.582158	9.985613	20
50	9.408254	9.422974	10.577026	9.985280	10
15	9.412996	9.428052	10.571948	9.984944	75
10	9.417684	9.433080	10.566920	9.984603	50
20	9.422318	9.438059	10.561941	9.984259	40
30	9.426899	9.442988	10.557012	9.983911	30
40	9.431429	9.447870	10.552130	9.983558	20
50	9.435908	9.452706	10.547294	9.983202	10
16	9.440338	9.457496	10.542504	9.982842	74
10	9.444720	9.462242	10.537758	9.982477	50
20	9.449054	9.466945	10.533055	9.982109	40
30	9.453342	9.471605	10.528395	9.981737	30
40	9.457584	9.476223	10.523777	9.981361	20
50	9.461782	9.480801	10.519199	9.980981	10
17	9.465935	9.485339	10.514661	9.980596	73
10	9.470046	9.489838	10.510162	9.980208	50
20	9.474115	9.494299	10.505701	9.979816	40
30	9.478142	9.498722	10.501278	9.979420	30
40	9.482128	9.503109	10.496891	9.979019	20
50	9.486075	9.507460	10.492540	9.978615	10
18	9.489982	9.511776	10.488224	9.978206	72
10	9.493851	9.516057	10.483943	9.977794	50
20	9.497682	9.520305	10.479695	9.977377	40
30	9.501476	9.524520	10.475480	9.976957	30
40	9.505234	9.528702	10.471298	9.976532	20
50	9.508956	9.532853	10.467147	9.976103	10
19	9.512642	9.536972	10.463028	9.975670	71
10	9.516294	9.541061	10.458939	9.975233	50
20	9.519911	9.545119	10.454881	9.974792	40
30	9.523495	9.549149	10.450851	9.974347	30
40	9.527046	9.553149	10.446851	9.973897	20
50	9.530565	9.557121	10.442879	9.973444	10
	Cosine.	Cotangent.	Tangent.	Sine.	Angle.

Angle.	Sine.	Tangent.	Cotangent.	Cosine.	
20°	9.534052	9.561066	10.438934	9.972986	70°
10′	9.537507	9.564983	10.435017	9.972524	50′
20	9.540931	9.568873	10.431127	9.972058	40
30	9.544325	9.572738	10.427262	9.971588	30
40	9.547689	9.576576	10.423424	9.971113	20
50	9.551024	9.580389	10.419611	9.970635	10
21	9.554329	9.584177	10.415823	9.970152	69
10	9.557606	9.587941	10.412059	9.969665	50
20	9.560855	9.591681	10.408319	9.969173	40
30	9.564075	9.595398	10.404602	9.968678	30
40	9.567269	9.599091	10.400909	9.968178	20
50	9.570435	9.602761	10.397239	9.967674	10
22	9.573575	9.606410	10.393590	9.967166	68
10	9.576689	9.610036	10.389964	9.966653	50
20	9.579777	9.613641	10.386359	9.966136	40
30	9.582840	9.617224	10.382776	9.965615	30
40	9.585877	9.620787	10.379213	9.965090	20
50	9.588890	9.624330	10.375670	9.964560	10
23	9.591878	9.627852	10.372148	9.964026	67
10	9.594842	9.631355	10.368645	9.963488	50
20	9.597783	9.634838	10.365162	9.962945	40
30	9.600700	9.638302	10.361698	9.962398	30
40	9.603594	9.641747	10.358253	9.961846	20
50	9.606465	9.645174	10.354826	9.961290	10
24	9.609313	9.648583	10.351417	9.960730	66
10	9.612140	9.651974	10.348026	9.960165	50
20	9.614944	9.655348	10.344652	9.959596	40
30	9.617727	9.658704	10.341296	9.959023	30
40	9.620488	9.662043	10.337957	9.958445	20
50	9.623229	9.665366	10.334634	9.957863	10
25	9.625948	9.668673	10.331328	9.957276	65
10	9.628647	9.671963	10.328037	9.956684	50
20	9.631326	9.675237	10.324763	9.956089	40
30	9.633984	9.678496	10.321504	9.955488	30
40	9.636623	9.681740	10.318260	9.954883	20
50	9.639242	9.684968	10.315032	9.954274	10
26	9.641842	9.688182	10.311818	9.953660	64
10	9.644423	9.691381	10.308619	9.953042	50
20	9.646984	9.694566	10.305434	9.952419	40
30	9.649527	9.697736	10.302264	9.951791	30
40	9.652052	9.700893	10.299107	9.951159	20
50	9.654558	9.704036	10.295964	9.950522	10
	Cosine.	Cotangent.	Tangent.	Sine.	Angle.

Angle.	Sine.	Tangent.	Cotangent.	Cosine.	
27°	9.657047	9.707166	10.292834	9.949881	63°
10′	9.659517	9.710282	10.289718	9.949235	50′
20	9.661970	9.713386	10.286614	9.948584	40
30	9.664406	9.716477	10.283523	9.947929	30
40	9.666824	9.719555	10.280445	9.947269	20
50	9.669225	9.722621	10.277379	9.946604	10
28	9.671609	9.725674	10.274326	9.945935	62
10	9.673977	9.728716	10.271284	9.945261	50
20	9.676328	9.731746	10.268254	9.944582	40
30	9.678663	9.734764	10.265236	9.943899	30
40	9.680982	9.737771	10.262229	9.943210	20
50	9.683284	9.740767	10.259233	9.942517	10
29	9.685571	9.743752	10.256248	9.941819	61
10	9.687843	9.746726	10.253274	9.941117	50
20	9.690098	9.749689	10.250311	9.940409	40
30	9.692339	9.752642	10.247358	9.939697	30
40	9.694564	9.755585	10.244415	9.938980	20
50	9.696775	9.758517	10.241483	9.938258	10
30	9.698970	9.761439	10.238561	9.937531	60
10	9.701151	9.764352	10.235648	9.936799	50
20	9.703317	9.767255	10.232745	9.936062	40
30	9.705469	9.770148	10.229852	9.935320	30
40	9.707606	9.773033	10.226967	9.934574	20
50	9.709730	9.775908	10.224092	9.933822	10
31	9.711839	9.778774	10.221226	9.933066	59
10	9.713935	9.781631	10.218369	9.932304	50
20	9.716017	9.784479	10.215521	9.931537	40
30	9.718085	9.787319	10.212681	9.930766	30
40	9.720140	9.790151	10.209849	9.929989	20
50	9.722181	9.792974	10.207026	9.929207	10
32	9.724210	9.795789	10.204211	9.928420	58
10	9.726225	9.798596	10.201404	9.927629	50
20	9.728227	9.801396	10.198604	9.926831	40
30	9.730217	9.804187	10.195813	9.926029	30
40	9.732193	9.806971	10.193029	9.925222	20
50	9.734157	9.809748	10.190252	9.924409	10
33	9.736109	9.812517	10.187483	9.923591	57
10	9.738048	9.815280	10.184720	9.922768	50
20	9.739975	9.818035	10.181965	9.921940	40
30	9.741889	9.820783	10.179217	9.921107	30
40	9.743792	9.823524	10.176476	9.920268	20
50	9.745683	9.826259	10.173741	9.919424	10
	Cosine.	Cotangent.	Tangent.	Sine.	Angle.

Angle.	Sine.	Tangent.	Cotangent.	Cosine.	
34°	9.747562	9.828987	10.171013	9.918574	56°
10′	9.749429	9.831709	10.168291	9.917719	50′
20	9.751284	9.834425	10.165575	9.916859	40
30	9.753128	9.837134	10.162866	9.915994	30
40	9.754960	9.839838	10.160162	9.915123	20
50	9.756782	9.842535	10.157465	9.914246	10
35	9.758591	9.845227	10.154773	9.913365	55
10	9.760390	9.847913	10.152087	9.912477	50
20	9.762177	9.850593	10.149407	9.911584	40
30	9.763954	9.853268	10.146732	9.910686	30
40	9.765720	9.855938	10.144062	9.909782	20
50	9.767475	9.858602	10.141398	9.908873	10
36	9.769219	9.861261	10.138739	9.907958	54
10	9.770952	9.863915	10.136085	9.907037	50
20	9.772675	9.866564	10.133436	9.906111	40
30	9.774388	9.869209	10.130791	9.905179	30
40	9.776090	9.871849	10.128151	9.904241	20
50	9.777781	9.874484	10.125516	9.903298	10
37	9.779463	9.877114	10.122886	9.902349	53
10	9.781134	9.879741	10.120259	9.901394	50
20	9.782796	9.882363	10.117637	9.900433	40
30	9.784447	9.884980	10.115020	9.899467	30
40	9.786089	9.887594	10.112406	9.898494	20
50	9.787720	9.890204	10.109796	9.897516	10
38	9.789342	9.892810	10.107190	9.896532	52
10	9.790954	9.895412	10.104588	9.895542	50
20	9.792557	9.898010	10.101990	9.894546	40
30	9.794150	9.900605	10.099395	9.893344	30
40	9.795733	9.903197	10.096803	9.892536	20
50	9.797307	9.905785	10.094215	9.891523	10
39	9.798872	9.908369	10.091631	9.890503	51
10	9.800427	9.910951	10.089049	9.889477	50
20	9.801973	9.913529	10.086471	9.888444	40
30	9.803511	9.916104	10.083896	9.887406	30
40	9.805039	9.918677	10.081323	9.886362	20
50	9.806557	9.921247	10.078753	9.885311	10
40	9.808067	9.923814	10.076186	9.884254	50
10	9.809569	9.926378	10.073622	9.883191	50
20	9.811061	9.928940	10.071060	9.882121	40
30	9.812544	9.931499	10.068501	9.881046	30
40	9.814019	9.934056	10.065944	9.879963	20
50	9.815485	9.936611	10.063389	9.878875	10
	Cosine.	Cotangent.	Tangent.	Sine.	Angle.

Angle.	Sine.	Tangent.	Cotangent.	Cosine.	
41°	9.816943	9.939163	10.060837	9.877780	49°
10′	9.818392	9.941713	10.058287	9.876678	50′
20	9.819832	9.944262	10.055738	9.875571	40
30	9.821265	9.946808	10.053192	9.874456	30
40	9.822688	9.949353	10.050647	9.873335	20
50	9.824104	9.951896	10.048104	9.872208	10
42	9.825511	9.954437	10.045563	9.871073	48
10	9.826910	9.956977	10.043023	9.869933	50
20	9.828301	9.959516	10.040484	9.868785	40
30	9.829683	9.962052	10.037948	9.867631	30
40	9.831058	9.964588	10.035412	9.866470	20
50	9.832425	9.967123	10.032877	9.865302	10
43	9.833783	9.969656	10.030344	9.864127	47
10	9.835134	9.972188	10.027812	9.862946	50
20	9.836477	9.974720	10.025280	9.861758	40
30	9.837812	9.977250	10.022750	9.860562	30
40	9.839140	9.979780	10.020220	9.859360	20
50	9.840459	9.982309	10.017691	9.858151	10
44	9.841771	9.984837	10.015163	9.856934	46
10	9.843076	9.987365	10.012635	9.855711	50
20	9.844372	9.989893	10.010107	9.854480	40
30	9.845662	9.992420	10.007580	9.853242	30
40	9.846944	9.994947	10.005053	9.851997	20
50	9.848218	9.997473	10.002527	9.850745	10
45	9.849485	10.000000	10.000000	9.849485	45
	Cosine.	Cotangent.	Tangent.	Sine.	Angle.

TABLE No. X.—RHUMBS, OR POINTS OF THE COMPASS.

Points.	Angles.	NORTH.	NORTH.	SOUTH.	SOUTH.
¼	2° 48′ 45″	N ¼ E	N ¼ W	S ¼ E	S ¼ W
½	5 37 30	N ½ E	N ½ W	S ½ E	S ½ W
¾	8 26 15	N ¾ E	N ¾ W	S ¾ E	S ¾ W
1	11 15 0	N by E	N by W	S by E	S by W
1¼	14 3 45	N by E ¼ E	N by W ¼ W	S by E ¼ E	S by W ¼ W
1½	16 52 30	N by E ½ E	N by W ½ W	S by E ½ E	S by W ½ W
1¾	19 41 15	N by E ¾ E	N by W ¾ W	S by E ¾ E	S by W ¾ W
2	22 30 0	NNE	NNW	SSE	SSW
2¼	25 18 45	NNE ¼ E	NNW ¼ W	SSE ¼ E	SSW ¼ W
2½	28 7 30	NNE ½ E	NNW ½ W	SSE ½ E	SSW ½ W
2¾	30 56 15	NNE ¾ E	NNW ¾ W	SSE ¾ E	SSW ¾ W
3	33 45 0	NE by N	NW by N	SE by S	SW by S
3¼	36 33 45	NE ¾ N	NW ¾ N	SE ¾ S	SW ¾ S
3½	39 22 30	NE ½ N	NW ½ N	SE ½ S	SW ½ S
3¾	42 11 15	NE ¼ N	NW ¼ N	SE ¼ S	SW ¼ S
4	45 0 0	NE	NW	SE	SW
4¼	47 48 45	NE ¼ E	NW ¼ W	SE ¼ E	SW ¼ W
4½	50 37 30	NE ½ E	NW ½ W	SE ½ E	SW ½ W
4¾	53 26 15	NE ¾ E	NW ¾ W	SE ¾ E	SW ¾ W
5	56 15 0	NE by E	NW by W	SE by E	SW by W
5¼	59 3 45	ENE ¾ N	WNW ¾ N	ESE ¾ S	WSW ¾ S
5½	61 52 30	ENE ½ N	WNW ½ N	ESE ½ S	WSW ½ S
5¾	64 41 15	ENE ¼ N	WNW ¼ N	ESE ¼ S	WSW ¼ S
6	67 30 0	ENE	WNW	ESE	WSW
6¼	70 18 45	ENE ¼ E	WNW ¼ W	ESE ¼ E	WSW ¼ W
6½	73 7 30	ENE ½ E	WNW ½ W	ESE ½ E	WSW ½ W
6¾	75 56 15	ENE ¾ E	WNW ¾ W	ESE ¾ E	WSW ¾ W
7	78 45 0	E by N	W by N	E by S	W by S
7¼	81 33 45	E ¾ N	W ¾ N	E ¾ S	W ¾ S
7½	84 22 30	E ½ N	W ½ N	E ½ S	W ½ S
7¾	87 11 15	E ¼ N	W ¼ N	E ¼ S	W ¼ S
8	90 0 0	EAST.	WEST.	EAST.	WEST.

TABLE No. XI.—RECIPROCALS OF NUMBERS FROM 1 TO 1000.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
1	1.000000	40	.025000	79	.012658	118	.008475
2	.500000	41	.024390	80	.012500	119	.008403
3	.333333	42	.023810	81	.012346	120	.008333
4	.250000	43	.023256	82	.012195	121	.008264
5	.200000	44	.022727	83	.012048	122	.008197
6	.166667	45	.022222	84	.011905	123	.008130
7	.142857	46	.021739	85	.011765	124	.008065
8	.125000	47	.021277	86	.011628	125	.008000
9	.111111	48	.020833	87	.011494	126	.007937
10	.100000	49	.020408	88	.011364	127	.007874
11	.090909	50	.020000	89	.011236	128	.007813
12	.083333	51	.019608	90	.011111	129	.007752
13	.076923	52	.019231	91	.010989	130	.007692
14	.071429	53	.018868	92	.010870	131	.007634
15	.066667	54	.018519	93	.010753	132	.007576
16	.062500	55	.018182	94	.010638	133	.007519
17	.058824	56	.017857	95	.010526	134	.007463
18	.055556	57	.017544	96	.010417	135	.007407
19	.052632	58	.017241	97	.010309	136	.007353
20	.050000	59	.016949	98	.010204	137	.007299
21	.047619	60	.016667	99	.010101	138	.007246
22	.045455	61	.016393	100	.010000	139	.007194
23	.043478	62	.016129	101	.009901	140	.007143
24	.041667	63	.015873	102	.009804	141	.007092
25	.040000	64	.015625	103	.009709	142	.007042
26	.038462	65	.015385	104	.009615	143	.006993
27	.037037	66	.015152	105	.009524	144	.006944
28	.035714	67	.014925	106	.009434	145	.006897
29	.034483	68	.014706	107	.009346	146	.006849
30	.033333	69	.014493	108	.009259	147	.006803
31	.032258	70	.014286	109	.009174	148	.006757
32	.031250	71	.014085	110	.009091	149	.006711
33	.030303	72	.013889	111	.009009	150	.006667
34	.029412	73	.013699	112	.008929	151	.006623
35	.028571	74	.013514	113	.008850	152	.006579
36	.027778	75	.013333	114	.008772	153	.006536
37	.027027	76	.013158	115	.008696	154	.006494
38	.026316	77	.012987	116	.008021	155	.006452
39	.025641	78	.012821	117	.008547	156	.006410

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
157	.006369	202	.004950	247	.004049	292	.003425
158	.006329	203	.004926	248	.004032	293	.003413
159	.006289	204	.004902	249	.004016	294	.003401
160	.006250	205	.004878	250	.004000	295	.003390
161	.006211	206	.004854	251	.003984	296	.003378
162	.006173	207	.004831	252	.003968	297	.003367
163	.006135	208	.004808	253	.003953	298	.003356
164	.006098	209	.004785	254	.003937	299	.003344
165	.006061	210	.004762	255	.003922	300	.003333
166	.006024	211	.004739	256	.003906	301	.003322
167	.005988	212	.004717	257	.003891	302	.003311
168	.005952	213	.004695	258	.003876	303	.003301
169	.005917	214	.004673	259	.003861	304	.003289
170	.005882	215	.004651	260	.003846	305	.003279
171	.005848	216	.004630	261	.003831	306	.003268
172	.005814	217	.004608	262	.003817	307	.003257
173	.005780	218	.004587	263	.003802	308	.003247
174	.005747	219	.004566	264	.003788	309	.003236
175	.005714	220	.004545	265	.003774	310	.003226
176	.005682	221	.004525	266	.003759	311	.003215
177	.005650	222	.004505	267	.003745	312	.003205
178	.005618	223	.004484	268	.003731	313	.003195
179	.005587	224	.004464	269	.003717	314	.003185
180	.005556	225	.004444	270	.003704	315	.003175
181	.005225	226	.004425	271	.003690	316	.003165
182	.005495	227	.004405	272	.003676	317	.003155
183	.005464	228	.004386	273	.003663	318	.003145
184	.005435	229	.004367	274	.003650	319	.003135
185	.005405	230	.004348	275	.003636	320	.003125
186	.005376	231	.004329	276	.003623	321	.003115
187	.005348	232	.004310	277	.003610	322	.003106
188	.005319	233	.004292	278	.003597	323	.003096
189	.005291	234	.004274	279	.003584	324	.003086
190	.005263	235	.004255	280	.003571	325	.003077
191	.005236	236	.004237	281	.003559	326	.003067
192	.005208	237	.004219	282	.003546	327	.003058
193	.005181	238	.004202	283	.003534	328	.003049
194	.005155	239	.004184	284	.003522	329	.003040
195	.005128	240	.004167	285	.003509	330	.003030
196	.005102	241	.004149	286	.003497	331	.003021
197	.005076	242	.004132	287	.003484	332	.003012
198	.005051	243	.004115	288	.003472	333	.003003
199	.005025	244	.004098	289	.003460	334	.002994
200	.005000	245	.004082	290	.003448	335	.002985
201	.004975	246	.004065	291	.003436	336	.002976

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
337	.002967	382	.002618	427	.002342	472	.002119
338	.002959	383	.002611	428	.002336	473	.002114
339	.002950	384	.002604	429	.002331	474	.002110
340	.002941	385	.002597	430	.002326	475	.002105
341	.002933	386	.002591	431	.002320	476	.002101
342	.002924	387	.002584	432	.002315	477	.002096
343	.002915	388	.002577	433	.002309	478	.002092
344	.002907	389	.002571	434	.002304	479	.002088
345	.002899	390	.002564	435	.002299	480	.002083
346	.002890	391	.002558	436	.002294	481	.002079
347	.002882	392	.002551	437	.002288	482	.002075
348	.002874	393	.002545	438	.002283	483	.002070
349	.002865	394	.002538	439	.002278	484	.002066
350	.002857	395	.002532	440	.002273	485	.002062
351	.002849	396	.002525	441	.002268	486	.002058
352	.002841	397	.002519	442	.002262	487	.002053
353	.002833	398	.002513	443	.002257	488	.002049
354	.002825	399	.002506	444	.002252	489	.002045
355	.002817	400	.002500	445	.002247	490	.002041
356	.002809	401	.002494	446	.002242	491	.002037
357	.002801	402	.002488	447	.002237	492	.002033
358	.002793	403	.002481	448	.002232	493	.002028
359	.002786	404	.002475	449	.002227	494	.002024
360	.002778	405	.002469	450	.002222	495	.002020
361	.002770	406	.002463	451	.002217	496	.002016
362	.002762	407	.002457	452	.002212	497	.002012
363	.002755	408	.002451	453	.002208	498	.002008
364	.002747	409	.002445	454	.002203	499	.002004
365	.002740	410	.002439	455	.002198	500	.002000
366	.002732	411	.002433	456	.002193	501	.001996
367	.002725	412	.002427	457	.002188	502	.001992
368	.002717	413	.002421	458	.002183	503	.001988
369	.002710	414	.002415	459	.002179	504	.001984
370	.002703	415	.002410	460	.002174	505	.001980
371	.002695	416	.002407	461	.002169	506	.001976
372	.002688	417	.002398	462	.002165	507	.001972
373	.002681	418	.002392	463	.002160	508	.001969
374	.002674	419	.002387	464	.002155	509	.001965
375	.002667	420	.002381	465	.002151	510	.001961
376	.002660	421	.002375	466	.002146	511	.001957
377	.002653	422	.002370	467	.002141	512	.001953
378	.002646	423	.002364	468	.002137	513	.001949
379	.002639	424	.002358	469	.002132	514	.001946
380	.002632	425	.002353	470	.002128	515	.001942
381	.002625	426	.002347	471	.002123	516	.001938

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
517	.001934	562	.001779	607	.001647	652	.001534
518	.001931	563	.001776	608	.001645	653	.001531
519	.001927	564	.001773	609	.001642	654	.001529
520	.001923	565	.001770	610	.001639	655	.001527
521	.001919	566	.001767	611	.001637	656	.001524
522	.001916	567	.001764	612	.001634	657	.001522
523	.001912	568	.001761	613	.001631	658	.001520
524	.001908	569	.001757	614	.001629	659	.001517
525	.001905	570	.001754	615	.001626	660	.001515
526	.001901	571	.001751	616	.001623	661	.001513
527	.001898	572	.001748	617	.001621	662	.001511
528	.001894	573	.001745	618	.001618	663	.001508
529	.001890	574	.001742	619	.001616	664	.001506
530	.001887	575	.001739	620	.001613	665	.001504
531	.001883	576	.001736	621	.001610	666	.001502
532	.001880	577	.001733	622	.001608	667	.001499
533	.001876	578	.001730	623	.001605	668	.001497
534	.001873	579	.001727	624	.001603	669	.001495
535	.001869	580	.001724	625	.001600	670	.001493
536	.001866	581	.001721	626	.001597	671	.001490
537	.001862	582	.001718	627	.001595	672	.001488
538	.001859	583	.001715	628	.001592	673	.001486
539	.001855	584	.001712	629	.001590	674	.001484
540	.001852	585	.001709	630	.001587	675	.001481
541	.001848	586	.001706	631	.001585	676	.001479
542	.001845	587	.001704	632	.001582	677	.001477
543	.001842	588	.001701	633	.001580	678	.001475
544	.001838	589	.001698	634	.001577	679	.001473
545	.001835	590	.001695	635	.001575	680	.001471
546	.001832	591	.001692	636	.001572	681	.001468
547	.001828	592	.001689	637	.001570	682	.001466
548	.001825	593	.001686	638	.001567	683	.001464
549	.001821	594	.001684	639	.001565	684	.001462
550	.001818	595	.001681	640	.001563	685	.001460
551	.001815	596	.001678	641	.001560	686	.001458
552	.001812	597	.001675	642	.001558	687	.001456
553	.001808	598	.001672	643	.001555	688	.001453
554	.001805	599	.001669	644	.001553	689	.001451
555	.001802	600	.001667	645	.001550	690	.001449
556	.001799	601	.001664	646	.001548	691	.001447
557	.001795	602	.001661	647	.001546	692	.001445
558	.001792	603	.001658	648	.001543	693	.001443
559	.001789	604	.001656	649	.001541	694	.001441
560	.001786	605	.001653	650	.001538	695	.001439
561	.001783	606	.001650	651	.001536	696	.001437

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
697	.001435	742	.001348	787	.001271	832	.001202
698	.001433	743	.001346	788	.001269	833	.001200
699	.001431	744	.001344	789	.001267	834	.001199
700	.001429	745	.001342	790	.001266	835	.001198
701	.001427	746	.001340	791	.001264	836	.001196
702	.001425	747	.001339	792	.001263	837	.001195
703	.001422	748	.001337	793	.001261	838	.001193
704	.001420	749	.001335	794	.001259	839	.001192
705	.001418	750	.001333	795	.001258	840	.001190
706	.001416	751	.001332	796	.001256	841	.001189
707	.001414	752	.001330	797	.001255	842	.001188
708	.001412	753	.001328	798	.001253	843	.001186
709	.001410	754	.001326	799	.001251	844	.001185
710	.001408	755	.001325	800	.001250	845	.001183
711	.001406	756	.001323	801	.001248	846	.001182
712	.001404	757	.001321	802	.001247	847	.001181
713	.001403	758	.001319	803	.001245	848	.001179
714	.001401	759	.001318	804	.001244	849	.001178
715	.001399	760	.001316	805	.001242	850	.001176
716	.001397	761	.001314	806	.001241	851	.001175
717	.001395	762	.001312	807	.001239	852	.001174
718	.001393	763	.001311	808	.001238	853	.001172
719	.001391	764	.001309	809	.001236	854	.001171
720	.001389	765	.001307	810	.001235	855	.001170
721	.001387	766	.001305	811	.001233	856	.001168
722	.001385	767	.001304	812	.001232	857	.001167
723	.001383	768	.001302	813	.001230	858	.001166
724	.001381	769	.001300	814	.001229	859	.001164
725	.001379	770	.001299	815	.001227	860	.001163
726	.001377	771	.001297	816	.001225	861	.001161
727	.001376	772	.001295	817	.001224	862	.001160
728	.001374	773	.001294	818	.001222	863	.001159
729	.001372	774	.001292	819	.001221	864	.001157
730	.001370	775	.001290	820	.001220	865	.001156
731	.001368	776	.001289	821	.001218	866	.001155
732	.001366	777	.001287	822	.001217	867	.001153
733	.001364	778	.001285	823	.001215	868	.001152
734	.001362	779	.001284	824	.001214	869	.001151
735	.001361	780	.001282	825	.001212	870	.001149
736	.001359	781	.001280	826	.001211	871	.001148
737	.001357	782	.001279	827	.001209	872	.001147
738	.001355	783	.001277	828	.001208	873	.001145
739	.001353	784	.001276	829	.001206	874	.001144
740	.001351	785	.001274	830	.001205	875	.001143
741	.001350	786	.001272	831	.001203	876	.001142

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
877	.001140	908	.001101	939	.001065	970	.001031
878	.001139	909	.001100	940	.001064	971	.001030
879	.001138	910	.001099	941	.001063	972	.001029
880	.001136	911	.001098	942	.001062	973	.001028
881	.001135	912	.001096	943	.001060	974	.001027
882	.001134	913	.001095	944	.001059	975	.001026
883	.001133	914	.001094	945	.001058	976	.001025
884	.001131	915	.001093	946	.001057	977	.001024
885	.001130	916	.001092	947	.001056	978	.001022
886	.001129	917	.001091	948	.001055	979	.001021
887	.001127	918	.001089	949	.001054	980	.001020
888	.001126	919	.001088	950	.001053	981	.001019
889	.001125	920	.001087	951	.001052	982	.001018
890	.001124	921	.001086	952	.001050	983	.001017
891	.001122	922	.001085	953	.001049	984	.001016
892	.001121	923	.001083	954	.001048	985	.001015
893	.001120	924	.001082	955	.001047	986	.001014
894	.001119	925	.001081	956	.001046	987	.001013
895	.001118	926	.001080	957	.001045	988	.001012
896	.001116	927	.001079	958	.001044	989	.001011
897	.001115	928	.001078	959	.001043	990	.001010
898	.001114	929	.001076	960	.001042	991	.001009
899	.001112	930	.001075	961	.001041	992	.001008
900	.001111	931	.001074	962	.001040	993	.001007
901	.001110	932	.001073	963	.001038	994	.001006
902	.001109	933	.001072	964	.001037	995	.001005
903	.001107	934	.001071	965	.001036	996	.001004
904	.001106	935	.001070	966	.001035	997	.001003
905	.001105	936	.001068	967	.001034	998	.001002
906	.001104	937	.001067	968	.001033	999	.001001
907	.001103	938	.001066	969	.001032	1000	.001000

WEIGHTS AND MEASURES.

WATER AND AIR AS STANDARDS FOR WEIGHT AND MEASURE.

WATER AS A STANDARD.

There are four notable temperatures for water, namely,

32° F., or	0° C.	=	the freezing point, under one atmosphere.
39°.1 or	4°	=	the point of maximum density.
62° or	16°.66	=	the British standard temperature.
212° or	100°	=	the boiling point, under one atmosphere.

The temperature 62° F. is the temperature of water used in calculating the specific gravity of bodies, with respect to the gravity or density of water as a basis, or as unity. In France, the temperature of maximum density, 39°.1 F., or 4° C., is used for this purpose, for solids.

Weight of one cubic foot of Pure Water.

At 32° F.		=	62.418 pounds.
At 39°.1		=	62.425– "
At 62°	(Standard temperature)	=	62.355 "
At 212°		=	59.640 "

The weight of a cubic foot of water is, it may be added, about 1000 ounces (exactly 998.8 ounces), at the temperature of maximum density.

The weight of water is usually taken in round numbers, for ordinary calculations, at 62.4 lbs. per cubic foot, which is the weight at 52°.3 F.; or it is taken at 62½ lbs. per cubic foot, where precision is not required, equal to $\frac{1000}{16}$ lbs.

The weight of a cylindrical foot of water at 62° F. is 48.973 pounds.

Weight of one cubic inch of Pure Water.

At 32° F.	= .03612 pound,	or 0.5779 ounce.		
At 39°.1	= .036125 "	" 0.5780 "		
At 62°	= .03608 "	" 0.5773 "	or 252.595 grains.	
At 212°	= .03451 "	" 0.5522 "		

The weight of one cylindrical inch of pure water at 62° F. is .02833 pound, or 0.4533 ounce.

Volume of one pound of Pure Water.

At 32° F.	= .016021 cubic foot,	or 27.684	cubic inches.
At 39°.1	= .016019 „	„ 27.680	„
At 62°	= .016037 „	„ 27.712	„
At 212°	= .016770 „	„ 28.978	„

The volume of one ounce of pure water at 62° F. is 1.732 cubic inches.

The Gallon.

The weight of one gallon of water at the standard temperature, 62° F., is 10 pounds, and the correct volume is 0.160372 cubic foot, or 277.123 cubic inches. But in an Act of Parliament, which came into force in 1825, the volume of one gallon is stated to be 277.274 cubic inches; this is the commonly accepted volume. See page 339.

The volume of 10 pounds of water at 62° F. is, therefore, to the volume of the imperial gallon, as 1 to 1.000545.

And, the weight of an imperial gallon of water at 62° F. is 10.00545 pounds avoirdupois; or 10 pounds, 38.15 grains.

One cubic foot of water contains 6.2355 gallons of 277.123 cubic inches, or 6.23208 gallons of 277.275 cubic inches, or approximately 6¼ gallons. One gallon is equal to .1604 cubic foot.

The volume of water at 62° F., in cubic inches, multiplied by .00036, gives the capacity in gallons.

The capacity of one gallon is equal to one square foot, two inches deep nearly (exactly 1.924 inches); or to one circular foot, 2½ inches deep nearly (exactly 2.45 inches).

One ton of water at 62° F. contains 224 gallons.

Other Measures of Water.

Volume of given weights of water, at 62.4 pounds per cubic foot:—

1 ton	35.90 cubic feet.
1 cwt	1.795 „
1 quarter	.449 „
1 pound	.016 cubic foot, or 27.692 cubic inches.
1 ounce	1.731 „
1 tonne, at 39°.1 F.	35.3156 cubic feet.
1 kilogramme, at 39°.1 F.	.0353 cubic foot, or 61.025 cubic inches.
1 tonne, at 52°.3 F. (62.4 pounds per cubic foot)	35.330 cubic feet.

Thirty-six cubic feet, or 1⅓ cubic yards, of water, at 62.4 pounds per cubic foot, being at the temperature 52°.3 F., weigh about one ton (exactly 6.4 pounds more).

One cubic yard, or twenty-seven cubic feet, of water weighs about 15 cwt., or ¾ ton (exactly 4.8 pounds more). It is equal to 168.36 gallons.

One cubic metre of water is equal in volume to 35.3156 cubic feet, or 1.308 cubic yards, or 220.09 gallons; and at 62.4 pounds per cubic foot, it weighs 1 ton nearly (exactly 36.3 pounds less). It is nearly equivalent

to the old English tun of 4 hogsheads—210 imperial gallons, and is a better unit for measuring sewage or water-supply than the gallon.

The cubic metre is generally used on the Continent for such measurements.

A pipe one yard long holds about as many pounds of water as the square of its diameter in inches (exactly 2 per cent. more).

Pressure of Water.

A pressure of one lb. per square inch is exerted by a column of water 2.3093 feet, or 27.71 inches high, at 62° F.; and a pressure of one atmosphere, or 14.7 lbs. per square inch, is exerted by a column of water 33.947 feet high, or 10.347 metres, at 62° F.

A column of water at 62° F., one foot high, presses on the base with a force of 0.433 lb., or 6.928 ounces per square inch. A column 100 feet high presses with a force of 43⅓ lbs. per square inch. A column one metre high presses with a force of 1.422 lbs. per square inch.

A column of water one inch high, presses on the base with a force of 0.5773 ounce per square inch, or 5.196 lbs. per square foot.

A column of water one mile deep, weighing 62.4 pounds per cubic foot, presses on the base with a force of about one ton per square inch (fresh water exactly 48 lbs. more; sea-water exactly 107.5 lbs. more).

Water is hardly compressible under pressure. Experiment appears to show that for each atmosphere of pressure it is condensed 47½ millionths of its bulk.

Sea-water.

One cubic foot of average sea-water, at 62° F., weighs 64 pounds, and the weight of fresh water is to that of sea-water as 39 to 40, or as 1 to 1.026.

Thirty-five cubic feet of sea-water weighs one ton.

One cubic yard of sea-water weighs 15½ cwt. nearly (8 lbs. less).

One cubic metre of sea-water weighs fully one ton (20 lbs. more).

Average sea-water is composed as follows:—

	Per 100 parts.	Per 100 parts.
Chloride of sodium (common salt),	2.50	
Sulphuret of magnesium,	0.53	
Chloride of magnesium,	0.33	
Carbonate of lime, Carbonate of magnesia,	0.02	
Sulphate of lime,	0.01	
Solid matter, say,		3.40
Water,		96.60
		100.00

showing that sea-water contains $\frac{1}{30}$th part of its weight of solid matter in solution.

According to Réclus, the mean specific gravity of sea-water is 1.028. In the Mediterranean Sea, it is 1.029; in the Black Sea, 1.016. The mean quantity of salts, or solid matter, in solution, is 3.44 per cent., three-fourths of which is common salt. In the Red Sea, the water contains 4.3 per cent.; in the Baltic Sea, 5 per cent.; and at Cronstadt, 2 per cent.

Ice and Snow.

One cubic foot of ice at 32° F. weighs 57.50 lbs.

One pound of ice at 32° F. has a volume of .0174 cubic foot, or 30.067 cubic inches.

The volume of water at 32° F. is to that of ice at 32° F., as 1.000 to 1.0855; the expansion in passing into the solid state being above 8½ per cent. of the volume of water.

The specific density of ice is 0.922, that of water at 62° F. being = 1.

The melting point of ice is 32° F., or 0° C., under the ordinary atmospheric pressure, of 14.7 lbs. per square inch. Under greater pressure the melting point is lower, being at the rate of .0133° F. for each additional atmosphere of pressure.

The specific heat of ice is .504, that of water being = 1.

One cubic foot of fresh snow weighs 5.20 lbs. Snow has 12 times the bulk of water, and its specific gravity is .0833.

French and English Measures of Water.

One litre of water is equal to 0.2201 gallon, or 1.761 pints: about 1¾ pints. One gallon is equal to 4.544 litres, and one pint is .568 litre.

One litre of water at 39°.1 F., or 4° C., the temperature of maximum density, weighs one kilogramme, or 2.2046 lbs.; at the temperature 62° F., or 16°.7 C., it weighs 2.202 lbs.

1000 litres = one cubic metre, equal to 35.3156 cubic feet; and, at 39°.1 F., or 4° C., weigh 1000 kilogrammes, or one ton nearly (35.4 lbs. less).

AIR AS A STANDARD.

The mean pressure of the atmosphere at the level of the sea, is equal to 14.7 lbs. per square inch, or 2116.4 lbs. per square foot; or to 1.0335 kilogrammes per square centimetre. This is called one atmosphere of pressure. The following are measures of pressures (see also pages 145, 158):—

One atmosphere of pressure:—(1.) A column of air at 32° F., 27,801 feet, or about 5¼ miles high, of uniform density equal to that of air at the level of the sea. (2.) A column of mercury at 32° F., 29.922 inches or 76 centimetres high; nearly 30 inches. At 62° F., the height is 30 inches. (3.) A column of water at 62° F., 33.947 feet or 10.347 metres high; nearly 34 feet.

A pressure of 1 lb. per square inch:—(1.) A column of air at 32° F., 1891 feet high, of uniform density as above. (2.) A column of mercury at 32° F., 2.035 inches or 51.7 millimetres high. At 62° F., the height is 2.04 inches. (3.) A column of water at 62° F., 2.31 feet or 27.72 inches high.

A pressure of 1 lb. per square foot:—(1.) A column of air at 32° F., 13.13 feet high, of uniform density as above. (2.) A column of mercury at 32° F., .0141 inch or .359 millimetre high. At 62° F., the height is .01417 inch. (3.) A column of water at 62° F., .1925 inch high.

The density, or weight of one cubic foot of pure air, under a pressure of one atmosphere, or 14.7 lbs. per square inch, is

At 32° F.,	=	.080728 pound, or	1.29 ounce, or	565.1 grains.
At 62° F.,	=	.076097 „ „	1.217 „ „	532.7 „

The weight of a litre of pure air, under one atmosphere, at 32° F., is 1.293 grammes, or 19.955 grains.

The weight of air, compared with that of water at three notable temperatures, and at 52°.3, under one atmosphere, is as follows:—

Weight of water at	32° F.,	773.2 times the weight of air at 32° F.			
„ „	39°.1,	773.27	„	„	„
„ „	62°,	772.4	„	„	„
„ „	62°,	819.4	„	„	62°.
„ „	52°.3,	820	„	„	„

The volume of one pound of air at 32° F., and under one atmosphere of pressure, is 12.387 cubic feet. The volume at 62° F., is 13.141 cubic feet.

The specific heat of air at constant pressure is .2377, and at constant volume .1688, that of water being = 1.

GREAT BRITAIN AND IRELAND.—IMPERIAL WEIGHTS AND MEASURES.

The origin of English measures is the grain of corn. Thirty-two grains of wheat, dried and gathered from the middle of the ear, weighed what was called one pennyweight; 20 pennyweights were called one ounce, and 20 ounces one pound. Subsequently, the pennyweight was divided into 24 grains. Troy weight was afterwards introduced by William the Conqueror, from Troyes, in France; but it gave dissatisfaction, as the troy pound did not weigh so much as the pound then in use; consequently, a mean weight was established, making 16 ounces equal to one pound, and called avoirdupois (*avoir du poids*).

Three grains of barleycorn, well-dried, placed end to end, made an inch —the basis of length. The length of the arm of King Henry I. was made the length of the *ulna*, or ell, which answers to the modern yard. The imperial standard yard is a solid square bar of gun-metal, kept in the office of the Exchequer at Westminster, 38 inches in length, 1 inch square, at the temperature 62° F., composed of copper 16 ounces, tin 2½ ounces, and zinc 1 ounce. Two cylindrical holes are drilled half through the bar, one near each end, and the centres of these holes are 36 inches, or 3 feet, apart—the length of the imperial standard yard. Compared with a pendulum vibrating seconds of mean time, at the level of the sea, in the latitude of London, in a vacuum, the yard is as 36 inches in length to 39.1393 inches, the length of the pendulum.

Measures of capacity were based on troy weight; it was enacted that 8 pounds troy of wheat, from the middle of the ear, well dried, should make 1 gallon of wine measure, and that 8 such gallons should make 1 bushel.

The imperial gallon is now the only standard measure of capacity, and it contains 277.274 cubic inches. It is said to be the volume of 10 pounds avoirdupois of distilled water, weighed in air, at 62° F.

Note.—The exact volume of 10 pounds of distilled water at 62° F. is 277.123 cubic inches.

Tables of weights and measures are conveniently classified thus—

1. Length; 2. Surface; 3. Volume; 4. Capacity; 5. Weight.

The following are some of the principal units of measurement:—

The *acre*, for land measure.
The *mile*, for itinerary measure.
The *yard*, for measure of drapery, &c.
The *coomb*, for capacity of corn, &c.
The *gallon*, for capacity of liquids.
The *grain*, for chemical analysis.
The *pound*, for grocers' ware, &c.
The *stone* of 8 pounds, for butchers' meat.
The *stone* of 14 pounds, for flour, oatmeal, &c.

I. Measures of Length.—Tables No. 12.

Lineal Measure.

3 barleycorns, or 12 lines, or 72 points, or 1000 mils	1 inch.
3 inches	1 palm.
4 inches	1 hand.
9 inches	1 span.
12 inches	1 foot.
18 inches	1 cubit.
3 feet	1 yard.
2½ feet	1 military pace.
5 feet	1 geometrical pace.
2 yards	1 fathom.
5½ yards	1 rod, pole, or perch.
40 poles, or 220 yards	1 furlong.
8 furlongs, or 1760 yards, or 5280 feet	1 mile.
3 miles	1 league.
2240 yards, or 1.272 miles	1 Irish mile.

The *inch* is also divided into halves, quarters, eighths, and sixteenths; sometimes into tenths.

The *hand* is used as a measure of the height of horses.

The *military pace* is the length of the ordinary step of a man.

The *geometrical pace* is the length of two steps. A thousand of such paces were reckoned to a mile.

The *fathom* is used in soundings to ascertain depths, and for measuring cordage and chains.

Land Measure.

7.92 inches	1 link.
100 links, or 66 feet, or 22 yards, or 4 poles	1 chain.
10 chains	1 furlong.
80 chains, or 8 furlongs	1 mile.

The *fen*, or *woodland pole* or *perch*, is 18 feet.
The *forest pole* is 21 feet.

Nautical Measure.

6086.44 feet, or 1000 fathoms, or 10 cables, or 1.1528 statute miles	1 nautical mile, or knot.
3 nautical miles	1 league.
60 nautical miles, or 69.168 statute miles or 20 leagues	1 degree.
360 degrees	Circumference of the earth at the equator.

The above value of the nautical mile is that which is commonly taken, and is the length of a minute of longitude at the equator. The mean length of a minute of latitude at the mean level of the sea is nearly 6076 feet, or 1.1508 statute miles.

The nautical fathom is the thousandth part of a nautical mile, and is, on an average, about $\frac{1}{80}$th longer than the common fathom.

Cloth Measure.

2¼ inches	1 nail.
2 nails	1 finger-length.
4 nails, or 9 inches	1 quarter.
4 quarters	1 yard.
5 quarters	1 ell.

Wire-Gauges.

The "Birmingham Wire-Gauge" is a scale of notches in the edge of a plate, of successively increasing or decreasing widths, to designate a set of arbitrary sizes or diameters of wire, ranging from about half an inch down to the smallest size easily drawn, say, four-thousands of an inch. The practical utility of such a gauge is obvious, when it is considered how far beyond the means supplied by the graduations of an ordinary scale of feet and inches is the measurement of the gradations of the wire-gauge. But the "Birmingham Wire-Gauge" is a variable measure. The principle, if there was any, on which it was originally constructed, is not known. Mr. Latimer Clark states that, when plotted, the widths of the gauge range in a curve approxi-

mating to a logarithmic curve, such as would be found by the successive addition of 10 or 12 per cent. to the width of the notches of the gauge. However that may be, there are many varieties of the wire-gauge in existence. The oldest and best-known gauge is that of which the numbers were carefully measured by Mr. Holtzapffel, and published by him in 1847. It has been, and still is, widely followed in the manufacture of wire; and also of tubes in respect of their thickness. It gives 40 measurements ranging from .454 inch to .004 inch, and is contained in Table No. 13. Although there are only 40 marks in the table, there are 60 different sizes of wire made, for which intermediate sizes have been added to the gauge. This table has also been used in rolling sheet iron, sheet steel, and other materials, and for joiners' screws; but it appears to be falling into disuse for these purposes.

BIRMINGHAM WIRE-GAUGE (*Holtzapffel's*).—Table No. 13.

For Wire and Tubes chiefly; and for Sheet Iron and Steel formerly.

Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.
No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
0000	.454	7	.180	17	.058	27	.016
000	.425	8	.165	18	.049	28	.014
00	.380	9	.148	19	.042	29	.013
0	.340	10	.134	20	.035	30	.012
1	.300	11	.120	21	.032	31	.010
2	.284	12	.109	22	.028	32	.009
3	.259	13	.095	23	.025	33	.008
4	.238	14	.083	24	.022	34	.007
5	.220	15	.072	25	.020	35	.005
6	.203	16	.065	26	.018	36	.004

BIRMINGHAM METAL-GAUGE, OR PLATE-GAUGE (*Holtzapffel's*).—
Table No. 14.

For Sheet Metals, Brass, Gold, Silver, &c.

Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.
No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
1	.004	10	.024	19	.064	28	.120
2	.005	11	.029	20	.067	29	.124
3	.008	12	.034	21	.072	30	.126
4	.010	13	.036	22	.074	31	.133
5	.012	14	.041	23	.077	32	.143
6	.013	15	.047	24	.082	33	.145
7	.015	16	.051	25	.095	34	.148
8	.016	17	.057	26	.103	35	.158
9	.019	18	.061	27	.113	36	.167

Another of Holtzapffel's tables, No. 14, the *Plate-Gauge*, has been, and may now, to some extent, be, employed for most of the sheet metals, except-

LANCASHIRE GAUGE (*Holtzapffel's*).—Table No. 15.

For Round Steel Wire, and for Pinion Wire.

Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.
No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
80	.013	57	.042	34	.109	11	.189	M	.295
79	.014	56	.044	33	.111	10	.190	N	.302
78	.015	55	.050	32	.115	9	.191	O	.316
77	.016	54	.055	31	.118	8	.192	P	.323
76	.018	53	.058	30	.125	7	.195	Q	.332
75	.019	52	.060	29	.134	6	.198	R	.339
74	.022	51	.064	28	.138	5	.201	S	.348
73	.023	50	.067	27	.141	4	.204	T	.358
72	.024	49	.070	26	.143	3	.209	U	.368
71	.026	48	.073	25	.146	2	.219	V	.377
70	.027	47	.076	24	.148	1	.227	W	.386
69	.029	46	.078	23	.150	A	.234	X	.397
68	.030	45	.080	22	.152	B	.238	Y	.404
67	.031	44	.084	21	.157	C	.242	Z	.413
66	.032	43	.086	20	.160	D	.246	A1	.420
65	.033	42	.091	19	.164	E	.250	B1	.431
64	.034	41	.095	18	.167	F	.257	C1	.443
63	.035	40	.096	17	.169	G	.261	D1	.452
62	.036	39	.098	16	.174	H	.266	E1	.462
61	.038	38	.100	15	.175	I	.272	F1	.475
60	.039	37	.102	14	.177	J	.277	G1	.484
59	.040	36	.105	13	.180	K	.281	H1	.494
58	.041	35	.107	12	.185	L	.290		

ing iron and steel: as copper, brass, gilding-metal, gold, silver, and platinum. The intervals are closer or smaller than those of the wire-gauge, and the maximum size, for No. 36, is $^1/_6$ inch. When thicker sheets are wanted, their measures are sought in the Birmingham wire-gauge.

The last table, No. 15, by Holtzapffel, the *Lancashire Gauge*, is employed exclusively for the bright steel wire prepared in Lancashire, and the steel pinion-wire for watch and clock makers. The larger sizes are marked by capital letters, to distinguish them from the others. This, the second part of the table, is known as the *Letter-Gauge.*

Needle-Gauge, for needle wire. The sizes correspond with some of those of the Holtzapffel wire-gauge. The following are the relative marks for equal sizes on the two gauges:—

Needle wire-gauge—Nos. 1, 2, 2½, 3, 4, 5, thence to 21,
corresponding to B. W.-G.—18½, 19, 19½, 20, 21, 22, thence to 38.

Music Wire-gauge, for the strings of pianofortes. The marks used are Nos. 6 to 20. The following are the relative marks for equal sizes with the Holtzapffel wire-gauge:—

Music wire-gauge—Nos. 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20,
corresponding to B. W.-G.—26, 25½, 25, 24½, 24, 23½, 23, 22, 21, 20, 19.

No. 6, the thinnest wire now used, measures about one fifty-fifth of an inch in diameter, and No. 20 about one twenty-fifth of an inch.

The preceding Tables of Gauges have been extracted from Holtzapffel's estimable work on *Turning and Mechanical Manipulation*, 1847.

Messrs. Rylands Brothers, of Warrington, manufacture iron wire according to the gauge in Table No. 16.

WARRINGTON WIRE-GAUGE (*Rylands Brothers*).—Table No. 16.

Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.
No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
7/0	1/2	0	.326	8	.159	15	.069
6/0	15/32	1	.300	9	.146	16	.0625, or 1/16
5/0	7/16	2	.274	10	.133	17	.053
4/0	13/32	3	.25, or ¼	10½	.125, or ⅛	18	.047
3/0	3/8	4	.229	11	.117	19	.041
2/0	11/32	5	.209	12	.10, or 1/10	20	.036
		6	.191	13	.090	21	.0315, or 1/32
		7	.174	14	.079	22	.028

For sheets, the wire-gauge that seems to be adhered to by the iron-sheet rollers of South Staffordshire, is a scale comprising 32 measurements, ranging from .3125 inch to .0125 inch, contained in Table No. 17.

BIRMINGHAM WIRE-GAUGE.—Table No. 17.

For Iron Sheets chiefly.

No.	Size.	No.	Size.	No.	Size.	No.	Size.
	Inch.		Inch.		Inch.		Inch.
1	.3125 (5/16)	9	.15625 (5/32)	17	.05625	25	.02344
2	.28125	10	.140625	18	.05 (1/20)	26	.021875
3	.25 (¼)	11	.125 (⅛)	19	.04375	27	.020312
4	.234375	12	.1125	20	.0375	28	.01875
5	.21875	13	.10 (1/10)	21	.034375	29	.01719
6	.203125	14	.0875	22	.03125 (1/32)	30	.015625
7	.1875 (3/16)	15	.075	23	.028125	31	.01406
8	.171875	16	.0625 (1/16)	24	.025 (1/40)	32	.0125 (1/80)

Sir Joseph Whitworth, in 1857, introduced his Standard Wire-Gauge, ranging from a half inch to a thousandth of an inch, and comprising 62 measurements, as given in Table No. 18. It commences with the smallest size, and increases by thousandths of an inch up to half an inch. The smallest size, 1/1000th of an inch, is No. 1; No. 2 is 2/1000ths of an inch, and so on, increasing up to No. 20 by intervals of 1/1000th of an inch; from No. 20 to No. 40 by 2/1000ths; from No. 40 to No. 100 by 5/1000ths of an inch. The sizes are designated or marked by their respective values in thousandths of an inch.

The Standard Imperial Wire Gauge came into force on the 1st March, 1884. It supersedes other gauges, which are rendered illegal.

SIR JOSEPH WHITWORTH & CO.'S STANDARD WIRE-GAUGE.—Table No. 18.

Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.
No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
1	.001	17	.017	55	.055	200	.200
2	.002	18	.018	60	.060	220	.220
3	.003	19	.019	65	.065	240	.240
4	.004	20	.020	70	.070	260	.260
5	.005	22	.022	75	.075	280	.280
6	.006	24	.024	80	.080	300	.300
7	.007	26	.026	85	.085	325	.325
8	.008	28	.028	90	.090	350	.350
9	.009	30	.030	95	.095	375	.375
10	.010	32	.032	100	.100	400	.400
11	.011	34	.034	110	.110	425	.425
12	.012	36	.036	120	.120	450	.450
13	.013	38	.038	135	.135	475	.475
14	.014	40	.040	150	.150	500	.500
15	.015	45	.045	165	.165		
16	.016	50	.050	180	.180		

STANDARD IMPERIAL WIRE-GAUGE.

Table No. 19.

Mark.	Size.	Mark.	Size.	Mark.	Size.
7/0	.500	16	.064	38	.0060
6/0	.464	17	.056	39	.0052
5/0	.432	18	.048	40	.0048
4/0	.400	19	.040	41	.0044
3/0	.372	20	.036	42	.0040
2/0	.348	21	.032	43	.0036
0	.324	22	.028	44	.0032
1	.300	23	.024	45	.0028
2	.276	24	.022	46	.0024
3	.252	25	.020	47	.0020
4	.232	26	.018	48	.0016
5	.212	27	.0164	49	.0012
6	.192	28	.0148	50	.0010
7	.176	29	.0136		
8	.160	30	.0124		
9	.144	31	.0116		
10	.128	32	.0108		
11	.116	33	.0100		
12	.104	34	.0092		
13	.092	35	.0084		
14	.080	36	.0076		
15	.072	37	0068		

Inches and their Equivalent Decimal Values in Parts of a Foot. —Table No. 20.

Inches.	Fraction of foot.	Foot.
1	1/12	.0833
2	1/6	.1667
3	1/4	.25
4	1/3	.3333
5	5/12	.4167
6	1/2	.5
7	7/12	.5833
8	2/3	.6667
9	3/4	.75
10	5/6	.8333
11	11/12	.9167
12	1	1.0

Fractional Parts of an Inch, and their Decimal Equivalents.—Tables No. 21.

Eighths.

Eighths.	Fractions.	Inch.
1	1/8	.125
2	1/4	.25
3	3/8	.375
4	1/2	.5
5	5/8	.625
6	3/4	.75
7	7/8	.875
8	1	1.0

Twelfths.

Twelfths.	Fractions.	Inch.
1	1/12	.08333
1½	1/8	.125
2	1/6	.16667
3	1/4	.25
4	1/3	.33333
5	5/12	.41667
6	1/2	.5
7	7/12	.58333
8	2/3	.66666
9	3/4	.75
10	5/6	.83333
11	11/12	.91667
12	1	1.0

Sixteenths and Thirty-seconds.—Tables No. 21 (*continued*).

Thirty-Seconds.	Sixteenths.	Fractions.	Inch.
1		1/32	.03125
2	1	1/16	.0625
3		3/32	.09375
4	2	1/8	.125
5		5/32	.15625
6	3	3/16	.1875
7		7/32	.21875
8	4	1/4	.25
9		9/32	.28125
10	5	5/16	.3125
11		11/32	.34375
12	6	3/8	.375
13		13/32	.40625
14	7	7/16	.4375
15		15/32	.46875
16	8	1/2	.5
17		17/32	.53125
18	9	9/16	.5625
19		19/32	.59375
20	10	5/8	.625
21		21/32	.65625
22	11	11/16	.6875
23		23/32	.71875
24	12	3/4	.75
25		25/32	.78125
26	13	13/16	.8125
27		27/32	.84375
28	14	7/8	.875
29		29/32	.90625
30	15	15/16	.9375
31		31/32	.96875
32	16	1	1.0

II. Measures of Surface.—Tables No. 22.

Superficial Measure.

144 square inches, or } 183.35 circular inches } .... 1 square foot.
9 square feet .... 1 square yard.
100 square feet .... 1 square.
272¼ square feet, or } 30¼ square yards } .... 1 rod.

The *square* is used in measuring flooring and roofing.
The *rod* is used in measuring brick-work.

Builders' Measurement.

1 superficial part	1 square inch.
12 parts	"1 inch" (12 square inches).
12 "inches"	1 square foot.

This table is employed in the superficial or flat measure of boards, glass, stone, artificers' work, &c.

Land Measure.

9 square feet	1 square yard.
30¼ square yards	1 square pole, rod, or perch.
16 square poles	1 square chain.
40 square poles, or 1210 square yards	1 rood.
4 roods, or 10 square chains, or 160 square poles, or 4,840 square yards, or 43,560 square feet	1 acre.*
640 acres, or 3,097,600 square yards	1 square mile.
30 acres	1 yard of land.
100 acres	1 hide of land.
40 hides	1 barony.

* The side of a square having an area of one acre is equal to 69.57 lineal yards.

III. Measures of Volume.—Tables No. 24.

Solid or Cubic Measure.

1728 cubic inches, 2200.15 cylindrical inches, 3300.23 spherical inches, 6600.45 conical inches	1 cubic foot.
27 cubic feet	1 cubic yard, or load.
35.3156 cubic feet, or 1.308 cubic yards	1 cubic metre.

Note.—The numbers of cylindrical, spherical, and conical inches in a cubic foot, are as 1, 1.5, 3.

Builders' Measurement.

1 solid part	12 cubic inches.
12 solid parts	1 "inch" (144 cubic inches).
12 "inches"	1 cubic foot.

This table is used in measuring square-sided timber, stone, &c.

Note.—The cubic contents of a piece,

6 inches square and		4 feet long is		1 cubic foot.		
7	,,	3	,,	1	,,	
8½	,,	2	,,	1	,,	
12	,,	1	,,	1	,,	
17	,,	1	,,	2	,,	
24	,,	1	,,	4	,,	

DECIMAL PARTS OF A SQUARE FOOT, IN SQUARE INCHES.—Table No. 23.

Hundredth Parts.	Square Inches.	Hundredth Parts.	Square Inches.	Hundredth Parts.	Square Inches.	Hundredth Parts.	Square Inches.
1	1.44	26	37.4	51	73.4	76	109.4
2	2.88	27	38.9	52	74.9	77	110.9
3	4.32	28	40.3	53	76.3	78	112.3
4	5.76	29	41.8	54	77.8	79	113.8
5	7.20	30	43.2	55	79.2	80	115.2
6	8.64	31	44.6	56	80.6	81	116.6
7	10.1	32	46.1	57	82.1	82	118.1
8	11.5	33	47.5	58	83.5	83	119.5
9	13.0	34	49.0	59	85.0	84	121.0
10	14.4	35	50.4	60	86.4	85	122.4
11	15.8	36	51.8	61	87.8	86	123.8
12	17.3	37	53.3	62	89.3	87	125.3
13	18.7	38	54.7	63	90.7	88	126.7
14	20.2	39	56.2	64	92.2	89	128.2
15	21.6	40	57.6	65	93.6	90	129.6
16	23.0	41	58.0	66	95.0	91	131.0
17	24.5	42	60.5	67	96.5	92	132.5
18	25.9	43	61.9	68	97.9	93	133.9
19	27.4	44	63.4	69	99.4	94	135.4
20	28.8	45	64.8	70	100.8	95	136.8
21	30.2	46	66.2	71	102.2	96	138.2
22	31.7	47	67.7	72	103.7	97	139.7
23	33.1	48	69.1	73	105.1	98	141.1
24	34.6	49	70.6	74	106.6	99	142.6
25	36.0	50	72.0	75	108.0	100	144.0

IV. MEASURES OF CAPACITY.—Tables No. 25.

Liquid Measure.

8.665 cubic inches .... 1 gill or quartern.
4 gills (34.659 cubic inches)......................... 1 pint.
2 pints .... 1 quart.
2 quarts .... 1 pottle.
4 quarts, or 8 pints (277.274 cubic inches)..... 1 gallon.

6.2355 gallons .... 1 cubic foot.

The *barn-gallon*, for milk, is equal to 2 imperial gallons.

Dry Measure.

2 pints	1 quart.
4 quarts	1 gallon.
2 gallons	1 peck.
4 pecks, or 8 gallons (1.28366 cubic feet)	1 bushel.
2 bushels	1 strike.
4 bushels	1 coomb.
5 bushels	1 sack.
8 bushels	1 quarter.
4 quarters (41.077 cubic feet)	1 chaldron.
5 quarters	1 wey or load.
2 loads	1 last.

In the Weights and Measures Act of 1878, it is only declared that the Imperial Standard Gallon contains 10 pounds of water at 62° F., and that 8 gallons shall be a bushel. Assuming that 1 cubic inch of water weighs 252.458 grains, the Imperial Standard Gallon has a capacity of 277.27384 cubic inches, or, say, 277.274 cubic inches, as before announced, page 125.

The Imperial Standard bushel, which is equal to 8 gallons, has a capacity of 2218.19072 cubic inches. The internal diameter of the standard bushel, 17.8 inches, is double its internal depth, 8.9 inches. Heaped measure, which was used for such goods as could not be stricken—as coals, potatoes, fruit, is now legally abandoned. Coals are sold by weight; and for other round goods, the measure is filled level with the brim as nearly as is practicable. The Market Garden bushel is made large enough to hold as much fruit as the heaped bushel held, filled level, so as to pack one on another.

Coal and Coke Measure.

3 bushels (heaped)	1 sack.
9 bushels	1 vat.
36 bushels, or 12 sacks (58.66 cubic feet)	1 chaldron.
5¼ chaldrons	1 room.
21 chaldrons	1 score.

Old Wine and Spirit Measure.

		Imperial Gallons.
4 gills or quarterns	1 pint.	
2 pints	1 quart.	
4 quarts (231 cubic inches)	1 gallon	= .8333
10 gallons	1 anker	= 8.333
18 gallons	1 runlet	= 15.
31½ gallons	1 barrel	= 26.250
42 gallons	1 tierce	= 35.
63 gallons, or 2 barrels	1 hogshead	= 52.5
84 gallons, or 1⅓ hogsheads	1 puncheon	= 70.
126 gallons, or 2 hogsheads, or 1½ puncheons	1 pipe or butt	= 105.
2 pipes, or 3 puncheons	1 tun	= 210.

By this measure wines, spirits, cider, perry, mead, vinegar, oil, &c., are measured; but the contents of every cask are reckoned in imperial gallons when sold. The imperial gallon is one-fifth larger than the old wine gallon.

Old Ale and Beer Measure.

			Imperial Gallons.
2 pints	1 quart.		
4 quarts (282 cubic inches)	1 gallon	=	1.017
9 gallons	1 firkin	=	9.153
2 firkins, or 18 gallons	1 kilderkin	=	18.306
2 kilderkins, or 36 gallons	1 barrel	=	36.612
1½ barrels, or 54 gallons	1 hogshead	=	54.918
3 barrels, or 108 gallons	1 butt	=	109.836

The imperial gallon is one-sixtieth smaller than the old beer gallon.

Apothecaries' Fluid Measure.

60 minims (♏)	1 fluid drachm	(*f* ʒ).
8 drachms (water, 1.732 cubic inches, 437½ grains)	1 fluid ounce	(*f* ℥).
20 ounces	1 pint	(O).
8 pints (water, 70,000 grains)	1 gallon	(*gall.*).

1 drop	1 grain.
60 drops	1 drachm.
4 drachms	1 tablespoonful.
2 ounces (water, 875 grains)	1 wineglassful.
3 ounces	1 teacupful.

V. Measures of Weight.—Tables No. 26.

Avoirdupois Weight.

16 drachms, or 437½ grains	1 ounce (*oz.*).
16 ounces, or 7000 grains	1 pound (imperial) (*lb.*).
8 pounds	1 stone (London meat market).
14 pounds	1 stone.
28 pounds, or 2 stones	1 quarter (*qr.*).
4 quarters, or 8 stones, or 112 pounds	1 hundredweight (*cwt.*).
20 hundredweights	1 ton.

The *grain* above noted, of which there are 7000 to the pound avoirdupois, is the same as the troy grain, of which there are 5760 to the troy pound.

Hence the troy pound is to the avoirdupois pound as 1 to 1.215, or as 14 to 17.

The troy ounce is to the avoirdupois ounce as 480 grains, the weight of the former, to 437½ grains, the weight of the latter; or, as 1 to .9115.

In Wales, the iron ton is 20 cwt. of 120 lbs. each.

Troy Weight.

24 grains	1 pennyweight (*dwt.*).
20 pennyweights, or 480 grains	1 ounce.
12 ounces, or 5760 grains	1 pound.
25 pounds	1 quarter.
4 quarters, or 100 pounds	1 hundredweight.

By troy weight are weighed gold, silver, jewels, and such liquors as are sold by weight.

Diamond Weight.

1 diamond grain	0.8 troy grain.
1 carat	4 diamond grains.
15½ carats	1 troy ounce.

Apothecaries' Weight.

The revised table of weights of the British Pharmacopeia is as follows: it is according to the avoirdupois scale:—

437½ grains	1 ounce.
16 ounces	1 pound.

In the old table of Apothecaries' Weight, superseded by the preceding table, the troy scale was followed, thus:—

Old Apothecaries' Weight.

20 grains	1 scruple (℈).
3 scruples, or 60 grains	1 drachm (ʒ).
8 drachms, or 480 grains	1 ounce (℥).
12 ounces, or 5760 grains	1 pound (*lb.*).

Weights of Current Coins.

1 farthing,	.8 inch diameter,	1/10 ounce.
1 halfpenny,	1.0 ”	1/5 ”
1 penny,	1.2 ”	1/3 ”
1 threepenny piece		1/20 ”
1 fourpenny piece		1/15 ”
1 sixpence		1/10 ”
1 shilling		1/5 ”
1 florin		2/5 ”
1 half-crown		1/2 ”
5 shillings or 10 sixpences		1 ”
1 sovereign		1/4 ounce, fully.

For the exact weight in grains of these coins, see Table of British Money.

Coal Weight.

14 pounds	1 stone.
28 pounds	1 quarter hundredweight.
56 pounds	1 half hundredweight.
88 pounds	1 bushel.*
1 sack, of 112 pounds	1 hundredweight.
1 double sack, of 224 pounds	2 hundredweights.
20 hundredweights, or 10 double sacks	1 ton.†
26½ hundredweights	1 chaldron (London).
53 hundredweights	1 chaldron (Newcastle).
7 tons	1 room.
21 tons 4 cwt.	1 barge or keel.

* *Sundry Bushel Measures.*

1 Cornish bushel of coal is 90 or 94 pounds; heaped, 101 pounds.
1 Welsh bushel, average weight 93 pounds.
1 Newcastle bushel is 80 or 84 pounds. Bradley Main, 92½ pounds.
1 London bushel, 80 or 84 pounds.

† In Wales the miners' coal-ton is 21 cwt. of 120 lbs. each.

Wool Weight.

7 pounds	1 clove.
2 cloves, or 14 pounds	1 stone.
2 stones	1 tod.
6½ tods	1 wey.
2 weys	1 sack.
12 sacks, or 39 hundredweight	1 last.
12 score, or 240 pounds	1 pack.

Hay and Straw Weight.

1 truss of straw	36 pounds.
1 load of straw	11 hundredweights, 64 pounds.
1 truss of old hay	56 pounds.
1 load of old hay	18 hundredweight.
1 cubic yard of old hay	15 stone.
1 truss of new hay	60 pounds.
1 load of new hay	19 hundredweights, 32 pounds.
1 cubic yard of new hay	6 stone.

Corn and Flour Weight.

1 peck, or stone of flour		14	pounds.
10 pecks	1 boll	= 140	,,
2 bolls	1 sack	= 280	,,
14 pecks	1 barrel	= 196	,,
1 bushel of wheat		60	,,
1 bushel of barley		47	,,
1 bushel of oats		40	,,

Six bushels of wheat should yield one sack of flour; 1 last of corn is 80 bushels.

Miscellaneous Tables.—No. 27.

Whatman's Drawing Papers.—Sizes of Sheets.

Antiquarian	53 inches long,	31 inches wide.
Double-elephant	40 „	27 „
Atlas	34 „	26 „
Colombier	34 „	23 „
Imperial	30 „	22 „
Elephant	28 „	23 „
Super-royal	27 „	19 „
Royal	23 „	19 „
Medium	22 „	17 „
Demy	20 „	15 „

Commercial Numbers and Stationery.

12 articles	1 dozen.
13 articles	1 long dozen.
12 dozen	1 gross.
20 articles	1 score.
5 score	1 common hundred.
6 score	1 great hundred.
30 deals	1 quarter.
4 quarters	1 hundred.
24 sheets of paper	1 quire.
20 quires	1 ream.
21½ quires	1 printers' ream.
5 dozen skins of parchment	1 roll.

Measures relating to Building.

Load of timber, unhewn or rough	40 cubic feet.
Load, hewn or squared	50 cubic feet, reckoned to weigh 20 cwt.
Stack of wood	108 cubic feet.
Cord of wood	128 „

(In dockyards, 40 cubic feet of hewn timber are reckoned to weigh 20 cwt.; 50 cubic feet is a load.)

100 superficial feet	1 square.
Hundred of deals	120 deals.
Load of 1-inch plank	600 square feet.

(Load of plank more than 1-inch thick = 600 ÷ thickness in inches.

Planks, section	11 by 3 inches.
Deals, section	9 by 3 „
Battens, section	7 by 2½ „

A reduced deal is 1½ inches thick, 11 inches wide, and 12 feet long.

Bundle of 4 feet oak-heart laths	120 laths.
Load of „ „	37½ bundles.
Bundle of 5 feet oak-heart laths	100 laths.
Load of „ „	30 bundles.

Measures relating to Building (continued.)

Load of statute bricks	500.
Load of plain tiles	1000.
Load of lime	32 bushels.
Load of sand	36 "
Hundred of lime	35 "
Hundred of nails, or tacks	120.
Thousand of nails, or tacks	1200.
Fodder of lead	19½ cwt.
Sheet lead	6 to 10 pounds per sq. ft.
Hundred of lead	112 pounds.
Table of glass	5 feet.
Case of glass	45 tables.
Case of glass	(Newcastle and Normandy glass, 25 tables).
Stone of glass	5 pounds.
Seam of glass	24 stone.

Sundry Commercial Measures.

Dicker of hides	10 skins.
Last of hides	20 dickers.
Weigh of cheese	256 pounds.
Barrel of herrings	26 ²/₃ gallons.
Cran of herrings	37½ "
Pocket of hops	1½ to 2 cwt.
Bag of hops	3½ cwt., nearly.
Last of potash, cod-fish, white herrings, meal, pitch, tar	12 barrels.
Barrel of tar	26½ gallons.
Barrel of anchovies	30 pounds.
Barrel of butter	224 "
Barrel of candles	120 "
Barrel of turpentine	2 to 2½ cwt.
Barrel of gunpowder	100 pounds.
Last of gunpowder	24 barrels.

Measures for Ships.

1 ton, displacement of a ship,	35	cubic feet.
1 ton, registered internal capacity of do.,	100	do.
1 ton, shipbuilders' old measurement,	94	do.

COMPARISON OF COMPOUND UNITS.—Tables No. 28.

Measures of Velocity.

1 mile per hour	1.467 feet per second. 88.0 feet per minute.
1 knot per hour	1.688 feet per second.
1 foot per second	.682 mile per hour.
1 foot per minute	.01136 mile per hour.

Measures of Volume and Time.

1 cubic foot per second...........	2.222 cubic yards per minute. 133.333 cubic yards per hour.
1 cubic foot per minute............	2.222 cubic yards per hour.
1 cubic yard per hour..............	.45 cubic foot per minute.
1 cubic inch per second...........	2.083 cubic foot per hour. 12.984 gallons per hour.
1 gallon per second.................	569.124 cubic feet per hour.
1 gallon per minute	9.485 cubic feet per hour.

Measures of Pressure and Weight. (See also page 127.)

1 lb. per square inch	144 lbs. per square foot. 1296 lbs. per square yard. .5786 ton per square yard.
1 atmosphere (14.7 lbs. per square inch)................	8.503 ton per square yard.
1 lb. per square foot..........	.00694 lb. per square inch. .1111 ounce per square inch. .0804 cwt. per square yard.
1 lb. per square inch	2.0355 inches of mercury at 32° F. 2.308 feet of water at 52°.3 F.
1 inch of mercury at 32° F.	.491 lb. per square inch. 1.133 feet of water at 52°.3 F.
1 foot of water, at 52°.3 F...	.4333 lb. per square inch. 62.4 lbs. per square foot. .8823 inch of mercury at 32° F.

Measures of Weight and Volume.

100 lbs. per cubic foot	405.1 grains per cubic inch. .926 ounce per cubic inch. 24.107 cwt. per cubic yard. 1.205 tons per cubic yard.
1 grain per cubic inch............	3.950 ounces per cubic foot. .247 pounds per cubic foot.
1 ounce per cubic inch...........	108 pounds per cubic foot.
1 cwt. per cubic yard..............	4.148 pounds per cubic foot.
1 ton per cubic yard...............	82.963 pounds per cubic foot.
1 grain per gallon (1 in 70,000 parts by weight, of water)......	1 pound for 1122 cubic feet. 1 pound for 41.5 cubic yards. 1 pound for 31.8 cubic metres. 220 grains for 1 cubic metre. .503 ounce for 1 cubic metre.

Measures of Power.

1 lb. of fuel per H.P. per hour.............	1,980,000 foot-pounds per lb. of fuel. 221.76 million foot-pounds per cwt. of fuel. 2,565 heat-units per lb. of fuel.
1,000,000 foot-pounds per lb. of fuel.......	1.98 pounds of fuel per H.P. per hour.

FRANCE.—THE METRIC STANDARDS OF WEIGHTS AND MEASURES.

The primary metric standards are:—the metre, the unit of length; and the kilogramme, the unit of weight, derived from the metre: being the two platinum standards deposited at the Palais des Archives at Paris.

The standard metre is defined to be equal to one ten-millionth part of the quadrant of the terrestrial meridian, that is to say, the distance from the equator to the pole, passing through Paris, which, by the latest and most authoritative measurement, is 39.3762 inches, in terms of the Imperial standard at 62° F. By the latest and most accurate measurement, the actual standard metre at 32° F. is, in terms of the Imperial standard at 62° F., 39.37043 inches; and its legal equivalent, declared in the Metric Act of 1864, is 39.3708 inches, being the same as that adopted in France.

The standard kilogramme (1000 grammes) is defined to be the weight of a cubic decimetre of distilled water at its maximum density, at 4°.0 C. or 39°.1 F. This is legally taken to be

2.20462125 lbs., or
2 lbs., 3 oz., 4.383 drachms, or
15,432.34874 grains.

There is in the Standard Department at Westminster a newly-constructed subdivided standard yard, laid down upon a bar of Baily's metal, upon which a subdivided metre has also been laid down.

The metric unit of capacity is the litre, defined to be equal to a cubic decimetre. Its Imperial equivalent is 0.22009 gallon.

There is no other official standard of weight and measure in France than the metre and the kilogramme; there is no standard litre or unit of capacity.

The metric system is not really founded on the length of a quadrant of the meridian, and although it is described as a scientific system, because of the simple and definite relation between the metre, which is its basis and unit of length, and the kilogramme and litre, which are the units of weight and capacity, it is admitted that it has been found impossible practically to carry it out with scientific accuracy. The standard kilogramme is admitted *not* to be actually the weight of a cubic decimetre of pure water at the specified temperature, nor the litre a measure of capacity holding a cubic decimetre of pure water. The real standard unit of weight is declared, even by men of science in France, to be merely the platinum kilogramme-weight deposited at the Palais des Archives, as the real standard unit and basis of the metric system is the platinum metre, also deposited there. It is an accomplished fact, however, that all civilized nations have tacitly agreed to recognize the metric system as affording for the future the advantages of a universal system of weights and measures, and to adopt the standards deposited at the Palais des Archives as the primary units of the system.

The French metric system has been adopted, and its use made compulsory by the following States:—France and Belgium, in 1801; Holland, in 1819; Greece, in 1836; Italy and Spain, in 1859; Portugal, in 1860–68; the German Empire, in 1872; Colombia, Venezuela, in 1872; Ecuador,

Brazil, Peru, and Chili, in 1860; also by the Argentine Confederation, and Uruguay.

Great Britain and Ireland, in 1864, adopted the metric system, so far as to render contracts in terms of the French metric system permissive.

The United States of North America, in 1866, legalized the French metric system concurrently with the old system; it was also legalized in British North America.

Switzerland, in 1856, legalized the foot of three decimetres as the unit of length, with a decimal scale; the unit of weight being the pound of 500 grammes, or half a kilogramme, with two distinct scales of multiples and parts, one decimal, the other according to the old custom.

Sweden, in 1855, by a law made compulsory in 1858, adopted a decimal system of weights and measures, having for the unit of length a foot of 0.297 metre, and the unit of weight a pound of 0.42 kilogramme:—being the original units decimally treated.

Denmark adopted the metric system so far as the pound of 500 grammes. The pound is decimally treated, and since 1863 the use of the greatest parts of the multiples of the pound not conformable to decimal subdivision has been prohibited.

Austria, in 1853, adopted a pound of 500 grammes, with decimal divisions, for customs and fiscal purposes.

Russia awaits the example of those countries with which she has commercial relations, especially of England.

In Morocco and Tunis, the weights and measures have no relation with the metric system.

On the 20th May, 1875, the international convention for the adoption of the French metrical system of weights and measures was signed at Paris by the plenipotentiaries of France, Austria, Germany, Italy, Russia, Spain, Portugal, Turkey, Switzerland, Belgium, Sweden, Denmark, the United States, the Argentine Republic, Peru, and Brazil. A special clause reserves to States not included in the above list the right of eventually adhering to the convention.

I. French Measures of Length.—Table No. 29.

10 millimetres	1 centimetre.
10 centimetres	1 decimetre.
10 decimetres, or 100 centimetres, or 1000 millimetres	1 METRE.
10 metres	1 decametre.
10 decametres	1 hectometre.
10 hectometres, or 1000 metres	1 KILOMETRE (*kilo.*)
10 kilometres	1 myriametre.

---

1 toise (old measure)		= 1.949 metres.
1000 toises	1 mille	= 1.949 kilometres.
2000 toises	1 itinerary league	= 3.898 ,,
2280.329 toises	1 terrestrial league	= 4.444 ,,
2850.411 toises	1 nautical league	= 5.555 ,,
1 nœud	(British nautical mile)	= 1.855 ,,

French Wire-Gauges (*Jauges de Fils de Fer*).

The French wire-gauge, like the English, has been subject to variation. Table No. 30 contains the values of the "points," or numbers, of the Limoges gauge; table No. 31 gives the values of a wire-gauge used in the manufacture of galvanized iron; and table No. 32 the values of a gauge which comprises wire and bars up to a decimetre in diameter.

French Wire-Gauge (*Jauge de Limoges*).—Table No. 30.

Number.	Diameter.		Number.	Diameter.		Number.	Diameter.	
	Millimetre.	Inch.		Millimetre.	Inch.		Millimetre.	Inch.
0	.39	.0154	9	1.35	.0532	18	3.40	.134
1	.45	.0177	10	1.46	.0575	19	3.95	.156
2	.56	.0221	11	1.68	.0661	20	4.50	.177
3	.67	.0264	12	1.80	.0706	21	5.10	.201
4	.79	.0311	13	1.91	.0752	22	5.65	.222
5	.90	.0354	14	2.02	.0795	23	6.20	.244
6	1.01	.0398	15	2.14	.0843	24	6.80	.268
7	1.12	.0441	16	2.25	.0886			
8	1.24	.0488	17	2.84	.112			

French Wire-Gauge for Galvanized Iron Wire.—Table No. 31.

Number.	Diameter.		Number.	Diameter.		Number.	Diameter.	
	M'metre.	Inch.		M'metre.	Inch.		M'metre.	Inch.
1	.6	.0236	9	1.4	.0551	17	3.0	.118
2	.7	.0276	10	1.5	.0591	18	3.4	.134
3	.8	.0315	11	1.6	.0630	19	3.9	.154
4	.9	.0354	12	1.8	.0709	20	4.4	.173
5	1.0	.0394	13	2.0	.0787	21	4.9	.193
6	1.1	.0433	14	2.2	.0866	22	5.4	.213
7	1.2	.0473	15	2.4	.0945	23	5.9	.232
8	1.3	.0512	16	2.7	.106			

French Wire- and Bar-Gauge.—Table No. 32.

Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.
	Millimetre.		Millimetre.		Millimetre.		Millimetre.
P	5	8	13	16	27	24	64
1	6	9	14	17	30	25	70
2	7	10	15	18	34	26	76
3	8	11	16	19	39	27	82
4	9	12	18	20	44	28	88
5	10	13	20	21	49	29	94
6	11	14	22	22	54	30	100
7	12	15	24	23	59		

II. French Measures of Surface.—Table No. 33.

100 square millimetres	1 square centimetre.
100 square centimetres	1 square decimetre.
100 square decimetres, or 10,000 square centimetres	1 square metre, or CENTIARE.
100 square metres, or centiares	1 square decametre, or ARE.
100 square decametres, or ares	1 square hectometre, or HECTARE.
100 square hectometres, or hectares	1 square myriametre.

Land is measured in terms of the *centiare*, the *are*, and the *hectare* or *arpent metrique* (metric acre). There is also the *decare*, of 10 ares.

III. French Measures of Volume.—Tables No. 34.

Cubic Measure.

1000 cubic millimetres	1 cubic centimetre.
1000 cubic centimetres	1 cubic decimetre.
1000 cubic decimetres	1 cubic metre.

Wood Measure.

10 decistères	1 stère* (1 cubic metre).
1 voie (Paris)	2 stères.
1 voie de charbon (charcoal)	0.2 stère ($^1/_5$ cubic metre).
1 corde	4 stères.

* The stère measures 1.14 metres × 0.88 metre × 1 metre, the billets of wood being 1.14 metres in length.

IV French Measures of Capacity.—Tables No. 35.

Liquid Measure.

10 cubic centimetres	1 centilitre.
10 centilitres	1 décilitre.
10 décilitres	1 LITRE.
10 litres	1 décalitre.

Dry Measure.

10 litres	1 décalitre.
10 décalitres, or 100 litres	1 hectolitre.
10 hectolitres, or 1000 litres	1 kilolitre (1 cubic metre).

The use of measures equal to a *double-litre*, a *half-litre*, a *double-décilitre*, a *half-décilitre*, is sanctioned by law.

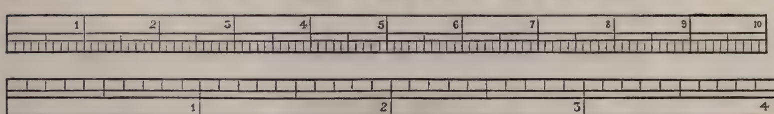
V. French Measures of Weight.—Table No. 36.

10 milligrammes	1 centigramme.
10 centigrammes	1 decigramme.
10 decigrammes	1 GRAMME.
10 grammes	1 decagramme.
10 decagrammes	1 hectogramme.
10 hectogrammes, or 1000 grammes	1 KILOGRAMME (*kil.*, *kilog.*)
10 kilogrammes	1 myriagramme.
10 myriagrammes, or 100 kilogrammes	1 quintal metrique.
10 quintaux, or 1000 kilogrammes	1 millier, tonneau de mer, or tonne (weight of 1 cubic metre of water at 39°.1).

EQUIVALENTS OF BRITISH IMPERIAL AND FRENCH METRIC WEIGHTS AND MEASURES.

I. Measures of Length.—Tables No. 37.

A DECIMETRE DIVIDED INTO CENTIMETRES AND MILLIMETRES.



INCHES AND TENTHS.

Metric Denominations and Values.		Equivalents in Imperial Denominations.			
	Metres.	Inches.	Feet.	Yards.	Miles.
1 millimetre	1/1000	= 0.03937	—	—	—
1 centimetre	1/100	= 0.39370	—	—	—
1 decimetre	1/10	= 3.93704	—	—	—
1 METRE	1	= 39.37043	= 3.28087	= 1.09362	—
1 dekametre	10	—	= 32.80869	= 10.93623	—
1 hectometre	100	—	—	= 109.36231	—
1 KILOMETRE	1,000	—	= 3280.87	= 1,093.6231	= 0.62138
1 myriametre	10,000	—	—	= 10,936.231	= 6.21377

Tables No. 37 (*continued*).

Imperial Denominations.	Equivalents in Metric Denominations.		
	Centimetres.	Metres.	Kilometres.
1 inch (25.4 millimetres)...........	= 2.53995	—	—
1 foot, or 12 inches	—	= 0.30480	—
1 yard, or 3 feet, or 36 inches....	—	= 0.91439	—
1 fathom, or 2 yards, or 6 feet....	—	= 1.82878	—
1 pole, or 5½ yards	—	= 5.02915	—
1 chain, or 4 poles, or 22 yards...	—	= 20.11662	—
1 furlong, 40 poles, or 220 yards	—	= 201.1662	= 0.20117
1 mile, 8 furlongs, or 1760 yards	—	= 1,609.3296	= 1.60933

Equivalent Values of Millimetres and Inches.—Tables No. 38.

Millimetres = Inches.

Millimetres.	Inches.	Millimetres.	Inches.	Millimetres.	Inches.	Millimetres.	Inches.
1	.0394	27	1.0630	53	2.0866	79	3.1103
2	.0787	28	1.1024	54	2.1260	80	3.1496
3	.1181	29	1.1417	55	2.1654	81	3.1890
4	.1575	30	1.1811	56	2.2047	82	3.2284
5	.1968	31	1.2205	57	2.2441	83	3.2677
6	.2362	32	1.2598	58	2.2835	84	3.3071
7	.2756	33	1.2992	59	2.3228	85	3.3465
8	.3150	34	1.3386	60	2.3622	86	3.3859
9	.3543	35	1.3780	61	2.4016	87	3.4252
10	.3937	36	1.4173	62	2.4410	88	3.4646
11	.4331	37	1.4567	63	2.4803	89	3.5040
12	.4724	38	1.4961	64	2.5197	90	3.5433
13	.5118	39	1.5354	65	2.5591	91	3.5827
14	.5512	40	1.5748	66	2.5984	92	3.6221
15	.5906	41	1.6142	67	2.6378	93	3.6614
16	.6299	42	1.6536	68	2.6772	94	3.7008
17	.6693	43	1.6929	69	2.7166	95	3.7402
18	.7087	44	1.7323	70	2.7559	96	3.7796
19	.7480	45	1.7717	71	2.7953	97	3.8189
20	.7874	46	1.8110	72	2.8347	98	3.8583
21	.8268	47	1.8504	73	2.8740	99	3.8977
22	.8661	48	1.8898	74	2.9134	100	3.9370
23	.9055	49	1.9291	75	2.9528	= 1 decimetre.	
24	.9449	50	1.9685	76	2.9922		
25	.9843	51	2.0079	77	3.0315		
26	1.0236	52	2.0473	78	3.0709		

Tables No. 38 (*continued*).

INCHES DECIMALLY = MILLIMETRES.

Inches.	Millimetres.	Inches.	Millimetres.	Inches.	Millimetres.	Inches.	Millimetres.
.01	.25	.26	6.60	.60	15.2	.94	23.9
.02	.51	.28	7.11	.62	15.7	.96	24.4
.03	.76	.30	7.62	.64	16.3	.98	24.9
.04	1.02	.32	8.13	.66	16.8	1.00	25.4
.05	1.27	.34	8.64	.68	17.3	2.00	50.8
.06	1.52	.36	9.14	.70	17.8	3.00	76.2
.07	1.78	.38	9.65	.72	18.3	4.00	101.6
.08	2.03	.40	10.2	.74	18.8	5.00	127.0
.09	2.29	.42	10.7	.76	19.3	6.00	152.4
.10	2.54	.44	11.2	.78	19.8	7.00	177.8
.12	3.05	.46	11.7	.80	20.3	8.00	203.2
.14	3.56	.48	12.2	.82	20.8	9.00	228.6
.16	4.06	.50	12.7	.84	21.3	10.00	254.0
.18	4.57	.52	13.2	.86	21.8	11.00	279.4
.20	5.08	.54	13.7	.88	22.4	12.00	304.8
.22	5.59	.56	14.2	.90	22.9	= 1 foot.	
.24	6.10	.58	14.7	.92	23.4		

INCHES IN FRACTIONS = MILLIMETRES.

Eighths.	Sixteenths.	Thirty-seconds.	Millimetres.	Eighths.	Sixteenths.	Thirty-seconds.	Millimetres.
		1	.79			17	13.5
	1	2	1.59		9	18	14.3
		3	2.38			19	15.1
1	2	4	3.17	5	10	20	15.9
		5	3.97			21	16.7
	3	6	4.76		11	22	17.5
		7	5.56			23	18.3
2	4	8	6.35	6	12	24	19.0
		9	7.14			25	19.8
	5	10	7.94		13	26	20.6
		11	8.73			27	21.4
3	6	12	9.52	7	14	28	22.2
		13	10.32			29	23.0
	7	14	11.11		15	30	23.8
		15	11.91			31	24.6
4	8	16	12.7	8	16	32	25.4

By means of the preceding tables of equivalent values of inches and millimetres, the equivalent values of inches in centimetres and decimetres, and even in metres, may be found by simply altering the position of the decimal point. This method naturally follows from the decimal subdivisions of French measure.

Take, for example, the tabular value of 1 millimetre, and shift the

decimal point successively, by one digit, towards the right-hand side; the values of a centimetre, a decimetre, and a metre are thereby expressed in inches, as follows:—

1 millimetre	.0394	inches.
1 centimetre	0.394	,,
1 decimetre	3.94	,,
1 metre	39.4	,,

At the same time, it appears that, by selecting the tabular value of 10 millimetres, the value of its multiples are given more accurately, thus,—

10 millimetres, or 1 centimetre	0.3937	inches.
1 decimetre	3.937	,,
1 metre	39.37	,,

Again:—

100 millimetres, or 1 decimetre = 3.937 inches.
1 metre = 39.37 ,,

Similarly, for example:—

.32 inch = 8.13 millimetres.
3.2 ,, = 81.3 ,,
32.0 ,, = { 813.0 ,, or .813 metre.

II. Square Measures, or Measures of Surface.—Tables No. 39.

Metric	=	Imperial Square Measures.
1 square millimetre	=	.00155 square inch.
1 square centimetre	=	.155 square inch.
1 square decimetre	=	15.5003 square inches.
1 square metre, or centiare	=	10.7641 square feet, or 1.1960 square yards.
1 ARE, or square dekametre, or 100 square metres	=	1076.41 square feet, or 119.60 square yards.
1 hectare, or metrical acre, or 100 ares, or 10,000 square metres	=	11,960.11 square yards, or 2.4711 acres, or 2 acres and 2280.1240 square yards.

Imperial = Metric Square Measures.

Imperial Measures.	Square Centimetres.	Square Metres.	Ares.	Hectares.
1 square inch	= 6.45148	—	—	—
1 square ft., or 144 sq. inches	—	= 0.092901	—	—
1 square yard, or 9 square feet, or 1296 sq. inches	—	= 0.836112	—	—
1 perch or rod, or 30¼ square yards	—	= 25.292	—	—
1 rood, or 40 perches, or 1210 square yards	—	= 1011.696	= 10.11696	—
1 acre, or 4 roods, or 4840 square yards	—	= 4046.782	= 40.4678	= 0.40468
1 square mile, or 640 acres	—	—	—	= 258.98944

III. Cubic Measures.—Tables No. 40.

Metric	=	Imperial Cubic Measures.
1 cubic centimetre	=	0.061025 cubic inch.
1 cubic decimetre	=	61.02524 cubic inches, or 0.0353156 cubic foot.
1 cubic metre	=	35.3156 cubic feet, or 1.308 cubic yards.

Imperial	=	Metric Cubic Measures.
1 cubic inch	=	16.387 cubic centimetres.
1 cubic foot..........................	=	28.3153 cubic decimetres, or 0.028315 cubic metre.
1 cubic yard	=	0.764513 cubic metre.

Wood Measure.

1 stère, or cubic metre	35.3156 cubic feet. 1.308 cubic yards.
1 decistère	3.5316 cubic feet.
1 voie de bois (wood), or 2 stères, Paris	70.6312 cubic feet, or 2.616 cubic yards.
1 voie de charbon (charcoal) = 1 sack = $^1/_5$ stère	5½ bushels, or 7.063 cubic feet.
1 corde of wood = 4 cubic metres......	141.26 cubic feet.

IV. Measures of Capacity.—Tables No. 41.

Metric Denominations and Values.		Equivalents in Imperial Denominations.					
	Litres.	Gills.	Pints.	Quarts.	Gallons.	Bushels.	Quarters.
Centilitre......	$^1/_{100}$	0.0704	0.0176	—	—	—	—
Decilitre.......	$^1/_{10}$	0.7043	0.1761	—	—	—	—
Litre (61.02524 c. in.)	1	—	1.7607	0.8804	0.2201	—	—
Dekalitre	10	—	—	—	2.2009	0.2751	—
Hectolitre.....	100	—	—	—	22.009	2.7511	0.344
Kilolitre	1000	—	—	—	220.09	27.511	3.439

Imperial Denominations.	Equivalents in Metric Denominations.		
	Litres.	Dekalitres.	Hectolitres.
1 gill....................................	= 0.1420	—	—
1 pint, or 4 gills....................	= 0.5679	—	—
1 quart, or 2 pints.................	= 1.1359	—	—
1 gallon, or 4 quarts	= 4.5435	—	—
1 peck, or 2 gallons..............	= 9.0869	= 0.9087	—
1 bushel, or 8 gallons............	= 36.3477	= 3.6348	—
1 quarter, or 8 bushels	= 290.7816	= 29.0782	= 2.9078

V. Measures of Weight.—Tables No. 42.

Metric Weights = Imperial Avoirdupois Weights.

1 kilogramme = 2 lbs. 3 oz. 4 drachms, 10.47374 grains.

Metric Weights.		Equivalents in Imperial Denominations.				
	Grammes.	Grains.	Ounces.	Pounds.	Hundred-weights.	Tons.
Milligramme.........	1/1000	0.0154	—	—	—	—
Centigramme........	1/100	0.1543	—	—	—	—
Decigramme.........	1/10	1.5432	—	—	—	—
GRAMME.............	1	15.4323	—	—	—	—
Dekagramme........	10	154.3235	0.3527	—	—	—
Hectogramme	100	1543.2349	3.5274	—	—	—
KILOGRAMME.......	1,000	15432.3487	35.2739	2.2046	—	—
Myriagramme.......	10,000	—	—	22.0462	—	—
Quintal, or 100 kilog.	100,000	—	—	220.4621	1.9684	—
Millier, or metric ton	1,000,000	—	—	2204.6212	19.6841	0.9842

Imperial Avoirdupois = Metric Weights.

Imperial Avoirdupois Weights.	Grammes.	Decigrammes.	Kilogrammes.	Millier, or Metric Ton.
1 drachm...............	= 1.77184	—	—	—
1 ounce, or 16 drams	= 28.34954	= 2.83495	—	—
1 pound, or 16 ounces	= 453.59265	= 45.35926	= 0.45359	—
1 hundredweight, or 112 pounds	—	—	= 50.80237	—
1 ton, or 20 hundredweights	—	—	= 1016.04754	= 1.01604

Metric Weights = Imperial Troy Weights.

1 kilogramme = 2 troy lbs. 8 oz. 3 dwts., .34874 grain.

Metric Weights.	Grains.	Pennyweights.	Ounces.	Troy Pound.
Milligramme ...	= 0.01543	—	—	—
Centigramme ...	= 0.15432	—	—	—
Decigramme ...	= 1.54323	—	—	—
GRAMME.........	= 15.43234	—	—	—
Dekagramme...	= 154.32349	= 0.64301	= 0.32151	—
Hectogramme..	= 1543.23487	= 6.43014	= 3.21507	—
KILOGRAMME...	= 15,432.34874	—	= 32.15073	= 2.67922

IMPERIAL TROY = METRIC WEIGHTS.

Imperial Troy Weights.	Equivalents in Metric Denominations.				
	Milligramme.	Gramme.	Dekagramme.	Hecto-gramme.	Kilo-gramme.
1 troy grain............	64.79895	0.06480	—	—	—
1 „ dwt., or 24 gr.	—	1.55517	—	—	—
1 „ oz., or 480 „	—	31.10349	3.11035	—	—
1 „ lb., or 5,760 „	—	373.24195	37.32419	3.73242	0.37324

APPROXIMATE EQUIVALENTS OF ENGLISH AND FRENCH MEASURES.

The following are approximately equal English and French measures of length:—

1 pole, or perch (5½ yards)... 5 metres (exactly 5.029 metres).
1 chain (22 yards)............... 20 metres (exactly 20.1166 metres).
1 furlong (220 yards) .... 200 metres (exactly 201.166 metres).
5 furlongs............................ 1 kilometre (exactly 1.0058 kilometres).
1 foot............................ { 3 decimetres (exactly 3.048 decimetres), or 30 centimetres.

One metre = 3.28 feet = 3 feet 3 inches and 3 eighths all but $^1/_{512}$ inch; = 40 inches nearly ($^1/_{64}$th or 1.6 per cent. less).

.100 metre (1 decimetre) = 4 inches nearly (exactly $3^{15}/_{16}$ inches).
.010 metre (1 centimetre) = .4 inch, or $^4/_{10}$ths inch, nearly.
.001 metre (1 millimetre) = .04 inch, or $^4/_{100}$ths inch, or two-thirds of $^1/_{16}$ inch, or $^1/_{25}$ inch, nearly.

One inch is about 2½ centimetres (exactly 2.54).
One inch is about 25 millimetres (exactly 25.4).
One yard is $^{11}/_{12}$ths of a metre. 11 metres are equal to 12 yards.
Approximate rule for converting metres, or parts of metres, into yards:—Add $^1/_{11}$th (¼ per cent. less).
For converting metres into inches:—Multiply by 40; and to convert inches into metres, or parts of metres, divide by 40.
One kilometre is about ⅝ mile (it is 0.6 per cent. less).
One mile is about 1.6 or 1 $^3/_5$ kilometres (it is 0.6 per cent. less) = 1610 metres, about.

With respect to superficial measures:—
One square centimetre is about $^1/_{6.5}$ part of a square inch.
One square inch is equal to about 6.5 square centimetres.
One square metre contains fully 10¾ square feet, or nearly $1^1/_5$ square yards.
One square yard is nearly $^6/_7$ ths of a square metre.
One acre is over 4000 square metres (about 1.2 per cent. more).
One square mile is nearly 260 hectares (about 0.4 per cent. less).

With respect to cubic measures, and to capacity:—

One cubic yard is about ¾ cubic metre (it is 2 per cent. more).

One cubic metre is nearly 1⅓ cubic yard (it is 1⅔ per cent. less).

One cubic metre is nearly 35 ⅓ cubic feet (it is .05 per cent. less).

One litre is over 1¾ pints (it is 0.57 per cent. more).

One gallon contains above 4½ litres (it holds about 1 per cent. more).

One kilolitre (a cubic metre) holds nearly 1 ton of water at 62° F. (1¾ per cent. less), or 220½ gallons.—One cubic foot contains 28.3 litres.

With respect to weights:—The ton and the gramme stand at nearly equal distances above and below the kilogramme, thus:—

1 ton is	1,016,047.5 grammes,
1 kilogramme is	1,000.0 grammes,
1 gramme	1.0 gramme,

in the ratio of about 1,000,000 : 1,000 : 1.

One gramme is nearly 15½ grains (about ½ per cent. less).

One kilogramme is about 2 ⅕ pounds avoirdupois (about ¼ per cent. more).

A thousand kilogrammes, or a metric ton, is nearly one English ton (about 1½ per cent. less).

One hundredweight is nearly 51 kilogrammes (⅖ per cent. less).

EQUIVALENTS OF FRENCH AND ENGLISH COMPOUND UNITS OF MEASUREMENT.

Weight, Pressure, and Measure.

1 kilogramme per metre	.672 pound per foot. 2.016 pounds per yard.
1 pound per foot	1.488 kilogrammes per metre.
1 pound per yard	.496 kilogramme per metre.
1000 kilogrammes per metre	.300 ton per foot.
1 ton per foot	3333.333 kilogrammes per metre.
1000 kilogrammes, or 1 tonne, per kilometre	1.584 tons per mile.
1 ton per mile	631.0 kilogrammes per kilometre.
1 kilogramme per square millimetre	1422.32 pounds per square inch. .635 tons per square inch.
1000 pounds per square inch	.703077 kilogramme per square millimetre.
1 ton per square inch	1.575 kilogrammes per square millimetre.
1 kilogramme per square centimetre	14.2232 pounds per square inch.
1.0335 kilogrammes per square centimetre (1 atmosphere)	14.7 pounds per square inch.
1 pound per square inch	.0703077 kilogramme per square centimetre.
1 pound per square foot	4.883 kilogrammes per square metre.

Weight, Pressure, and Measure (continued).

1 kilogramme per square metre	.205 pounds per square foot.
1 centimetre of mercury	.394 inch of mercury.
1 inch of mercury	2.540 centimetres of mercury.
1 centimetre of mercury	.193 pound per square inch.
1 pound per square inch	5.170 centimetres of mercury.
1 gramme per litre	70.116 grains per gallon.
1 grain per gallon	.0143 gramme per litre.
1 kilogramme per cubic metre	.0624 pound per cubic foot.
1 pound per cubic foot	16.020 kilogrammes per cubic metre.
1 tonne per cubic metre	.984 ton per cubic metre. .752 ton per cubic yard.
1 kilogramme per litre	10.016 pounds per gallon.
1 pound per gallon	.998 kilogramme per litre.
1 ton per cubic metre	1.016 tonnes per cubic metre.
1 ton per cubic yard	1.329 tonnes per cubic metre.
1 cubic metre per kilogramme	16.019 cubic feet per pound.
1 cubic foot per pound	.0624 cubic metre per kilogramme.
1 cubic metre per tonne	1.329 cubic yards per ton. 1.794 cubic feet per cwt. 35.882 cubic feet per ton.
1 cubic yard per ton	.752 cubic metre per tonne.
1 cubic foot per cwt.	.557 cubic metre per tonne.
1 cubic foot per ton	.0279 cubic metre per tonne.

Volume, Area, and Length.

1 cubic metre per lineal metre	1.196 cubic yards per lineal yard.
1 cubic yard per lineal yard	.836 cubic metre per lineal metre.
1 cubic metre per square metre	3.281 cubic feet per square foot.
1 cubic foot per square foot	3.048 cubic metres per square metre.
1 litre per square metre	.0204 gallon per square foot.
1 gallon per square foot	48.905 litres per square metre.
1 cubic metre per hectare	.405 cubic metre per acre. .529 cubic yard per acre. 89.065 gallons per acre.
1 cubic metre per acre	2.471 cubic metres per hectare.
1 cubic yard per acre	1.902 cubic metres per hectare.
1000 gallons per acre	11.226 cubic metres per hectare.

Work.

1 kilogrammetre ($k \times m$)	7.233 foot-pounds.
1 foot-pound	.1382 kilogrammetre.
1 cheval-vapeur, or cheval ($75\ k \times m$ per second)	.9863 horse-power.
1 horse-power	1.0139 chevaux.
1 kilogramme per cheval	2.235 pounds per horse-power.
1 pound per horse-power	.447 kilogramme per cheval.
1 square metre per cheval	10.913 square feet per horse-power.
1 square foot per horse-power	.0916 square metre per cheval.
1 cubic metre per cheval	35.806 cubic feet per horse-power.
1 cubic foot per horse-power	.0279 cubic metre per cheval.

Heat.

1 calorie, or French unit	3.968 English heat-units.
1 English heat-unit	.252 calorie.
French mechanical equivalent (425 kilogrammetres)	3074 foot-pounds = 774.70 foot-pounds per English unit.
English mechanical equivalent (772 foot-pounds)	10.67 kilogrammetres.
1 calorie per square metre	.369 heat-unit per square foot.
1 heat-unit per square foot	2.713 calories per square metre.
1 calorie per kilogramme	1.800 or 9/5 heat-units per pound.
1 heat-unit per pound	.5555 or 5/9 calorie per kilogramme.

Speed, &c.

1 metre per second	3.281 feet per second. 196.860 feet per minute. 2.236 miles per hour.
1 kilometre per hour	.621 mile per hour.
1 foot per second, or per minute	.305 metre per second, or per minute.
1 mile per hour	.447 metre per second. 1.609 kilometres per hour.
1 cubic metre per second	35.316 cubic feet per second. 2119 cubic feet per minute.
1 cubic foot per second, or per minute	.0283 cubic metre per second, or per minute.
1 cubic metre per minute	1.308 cubic yards per minute.
1 cubic yard per minute	.765 cubic metre per minute.

Money.

1 franc per kilogramme	4.320 pence per pound. .360 shilling per pound. 40.320 shillings per cwt., or £40.32 per ton.
1 penny per pound	.231 franc per kilogramme.
1 shilling per pound	2.772 franc per kilogramme.
1 shilling per cwt., or £1 per ton	24.802 francs per tonne. 2.48 francs per quintal.
1 franc per quintal	.403 shilling per cwt.
1 franc per tonne	.484 penny per cwt. .806 shilling per ton.
1 franc per metre	.726 shilling per yard. 8.709 pence per yard.
1 shilling per yard	1.378 francs per metre.
1 franc per kilometre	.06386 £ per mile. 15.326 pence per mile.
£1 per mile	15.660 francs per kilometre.
1 penny per mile	.0652 franc per kilometre.
1 franc per square metre	7.963 pence per square yard. .6636 shilling per square yard.

1 shilling per square yard	1.510 francs per square metre.
£1 per square yard	30.194 francs per square metre.
1 franc per cubic metre	.270 penny per cubic foot. 7.281 pence per cubic yard. .607 shilling per cubic yard. .0303 £ per cubic yard.
1 penny per cubic foot	3.708 francs per cubic metre.
1 penny per cubic yard	.137 franc per cubic metre.
1 shilling per cubic yard	1.648 francs per cubic metre.
£1 per cubic yard	32.962 francs per cubic metre.
1 franc per litre	43.270 pence per gallon. 3.606 shillings per gallon.
1 franc per hectolitre	1.893 shillings per hogshead (wine).
1 shilling per hogshead	.528 franc per hectolitre.

GERMAN EMPIRE.—WEIGHTS AND MEASURES.—Tables No. 43.

From the 1st January, 1872, the French metric system of weights and measures became compulsory throughout the German Empire, as follows:—

I. GERMAN MEASURES OF LENGTH.

			French Measure.
	1 Strich	=	1 millimetre.
10 Strichs	1 New-Zoll	=	1 centimetre.
100 New-Zolls	1 Stab	=	1 metre.
10 Stabs	1 Kette	=	1 dekametre.
100 Kettes	1 Kilometre	=	1 kilometre.
7 Kilometres	1 Mile	=	7000 metres, or 4.35 English miles.

II. GERMAN MEASURES OF SURFACE.

	1 Quadrat-Stab	=	1 square metre.
100 Quadrat-Stabs	1 Ar	=	100 square metres.
100 Ars	1 Hectar	=	10,000 square metres, or 2.47 acres.

III. GERMAN MEASURES OF CAPACITY.

	1 Schoppen (Beer Measure.)	=	½ litre.
2 Schoppens	1 Kanne	=	1 litre.
50 Kannes	1 Scheffel (bushel)	=	50 litres, or 1.376 imperial bushels.
2 Scheffels	1 Fass (cask)	=	1 hectolitre, or 22.01 gallons.

The kanne is further divided into measures of ¼ kanne, ⅛ kanne, and 1/16 kanne.

IV. German Measures of Weight.

	1 Milligramm	=	1 milligramme.
10 Milligramms......	1 Centigramm	=	1 centigramme.
10 Centigramms.....	1 Dezigramm	=	1 decigramme.
100 Dezigramms	1 New-Loth	=	10 grammes, or .35273 ounce.
50 New-Loths........	1 Pfund	=	500 grammes, or ½ kilogramme, or 1.1023 pounds avoirdupois.
100 Pfunds	1 Centner	=	50 kilogrammes, or 110.23 pounds avoirdupois.
20 Centners, or 2000 Pfunds. ...	1 Tonne	=	1000 kilogrammes, or 2204.6 pounds avoirdupois.

OLD WEIGHTS AND MEASURES OF THE GERMAN STATES.

These vary for every state. The chief measures of length are the Fuss, and the Elle, of which the second is in general twice the first. The following are the values of the Fuss, which is the German foot, in the principal states.

Values of the German Fuss in the States and Free Towns of the German Empire.—Table No. 44.

Prussia	12.356 inches.
Bavaria	11.491 ,,
Würtemberg	11.279 ,,
Saxony..	11.149 ,,
Baden	11.811 ,,
Mecklenburg-Schwerin	11.457 ,,
Hesse-Darmstadt	9.843 ,,
Hesse-Cassel	11.328 ,,
Oldenburg	11.649 ,,
Brunswick	11.235 ,,
Hanover	11.500 ,,
Mecklenburg-Strelitz.....................	11.457 ,,
Anhalt...	12.356 ,,
Saxe-Coburg-Gotha	11.324 ,,
Saxe-Altenburg	11.122 ,,
Waldeck..	11.512 ,,
Lippe	11.398 ,,
Schwarzburg-Rudolstadt..................	15.047 ,,
Schwarzburg-Sondershausen:—	
(1) High Sovereignty and Arnstadt ...	11.149 ,,
(2) Low Sovereignty and Sondershausen	11.331 ,,
Reuss	11.280 ,,
Schaumburg-Lippe...........................	11.421 ,,
Hamburg	11.283 ,,
Lübeck	11.324 ,,
Bremen	11.392 ,,

KINGDOM OF PRUSSIA.—Old Weights and Measures.—Tables No. 45.

I. Prussian Measures of Length.

			English Measure.
	1 Linie	=	.0858 inch.
12 Linien	1 Zoll	=	1.0297 inches.
12 Zoll	1 Fuss	=	12.356 inches, or 1.0297 feet.
2 Fuss	1 Elle	=	2.0596 feet.
6 Ellen, or 12 Fuss	1 Ruthe	=	4.1192 yards.
2000 Ruthen	1 Meile	=	8238.4 yards, or 4.6809 miles.

Used by Miners.

	1 Lachterlinie	=	.0927 inch.
10 Lachterlinien	1 Lachterzoll	=	.9268 inch.
10 Lachterzoll	1 Achtel	=	.7723 foot.
8 Achtels, or 6 Fuss	1 Lachter	=	2.0596 yards.
9 Fuss	1 Spanne	=	6.1788 yards.

Surveyors' Measure.

	1 Scrupel	=	.0148 inch.
10 Scrupel	1 Linie	=	.1483 inch.
10 Linien	1 Zoll	=	1.4828 inches.
10 Zoll	1 Land-Fuss	=	1.2356 feet.
10 Land-Fuss	1 Ruthe	=	4.1192 yards.
2000 Ruthen	1 Meile	=	4.6809 miles.

II. Prussian Measures of Surface.

	1 Square Linie	=	.00736 square inch.
144 Square Linien	1 Square Zoll	=	1.0603 square inches.
144 Square Zoll	1 Square Fuss	=	1.0603 square feet.
144 Square Fuss	1 Square Ruthe	=	16.967 square yards.
180 Square Ruthen	1 Morgen	=	.63103 acre.
30 Morgen	1 Hufe	=	18.931 acres.

III. Prussian Measures of Volume.

Cubic Measure.

	1 Cubic Linie	=	.000632 cubic inch.
1728 Cubic Linien	1 Cubic Zoll	=	1.092 cubic inches.
1728 Cubic Zoll	1 Cubic Fuss	=	1.092 cubic feet.
1728 Cubic Fuss	1 Cubic Ruthe	=	69.893 cubic yards.

For measuring stone and brickwork, earth, peat, fascines, and firewood, the following are used:—

	1 Cubic Klafter, or 108 Cubic Fuss	= 117.93 cubic feet.
4½ Klafters	1 Haufe	= 530.70 „
1 Schachruthe (in architecture)	144 Cubic Fuss	= 157.25 „

IV. Prussian Measures of Capacity.

Dry Measure.

	1 Maasche	=	.7560 quart.
4 Mässchen, or 3 Quarts	1 Metze	=	3.0242 quarts.
4 Metzen	1 Viertel	=	3.0242 gallons.
4 Viertel, or 48 Quarts	1 Scheffel	=	1.5121 bushels, or 1.941 cubic feet.
4 Scheffeln	1 Tonne	=	6.0484 bushels.
3 Tonnes, or 12 Scheffeln	1 Malter	=	2.26815 quarters.
5 Malters, or 60 Scheffeln	1 Last	=	11.3407 quarters.

The Tonne in the table is the measure for salt, lime, and charcoal. A Tonne of flax-seed is 2.354 Scheffeln.

Liquid Measure (for Wine and Spirits).

32 Cubic Zoll	1 Ossel	=	1.0079 pints.
2 Ossel	1 Quart	=	1.0079 quarts.
30 Quarts, or 60 Ossel	1 Anker	=	7.559 gallons.
2 Ankers	1 Eimer	=	15.118 „
2 Eimers	1 Ohm	=	30.237 „
3 Eimers, or 1½ Ohm	1 Oxhoft	=	45.355 „
4 Oxhoft, or 6 Ohm	1 Fuder	=	181.422 „

V. Prussian Measures of Weight.

	1 Corn	=	4.115 grains.
10 Corns...........	1 Cent	=	.09406 dram.
10 Cents...........	1 Quentche	=	.9406 dram.
10 Quentchen.....	1 Loth	=	.588 ounce.
30 Loth	1 Zollpfund	=	1.1023 pounds.
100 Zollpfund	1 Centner	=	110.23 pounds.
20 Zollpfund	1 Stein	=	22.046 pounds.
3 Centners	1 Schiffspfund	=	330.69 pounds, or 2.506 hundredweights.
40 Centners	1 Schiffslast	=	4409.2 pounds, or 1.9684 tons.

The Tonne of coals is 2270 pounds avoirdupois, or 1.013 tons.

KINGDOM OF BAVARIA.—Old Weights and Measures.—Tables No. 46.

I. Bavarian Measures of Length.

	1 Linie	= .0798 inch.
12 Linien	1 Zoll	= .95756 inch.
12 Zoll	1 Fuss	= .95756 foot.
6 Fuss	1 Klafter	= 5.74536 feet.
10 Fuss	1 Ruthe	= 9.5756 feet.

In surveying, the Fuss is divided into 10 Zoll, and 1 Zoll into 10 Linien. The Elle contains 2 Fuss 10¼ Zoll, = 2.733 feet.

II. Bavarian Measures of Surface.

	1 Square Zoll	= .91692 square inch.
144 Square Zoll	1 Square Fuss	= .91692 square foot.
100 Square Fuss	1 Square Ruthe	= 10.188 square yards.
400 Square Ruthen	1 Tagwerk, Morgen, or Juchert	= 4075.188 square yards, or .842 acre.

III. Bavarian Measures of Volume.

	1 Cubic Zoll	= .878 cubic inch.
1728 Cubic Zoll	1 Cubic Fuss	= .878 cubic foot.
126 Cubic Fuss (6 × 6 × 3½ Fuss)	1 Klafter	= 110.628 cubic feet, or 4.097 cubic yards.

IV. Bavarian Measures of Capacity.

Dry Measure.

	1 Dreisiger	= .12745 peck.
4 Dreisigers	1 Maassl	= .12745 bushel.
4 Maassls	1 Viertel	= .5098 bushel.
2 Viertel	1 Metze	= 1.0196 bushels.
6 Metzen	1 Schäffel	= 6.1176 bushels.
4 Schäffel	1 Muth	= 3.0588 quarters.

Liquid Measure.

	1 Maaskanne	= .23529 gallon.
64 Maaskannen	1 Eimer	= 15.05856 gallons.
25 Eimer	1 Fass	= 376.464 gallons.

The Schenk-Eimer, ordinarily used in the Wine trade, contains only 60 Maaskannen, equal to 14.1174 imperial gallons.

V. Bavarian Measures of Weight.

	1 Quentchen	= .15433 ounce.
4 Quentchen	1 Loth	= .6173 ounce.
32 Loth	1 Pfund	= 1.23457 pounds.
100 Pfund	1 Centner	= 123.457 pounds, or 1.102 hundredweights.

KINGDOM OF WÜRTEMBERG.—OLD WEIGHTS AND MEASURES.—Tables No. 47.

I. WÜRTEMBERG MEASURES OF LENGTH.

	1 Punkte	=	.01128 inch.
10 Punkte	1 Linie	=	.1128 inch.
10 Linien	1 Zoll	=	1.128 inches.
10 Zoll	1 Fuss	=	.93995 foot.
10 Fuss	1 Ruthe	=	9.3995 feet.
2.144 Fuss	1 Elle	=	2.015 feet.
6 Fuss	1 Klafter	=	5.6397 feet.
26,000 Fuss	1 Meile	=	8146.25 yards, or 4.6285 miles.

II. WÜRTEMBERG MEASURES OF SURFACE.

	1 Square Zoll	=	1.272 square inches.
100 Square Zoll	1 Square Fuss	=	.8835 square foot.
100 Square Fuss	1 Square Ruthe	=	88.3506 square feet.
384 Square Ruthen	1 Morgen	=	3769.626 square yards, or .779 acre.

III. WÜRTEMBERG MEASURES OF VOLUME.

	1 Cubic Linie	=	.001434 cubic inch.
1000 Cubic Linien	1 Cubic Zoll	=	1.434 cubic inches.
1000 Cubic Zoll	1 Cubic Fuss	=	.83045 cubic foot.
144 Cubic Fuss	1 Cubic Klafter	=	119.583 cubic feet.

IV. WÜRTEMBERG MEASURES OF CAPACITY.

Dry Measure.

	1 Viertlein	=	.305 pint.
4 Viertlein	1 Ecklein	=	1.219 pints.
8 Ecklein	1 Vierling	=	1.219 gallons.
4 Vierling	1 Simri	=	4.876 gallons.
8 Simri	1 Scheffel	=	4.876 bushels.

Liquid Measure.

	1 Quart or Schoppen	=	.4043 quart.
4 Quarts	1 Helleich Maass	=	1.6173 quarts.
10 Helleich Maass	1 Imi	=	4.0433 gallons.
16 Imi	1 Eimer	=	64.6928 gallons.
6 Eimer	1 Fuder	=	388.1568 gallons.

V. WÜRTEMBERG MEASURES OF WEIGHT.

	1 Quentchen	=	.1289 ounce.
4 Quentchen	1 Loth	=	.5156 ounce.
32 Loth	1 Light Pfund	=	1.03115 pounds.
100 Heavy Pfund, or 104 Light Pfund	1 Centner	=	107.2396 pounds.
100 Light Pfund		=	103.115 pounds.

KINGDOM OF SAXONY.—Old Weights and Measures.—Tables No. 48.

I. Saxon Measures of Length.

	1 Linie	=	.07742 inch.
2 Linien	1 Zoll	=	.9291 inch.
12 Zoll	1 Fuss	=	.9291 foot.
2 Fuss	1 Elle	=	1.8582 feet.
2 Ellen	1 Stab	=	3.7165 feet.

15 Fuss, 2 Zoll	1 Ruthe (Land Measure)	=	4.6972 yards.
16 Fuss	1 Ruthe (Road Measure)	=	4.9553 yards.
	1 Lachter (Mining)	=	2.1873 yards.
1324.987 Ellen	1 Meile Post	=	4.6604 miles.

II. Saxon Measures of Surface.

	1 Square Zoll	=	.8632 square inch.
144 Square Zoll	1 Square Fuss	=	.8632 square foot.
300 Square Ruthen	1 Acker	=	1.4865 acres.

III. Saxon Measures of Volume.

	1 Cubic Zoll	=	.8021 cubic inch.
1728 Cubic Zoll	1 Cubic Fuss	=	.8021 cubic foot.
108 Cubic Fuss	1 Klafter	=	86.624 cubic feet.
3 Klafter	1 Schragen	=	259.873 cubic feet.

The Klafter is 6 Fuss by 6 Fuss by 3 Fuss. The Schragen is used in the measurement of firewood.

IV. Saxon Measures of Capacity.

Dry Measure.

	1 Mäasche	=	1.4463 quarts.
4 Mäaschen	1 Metze	=	1.4463 gallons.
4 Metzen	1 Viertel	=	5.7852 gallons.
4 Viertel	1 Scheffel	=	2.8926 bushels.
12 Scheffel	1 Malter	=	34.7124 bushels.
2 Malter	1 Wispel	=	69.4249 bushels.

Liquid Measure.

	1 Quartier	=	.2059 pint.
4 Quartier	1 Nossel	=	.8237 pint.
2 Nossel	1 Kanne	=	1.6474 pints.
36 Kannen	1 Anker	=	7.4237 gallons.
2 Anker	1 Eimer	=	14.8262 gallons.
3 Eimer	1 Oxhoft	=	44.4687 gallons.
6 Eimer	1 Fass or Barrel	=	88.9374 gallons.

V. Saxon Measures of Weight.

The old Saxon measures of weight are the same as those of Prussia.

GRAND DUCHY OF BADEN.—Old Weights and Measures.—Tables No. 49.

I. Baden Measures of Length.

	1 Punkte	=	.0118 inch.
10 Punkte	1 Linie	=	.118 inch.
10 Linien	1 Zoll	=	1.181 inches.
10 Zoll	1 Fuss	=	.9842 foot.
2 Fuss	1 Elle	=	1.9685 feet.
10 Fuss	1 Ruthe	=	9.8427 feet.
6 Fuss	1 Klafter	=	5.9055 feet.
14814.815 Fuss	1 Stunde	=	4860.59 yards.
2 Stunden	1 Meile	=	5.5234 miles.

II. Baden Measures of Surface.

	1 Square Zoll	=	1.3951 square inches.
100 Square Zoll	1 Square Fuss	=	.9688 square foot.
100 Square Fuss	1 Square Ruthe	=	10.7643 square yards.
100 Square Ruthen	1 Viertel	=	1076.43 square yards.
4 Viertel	1 Morgen	=	4305.72 square yards, or .8896 acre.

III. Baden Measures of Volume.

	1 Cubic Fuss	=	.95335 cubic foot.
144 Cubic Fuss	1 Klafter	=	137.28 cubic feet.

IV. Baden Measures of Capacity.

Liquid Measure.

	1 Glass	=	1.0563 gills.
10 Glass	1 Maass	=	1.3204 quarts.
10 Maass	1 Stutze	=	3.3014 gallons.
10 Stutzen	1 Ohm	=	33.014 gallons.
10 Ohm	1 Fuder	=	330.14 gallons.

Dry Measure.

	1 Becher	=	.2643 pint.
10 Becher	1 Mäasslein	=	.1652 peck.
10 Mäasslein	1 Sester	=	.4127 bushel.
10 Sester	1 Malter	=	4.1268 bushels.
10 Malter	1 Zuber	=	41.2679 bushels.

V. Baden Measures of Weight.

	1 As	=	.7716 grain.
10 As	1 Pfennig	=	7.716 grains.
10 Pfennig	1 Centas	=	.1764 ounce.
10 Centas	1 Zehnling	=	1.7637 ounces.
10 Zehnling	1 Pfund	=	1.1023 pounds.
100 Pfund	1 Centner	=	110.230 pounds.

THE HANSE TOWNS.—OLD WEIGHTS AND MEASURES.— Tables No. 50.

HAMBURG.—WEIGHTS AND MEASURES.

I. HAMBURG MEASURES OF LENGTH.

	1 Achtel	=	.1175 inch.
8 Achtel	1 Zoll	=	.9402 inch.
12 Zoll	1 Fuss	=	.9402 foot.
2 Fuss	1 Elle	=	1.8804 feet.
6 Fuss	1 Klafter, or Faden	=	5.6413 feet.
14 Fuss	1 Marsch-Ruthe	=	13.1629 feet.
16 Fuss	1 Geest-Ruthe	=	15.0434 feet.

The Hamburg Elle above is used for silk, linen, and cotton goods. The Brabant Elle is equal to 1 1/5 Hamburg Elle; and 4 of them are reckoned equal to 3 yards. The Prussian Ruthe is also used. The Prussian Fuss is used in surveying.

II. HAMBURG MEASURES OF SURFACE.

	1 Square Zoll	=	.8840 square inch.
144 Square Zoll	1 Square Fuss	=	.8840 square foot.
196 Square Fuss	1 Square Marsch-Ruthe	=	173.26 square feet.
256 Square Fuss	1 Square Geest-Ruthe	=	226.30 square feet.
200 Square Geest-Ruthen	1 Scheffel Geest-Land	=	5028.98 square yards, or 1.039 acres.
600 Sq. Marsch-Ruthen	1 Morgen	=	11550.93 square yards, or 2.386 acres.

III. HAMBURG MEASURES OF VOLUME.

	1 Cubic Zoll	=	.8311 cubic inch.
1728 Cubic Zoll	1 Cubic Fuss	=	.8311 cubic foot.
88.9 Cubic Fuss	1 (Cubic) Klafter	=	73.88 cubic feet.
120 Cubic Fuss	1 Tehr	=	99.73 cubic feet.

IV. HAMBURG MEASURES OF CAPACITY.

Liquid Measure.

	1 Ossel	=	.09965 gallon.
2 Ossel	1 Quartier	=	.1993 gallon.
2 Quartier	1 Kanne	=	.3987 gallon.
2 Kannen	1 Stubchen	=	.7974 gallon.
1 Stubchen	1 Viertel	=	1.5947 gallons.
4 Viertel	1 Eimer	=	6.3788 gallons.
5 Viertel	1 Anker	=	7.9735 gallons.
6 Eimer	1 Tonne	=	38.2728 gallons.
4 Anker	1 Ohm	=	31.8940 gallons.
6 Anker	1 Oxhoft	=	47.8410 gallons.
6 Ohm	1 Fuder, or Tonneau	=	191.3640 gallons.

The above are measures for Wines and Spirits. For Beer, there are three sizes of Tonne, containing respectively 48, 40, and 32 Stubchen.

Dry Measure.

	1 Small Maass	=	.0236 bushel.
2 Small Maass.........	1 Large Maass	=	.0473 bushel.
4 Large Maass.........	1 Spint	=	.1890 bushel.
4 Spint..................	1 Himten	=	.7560 bushel.
2 Himten................	1 Fass	=	1.5121 bushels.
2 Fass....................	1 Scheffel	=	3.0242 bushels.
10 Scheffeln	1 Wispel	=	30.2416 bushels.
3 Wispel	1 Last	=	90.7248 bushels.

For barley and oats, the Scheffel contains 3 Fass.

V. Hamburg Measures of Weight.

	1 Half Gramme	=	.0011 pound	=	.5 gramme.
10 Half Grammen	1 Quint	=	.01102 pound	=	5 grammes.
10 Quinten	1 (New) Unze	=	.11023 pound	=	50 „
10 (New) Unzen..	1 (New) Pfund	=	1.10232 pounds	=	500 „
100 (New) Pfund	1 Centner	=	110.232 pounds	=	50 kilog.
60 Centners........	1 (Commercial) Last	=	6613.92 pounds, or 2.953 tons	=	3000 kilog.

This, it is apparent, is a metric system of weights, which was comparatively recently introduced and adopted at Hamburg. It is now, of course, overruled by the French metric system enforced for the German Empire.

BREMEN.—Old Weights and Measures.

The Fuss is equal to 11.392 inches, and the Klafter is equal to 5.696 feet. The Morgen = .6368 acre. The principal measures for wines and spirits are the Viertel = 1.56 gallons; the Anker = 5 Viertels = 7.80 gallons; the Oxhoft = 46.80 gallons. The Scheffel, for dry goods = 2.0388 bushels. The old weights are the same as those of Hamburg.

LUBEC.—Old Weights and Measures.

The Fuss is equal to 11.324 inches. The Viertel = 1.60 gallons; the Anker = 8 gallons; the Oxhoft = 48.04 gallons. The Scheffel, for dry goods, = .9545 bushel. The old Pfund = 1.0725 pounds, and the Centner = 1.0725 cwts.

GERMAN CUSTOMS UNION.—Old Weights and Measures.—Table No. 51.

Centner........................	110.23 pounds (50 kilogrammes).
Ship-Last of timber.......	about 80 cubic feet.
Scheffel........................	1.512 bushels.
Klafter	6 feet.

In Oldenburg, Hanover, Brunswick, Saxe-Altenbourg, Birkenfeld, Anhalt, Waldeck, Reuss, and Schaumburg-Lippe, the old system of weights is the same as that of Prussia.

AUSTRIAN EMPIRE.—Weights and Measures.—Tables No. 52.

I. Austrian Measures of Length.

	1 Punkte	=	.0072 inch.
12 Punkte	1 Linie	=	.0864 inch.
12 Linien	1 Zoll	=	1.0371 inches.
12 Zoll	1 Fuss	=	1.0371 feet.
2 Fuss	1 Elle	=	2.0742 feet.
6 Fuss	1 Klafter	=	6.2226 feet.
4000 Klafter	1 Meile (post)	=	8297 yards, or 4.714 miles.

II. Austrian Measures of Surface.

	1 Square Zoll	=	1.0756 square inches.
144 Square Zoll	1 Square Fuss	=	1.0756 square feet.
36 Square Fuss	1 Square Klafter	=	38.7225 square feet, or 4.3025 square yards.
8⅓ Square Klafter, or 300 Square Fuss	1 Square Ruthe	=	35.854 square yards.
64 Square Ruthen	1 Metze	=	2294.7 square yards.
3 Metzen, or 1600 Square Klafter	1 Joch	=	6884 square yards, or 1.4223 acres.

III. Austrian Measures of Volume.

Cubic Measure.

	1 Cubic Zoll	=	1.1155 cubic inches.
1728 Cubic Zoll	1 Cubic Fuss	=	1.1155 cubic feet.
216 Cubic Fuss	1 Cubic Klafter	=	240.94 cubic feet, or 8.924 cubic yards.

IV. Austrian Measures of Capacity.

Dry Measure.

	1 Probmetzen	=	.10575 pint, or 3.665 cubic inches.
8 Probmetzen	1 Becher	=	.8460 pint.
4 Becher	1 Futtermassel	=	1.6920 quarts.
2 Futtermassel	1 Muhlmassel	=	3.3840 quarts, or .8460 gallon.
2 Muhlmassel	1 Achtel	=	1.6920 gallons.
2 Achtel	1 Viertel	=	3.3840 gallons, or .4230 bushel.
4 Viertel	1 Metze	=	1.6918 bushels.
30 Metzen	1 Muth	=	50.7536 bushels, or 8.3442 quarters.

Liquid Measure.

	1 Pfiff	=	1.246 gills, or 10.781 cubic inches.
2 Pfiff	1 Seidel	=	2.491 cubic inches, or .6229 pint.
2 Seidel	1 Kanne	=	1.2457 pints.
2 Kannen	1 Mass	=	1.2457 quarts.
10 Mass	1 Viertel	=	3.1143 gallons.
4 Viertel	1 Eimer	=	12.4572 gallons.
32 Eimer	1 Fuder	=	398.6304 gallons.

V. Austrian Measures of Weight.

	1 Pfenning	=	270.1 grains, or .6173 dram.
4 Pfenning	1 Quentchen	=	2.4694 drams.
4 Quentchen	1 Loth	=	9.8776 drams, or .6173 ounce.
2 Loth	1 Unze	=	1.2347 ounces.
4 Unzen	1 Vierdinge	=	4.9388 ounces.
2 Vierdinges	1 Mark	=	9.8776 ounces, or .6173 pound avoirdupois.
2 Marks, or 16 Unzen	1 Pfund	=	1.2347 pounds avoirdupois.
100 Pfund	1 Centner	=	123.47 pounds avoirdupois, or 1.1024 hundredweights.

In 1853, a pfund of 500 grammes, with decimal subdivisions, was adopted for customs and fiscal purposes.

RUSSIA.—Weights and Measures.—Tables No. 53.

I. Russian Measures of Length.

			English Equivalent.
	1 Vershok	=	1.75 inches.
16 Vershoks	1 Arschine	=	28 "
3 Arschines	1 Sajene	=	7 feet.
500 Sajenes	1 Verst	=	3500 feet, or 1166⅔ yards, or 0.6629 mile.

The Fuss, or Russian foot, is 13.75 inches; but, since 1831, the English foot of 12 inches has been used as the ordinary standard of length, each inch being divided into 12 parts.

1 Lithuanian Meile	5.5574 English miles.
1 Rhein Fuss, used in calculating export duties on timber	1.03 English feet.

II. Russian Measures of Surface.

	1 Square Arschine	=	784 square inches, or 5.444 square feet.
9 Square Arschines..	1 Square Sajene	=	49 square feet, or 5.444 square yards.
2400 Square Sajenes.....	1 Desatine	=	13,066 square yards, or 2.70 acres.

For earthworks, masonry, &c., the Sajene is divided into *tenths* (dessiatka), *hundredths* (sotka), and *thousandths* (tisiatchka), which are used as a basis for lineal, superficial, and cubic measurements, similarly to the French metre with its sub-multiples.

III. Russian Measures of Capacity.

Liquid Measure.

	1 Tscharkey	=	.8656 gill, or .2164 pint.
10 Tscharkeys......	1 Kruschka	=	1.0820 quarts.
100 Tscharkeys......	1 Vedro	=	2.7049 gallons.
3 Vedros............	1 Anker	=	8.1147 ,,
40 Vedros, or 13 1/3 Ankers	1 Sarokowaja Boshka	=	108.196 ,,

Dry Measure (Grain).

	1 Garnietz	=	2.885 quarts.
2 Garnietz	1 Tschetwerka	=	1.4424 gallons.
4 Tschetwerkas...	1 Tschetwerik	=	.7213 bushel.
2 Tschetweriks....	1 Pajak	=	1.4426 bushels.
2 Pajaks............	1 Osmin	=	2.8852 ,,
2 Osmins	1 Tschetwert*	=	5.7704 ,,
16 Tschetwerts.....	1 Last	=	11.5408 quarters, or 1.154 imperial lasts.

* A Tschetwert is usually reckoned as 5¾ bushels, and 100 Tschetwerts as 72 quarters, though they are more exactly 72.1308 quarters.
100 quarters are equal to 138.637 Tschetwerts.

IV. Russian Measures of Weight.

	1 Dolis	=	.68576 grain.
96 Dolis.........	1 Zolotnick	=	65.833 grains, or .1505 ounce.
3 Zolotnicks...	1 Lotti	=	.4514 ,,
8 Zolotnicks...	1 Lana	=	1.2037 ounces.
12 Lanas, or 32 Lottis, or 96 Zolotnicks	1 Funt, or pound	=	.90285 pound avoirdupois, or 14.446 ounces, or 6320 grains.
40 pounds	1 Pood	=	36.114 pounds avoirdupois.
10 Poods.........	1 Berkovitz	=	361.14 pounds avoirdupois, or 3.224 hundredweights.
3 Berkovitz.....	1 Packen	=	9.672 hundredweights.

62.0257 Poods	1 English ton.
2481.0268 Russian pounds	1 „

The Pood is commonly estimated at 36 pounds avoirdupois.

The Nuremberg pound, used for apothecaries' weight, weighs 5527 grains, or about .96 pound troy.

The Ship-Last is equal to 2 tons nearly.

The Carat, for weighing pearls and precious stones, is about 3 $^1/_6$ grains.

HOLLAND.

The metric system was adopted in Holland in 1819; the denominations corresponding to the French are as follows:—

Length.—Millimetre, Streep; centimetre, Duim; decimetre, Palm; metre, El; decametre, Roede; kilometre, Mijle.

Surface.—Square millimetre, Vierkante Streep; square centimetre, Vierkante Duim; and so on. Hectare, Vierkante Bunder.

Cubic Measure.—Millistere, Kubicke Streep, and so on.

Capacity.—Centilitre, Vingerhoed; decilitre, Maatje; liquid litre, Kan; dry litre, Kop; decalitre, Schepel; liquid hectolitre, Vat or Ton; dry hectolitre, Mud or Zak; 30 hectolitres = 1 Last = 10.323 quarters.

Weight.—Decigramme, Korrel; gramme, Wigteje; decagramme, Lood; hectogramme, Onze; kilogramme, Pond.

BELGIUM.

The French metric system is used in Belgium. The name *Livre* is substituted for kilogramme, *Litron* for litre, and *Aune* for metre.

DENMARK.

WEIGHTS AND MEASURES.—Tables No. 54.

I. DANISH MEASURES OF LENGTH.

	1 Linie	=	.0858 inch.
12 Linier	1 Tomme	=	1.0297 inches.
12 Tommer	1 Fod	=	1.0297 feet.
2 Fod	1 Alen	=	2.0594 „
3 Alen, or 6 Fod	1 Favn	=	6.1783 „
2 Favn, or 12 Fod	1 Rode	=	12.3567 „
2,000 Roder, or 24,000 Fod	1 Mil	=	8237.77 yards, or 4.68055 miles.
23,642 Fod	1 nautical mile	=	4.61072 English miles.

II. DANISH MEASURES OF SURFACE.

144 Square Linie	1 Square Tomme	=	1.0603 square inches.
144 Square Tomme	1 Square Fod	=	1.0603 square feet.
144 Square Fod	1 Square Rode	=	16.966 square yards.

III. Danish Measures of Volume.

1728 Cubic Linier......	1 Cubic Tomme	=	1.0918 cubic inches.
1728 Cubic Tomme....	1 Cubic Fod	=	1.0918 cubic feet.

The Favn of firewood measures $6 \times 6 \times 2$ Fod = 72 cubic Fod = 78.60 cubic feet. In forest measure it is $6\frac{1}{2} \times 6\frac{1}{2} \times 2$ Fod = $84\frac{1}{2}$ cubic Fod = 92.26 cubic feet.

IV. Danish Measures of Capacity.

Liquid Measure.

	1 Paegle	=	.4248 pint.
4 Paegle	1 Pot	=	1.6991 pints.
2 Potter	1 Kande	=	3.3983 ,,
38 Potter	1 Anker	=	8.0709 gallons.
136 Potter	1 Tonde	=	28.885 ,,
6 Ankerne	1 Oxehoved	=	48.4256 ,,
4 Oxehoveder..........	1 Fad	=	193.7027 ,,

Dry Measure.

	1 Pot	=	1.6991 pints.
18 Potter...............	1 Skeppe	=	3.8232 gallons.
2 Skepper............	1 Fjerdingkar	=	.9558 bushel.
4 Fjerdingkar	1 Tönde	=	3.8231 bushels.
12 Tönder	1 Laest	=	45.8769 ,,

V. Danish Measures of Weight.

	1 Ort	=	7.7163 grains.
10 Ort	1 Kvint	=	77.163 ,,
100 Kvinten	1 Pund	=	1.1023 pounds.
100 Pund............	1 Centner	=	110.23 ,,
40 Centner.........	1 Last	=	1.9684 tons.
52 Centner.........	1 Skip-Last	=	2.5590 ,,
16 Pund	1 Lispund	=	17.637 pounds.
320 Pund	1 Skippund	=	3.149 cwts.

SWEDEN.—Weights and Measures.—Tables No. 55.

I. Swedish Measures of Length.

	1 Linie	=	.1169 inch.
10 Linier.........	1 Tum	=	1.1689 inches.
10 Tumer	1 Fot	=	11.6892 ,,
10 Fot............	1 Stang	=	9.7411 feet.
10 Stanger	1 Ref	=	32.4703 yards.
360 Ref............	1 Meile	=	11,689.308 yards, or 6.6417 miles.
2 Fot............	1 Aln	=	1.942 feet.
6 Fot............	1 Faden	=	5.845 ,,

II. Swedish Measures of Surface.

100 Square Linier...	1 Square Tum	=	1.3666 square inches.
100 Square Tumer..	1 Square Fot	=	.9489 square foot.
100 Square Fot......	1 Square Stang	=	3.5146 square yards.
100 Square Stanger	1 Square Ref	=	{ 1054.2 square yards, or .2178 acre.
4 Square Fot......	1 Square Aln	=	3.7956 square feet.
5.6 Square Ref....	1 Tunnland	=	{ 5798.1 square yards, or 1.2198 acres.

III. Swedish Measures of Volume.

Cubic Measure.

	1 Cubic Tum	=	1.5972 cubic inches.
1000 Cubic Tumer.........	1 Cubic Fot	=	.9263 cubic foot.
8 Cubic Fot............	1 Cubic Aln	=	7.4104 cubic feet.

Liquid and Dry Measure.

1000 Cubic Linier	1 Cubic Tum	=	.1843 gill.
100 Cubic Tumer	1 Kanna	=	2.3096 quarts.
10 Kanna	1 Cubic Fot	=	5.774 gallons.
8 Cubic Fot.............	1 Cubic Aln	=	46.192 ,,

IV. Swedish Measures of Weight.

	1 Korn	=	.6564 grain.
100 Korn..............	1 Ort	=	2.4005 drams.
100 Ort.................	1 Skälpund	=	.9377 pound.
100 Skälpund	1 Centner	=	{ 93.7729 pounds, or .8372 cwt.
100 Centner............	1 Ny-Läst	=	4.1892 tons.

The metric system will become obligatory in 1889.

NORWAY.

The French metric system is in force in Norway.

SWITZERLAND.—Weights and Measures.—Tables No. 56.

I. Swiss Measures of Length.

	1 Striche....................	=	.01181 inch.
10 Striche......	1 Linie	=	.11811 ,,
10 Linien......	1 Zoll.....................	=	1.18112 inches.
10 Zoll	1 Fuss (3 decimetres)....	=	11.81124 ,,
2 Fuss.........	1 Elle	=	1.9685 feet.
6 Fuss.........	1 Klafter	=	5.9056 ,,
10 Fuss.........	1 Ruthe	=	9.8427 ,,
1600 Ruthen	1 Schweizer-stunde, or Lien	=	{ 5249.44 yards, or 2.9826 miles.

II. Swiss Measures of Surface.

	1 Square Zoll	=	1.3947 square inches.
100 Square Zoll......	1 Square Fuss	=	.9688 square foot.
36 Square Fuss......	1 Square Klafter	=	34.8768 square feet.
100 Square Fuss......	1 Square Ruthe	=	10.7643 square yards
400 Square Ruthen..	1 Juchart	=	.8694 acre.
6400 Jucharten........	1 Square Stunde	=	5693.52 acres.

350 Square Ruthen.......... 1 Juchart, of meadow land.
450 Square Ruthen........... 1 Juchart, of woodland.

III. Swiss Measures of Volume.

	1 Cubic Zoll	=	1.6476 cubic inches.
1000 Cubic Zoll........	1 Cubic Fuss	=	.9535 cubic foot.
216 Cubic Fuss.......	1 Cubic Klafter	=	7.6172 cubic yards.
1000 Cubic Fuss.......	1 Cubic Ruthe	=	35.3166 ,,

IV. Swiss Measures of Capacity.

Dry Measure.

	1 Imi	=	1.3206 quarts.
10 Imi	1 Maass	=	.4127 bushel.
10 Maass	1 Malter	=	4.1268 bushels.

Liquid Measure.

2 Halbschoppen..........	1 Schoppen	=	2.6412 gills.
2 Schoppen	1 Halbmaass	=	1.3206 pints.
2 Halbmaass	1 Maass	=	2.6412 ,,
100 Maass	1 Saum	=	33.015 gallons.

V. Swiss Measures of Weight.

	1 Quntli	=	2.2048 drams.
4 Quntli.........	1 Loth	=	.5511 ounce.
2 Loth	1 Unze	=	1.1023 ounces.
16 Unzen	1 Pfund	=	1.1023 pounds.
100 Pfund..........	1 Centner	=	110.233 pounds, or .9842 cwt.

The Pfund is divided into halves, quarters, and eighths. It is also divided into 500 Grammes, and decimally into Decigrammes, Centigrammes, and Milligrammes.

SPAIN.—Weights and Measures.—Tables No. 57.

The French metric system was established in Spain in 1859. The metre is named the Metro; the litre, Litro; the gramme, Grammo; the are, Area; the tonne, Tonelada. The metric system is established likewise in the Spanish colonies. The old weights and measures are still largely used.

I. Old Spanish Measures of Length.

	1 Punto	=	.00644 inch.
12 Puntos	1 Linea	=	.07725 inch.
12 Lineas	1 Pulgada	=	.927 inch.
6 Pulgadas	1 Sesma	=	5.564 inches.
2 Sesmas	1 Pies de Burgos	=	.9273 foot.
3 Pies de Burgos	1 Vara	=	2.782 feet.
2 Varas	1 Estado	=	5.564 feet.
4 Varas	1 Estadal	=	11.128 feet.
5000 Varas	1 Legua (Castilian)	=	2.6345 miles.
8000 Varas	1 Legua (Spanish)	=	4.2151 miles.

II. Old Spanish Measures of Surface.

	1 Square Pies	=	.860 square foot.
9 Square Pies	1 Square Vara	=	.860 square yard.
16 Square Varas.....	1 Square Estadal	=	13.759 square yards.
50 Square Varas	1 Estajo	=	42.997 square yards.
576 Square Estadals.	1 Fanegada	=	1.6374 acres.
50 Fanegadas........	1 Yugada	=	81.870 acres.

III. Old Spanish Measures of Capacity.

Liquid Measure.

	1 Capo	=	.888 gill.
4 Capos	1 Cuartillo	=	.111 gallon.
4 Cuartillos	1 Azumbre	=	.444 gallon.
2 Azumbres......	1 Cuartilla	=	.888 gallon.
4 Cuartillas...	1 Arroba Mayor, or Cantara (for wine)	=	3.552 gallons.
16 Cantaras.......	1 Mayo	=	56.832 gallons.

The old measure for oil is the Arroba Menor = 2.7652 gallons.

Dry Measure.

	1 Ochavillo	=	.00785 peck.
4 Ochavillos.........	1 Racion	=	.0314 peck.
4 Raciones...........	1 Quartillo	=	.0314 bushel.
2 Quartillos..........	1 Medio	=	.0628 bushel.
2 Medios	1 Almude	=	.1256 bushel.
12 Amuerzas	1 Fanega	=	1.5077 bushels.
12 Fanegas............	1 Cahiz	=	18.0920 bushels.

IV. Old Spanish Weights.

	1 Grano	=	.771 grain.
12 Granos	1 Tomin	=	9.247 grains.
3 Tomines	1 Adarme	=	27.74 grains.
2 Adarmes	1 Ochavo, or Drachma	=	.1268 ounce.
8 Ochavos	1 Onza	=	1.0144 ounces.
8 Onzas	1 Marco	=	8.1154 ounces.
2 Marcos......	1 Libra (Castilia a)	=	1.0144 pounds.
100 Libras	1 Quintal	=	101.442 pounds.
10 Quintals.....	1 Tonelada	=	1014.42 pounds.

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PORTUGAL.

The French metric system of weights and measures was adopted in its entirety during the years 1860–63, and was made compulsory from the 1st October, 1868. The chief old measures still in use are, the Libra = 1.012 pounds; Almude, of Lisbon = 3.7 gallons; Almude, of Oporto = 5.6 gallons; Alquiere = 3.6 bushels; Moio = 2.78 quarters.

ITALY.

The French metric system is used in Italy. The metre is named the Metra; the are, Ara; the stère, Stero; the litre, Litro; the gramme, Gramma; the tonneau métrique, Tonnelata de Mare. The various old weights and measures of the different Italian States are still occasionally used.

TURKEY.

Length.—1 Pike or Drâ = 27 inches, divided into 24 Kerâts; 1 Forsang = 3.116 miles, divided into 3 Berri; the Surveyor's Pik, or the Halebi = 27.9 inches; and 5½ Halebis = 1 reed.

Surface.—The squares of the Kerât, the Pike, and the Reed. The Feddan is an area equal to as much as a yoke of oxen can plough in a day.

Capacity, Dry.—The Rottol = 1.411 quarts, contains 900 Dirhems; 22 Rottols = 1 Killow = 7.762 gallons, or .97 bushel, the chief measure for grain.

Liquid.—1 Oka = 1.152 pints; 8 Oke = 1 Almud = 1.152 gallons; 1 Rottol = 2.5134 pints; 100 Rottols = 1 Cantar = 31.417 gallons.

Weights.—The Oke = 2.8342 pounds, divided into 4 Okiejehs, or 400 Dirhems of 1.81 drams; 1 Rottolo = 1.247 pounds; 100 Rottolos = 1 Cantar = 124.704 pounds.

GREECE AND IONIAN ISLANDS.

The French metric system is employed in Greece. The metre is named the Pecheus; kilometre, Stadion; are, Stremma; litre, Litra; gramme, Drachmé. 1½ kilogrammes = 1 Mnâ; 1½ Quintals = 1 Tolanton 1½ Tonneaux = 1 Tonos = 29.526 cwts.

In the Ionian Islands, whilst they were under the protection of Great Britain (1830 to 1864), the British weights and measures were those in use, with Italian names. The foot was named the Piede; the yard, the Jarda; the pole, the Carnaco; the furlong, the Stadio; the mile, the Miglio. The gallon was the Gallone; the bushel, the Chilo; the pint, the Dicotile; the pound avoirdupois, the Libra Grossa; the pound troy, the Libra Sottile. The Talanto consisted of 100 pounds, and the Miglio of 1000 pounds.

MALTA.

In round numbers, 3½ Palmi = 1 yard; 1 Canna = 2 $^2/_7$ yards.

The Salma = 4.964 acres. Approximately, 543 Square Palmi = 400 square feet; 16 Salmi = 71 acres.

1 Cubic Tratto = 8 cubic feet; 144 Cubic Palmi = 96 cubic feet; 1 Cubic Canna = 343 cubic feet.

Approximate weights:—15 Oncie = 14 ounces; 1 Rotolo = 1¾ pounds; 4 Rotoli = 7 pounds; 64 Rotoli = 1 cwt.; 1 Cantaro = 175 pounds; 1 Quintal = 199 pounds; 64 Cantari = 5 tons.

EGYPT.—Weights and Measures.—Tables No. 58.

I. Egyptian Measures of Length.

Pik, or cubit of the Nilometre	20.65 inches.
Pik, indigenous	22.37 ,,
Pik, of merchandise	25.51 ,,
Pik, of construction	29.53 ,,

6 Palms	1 Pik.	
24 Kirats	1 Pik or Drâa.	
4.73 Piks of construction	1 Kassaba in surveying,	= 11.65 feet.

II. Egyptian Measures of Surface.

	1 Square Pik	= 6.055 square feet.
22.41 Square Piks	1 Square Kassaba	= 15.07 square yards
333.33 Square Kassaba,	1 Feddan	= .9342 acre.

III. Egyptian Measures of Capacity.

	1 Kadah	= 1.684 pints.
2 Kadahs	1 Milwah	= 6.735 ,,
2 Milwahs	1 Roobah	= 1.684 gallons.
2 Roobahs	1 Kelah	= 3.367 ,,
2 Kelehs	1 Webek	= 6.734 ,,
6 Webeks	1 Ardeb	= { 40.404 gallons, or 6.48 cubic feet.

The Guirbah of water (a government measure) is $^1/_{15}$ cubic metre = 66⅔ litres, or 11.772 cubic feet.

IV. Egyptian Measures of Weight.

	1 Kamhah	= .746 grain.
4 Kamhahs	1 Kerat.	
16 Kerats	1 Dirhem	= 1.792 drachms.
24 Kerats	1 Mitkal.	
8 Mitkals	1 Okieh.	
12 Okiehs, or 144 Dirhems	1 Rottol	= .9821 pound.
100 Rottols	1 Kantar	= 98.207 pounds.

400 Dirhems	1 Oke	= 2.728 ,,
36 Okes	1 Kantar	= 98.207 ,,

MOROCCO.

Length.—The Tomin = 2.81025 inches; the Dra'a = 8 Tomins = 22.482 inches.

Capacity.—The Muhd = 3.08135 gallons; the Saâ = 4 Muhds = 12.3254 gallons.

Weights.—The Uckia = 392 grains; the Rotal or Artal = 20 Uckieh = 1.12 pounds; the Kintar = 100 Rotales = 112 pounds.

Oil is sold by the Kula = 3.3356 gallons. Other liquids are sold by weight.

TUNIS.

Length.—The Dhraâ, or Pike, is the unit of length. The Arabian Dhraâ, for cotton goods = 19.224 inches; the Turkish Dhraâ, for lace = 25.0776 inches; the Dhraâ Endaseh, for woollen goods = 26.4888 inches.

The Mil Sah'ari = .9149 mile.

Capacity.—For dry goods the Saâ = 1.2743 pint; 12 Saâ = 1 Hueba = 6.8228 gallons.

For liquids, the Pichoune = .4654 pint; 4 Pichounes = 1 Pot = 1.8616 pints; 15 Pots = 1 Escandeau, and 4 Escandeaux = 1 Millérole = 13.9623 gallons.

ARABIA.

The weights and measures of Egypt are used in Arabia.

CAPE OF GOOD HOPE.

The standard weights and measures are British, with the exception of the land measure. To some extent, the old British and the Dutch measures are in use. The general measure of surface is the old Amsterdam *Morgen*, reckoned equal to 2 acres; though the exact value is equal to 2.11654 acres. 1000 Cape feet are equal to 1033 British feet.

INDIAN EMPIRE.—Weights and Measures.

An Act "to provide for the ultimate adoption of an uniform system of weights and measures of capacity throughout British India" was passed in October, 1871. The *ser* is adopted under the Act as the primary standard or unit of weight, and is a weight of metal in the possession of the Government, equal, when weighed in a vacuum, to one kilogramme. The unit of capacity is the volume of one *ser* of water at its maximum density, equivalent to the litre. Other weights and measures are to be multiples or submultiples of the ser, and of the volume of one ser of water.

The following are the weights and measures in common use in India:—

BENGAL.—Weights and Measures.—Tables No. 59.

I. Bengal Measures of Length.

	1 Jow, or Jaub	=	¼ inch.
3 Jow	1 Ungulee...........	=	¾ "
4 Ungulees	1 Moot	=	3 inches.
3 Moots	1 Big'hath, or Span	=	9 "
2 Big'haths	1 Hât'h, or Cubit...	=	18 "
2 Hât'h	1 Guz	=	1 yard.
2 Guz	1 Danda, or Fathom	=	2 yards.
1000 Dandas	1 Coss	=	2000 yards, or 1.1364 miles.
4 Coss	1 Yojan	=	4.5454 miles.

II. Bengal Measures of Surface.

	1 Square Hât'h	=	2.25 square feet.
4 Square Hât'hs.....	1 Cowrie	=	1 square yard.
4 Cowries	1 Gunda	=	4 square yards.
20 Gundas	1 Cottah	=	80 "
20 Cottahs	1 Beegah	=	1600 square yards, or .3306 acre.

For land measure, the following table is used for Government surveys:—

	1 Guz	=	33 lineal inches.
3 Guz	1 Baus, or Rod	=	8¼ lineal feet.
9 Square Guz......	1 Square Rod	=	68 $^1/_{16}$ square feet.
400 Square Rods.....	1 Beegah	=	3025 square yards, or .625 acre.

III. Bengal Measures of Capacity.

The Seer is a measure common to liquids and dry goods. It is taken at 68 cubic inches, or 1.962 pints, in volume. But it varies in different localities. 5 Seer = 1 Palli, and 8 Palli = 1 Maund, or 9.81 gallons. The Sooli = 3.065 bushels, and 16 Soolis = 1 Khahoon, or 49.05 bushels.

IV. Bengal Measures of Weight.

The Tolah, or weight of a Rupee, 180 grains, is the unit of weight.

	1 Tolah	=	180 grains.
5 Tolahs	1 Chittâk	=	900 "
16 Chittâks	1 Seer	=	2.057 pounds.
5 Seers	1 Passeeree	=	10.286 "
8 Passeerees................	1 Maund	=	82.286 "

MADRAS.—Weights and Measures.—Tables No. 60.

I. Madras Measures of Length.

The English foot and yard are used. The Guz is 33 inches. The Baum or fathom is about 6½ feet. A Nâlli-Valli is a little under 1½ miles. 7 Nâlli-Valli = 1 Kâdam, or about 10 miles. The following are native measures:—

8 Torah	1 Vurruh	= .4166 inch.
24 Vurruh	1 Mulakoli	= 10 inches.
4 Mulakoli	1 Dumna	= 40 ,,

II. Madras Measures of Surface.

The English acre is generally known. The native measures are uncertain. In Madras and some other districts, the following native measures are used:—

	1 Coolie =	64 square yards.
4 1/6 Coolies	1 Ground =	266⅔ square yards.
24 Grounds, or 100 Coolies	1 Cawnie =	6400 square yards, or 1.3223 acres.
16 Annas (each 400 yards),	1 Cawnie.	

III. Madras Measures of Capacity.

	1 Olluck	= .361 pint.
8 Ollucks	1 Puddee	= 1.442 quarts.
8 Puddees	1 Mercâl	= 2.885 gallons.
5 Mercâls	1 Parah	= 14.426 ,,
80 Parahs	1 Garce	= 18.033 quarters.

This, though the legal system, is not used. The "customary" Puddee is still in general use; it has, when slightly heaped, a capacity of 1.504 quarts. The Mercâl has a capacity of 3.0006 gallons; but, when heaped, it is equal to 8 heaped Puddees. The Seer-measure is the most common; its cubic contents are from 66½ to 67 cubic inches.

IV. Madras Measures of Weight.

	1 Tola	= 180 grains.
3 Tolas	1 Pollum	= 1.234 ounces.
8 Pollums	1 Seer	= 9.874 ,,
5 Seers	1 Viss	= 3.086 pounds.
8 Viss	1 Maund	= 24.686 ,,
20 Maunds	1 Candy	= 493.714 pounds, or 4.408 cwts.

In commerce, the Viss is reckoned as 3¼ pounds; the Maund, 25 pounds; and the Candy, 500 pounds.

BOMBAY.—Weights and Measures.—Tables No. 61.

I. Bombay Measures of Length.

	1 Ungulee	= 9/16 inch.
2 Ungulee	1 Tussoo	= 1⅛ inches.
8 Tussoos	1 Vent'h	= 9 ,,
16 Tussoos	1 Hat'h	= 18 ,,
24 Tussoos	1 Guz	= 27 ,,

The Builder's Tussoo = 2.3625 inches in Bombay; and 1 inch in Surat.

II. Bombay Measures of Surface.

34 1/6 Square Hat'h...	1 Kutty =	9.8175 square yards.
20 Kutties.............	1 Pund =	196.35 „
20 Pund...............	1 Beegah =	3927 square yards, or .8114 acre.
120 Beegah.............	1 Chahur =	97.368 acres.

In the Revenue Field Survey, the English acre is used.

III. Bombay Measures of Capacity.

	1 Tippree =	.2800 pint.
2 Tipprees...............	1 Seer =	.5600 „
4 Seers	1 Pylee =	2.2401 pints.
16 Pylees..................	1 Parah =	4.4802 gallons.
8 Parahs..................	1 Candy =	35.8415 „
25 Parahs..................	1 Mooda =	112.0045 gallons, or 1.7501 quarters.

Another liquid measure is the Seer of 60 Tolas = 1.234 pints.

In timber measurement in the Bombay dockyards, a Covit or Candi = 12.704 cubic feet.

CEYLON.

The British weights and measures are used.

BURMAH.

The English yard, foot, and inch are being adopted; also the English Measures of Capacity. *Weights.*—The Piakthah or Viss is 3.652 pounds, and contains 100 Kyats of 252 grains each.

CHINA.—Weights and Measures.—Tables No. 62.

I. Chinese Measures of Length.

	1 Fên (line) =	.141 inch.
10 Fên..................	1 Ts'un (punto or inch) =	1.41 inches.
10 Ts'un...............	1 Ch'ih (covid or foot) =	14.1 „
10 Ch'ih...............	1 Chang (rod) =	141 „ or 11.75 feet.
10 Chang..............	1 Yin =	39.17 yards.

The Ch'ih of 14.1 inches is the legal measure at all the ports of trade.

At Canton, the values of the Ch'ih are as follows:—

Tailor's Ch'ih..................................	14.685 inches.
Mercer's Ch'ih (wholesale)....................	14.66 to 14.724 inches.
Mercer's Ch'ih (retail)........................	14.37 to 14.56 „
Architect's Ch'ih...............................	12.7 inches.

At Pekin there are thirteen different Ch'ihs.

Itinerary Measure.

5 Ch'ih (covids)..................	1 Pú (pace).	
360 Pú..................................	1 Lí	= about ⅓ mile.
250 Lí (geographical)..	1 Tú (degree) =	„ 83 miles.

II. Chinese Land Measure.

25 Ch'ih (covids)..........	1 Kung (bow) =	30¼ square feet.
240 Kung....................	1 Mou (rood) =	7260 „ or 806¾ square yards.
100 Mou......................	1 King =	16⅔ acres.

The principal land measure is the Mou.

III. Chinese Cubic Measure, and Measures of Capacity.

100 Cubic Ch'ih (covids)..................	1 Fang or Ma.	
10 Ho (gills).....................	1 Shêng (pint) =	about 2 pints.
10 Shêng	1 Tou (peck) =	„ 2½ gallons.
5 Tou...........................	1 Hu (bushel) =	„ 12½ „

Liquids are measured by vessels containing definite weights, as 1, 2, 4, and 8 Taels; also large earthen vessels holding 15, 30, and 60 Catties.

IV. Chinese Measures of Weight.

	1 Liang or Tael =	$1^1/_3$ ounces.
16 Liang...........................	1 Chin or Catty =	$1^1/_3$ pounds.
100 Chin...........................	1 Tan or Picul =	$133^1/_3$ „

COCHIN-CHINA.

Length.—The Thuoc, or cubit, 19.2 inches, is the chief unit of measure of length. It varies considerably for different places. The Lí or mile is 486 yards; 2 Lí make 1 Dam; and 5 Dam make 1 League = 2.761 miles.

Surface.—9 Square Ngu make 1 Square Saö = 64 square yards. 100 Square Saö make 1 Square Maö = 6400 square yards, or 1.32 acres.

Weights.—The smallest weight is the Ai = .0000006 grain. The weights ascend by a decimal scale, until 10,000,000,000 Ai are accumulated = 1 Nen = .8594 pound. The greatest weight is the Quan = 687½ pounds.

Capacity for Grain.—1 Hao = $6^2/_9$ gallons. 2 Hao = 1 Shita = $12^4/_9$ gallons.

PERSIA.

Length.—The Gereh = 2⅜ inches; 16 Gerehs = 1 Zer = ·38 inches. The Kadam or Step = about 2 feet; 12,000 Kadam = 1 Fersakh = about 4½ miles.

Surface and Cubic Measures.—These are the squares and cubes of the lengths.

Capacity (Dry Goods).—The Sextario = .07236 gallon. 4 Sextarios = 1 Chenica; 2 Chenicas = 1 Capicha; 3⅛ Capichas = 1 Collothun; 8 Collothun = 1 Artata = 1.809 bushels.

Liquids are sold by weight.

Weights.—The Miscal = 71 grains; 16 Miscals = 1 Sihr; 100 Miscals = 1 Ratel = 1.014 pounds; 40 Sihrs = 1 Batman (Maund) = 6.49 pounds; 100 Batman (of Tabreez) = 1 Karwar = 649.142 pounds.

JAPAN.—Weights and Measures.—Tables No. 63.

I. Japanese Measures of Length.

	1 Rin	=	.012 inch.
10 Rin..................	1 Boo	=	.120 inch.
10 Boo..................	1 Sun	=	1.20 inches.
10 Sun..................	1 Shaku	=	$23^{15}/_{16}$ inches.
10 Shaku	1 Jô	=	9 feet $11^{5}/_{16}$ inches.
6 Shaku..............	1 Ken	=	5 feet $11\frac{5}{8}$ inches.
60 Ken..................	1 Chô	=	119.4 yards.
36 Chu..................	1 Ri	=	4298.4 yards, or 2.442 miles.

Rough timber is sold by the Yama-Ken-Zaü = 63 Sun. Cloth is measured by the Shaku of 15 inches, with decimal sub-multiples.

II. Japanese Measures of Land.

	1 Shaku	=	.9885 square foot.
36 Square Shaku...........	1 Tsubo	=	3.954 square yards.
30 Tsubo.....................	1 Se	=	118.615 square yards.
10 Se..........................	1 Tan	=	39.212 square poles.
10 Tan........................	1 Chō	=	2.451 acres.

III. Japanese Measures of Capacity.

	1 Kei	=	.0000318 pint.
10 Kei........................	1 Sat	=	.000318 pint.
10 Sats......................	1 Sai	=	.00318 pint.
10 Sai........................	1 Shaku	=	.0318 pint.
10 Shaku....................	1 Gō	=	.3178 pint.
10 Gō........................	1 Shō	=	.3973 gallon.
10 Shō.......................	1 To	=	3.970 gallons.
10 To.........................	1 Koku	=	39.703 gallons.

IV. Japanese Measures of Weight.

	1 Shi	=	.0058 grain.
10 Shi....................	1 Mo	=	.058 „
10 Mo....................	1 Rin	=	.5797 „
10 Rin....................	1 Fun	=	5.7972 grains.
10 Fun....................	1 Momme	=	57.972 „
100 Momme............	1 Hiyaku-me	=	.8282 pound.
1000 Momme............	1 Kwam-me	=	8.2817 pounds.
160 Momme............	1 Kin	=	$1\frac{1}{3}$ „
100 Kin..................	1 Hiyak-kin	=	$132\frac{1}{2}$ „

STRAITS SETTLEMENTS.

The unit measure of length is the yard; land is measured by the acre. The Chupack or quart of 4 Paus = 8 imperial gills; 4 quarts = 1 Gantang or gallon = 32 gills. The Kati = $1^1/_3$ pounds; 100 Kati = 1 Picul = $133^1/_3$ pounds; 40 Picul = 1 Koyan = $5333^1/_3$ pounds.

JAVA.

Length.—The Duim = 1.3 inches. 12 Duims = 1 foot. The Ell = 27.08 inches.

Surface.—The Djong of 4 Bahu = 7.015 acres.

Capacity, for rice and grain.—The measures are in fact measures of definite weights. 1 sack = 61.034 pounds; 2 sacks = 1 Pecul; 5 Peculs = 1 Timbang = 5.45 cwts.; 6 Timbang = 1 Coyau = 32.7 cwts. For liquids: The Kan = .328 gallon; 388 Kans = 1 Leager = 127.34 gallons.

Weights.—The Tael = 593.6 grains; 16 Taels = 1 Catty = 1.356 pounds: 100 Catties = 1 Pecul = 135.63 pounds.

UNITED STATES OF AMERICA.

Length.—The measures are the same as those of Great Britain.

In Land Surveying, the unit of measurement is the chain, and it is decimally subdivided.

In City Measurements, the unit is the foot, and it is decimally subdivided.

In Mechanical Measurements, the unit is the inch, and it is divided into a hundred parts.

Surface.—The measures are the same as those of Great Britain.

Capacity.—The measures of capacity for dry goods and for liquids are the same as the old English measures. The standard U. S. gallon is equal to the old English wine gallon, or 231 cubic inches; it contains 8⅓ pounds of pure water at 62° F.

Dry Measure.—Table No. 64.

	1 gill.	=	.96945	imperial	gill.
4 gills...........	1 pint	=	.96945	imperial	pint.
2 pints	1 quart	=	1.9388	,,	pints.
4 quarts	1 gallon	=	.96945	,,	gallon.
2 gallons........	1 peck	=	1.9388	,,	gallons.
4 pecks.........	1 bushel	=	.96945	,,	bushel.
4 bushels.......	1 coomb	=	3.8777	,,	bushels.
2 coombs.......	1 quarter	=	.96945	,,	quarter.
5 quarters......	1 wey or load	=	4.8472	,,	quarters.
2 weys..........	1 last	=	9.6945	,,	quarters.

For the Wine and Spirit Measures, and the Ale and Beer Measures, see the Old Measures of Great Britain, page 139.

1 cord of wood = 128 cubic feet = (4 feet × 4 feet × 8 feet).

Weights.—The Weights are the same as those of Great Britain. (See page 140.)

There are, in addition, the Quintal or Centner of 100 pounds; and the New York ton of 2000 pounds, which is also used, for retail purposes especially, in most of the States. The old hundredweight and old ton are, for the most part, superseded by the quintal and the New York ton. The wholesale coal and iron ton is 2240 pounds. The French metric system of weights and measures was legalized in 1866 concurrently with the old system.

BRITISH NORTH AMERICA.—Weights and Measures.

Until the 23d May, 1873, the standard measures of length and surface, and the weights, were the same as those of Great Britain; whilst the measures of capacity were the old British measures for dry goods, for wine, and for ale and beer. At the above-named date a new and uniform system of weights and measures came into force, in which the imperial yard, pound avoirdupois, gallon, and bushel, became the standard units, and the imperial system was adopted in its integrity, with two important exceptions: that the hundredweight of 112 pounds, and the ton of 2240 pounds were abolished; and the hundredweight was declared to be 100 pounds, and the ton 2000 pounds avoirdupois,—thus assimilating the weights of Canada to those of the United States.

The French metric system of weights and measures has been made permissive concurrently with the standard weights and measures.

MEXICO.

The weights and measures are the old weights and measures of Spain.

CENTRAL AMERICA AND WEST INDIES.

WEST INDIES (British).

The weights and measures are the same as those of Great Britain.

CUBA.

The old weights and measures of Spain are in general use. For engineering and carpentry work the Spanish, English, and French measures are in use. The French metric system of weights and measures is legalized, and is used in the customs departments.

GUATEMALA AND HONDURAS.

The weights and measures are the old weights and measures of Spain.

BRITISH HONDURAS.

In British Honduras, the British weights and measures are in use.

COSTA RICA.

The old weights and measures of Spain are in general use. But the introduction of the French metric system is contemplated.

ST. DOMINGO.

The old Spanish weights and measures are in general use. The French metric system is coming into use.

SOUTH AMERICA.

COLOMBIA.

The French metric system was introduced into the Republic in 1857, and is the only system of weights and measures recognized by the government. In ordinary commerce, the Oncha, of 25 lbs., the Quintal, of 100 lbs., and the Carga, of 250 lbs., are generally used. The Libra is 1.102 pounds. The yard is the usual measure of length.

VENEZUELA.

The system and practice are the same as those of Colombia.

ECUADOR.

The French metric system became the legal standard of weights and measures on the 1st January, 1858.

GUIANA.

In British Guiana, the weights and measures are those of Great Britain. In French Guiana or Cayenne, the ancient French system is practised. In Dutch Guiana, the weights and measures of Holland are employed.

BRAZIL.

The French metric system, which became compulsory in 1872, was adopted in 1862, and has since been used in all official departments. But the ancient weights and measures are still partly employed. They are, with some variations, those of the old system of Portugal.

Length.—The Line = .0911 inch, and is divided into tenths. The Pollegada = 1.0936 inches. The Pé = 13.1236 inches, or $^1/_3$ metre. The Vara = 1.215 yards; and 1½ Varas = the geometrical pace = 1.8227 yards. The Milha = 1.2965 miles; and 3 Milhas = 1 Legoa = 3.8896 miles.

6 yards are reckoned equal to 5 Varas.

Surface.

64 Square Pollegadas...	1 Square Palmo	= .5315 square foot.
25 Square Palmos	1 Square Vara	= 1.4766 square yards.
4 Square Varas.........	1 Square Braça	= 5.9063 ,,
4840 Square Varas.........	1 Geira	= 1.4766 acres.

Capacity (Dry Goods).—The Salamine = .3808 gallon; 2 Salamines = 1 Oitavo; 2 Oitavo = 1 Quarto; 4 Quartas = 1 Alqueiro = .3808 bushel; 4 Alqueiras = 1 Fangas; 15 Fangas = 1 Moio = 2.8560 quarters.

Liquids.—The Quartilho = .6141 pint; 4 Quartilhos = 1 Canada; 6 Canadas = 1 Pota or Cantaro; 2 Potas = 1 Almuda = 3.6846 gallons.

Weights.—The Arratel = 1.0119 pounds, is divided into 16 Onças, and then into 8 Oitavos. 32 Arratels = 1 Arroba; 4 Arrobas = 1 Quintal = 129.5181 pounds; and 13½ Quintals = 1 Tonelada = 15.6116 cwts.

There is also the Quintal of 100 Arratels. Ships' freight is reckoned by the English ton = 70 Arrobas.

PERU.

The French metric system was established in 1860, but is not yet generally used. The weights and measures in common use are:—The ounce = 1.014 ounce; the Libra = 1.014 pound; the Quintal = 101.44 pounds; the Arroba = 25.36 pounds, or 6.70 gallons; the gallon = .74 imperial gallon; the Vara = .927 yard; the square Vara = .859 square yard.

CHILI.

The French metric system has been legally established; but the old weights and measures are still in general use. These are the same as those of Peru.

BOLIVIA.

The weights and measures are the same as the old weights and measures of Peru and Chili.

ARGENTINE CONFEDERATION.

The French metric system has recently been established. The old weights and measures are commonly used:—the Castilian standards of the old Spanish system. The Quintal = 101.4 pounds; the Arroba = 25.35 pounds; the Fanega = 1.5 bushels.

URUGUAY.

The French metric system was established in 1864. The old weights and measures are the same as those of the Argentine Confederation. The weights and measures of Brazil are in general use.

PARAGUAY.

The weights and measures are the same as the old ones of the Argentine Confederation.

AUSTRALASIA.

In New South Wales, Queensland, Victoria, South Australia, West Australia, Tasmania, and New Zealand, the legal weights and measures are the same as those of Great Britain. But the old British measures of capacity are also much used.

In land measurement, a "section" is an area equal to 80 acres.

MONEY.

GREAT BRITAIN AND IRELAND.

	Coins.	Material.	Weight. Grains.
¼*d.*	farthing	bronze	43.750
½*d.*	halfpenny	do.	87.500
4 farthings	1 penny	do.	145.833
3*d.*	threepenny piece	silver	21.818
4*d.*	groat, or fourpenny piece	do.	29.091
6*d.*	sixpence	do.	43.636
12 pence	1 shilling	do.	87.273
2 shillings	1 florin	do.	174.545
2½*s.*	1 half-crown	do.	218.182
10*s.*	1 half-sovereign	gold	61.6372
20*s.*	1 sovereign, or pound sterling	do.	123.2745

The bronze coins are made of an alloy of copper, tin, and zinc; the silver coins contain 92½ per cent. of fine silver, and 7½ per cent. of alloy; the gold coins, 91⅔ per cent. of fine gold, and 8⅓ per cent. of alloy.

The Mint price of standard gold is £3, 17*s.* 10½*d.* per ounce.

One pound weight of silver is coined into 66 shillings. The intrinsic value of 22 shillings is equal to £1 sterling.

The intrinsic value of 480 pence is equal to £1 sterling.

FRANCE.—Money.

Copper.

	Coins.	Weight. Grammes.	Value in English Money. £	s.	d.
1/100 franc	1 centime	1	0	0	1/10
1/50 franc	2 centimes	2	0	0	1/5
1/20 franc	5 centimes (*sou*)	5	0	0	½
1/10 franc	10 centimes (*gros sou*)	10	0	0	1

Silver.

	Coins.	Weight. Grammes.	£	s.	d.
1/5 franc	20 centimes	1	0	0	2
½ franc	50 centimes	2.5	0	0	4¾
1 franc	100 centimes	5	0	0	9½
		more exactly			9.524*d.*
2 francs		10	0	1	7
5 francs		25	0	3	11⅝

Gold.

	Grammes.	£	s.	d.
5 francs	1·61290	0	3	$11\frac{5}{8}$
10 francs	3·22580	0	7	$11\frac{1}{4}$
20 francs (Napoleon)	6·45161 (99·56 grains)	0	15	$10\frac{1}{2}$
50 francs	16·12902	1	19	$8\frac{1}{5}$
100 francs	32·25805	3	19	$4\frac{4}{10}$

The English value is calculated at the rate of 25 francs 20 centimes to £1. The bronze coins consist of an alloy of 95 parts of copper, 4 of tin, 1 of zinc. The standard fineness of the gold pieces, and of the silver 5-franc pieces is 90 per cent., with 10 per cent. of copper; of the other silver coins, 83.5 per cent.; and of the bronze coins, 95 per cent.

GERMANY.—MONEY.

The following system of currency was established throughout the German Empire in 1872:—

			English Value. s.	d.
	1 Pfennig	=	0	.1175
10 Pfennig	1 Groschen	=	0	1.175
10 Groschen	1 Mark	=	0	$11\frac{3}{4}$
10 Marks (gold)		=	9	$9\frac{1}{2}$
20 Marks (gold)		=	19	7

The 20-mark gold piece weighs 122.92 grains, and the standard fineness of the gold pieces is 90 per cent. of gold.

Before 1872, accounts were reckoned in the following currency in North Germany:—

			s.	d.
12 Pfennig	1 Silbergroschen	=	1	$1\frac{1}{5}$
30 Silbergroschen	1 Thaler	=	3	0

In South Germany:—

4 Pfennig	1 Kreutzer	=	0	$\frac{1}{3}$
60 Kreutzers	1 Florin	=	1	8

HANSE TOWNS.—MONEY.

The monetary system is that of the German Empire.

Hamburg.—According to the old monetary system, in which silver was the standard, 12 Pfennig = 1 Schilling = $\frac{5}{6}$ *d.*; and 16 Schillings = 1 Mark = $13\frac{1}{3}$*d.*

Bremen.—Old system:—5 Schmaren = 1 Groot = $\frac{11}{20}$*d.*; and 71 Groots = 1 Rix-dollar = 3*s.* $3\frac{3}{5}$ *d.* The Rix-dollar, or Thaler, was a money of account.

Lubec.—The old system was the same as that of Hamburg, and, in addition, 3 Marks = 1 Thaler = 3*s.* 4*d.*

AUSTRIA.—Money.

	s.	d.
1 Kreutzer (copper)	0	1/5
4 Kreutzers (do.)	0	4/5
10 Kreutzers (silver)	0	2⅜
20 Kreutzers (do.)	0	4¾
¼ Florin (do.)	0	5¾
1 Florin (do.)	1	11½
2 Florins (do.)	3	11½
4 Florin piece (gold)	7	11
8 Florin piece (do.)	15	10

100 Kreutzers make 1 Florin.

The 4-florin gold piece weighs 49.92 grains, and the standard of fineness is 90 per cent. of gold.

RUSSIA.—Money.

			s.	d.
	1 Copeck	=	0	.38
100 Copecks	1 Silver Rouble	=	3	2

The copper coins are pieces of ¼, ½, 1, 2, 3, 5 Copecks. The silver coins are pieces of 5, 10, 15, 20, 25 Copecks, the Half Rouble, and the Rouble; the gold coins are the Three-rouble piece, the Half Imperial of five Roubles, and the Imperial of 10 Roubles. The 5-rouble gold piece weighs 101 grains, and the standard of fineness is 91⅔ per cent. of gold. Paper currency:—1, 3, 5, 10, 25, 50, 100 Roubles.

HOLLAND.—Money.

			s.	d.
	1 Cent	=	0	1/5
100 Cents	1 Guilder or Florin	=	1	8

BELGIUM.—Money.

The monetary system is exactly the same as that of France.

DENMARK.—Money.

			s.	d.
	1 Skilling	=	0	.2745
16 Skillings	1 Mark	=	0	4.392
96 Skillings, or 6 Marks	1 Rigsdaler, or Daler	=	2	2 7/20

SWEDEN.—Money.

			s.	d.
	1 Öre	=	0	.133
100 Öre	1 Riksdaler	=	1	1⅓

NORWAY.—Money.

			s.	d.
	1 Skilling	=	0	.444
24 Skillingen	1 Ort or Mark	=	0	10⅔
5 Ort	1 Species-Daler	=	4	5⅓

SWITZERLAND.—Money.

The monetary system of Switzerland is the same as that of France. The Centime is called a Rappe.

SPAIN.—Money.

				d.
	1 Centimo	=		·95
100 Centimos	1 Peseta	=	1 franc, or	9½

The bronze coins are pieces of 1, 2, 5, and 10 centimos. The silver coins are pieces of 20 and 25 centimos, and 1, 2, and 5 pesetas. The gold coins are pieces of 5, 10, 20, 25, 50, and 100 pesetas. The piece of 5 pesetas is 3*s.* 11½*d.*, English value. The 25 peseta piece is 19*s.* 9½*d.*, English value.

The old monetary system was based on the Real-Vellon, 2½*d.* English value; it was the 20th part of the Silver Hard Dollar, 4*s.* 2*d.* English value, and of the Gold Dollar or Coronilla. The Duro was identical with the American Dollar.

PORTUGAL.—Money.

The unit of account is the Rei, of which 18¾ Reis make 1 penny; and 4500 Reis make 1 sovereign. The Milreis is 1000 Reis, 4*s.* 5⅓*d.* English value. The Corda is the heaviest gold coin, of 10,000 Reis, £2, 4*s.* 5½*d.* English value, and weighs 17.735 grammes.

ITALY.—Money.

			d.
	1 Centime	=	·95
100 Centimes	1 Lira	=	1 franc, or 9½

Copper coins are pieces of 1, 3, and 5 Centimes; silver coins, 20 and 50 Centimes, and 1, 2, and 5 Lire; gold coins, 5, 10, 20, 50, and 100 Lire. These coins are the same in weight and fineness as the coins of France.

TURKEY.—Money.

			s.	*d.*
	1 Para	=	0	$^1/_{18.5}$
40 Paras	1 Piastre	=	0	2.16
100 Piastres	1 Medjidie, or Lira Turca	=	18	0

The Piastre is roughly taken equal to 2*d.* sterling.

GREECE AND IONIAN ISLANDS.—Money.

100 Lepta............1 Drachma.......... = 1 franc, or 9½*d.*

The currency of Greece is the same as that of France.

In the Ionian Islands, whilst they were under British protection (1830–1864), accounts were kept by some persons in Dollars, of 100 Oboli = 4*s.* 2*d.*; by others in Pounds, of 20 shillings, of 12 pence, Ionian currency; the Ionian Pound being equal to 20*s.* 9.6*d.* sterling. By other persons accounts were kept in Piastres of 40 Paras = 2 $^4/_{45}$*d.*

MALTA.—MONEY.

			s.	d.
	1 Grano	=	0	1/12
20 Grani	1 Taro	=	0	1⅔
12 Tari	1 Scudo	=	1	8
	Or,			
60 Piccioli	1 Carlino	=	0	.185
9 Carlini	1 Taro	=	0	1⅔
12 Tari	1 Scudo	=	1	8

British money is in general circulation. The Sovereign = 12 Scudi; the Shilling = 7 Tari 4 Grani.

EGYPT.—MONEY.

			£	s.	d.
	1 Para	=	0	0	.0615
40 Paras	1 Piastre (Tariff)	=	0	0	2.461
100 Piastres	1 Egyptian Guinea	=	1	0	6.84
5 Egyptian Guineas	1 Kees, or Purse	=	5	2	10.2
1000 Purses	1 Khuzneh, or Treasury	=	5142	10	0
97.22 Piastres	1 English Sovereign.				

The Egyptian guinea weighs 132 grains, and the standard of fineness is 87½ per cent. of gold.

Two piastres (current) are equal to one piastre (tariff).

MOROCCO.—MONEY.

			s.	d.
	1 Flue	=	0	37/960
24 Flues	1 Blankeel	=	0	37/40
4 Blankeels	1 Ounce	=	0	3.7
10 Ounces	1 Mitkul	=	3	1

TUNIS.—MONEY.

			s.	d.
	1 Fel	=	0	35/288
3 Fels	1 Karub	=	0	35/96
16 Karubs	1 Piastre	=	0	5 5/6

ARABIA.—MONEY.

			s.	d.
80 Caveers	1 Piastre or Mocha Dollar	=	3	5

CAPE OF GOOD HOPE.—MONEY.

Public accounts are kept in English money; but private accounts are often kept in the old denominations, as follows:—

			s.	d.
	1 Stiver	=	0	3/8
6 Stivers	1 Schilling	=	0	2¼
8 Schilling	1 Rix-dollar	=	1	6

The Guilder is equal to 6*d.*

INDIAN EMPIRE.—Money.

Throughout India, accounts are kept in the following moneys:—

		s.	d.	
	1 Pie	= 0	0⅛	nominal value.
12 Pies	1 Anna	= 0	1½	do.
16 Annas	1 Rupee	= 2	0	do.

The intrinsic value of the Rupee is 1*s.* 10½*d.*; it weighs 180 grains. The English Sovereign is equal to 10 Rupees 4 Annas.

1 Lac of Rupees = 100,000 rupees = £10,000.
1 Crore of Rupees = 100 lacs = £1,000,000.

In Ceylon, the Rupee is divided into 100 Cents.

The gold coin, Mohur, is equal to 15 rupees; it weighs 180 grains, and the standard fineness is 91.65 per cent. of gold.

CHINA.—Money.

		s.	d.
	1 Cash (Le)	= 0	7/100
10 Cash	1 Candareen (Fun)	= 0	7/10
10 Candareens	1 Mace (Tsien)	= 0	7
10 Mace	1 Tael (Lëang)	= 5	10

COCHIN-CHINA.—Money.

		s.	d.
	1 Sapek, or Dong, or Cash	= 0	1/18
60 Sapeks	1 Mas, or Mottien	= 0	3⅓
10 Mas	1 Quan, or String	= 2	9⅓

PERSIA.—Money.

		s.	d.
	1 Dinar	= 0	1/80
50 Dinars	1 Shahi	= 0	⅝
20 Shahis	1 Keran	= 0	11¼
10 Kerans	1 Toman	= 9	3½

JAPAN.—Money.

		s.	d.
10 Rin	1 Sen	=	½
100 Sen	1 Yen	= 4	2

There are gold coins of the value of 1, 2 and 5 yen, with a standard fineness of 90 per cent. The 5-yen piece weighs 128.6 grains. The silver yen weighs 416 grains, with the same standard of fineness.

JAVA.—Money.

The money account of Java is the same as that of Holland.

UNITED STATES OF AMERICA.—Money.

		s.	d.
	1 Cent	= 0	½
10 Cents	1 Dime	= 0	5
100 Cents	1 Dollar	= 4	2

CANADA.—BRITISH NORTH AMERICA.—MONEY.

			s.	d.	
	1 Mil	=	0	$^1/_{20}$	sterling.
10 Mils	1 Cent	=	0	½	do.
100 Cents	1 Dollar	=	4	1¼	do.
4 Dollars		=	20	0	currency.

Or,

	1 Penny currency	=	0	¾	sterling.
12 Pence	1 Shilling do.	=	0	9 $^4/_5$	do.
20 Shillings	1 Pound do.	=	16	5¼	do.

The Dollar of Nova Scotia, New Brunswick, and Newfoundland, is equal to 4*s.* 2*d.* sterling. In the Bermudas, accounts are kept in sterling money.

MEXICO.—MONEY.

Accounts are kept in dollars of 100 cents. The dollar is equal to 4*s.* 2*d.* sterling.

CENTRAL AMERICA AND WEST INDIES.—MONEY.

WEST INDIES (British).

Accounts are kept in English money; and sometimes in dollars and cents. 1 dollar = 4*s.* 2*d.*

CUBA.—MONEY.

The moneys of various nations were in circulation before the current war (1875). But the principal silver currency was the 10 cent and 5 cent pieces of the United States. The gold currency consists of the Ounce, of the value of 16 dollars, ½ ounce, ¼ ounce, ⅛ ounce.

GUATEMALA, HONDURAS, COSTA RICA.

The moneys of account are the same as those of Mexico.

ST. DOMINGO.

Accounts are kept in current dollars (called *Gourdes*) and cents. The cent = $^1/_{32}$*d.*; and 100 cents = 1 dollar = 3⅛*d.*

SOUTH AMERICA.—MONEY.

COLOMBIA, VENEZUELA, ECUADOR.

The moneys of account are, the Centavo = ½*d.*; and 100 Centavos = 1 Peso = 4*s.* 2*d.*

GUIANA.

In British Guiana the dollar of 4*s.* 2*d.* is used, divided into 100 cents. In French Guiana, French money is used. In Dutch Guiana, the money of Holland is used.

BRAZIL.—Money.

			s.	d.
	1 Rei	=	0	2.7/100
1000 Reis	1 Milreis	=	2	3

PERU.—Money.

			s.	d.
	1 Centesimo	=	0	·37
100 Centesimos	1 Dollar, or Peso	=	3	1

CHILI.—Money.

			s.	d.
	1 Centavo	=	0	·45
100 Centavos	1 Dollar, or Peso	=	3	9

BOLIVIA.

	1 Centena	=	0	·37
100 Centenas	1 Dollar	=	3	1

ARGENTINE CONFEDERATION.

	1 Centesimo	=	0	.25
100 Centesimos	1 Dollar, or Patercon	=	2	1

URUGUAY.

	1 Centime	=	0	0½
100 Centimes	1 Dollar	=	4	2

PARAGUAY.

	1 Centena	=	0	·37
100 Centenas	1 Dollar	=	3	1

AUSTRALASIA.

Accounts are kept in pounds, shillings, and pence sterling.

WEIGHT AND SPECIFIC GRAVITY.

The specific gravity, or specific weight of a body, is the ratio which the weight of the body bears to the weight of another body of equal volume adopted as a standard for comparison of the weights of bodies. For solids and liquids, pure water at the mean temperature 62° F., is adopted as the standard body for comparative weight. For gases, dry air at 32° F., and under one atmosphere of pressure, or 14.7 lbs. per square inch, is the body with which they are compared.

The specific gravity of bodies is found by weighing them in and out of water, according to the following rules.

RULE 1.—*To find the specific gravity of a solid body heavier than water.* Weigh it in pure water at 62° F., and divide its weight out of water by the loss of weight in the water. The quotient is the specific gravity.

Note.—The loss of weight in water is the difference of the weight in air and the weight in water, and it is equal to the weight of the quantity of water displaced, which is equal in volume to the body.

RULE 2.—*To find the specific gravity of a solid body lighter than water.* Load it so as to sink it in pure water at 62° F., and weigh it and the load together, out of water, and in water; weigh the load separately in and out of water; deduct the loss of weight of the load singly from that of the combined body and load; the remainder is the loss of weight of the body singly, by which its weight out of water is to be divided. The quotient is the specific gravity.

RULE 3.—*To find the specific gravity of a solid body which is soluble in water.* Weigh it in a liquid in which it is not soluble; divide the weight out of the liquid by the loss of weight in the liquid, and multiply by the specific gravity of the liquid. The product is the specific gravity of the body.

RULE 4.—*To find the specific gravity of a liquid.* Weigh a solid body in the liquid and in water, as well as in the air, and divide the loss of weight in the liquid by the loss of weight in water. The quotient is the specific gravity.

RULE 5.—*To find the weight of a body when the specific gravity is given.* Multiply the specific gravity by

MULTIPLIER.		WEIGHT OF
62.355 (the weight in pounds of a cubic foot of pure water at 62° F.)	= 1	cubic foot, in lbs.
1683.60	= 1	cubic yard, in lbs.
15.0	= 1	,, in cwts.
·75	= 1	,, in tons.

Note.—As one cubic foot of water at 62° F. weighs about 1000 ounces (exactly 997.68 ounces), the weight in ounces of a cubic foot of any other substance will represent, approximately, its specific gravity, supposing water = 1000.

If the last three places of figures be pointed off as decimals, the result will be the specific gravity approximately, water being = 1.

In France, the standard temperature for comparison of the density of bodies, and the determination of their specific gravities, is that of the maximum density of water,—about 4° C., or 39°.1 F., for solid bodies; and 32° F., or 0° C., for gases and vapours, under one atmosphere or .76 centimetres of mercury. In practice, it is usual to adopt the cubic decimetre or litre as the unit of volume, since the cubic decimetre of distilled water, at 4° C. weighs, by the definition, 1 kilogramme. Consequently the specific gravity of a body is expressed by the weight in kilogrammes of a cubic decimetre of that body.

The densities of the metals vary greatly. Potassium and one or two others are lighter than water. Platinum is more than twenty times as heavy. Lead is over eleven times as heavy; and the majority of the useful metals are from seven to eight times as heavy as water.

Stones for building or other purposes vary in weight within much narrower limits than metals. With one exception, they vary from basalt and granite, which are three times the weight of water, to volcanic scoriæ which are lighter than water. The exception referred to is barytes, which is conspicuously the heaviest stone, being 4½ times as heavy as water. The sulphate of baryta is known as *heavy spar*.

Amongst other solids, flint-glass has three times the weight of water; clay and sand, twice as much; coal averages one and a half times the weight of water; and coke from one to one and a half times. Camphor has about the same weight as water.

Of the precious stones, zircon is the heaviest, having four and a half times the weight of water; garnet is four times as heavy, diamond three and a half times as heavy, and opal, the lightest of all, has just twice the weight of water.

Peat varies in weight from one-fifth to a little more than the weight of water.

The heaviest wood is that of the pomegranate, which has one and a third times the weight of water. English oak is nearly as heavy as water, and heart of oak is heavier; the densest teak has about the same weight as water; mahogany averages about three-fourths, elm over a half, pine from a half to three-fourths, and cork one-fourth of the weight of water. Of the colonial woods, the average of 22 woods of British Guiana weighs 74 per cent. of the weight of water; of 36 woods of Jamaica, 83 per cent.; and of 18 woods of New South Wales, 96 per cent.

Wood-charcoal in powder averages one and a half times the weight of water; in pieces heaped, it averages only two-fifths. Gunpowder has about twice the weight of water.

Of animal substances, pearls weigh heaviest, two and three-quarter times the weight of water; ivory and bone twice, and fat over nine-tenths the weight of water.

Of vegetable substances, cotton weighs about twice as much as water; gutta-percha and caoutchouc nearly the same weight as water.

Mercury, the heaviest liquid at ordinary temperatures, has over thirteen and a half times the weight of water; and bromine nearly three times the weight. The water of the Dead Sea is a fourth heavier, and ordinary sea-water two and a half per cent. heavier than water; whilst olive-oil is about one-tenth lighter, and pure alcohol and wood-spirit a fifth lighter than water.

Turning to gaseous bodies, water at 62° F. has 772.4 times the weight of air at 32° F., under a pressure of one atmosphere; and the specific gravity of air at 32° F. is .001293, that of water at 62° F. being = 1. Oxygen gas weighs a tenth more than air, gaseous steam weighs only five-eighths of air, and hydrogen, the most perfect type of gaseity, has only seven per cent. of the weight of air. Water has upwards of 11,000 times the weight of hydrogen.

One pound of air at 62° F. has the same volume as a ton of quartz.

The following Tables, Nos. 65 to 69, contain the weights and specific gravities of solids, liquids, and gases and vapours. The specific gravities have been derived from the works of Rankine, Ure, Wilson, Claudel, and Peclet, Delabèche and Playfair, Fowke, and others whose names are mentioned in the body of the tables. Columns containing the bulks of bodies have been added to the tables.

The specific gravity of alloys does not usually follow the ratios of those of their constituents; it is sometimes greater and sometimes less than the mean of these. Ure gives the specific gravities of some alloys of copper, tin, zinc, and lead, examined by Crookewitt. The following are the specific gravities of the alloys, as ascertained by Crookewitt; and, for the purpose of comparison, they are preceded by the specific gravities of the particular samples of the elementary metals employed.

	Specific gravity.
Copper	8.794
Tin	7.305
Zinc	6.860
Lead	11.354
Alloys:—Copper 2, tin 5	7.652
Copper 1, tin 1	8.072
Copper 2, tin 1	8.512
Copper 3, zinc 5	7.939
Copper 3, zinc 2	8.224
Copper 2, zinc 1	8.392
Copper 2, lead 3	10.753
Copper 1, lead 1	10.375
Tin 1, zinc 2	7.096
Tin 1, zinc 1	7.115
Tin 3, zinc 1	7.235
Tin 1, lead 2	9.965
Tin 1, lead 1	9.394
Tin 2, lead 1	9.025

The following binary alloys have, on the one side, a density greater than the mean density of their constituents; and, on the other side, a density less than the mean density of the constituents.

Alloys having a density greater than the mean.	Alloys having a density less than the mean.
Gold and zinc.	Gold and silver.
Gold and tin.	Gold and iron.
Gold and bismuth.	Gold and lead.
Gold and antimony.	Gold and copper.
Gold and cobalt.	Gold and iridium.
Silver and zinc.	Gold and nickel.
Silver and lead.	Silver and copper.
Silver and tin.	Iron and bismuth.
Silver and bismuth.	Iron and antimony.
Silver and antimony.	Iron and lead.
Copper and zinc.	Tin and lead.
Copper and tin.	Tin and palladium.
Copper and palladium.	Tin and antimony.
Copper and bismuth.	Nickel and arsenic.
Lead and antimony.	Zinc and antimony.
Platinum and molybdenum.	
Palladium and bismuth.	

TABLE No. 65.—VOLUME, WEIGHT, AND SPECIFIC GRAVITY OF SOLID BODIES.

FAMILIAR METALS.	Weight of one cubic foot.	Specific Gravity.
	pounds.	Water = 1.
Platinum	1342	21.522
Gold	1200	19.245
Mercury, fluid	849	13.596
Lead, milled sheet	712	11.418
Do. wire	704	11.282
Silver	655	10.505
Bismuth	617	9.90
Copper, sheet	549	8.805
Do. hammered	556	8.917
Do. wire	554	8.880
Bronze:—84 copper, 16 tin, gun metal	534	8.56
83 „ 17 „ „	528	8.46
81 „ 19 „ „	528	8.46
79 „ 21 „ mill-bearings	544	8.73
35 „ 65 „ small bells	503	8.06
21 „ 79 „ „	461	7.39
15 „ 85 „ speculum metal	465	7.45
Nickel, hammered	541	8.67
Do. cast	516	8.28
Brass:—cast	505	8.10
75 copper, 25 zinc, sheet	527	8.45
66 „ 34 „ yellow	518	8.30
60 „ 40 „ Muntz's metal,	511	8.20
Brass, wire	533	8.548
Manganese	499	8.00
Steel:—Least and greatest density	435 to 493	7.729 to 7.904
Homogeneous metal	493	7.904
Blistered steel	488	7.823
Crucible steel	488 to 490	7.825 to 7.859
Do. average	489	7.842
Cast steel,	489 to 489.5	7.844 to 7.851
Do. average	489.3	7.848
Bessemer steel	489 to 490	7.844 to 7.857
Do. average	489.6	7.852
Mean for ordinary calculations	489.6	7.852
Iron, wrought:—Least and greatest density	466 to 487	7.47 to 7.808
Common bar	471	7.55
Puddled slab	469.5 to 474	7.53 to 7.60
Various—Irons tested by Mr. Kirkaldy	468 to 486	7.5 to 7.8
Do. average	477	7.65
Common rails	466 to 476	7.47 to 7.64
Do. average	470	7.54
Yorkshire iron bar	484	7.758
Lowmoor plates, 1½ to 3 ins. thick	487	7.808
Beale's rolled iron	476	7.632
Pure iron (exceptional), by electro-deposit (Dr. Percy)	508	8.140
Mean, for ordinary calculations	480	7.698

FAMILIAR METALS (*continued*).	Weight of one cubic foot.	Specific Gravity.
	pounds.	Water = 1.
Iron, cast:—Least and greatest density	378.25 to 467.66	6.900 to 7.500
White	468	7.50
Gray	449	7.20
Eglinton hot-blast, 1st melting	435	6.969
2d do.	435	6.970
14th do.	470	7.530
Rennie	435 to 444	6.977 to 7.113
Mallett	442	7.094
Mean, for ordinary calculations	450	7.217
Tin	462	7.409
Zinc, sheet	449	7.20
Do. cast	428	6.86
Antimony	418	6.71
Aluminium, wrought	167	2.67
Do. cast	160	2.56
Magnesium	108.5	1.74
OTHER METALS.		
Iridium	1165.0	18.68
Uranium	1147.0	18.40
Tungsten	1097.0	17.60
Thallium	742.6	11.91
Palladium	735.8	11.80
Rhodium	660.9	10.60
Osmium	623.6	10.00
Cadmium	542.5	8.70
Molybdenum	537.5	8.62
Ruthenium	536.2	8.60
Cobalt	530.0	8.50
Tellurium	381.0	6.11
Chromium	374.1	6.00
Arsenic	361.5	5.80
Titanium	330.5	5.30
Strontium	158.4	2.54
Glucinum	131.0	2.10
Calcium	98.5	1.58
Rubidium	94.8	1.52
Sodium	60.5	0.97
Potassium	53.6	0.86
Lithium	37.0	0.59

PRECIOUS STONES.

	Specific Gravity.		Specific Gravity.
Zircon	4.50	Diamond, Pure	3.52
Garnet	3.60 to 4.20	Boart	3.50
Malachite	4.01	Topaz	3.50
Sapphire	3.98	Tourmaline	3.07
Emerald	3.95	Lapis lazuli	2.96
Do. Aqua marine	2.73	Turquoise	2.84
Amethyst	3.92	Jasper, Onyx, Agate	2.6 to 2.7
Ruby	3.95	Beryl	2.68
Diamond	3.50 to 3.53	Opal	2.09

STONES.	Cubic feet to one ton, solid.	Weight of one cubic foot, solid.	Specific Gravity.
	cubic feet.	pounds.	Water = 1.
Specular, or red iron ore	6.84	327.4	5.251
Magnetic iron ore	7.05	317.6	5.094
Brown iron ore	9.16	244.6	3.922
Spathic iron ore	9.38	238.8	3.829
Clydesdale iron ores	11.76	190.5	3.055 to 3.380
Barytes	8.07	277.5	4.45
Basalt	14.7 to 12.0	152.8 to 187.1	2.45 to 3.00
Mica	14.0 to 12.3	160.3 to 182.7	2.57 to 2.93
Limestone, Magnesian	12.6	178.3	2.86
Do. Carboniferous	13.3	168.0	2.69
Marble:—Paros	12.7	177.1	2.84
African	12.8	174.6	2.80
Siberian	13.2	170.2	2.73
Pyrenean	13.2	170.2	2.73
Carrara	13.2	169.6	2.72
Egyptian, green	13.5	166.5	2.67
French	13.6	165.2	2.65
Florentine, Sienna	14.3	157.1	2.52
Trap, touchstone	13.2	169.6	2.72
Granite, Sienite, gneiss	15.2 to 12.1	147.1 to 184.6	2.36 to 2.96
Do. Gray	12.8 to 11.8	174.6 to 190.8	2.80 to 3.06
Porphyry	13.5 to 13.1	166.5 to 171.5	2.67 to 2.75
Alabaster, Calcareous	13.0	172.1	2.76
Do. Gypseous	15.6	144.0	2.31
Chalk, Air-dried	14.9 to 14.1	150 to 159	2.46 to 2.55
Slate	13.8 to 12.6	162.1 to 177.7	2.60 to 2.85
Serpentine	12.8	175.2	2.81
Potter's Stone	12.8	174.6	2.80
Schist, Slate	12.8	174.6	2.80
Do. Rough	19.9 to 12.9	112.8 to 173.3	1.81 to 2.78
Lava, Vesuvian	21.0 to 12.8	106.6 to 175.2	1.71 to 2.81
Talc, Steatite	13.3	168.4	2.70
Rock Crystal	13.6	165.2	2.65
Quartz	13.8 to 13.3	162.8 to 169.0	2.61 to 2.71
Do. Crystalline	13.6	165.2	2.65
Do. for paving	14.4	155.9	2.50
Do. porous, for millstones	28.5	78.6	1.26
Do. flaky, for do.	14.1	159.0	2.55
Flint	13.7	164.0	2.63
Felspar	13.8	162.1	2.60
Gypsum	15.6	143.4	2.30
Lias	16.0 to 14.7	140.3 to 152.8	2.25 to 2.45
Graphite	16.3	137.2	2.20
Sandstone	17.3 to 14.3	129.7 to 157.1	2.08 to 2.52
Tufa, volcanic	29.7 to 26.1	75.4 to 86.0	1.21 to 1.38
Scoria, do.	43.3	51.7	.83

SUNDRY MINERAL SUBSTANCES.	Cubic feet to one ton, solid.	Weight of one cubic foot, solid.	Specific Gravity.
	cubic feet.	pounds.	Water = 1.
Glass:—Flint	... ...	... 187.0 ...	... 3.00
Green		168.4	2.70
Plate	... ...	... 168.4 ...	... 2.70
Crown		155.9	2.50
St. Gobain	... ...	... 155.3 ...	... 2.49
Common, with base of potash		153.4	2.46
Fine, do. do.	... ...	... 152.8 ...	... 2.45
Common, with base of soda...		152.8	2.45
Fine, do. do. ...	... ...	... 152.1 ...	... 2.44
Soluble		77.9	1.25
Porcelain:—China	... ...	... 148.4 ...	... 2.38
Sevres		139.7	2.24
Portland Cement	28.7 to 23.8	... 78 to 94 ...	1.25 to 1.51
Concrete:—			
P. cement 1, and shingle 10	... 16.1 ...	... 139 ...	... 2.23
P. cement, rubble, and sand	16.6 to 16.0	135 to 140	2.17 to 2.25
P. cement 1, and sand 2	... 17.6 ...	... 127 ...	... 2.04
Roman cement 1, and sand 2	18.7	120	1.92
Mortar	... 20.6	... 109 ...	... 1.75
Brick	18.1 to 16.0	124.7 to 135.3	2.00 to 2.17
Brickwork	20.4 to 19.5	...110 to 115...	1.76 to 1.84
Masonry, Rubble	19.4 to 15.6	115.3 to 143.4	1.85 to 2.30
Marl	22.4 to 18.9	99.8 to 118.5	1.60 to 1.90
Do. very tough	15.3	146	2.34
Potash	... 17.1 ...	... 131 ...	... 2.10
Sulphur	18.0	124.7	2.00
Tiles	... 18.0 ...	... 124.7 ...	... 2.00
Rock Salt	17.1 to 15.9	131 to 140.7	2.100 to 2.257
Common Salt, as a solid	... 18.7 ...	... 119.7 ...	... 1.92
Clay	18.7	119.7	1.92
Sand, pure	... 18.9 ...	... 118.5 ...	... 1.90
earthy	21.1	106.0	1.70
Earth:—Potter's	... 18.9 ...	... 118.5 ...	... 1.90
Argillaceous	22.4	99.8	1.60
Light vegetable	... 25.7 ...	... 87.3 ...	... 1.40
Mud	22.	101.6	1.63
Materials in the bed of the Clyde:—			
Fine sand and a few pebbles, laid in a box, loose, not pressed, nearly dry	... 26 ...	... 87 ...	... 1.39
Pressed	24	92	1.48
Mud at Whiteinch, dry, and firmly packed, containing very fine sand and mica	... 23 ...	... 97 ...	... 1.56
Wet mud, rather compact and firm, well pressed into the box	... 19 ...	... 115 ...	... 1.95
Wet, fine, sharp gravel, well pressed	... 18 ...	... 124 ...	... 1.99
Wet, running mud	18.1	122½	1.97
Sharp dry sand deposit, in harbour	... 24.3 ...	... 92 ...	... 1.48
Port-Glasgow bank (sand), wet, pressed into a box	... 18.6 ...	... 120.5 ...	... 1.93

MINERAL SUBSTANCES (*continued*).	Cubic feet to one ton, solid.	Weight of one cubic foot, solid.	Specific Gravity.
	cubic feet.	pounds.	Water = 1.
Materials in the bed of the Clyde:—			
Sand opposite Erskine House, wet, pressed	19.3	116	1.86
Alluvial earth, pressed	24	93	1.49
Do. do. loose	33	67	1.08
Plaster:—24 hours after using	22.6	99.2	1.59
2 months after using	25.7	87.3	1.40
Coal, Anthracite (see Sect. COAL)	26.2 to 22.6	85.4 to 99.1	1.37 to 1.59
Bituminous do. do.	30 to 28.1	74.8 to 81.7	1.20 to 1.31
Boghead (cannel) do. do.	30	74.8	1.20
Coke	39 to 21.6	57.4 to 103.5	.92 to 1.66
Phosphorus	20.3	110.4	1.77
Alum	20.9	107.2	1.72
Camphor	36.3	61.7	.99
Melting Ice	39	57.5	.922

COALS.	Cubic feet in a ton.	Weight of one cubic foot.		Specific Gravity.
	Heaped.	Solid.	Heaped.	
(*Delabeche and Playfair.*)	cubic feet.	pounds.	pounds.	Water = 1.
Welsh:—Anthracite	38.4	85.4	58.3	1.37
Porth Mawr (highest)	42.0	86.7	53.3	1.39
Llynvi (one of the lowest)	42.0	80.3	53.3	1.28
Average of 37 samples	42.7	82.3	53.1	1.315
Newcastle:—Hedley's Hartley (highest)	43.1	81.8	52.0	1.31
Original Hartley (one of the lowest)	45.6	78.0	49.1	1.25
Average of 18 samples	45.3	78.3	49.8	1.256
Derbyshire and Yorkshire:—Elsecar	47.4	80.8	47.2	1.296
Butterley	47.3	79.8	47.4	1.28
Stavely	44.9	79.8	49.9	1.27
Loscoe, soft	48.8	79.6	45.9	1.285
Average of 7 samples	47.4	79.6	45.9	1.292
Lancashire:—Laffack Bushy Park (highest)	42.6	84.1	52.6	1.35
Cannel, Wigan (lowest)	46.4	76.8	48.3	1.23
Average of 28 samples	45.2	79.4	49.7	1.273
Scotch:—Grangemouth (highest)	40.1	80.5	54.3	1.29
Wallsend Elgin	41.0	74.8	54.6	1.20
Average of 8 samples	42.0	78.6	50.0	1.259
Irish:—Slievardagh Anthracite	35.7	99.6	62.8	1.59
Warlich's artificial fuel	32.4	72.2	69.6	1.15
(*Nicoll and Lynn.*)				
South Lancashire and Cheshire Coals, average of 14 samples	42	—	—	—

PEAT.	Cubic feet per ton, stalked.	Weight of one cubic foot, stalked.	Weight of one cubic foot, solid.	Specific Gravity.
(Dr. Sullivan.)				
Irish peat (comprising an average amount of water from 20 to 25 per cent):—	cubic feet.	pounds.	pounds.	Water = 1.
Lightest upper moss peat ...	...369.60...	... 6.06		
Average light moss peat	254.20	8.81		
Average brown peat..........	...147.00...	... 15.13		
Compact black peat..........	131.28	17.06		
Densest peat....................	... 99.36...	... 22.54		
Mean of five samples.........	200.29	11.18		
(Another observation.)				
Average upper brown peat ..	...188.0 ...	... 11.92		
Moderately compact lower brown turf..................	155.5	14.40		
Mean of two classes..........	...141.75...	... 15.80		
Condensed peat................	51.2 to 40.0	43.75 to 56.8	62.5 to 81.1	1.0 to 1.3
(Kane and Sullivan.)				
Excessively light, spongy surface peat..................	... ...	... ...	13.7 to 21.0	.219 to .337
Light surface peat.............			20.9 to 25.3	.335 to .405
Rather dense peat.............	... ...	... ...	29.7 to 41.7	.476 to .669
Very dense dark brown peat			40.5 to 44.5	.650 to .713
Very dense blackish brown compact peat..............	... ...	... ...	45.1 to 61.3	.724 to .983
Exceedingly dense jet black peat..........................	... ...	... ...	53.2 to 61.8	.725 to .991
Exceedingly dense, dark, blackish brown peat......	... ...	... ...	... 66.0 ...	... 1.058
(Karmarsch.)				
Turfy peat, Hanover..........	... ...	... ...	6.9 to 16.2	.11 to .26
Fibrous peat, do.			15.0 to 41.8	.24 to .67
Earthy peat, do.	... ...	... ...	25.6 to 56.1	.41 to .90
Pitchy peat, do.			38.7 to 64.2	.62 to 1.03

FUEL IN FRANCE.	Weight of one cubic foot.	Specific Gravity.
(Claudel.)	pounds.	Water = 1.
Pure Graphite..	... 145.3 ...	... 2.33
Anthracite..	83.5 to 91.0	1.34 to 1.46
Rich coal, with a long flame...........................	79.8 to 84.8	1.28 to 1.36
Dry coal, with a long flame...........................	84.8	1.36
Rich and hard coal......................................	... 82.3 ...	... 1.32
Smithy coal..	79.8 to 81.1	1.28 to 1.30
Lignite..	77.9 to 84.2	1.25 to 1.35
Do. bituminous..	72.3 to 74.8	1.16 to 1.20
Do. imperfect	68.6 to 74.2	1.10 to 1.19
"Jayet"..	81.7	1.31
Bitumen, red	... 72.3 ...	... 1.16
Do. black	66.7	1.07
Do. brown	... 51.7 ...	... 0.83
Asphalte	66.1	1.06

WOODS.	Weight of one cubic foot.	Specific Gravity.
	pounds.	
Pomegranate	84.2	1.35
Boxwood	64.8	1.04
Do. of Holland	82.3	1.32
Do. of France	56.7	0.91
Lignum vitæ	40.5 to 82.9	.65 to 1.33
Ebony	70.5	1.13
Do. Green	75.5	1.21
Do. Black	74.2	1.19
Oak, Heart of	73.0	1.17
Do. English	58.0	0.93
Do. European	43.0 to 61.7	.69 to .99
Do. American, Red	54.2	.87
Lancewood	41.8 to 63.0	.67 to 1.01
Rosewood	64.2	1.03
Satin-wood	59.9	0.96
Walnut, Green	57.4	0.92
Do. Brown	42.4	0.68
Laburnum	57.4	0.92
Hawthorn	56.7	0.91
Mulberry	55.5	0.89
Plum-tree	54.2	0.87
Teak, African		.98
Mahogany, Spanish	53.0	0.85
Do. St. Domingo	46.8	0.75
Do. Cuba	34.9	0.56
Do. Honduras	34.9	0.56
Beech	46.8 to 53.0	0.75 to 0.85
Do. with 20 per cent. moisture	51.1	0.82
Do. cut one year	41.2	0.66
Ash	52.4	0.84
Do. with 20 per cent. moisture	43.7	0.70
Acacia	51.1	0.82
Do. with 20 per cent. moisture	44.9	0.72
Holly	47.5	0.76
Hornbeam	47.5	0.76
Yew	46.1 to 50.5	0.74 to 0.81
Birch	44.9 to 46.1	0.72 to 0.74
Elm	34.3	0.55
Do. Green	47.5	0.76
Do. with 20 per cent. moisture	44.9	0.72
Yoke-Elm do. do.	47.5	0.76
Rock-Elm	50.0	0.80
Fir, Norway pine	46.1	0.74
Do. Red pine	29.9 to 43.7	0.48 to 0.70
Do. Spruce	29.9 to 43.7	0.48 to 0.70
Do. Larch	31.18 to 39.9	0.50 to 0.64
Do. White pine, English	34.3	0.55
Do. do. Scotch	34.3	0.53
Do. do. do. 20 per cent. moisture	30.6	0.49
Do. Yellow pine	41.2	0.66
Do. do. American	28.7	0.46
American Pine-wood, in cord (heaped)	21	0.34
Apple-tree	45.5	0.73

	Weight of one cubic foot.	Specific Gravity.
	pounds.	
Pear-tree	45.5	0.73
Orange-tree	44.3	0.71
Olive-tree	42.4	0.68
Maple	40.5	0.65
Do. 20 per cent. moisture	41.8	0.67
Service-tree	41.8	0.67
Cypress, cut one year	41.2	0.66
Plane-tree	40.5	0.65
Vine-tree	37.4	0.60
Aspen-tree	37.4	0.60
Alder-tree	34.9	0.56
Do. 20 per cent. moisture	37.4	0.60
Sycamore	36.8	0.59
Cedar of Lebanon	30.6 to 35.5	0.49 to 0.57
Bamboo	19.5 to 24.9	0.31 to 0.40
Poplar	24.3	0.39
Do. White	20.0 to 31.8	0.32 to 0.51
Do. 20 per cent. moisture	29.9	0.48
Willow	30.6	0.49
Cork	15.0	0.24
Elder pith	4.74	0.076
INDIAN WOODS.		
(*Berkley.*)		
Northern Teak	55	0.882
Southern Teak	48	0.770
Jungle Teak	41	0.658
Blackwood	56	0.898
Khair	73	1.171
Erroul	63	1.014
Red Eyne	68	1.091
Bibla	56	0.898
Poon	39	0.625
Kullum	41	0.658
Hedoo	39	0.625
COLONIAL WOODS.		
(*Fowke.*)		
JAMAICA:—		
Black heart ebony	74.2	1.19
Lignum vitæ	40.5 to 73.0	0.65 to 1.17
Small leaf	73.0	1.17
Neesberry bullet-tree	65.5	1.05
Red bully-tree	62.36	1.00
Iron wood	61.7	0.99
Sweet wood	60.5	0.97
Fustic	60.5	0.97
Satin candlewood	59.9	0.96
Bastard cabbage bark	58.6	0.94
White dogwood	58.6	0.94
Black do.	58.0	0.93
Gynip	58.0	0.93

Colonial Woods (*continued*).	Weight of one cubic foot.	Specific Gravity.
	pounds.	
Jamaica (*continued*):—		
Wild mahogany	57.4	0.92
Cashaw	57.4	0.92
Wild orange	53.0 to 56.7	0.85 to 0.91
Sweet do.	49.3	0.79
Bullet-tree (bastard)	56.1	0.90
Tamarind	54.2	0.87
Do. wild	46.8	0.75
Prune	53.6	0.86
Yellow Sanders	53.6	0.86
Beech	52.4	0.84
French Oak	48.0	0.77
Broad Leaf	48.0	0.77
Fiddle Wood	44.3	0.71
Prickle Yellow	43.0	0.69
Boxwood	43.0	0.69
Locust-tree	42.4	0.68
Lancewood	42.4	0.68
Green Mahogany	41.2	0.66
Yacca	39.3	0.63
Cedar	36.2	0.58
Calabash	34.9	0.56
Bitter Wood	34.3	0.55
Blue Mahoe	33.7	0.54
Average of 36 woods of Jamaica	52.1	0.835
New South Wales:—		
Box of Ilwarra	73.0	1.17
Do. Bastard	69.8	1.12
Do. True, of Camden	60.5	0.97
Mountain Ash	69.2	1.11
Kakaralli	68.6	1.10
Iron Bark	64.2	1.03
Do. broad-leaved	63.6	1.02
Woolly Butt	63.0	1.01
Black Do	55.5	0.89
Water Gum	63.6	1.00
Blue Do.	52.4	0.84
Cog Wood	59.9	0.96
Mahogany	59.2	0.95
Do. swamp	53.6	0.86
Gray Gum	58.0	0.93
Stringy Bark	53.6	0.86
Hickory	46.8	0.75
Forest Swamp Oak	41.2	0.66
Mean of 18 woods of New South Wales	59.9	0.96
British Guiana:—		
Sipiri, or Greenheart	65.5 to 68.0	1.05 to 1.09
Wallaba	64.8	1.04
Brown Ebony	64.2	1.03
Letter Wood	62.36	1.00
Cuamara or Tonka	61.7	0.99
Monkey Pot	58.6	0.94
Mora	57.4	0.92

Colonial Woods (*continued*).	Weight of one cubic foot.	Specific Gravity.
	pounds.	
British Guiana (*continued*):—		
Ducaballi	56.7	0.91
Cabacalli	55.5	0.89
Kaieeri-balli	54.2	0.87
Sirabuliballi	52.4	0.84
Buhuradda	50.5	0.81
Buckati	50.5	0.81
Houbaballi	50.5	0.81
Baracara	50.5	0.81
White Cedar	48.0	0.77
Locust-tree	44.3	0.71
Cartan	43.7	0.70
Purple Heart	42.4	0.68
Bartaballi	39.9	0.64
Crabwood	37.4	0.60
Silverballi	34.3	0.55
Mean of 22 woods of British Guiana	46.1	0.74
WOOD-CHARCOAL (as powder).		
(*Claudel.*)		
Willow	96.7	1.55
Oak	95.4	1.53
Alder	92.9	1.49
Lime-tree	91.0	1.46
Poplar	90.4	1.45
Average of 5 charcoals	93.5	1.50
WOOD-CHARCOAL (in small pieces, heaped).		
(*Claudel.*)		
Walnut	39.3	0.63
Ash	34.3	0.55
Beech	32.5	0.52
Yoke-elm	28.7	0.46
Apple-tree	28.7	0.46
White Oak	26.2	0.42
Cherry-tree	25.6	0.41
Birch	22.5	0.36
Elm	22.5	0.36
Yellow Pine	20.6	0.33
Chestnut-tree	17.5	0.28
Poplar	15.6	0.25
Cedar	15.0	0.24
Average of 13 charcoals	25.3	0.405
Gunpowder	109.1 to 114.7	1.75 to 1.84
WOOD-CHARCOAL (as made, heaped).		
Oak and Beech	15 to 15.6	0.24 to 0.25
Birch	13.7 to 14.3	0.22 to 0.23
Pine	12.5 to 13.1	0.20 to 0.21
Average	14	0.225

	Weight of one cubic foot.	Specific Gravity.
ANIMAL SUBSTANCES.	pounds.	
(Claudel.)		
Pearls	169.6	2.72
Coral	167.7	2.69
Ivory	119.7	1.92
Bone	112.2 to 124.7	1.80 to 2.00
Wool	100.4	1.61
Tendon	69.8	1.12
Cartilage	68.0	1.09
Crystalline humour	67.3	1.08
Human body	66.7	1.07
Nerve	64.9	1.04
Wax	59.9	0.96
White of whalebone	58.7	0.94
Butter	58.7	0.94
Pork fat	58.7	0.94
Mutton fat	57.4	0.92
Animal charcoal, in heaps	50 to 52	0.80 to 0.83
VEGETABLE SUBSTANCES.		
(Claudel.)		
Cotton	121.6	1.95
Flax	111.6	1.79
Starch	95.4	1.53
Fecula	93.5	1.50
Gum—Myrrh	84.8	1.36
Do. Dragon	82.3	1.32
Do. Dragon's blood	74.8	1.20
Do. Sandarac	68.0	1.09
Do. Mastic	66.7	1.07
Resin—Jalap	76.1	1.22
Do. Guayacum	74.8	1.20
Do. Benzoin	68.0	1.09
Do. Colophany	66.7	1.07
Amber, Opaque	68.0	1.09
Do. Transparent	67.3	1.08
Gutta-percha	60.5	0.97
Caoutchouc	58.0	0.93
Grain, Wheat, heaped	46.7	0.75
Do. Barley, do.	36.6	0.59
Do. Oats, do.	31.2	0.50

TABLE No. 66.—WEIGHT AND VOLUME OF VARIOUS SUBSTANCES. (*Tredgold.*)

SUBSTANCE.	Cubic feet per ton, in bulk.	Weight of one cubic foot, in bulk.
	cubic feet.	lbs.
Lead (cast in pigs)	4	567
Iron (cast in pigs)	6.25	360
Limestone or marble (in blocks)	13	172
Granite (Aberdeen, in blocks)	13.5	166
Granite (Cornish, in blocks)	14	164
Sandstone (in blocks)	16	141
Portland stone (in blocks)	17	132
Potter's clay	17	130
Loam or strong soil	18	126
Bath stone (in blocks)	18	123.5
Gravel	21	109
Sand	23.5	95
Bricks (common stocks, dry)	24	93
Culm	36	63
Water (river)	36	62.5
Splint coal	39.5	57
Oak (seasoned)	43	52
Coal (Newcastle caking)	45	50
Wheat	47	48
Barley	59	38
Red fir	59	38
Hay (compact, old)	280	8

TABLE No. 67.—WEIGHT AND VOLUME OF GOODS CARRIED ON THE BOMBAY, BARODA, AND CENTRAL INDIA RAILWAY.

By Colonel J. P. KENNEDY, Consulting Engineer of the Railway.

No. of kind.	CLASSIFICATION OF GOODS CONVEYED.	Cubic feet per ton.	Weight per cubic foot.	Cubic feet per ton, in bulk (estimated).
		cubic feet.	lbs.	cubic feet.
1	Unpressed cotton	224	10	280
2	Furniture	200	11	250
3	Half-pressed cotton	186	12	233
4	Cotton seeds	186	12	233
5	Wool	140	16	175
6	Fruit and vegetables	100	22	125
7	Eggs	90	25	113
Class 1.	Averages	174	13	217

GOODS CONVEYED OVER THE INDIAN RAILWAY (*continued*).

No. of kind.	CLASSIFICATION OF GOODS CONVEYED.	Cubic feet per ton.	Weight per cubic foot.	Cubic feet per ton, in bulk (estimated).
		cubic feet.	lbs.	cubic feet.
8 ...	Grass	... 80 ...	... 28 ...	...100
9	Sundries	80	28	100
10 ...	Bagging	... 70 ...	... 32 ...	... 87
11	Commissariat stores	70	32	87
12 ...	Full-pressed cotton	... 70 ...	... 32 ...	... 87
13	Flax and hemp	70	32	87
14 ...	Groceries	... 60 ...	... 37 ...	... 75
15	Grains and seed	60	37	75
16 ...	Twist	... 60 ...	... 37 ...	... 75
17	Sugar	56	40	70
18 ...	Soap	... 56 ...	... 40 ...	... 70
19	Firewood	56	40	70
20 ...	Salt	... 51 ...	... 44 ...	... 64
21	Lime	51	44	64
22 ...	Dry Fruits	... 50 ...	45	... 63
Class 2.	Averages	... 60 ...	... 37 ...	... 75
23 ...	Jagree (Molasses)	... 45 ...	... 50 ...	... 56
24	Kupas (Seed cotton)	45	50	56
25 ...	Mowra (flowers which produce spirit)	... 45 ...	... 50 ...	... 56
26	Timber	45	50	56
27 ...	Ghee (clarified butter)	... 40 ...	... 56 ...	... 50
28	Oil	40	56	50
29 ...	Piece goods	... 40 ...	... 56 ...	... 50
30	Rape	40	56	50
31 ...	Beer and Spirits	... 36 ...	... 62 ...	... 45
32	Coal	28	80	35
33 ...	Paper	... 28 ...	... 80 ...	... 35
34	Tobacco	28	80	35
35 ...	Opium	... 26 ...	... 86 ...	... 33
36	Machinery	25	90	31
Class 3.	Averages	... 41 ...	... 54 ...	... 51
37 ...	Cutlery	... 20 ...	... 112 ...	... 25
38	Potash	20	112	25
39 ...	Sand	... 20 ...	... 112 ...	... 20
40	Colour	18	124	22
41 ...	Bricks	... 17 ...	... 132 ...	... 21
42	Stone	15	148	19
43 ...	Metal	... 5 ...	... 443 ...	... 6¼
Class 4.	Averages	... 11 ...	... 203 ...	... 14
	Averages of all classes	...64.4...	...35.4...	... 80

Note.—The last column has been added by the author; the quantities are calculated by adding one-fourth to the quantities in the third column, to give approximate estimate of the volume occupied in waggons by the goods, or the space required to load a ton of each kind. Sand, No. 39, lies solid in any situation.

TABLE No. 68.—WEIGHT AND SPECIFIC GRAVITY OF LIQUIDS.

LIQUIDS AT 32° F.	Weight of one cubic foot.	Weight of one gallon.	Specific Gravity.
	pounds.	pounds.	Water = 1.
Mercury	848.7	136.0	13.596
Bromine	185.1	29.7	2.966
Sulphuric acid, maximum concentration	114.9	18.4	1.84
Nitrous acid	96.8	15.5	1.55
Chloroform	95.5	15.3	1.53
Water of the Dead Sea	77.4	12.4	1.24
Nitric acid, of commerce	76.2	12.2	1.22
Acetic acid, maximum concentration	67.4	10.8	1.08
Milk	64.3	10.3	1.03
Sea water, ordinary	64.05	10.3	1.026
Pure water (distilled) at 39°.1 F.	62.425	10.0	1.000
Wine of Bordeaux	62.1	9.9	0.994
Do. Burgundy	61.9	9.9	0.991
Oil, lintseed	58.7	9.4	0.94
Do. poppy	58.1	9.3	0.93
Do. rape-seed	57.4	9.2	0.92
Do. whale	57.4	9.2	0.92
Do. olive	57.1	9.15	0.915
Do. turpentine	54.3	8.7	0.87
Do. potato	51.2	8.2	0.82
Petroleum	54.9	8.8	0.88
Naphtha	53.1	8.5	0.85
Ether, nitric	69.3	11.1	1.11
Do. sulphurous	67.4	10.8	1.08
Do. nitrous	55.6	8.9	0.89
Do. acetic	55.6	8.9	0.89
Do. hydrochloric	54.3	8.7	0.87
Do. sulphuric	44.9	7.2	0.72
Alcohol, proof spirit	57.4	9.2	0.92
Do. pure	49.3	7.9	0.79
Benzine	53.1	8.5	0.85
Wood spirit	49.9	8.0	0.80

TABLE No. 69.—WEIGHT AND SPECIFIC GRAVITY OF GASES AND VAPOURS.

GASES AT 32° F. AND UNDER ONE ATMOSPHERE OF PRESSURE.	Volume of one pound weight.	Weight of one cubic foot.		Specific Gravity.
	cubic feet.	in pounds.	in ounces.	Air = 1.
Vapour of mercury (ideal)	1.776	0.563	9.008	6.9740
Vapour of bromine	2.236	0.447	7.156	5.5400
Chloroform	2.337	0.428	6.846	5.3000
Vapour of turpentine	2.637	0.378	6.042	4.6978
Acetic ether	4.075	0.245	3.927	3.0400
Vapour of benzine	4.598	0.217	3.480	2.6943
Vapour of sulphuric ether	4.790	0.209	3.340	2.5860
Vapour of ether (?)	4.777	0.206	3.302	2.5563
Chlorine	5.077	0.197	3.152	2.4400
Sulphurous acid	5.513	0.1814	2.902	2.2470
Alcohol	7.679	0.1302	2.083	1.6130
Carbonic acid (actual)	8.101	0.12344	1.975	1.5290
Do. (ideal)	8.157	0.12259	1.961	1.5186
Oxygen	11.205	0.089253	1.428	1.1056
Air	12.387	0.080728	1.29165	1.0000
Nitrogen	12.723	0.078596	1.258	0.9736
Carbonic oxide	12.804	0.0781	1.250	0.9674
Olefiant gas	12.580	0.0795	1.272	0.9847
Gaseous steam	19.913	0.05022	0.8035	0.6220
Ammoniacal gas	21.017	0.04758	0.7613	0.5894
Light carburetted hydrogen	22.412	0.04462	0.7139	0.5527
Coal-gas (page 458)	28.279	0.03536	0.5658	0.4381
Hydrogen	178.83	0.005592	0.0895	0.0692

TABLES OF THE WEIGHT OF IRON AND OTHER METALS.

Wrought Iron.—According to Table No. 65 of the Weight and Specific Gravity of Solids, the weight of a cubic foot of wrought iron varies, for various qualities, from 466 pounds to 487 pounds per cubic foot, and the average weight, taken for purposes of general calculation, is 480 pounds per cubic foot. This average weight is equivalent to a weight of 40 pounds per square foot, 1 inch in thickness—a convenient unit, which is usually employed in the development of tables of weights of iron for engineering and manufacturing purposes. The extremes of variation from this medium unit, extend from ⅞ pound less, to about ⅝ pound more than 40 pounds per square foot, or from 2.2 to 1.5 per cent. either way—a deviation, the extent of which is of little or no practical consequence, and which, at all events, is comprehended in the percentages allowed in the framing of estimates.

The average weight of a cubic inch of wrought iron is

$$\frac{480}{1728} = .27\dot{7} \text{ pound,}$$

or one-tenth more than a quarter of a pound. For a round number, when cubic inches are dealt with, it may be, and is usually, taken as .28 pound, which is only four-fifths of 1 per cent. more than the medium weight, and corresponds to a weight of 483.84 pounds per cubic foot, or to 40.32 pounds per square foot, 1 inch thick, or to 10 pounds per lineal yard, 1 inch square.

The volume of 1 pound of wrought iron is 3.6 cubic inches.

Steel.—The weight of a cubic foot of steel varies from 435 pounds to 493 pounds per cubic foot, and the average weight is about 490 pounds per cubic foot. For convenience of calculation, the average weight is taken in the following tables, as 489.6 pounds per cubic foot, for which the specific weight is 1.02, when that of wrought iron = 1.00. The weight of a square foot, 1 inch thick, is 40.8 pounds; of a lineal yard, 1 inch square, 10.2 pounds; and of a cubic inch, .283 pound.

The volume of 1 pound of steel is 3.53 cubic inches.

Cast Iron.—The weight of a cubic foot of cast iron varies from 378¼ pounds to 467⅔ pounds per cubic foot, and the average weight is taken as 450 pounds. The weight of a square foot, 1 inch thick is, therefore, 37.5 pounds; of a lineal yard, 1 inch square, 9.375 pounds; and a cubic inch, .26 pound. The specific weight is .9375.

The volume of 1 pound of cast iron is 3.84 cubic inches.

The following data, for the weight of iron, are abstracted for readiness of reference:—

Wrought Iron, Rolled.

1 cubic foot,...............................	480 pounds, or 4.29 cwts.
1 square foot, 1 inch thick,...........	40 pounds.
1 square foot, 3 inches thick, ...	120 pounds, or 1.07 cwts.
3 square feet, 1 inch thick, ...	120 pounds, or 1.07 cwts.
1 lineal foot, 1 inch square,...........	3⅓ pounds, or .03 cwt.
1 cubic inch, say.........................	0.28 pound.
3.6 cubic inch,..........................	1 pound.
1 lineal yard, 1 inch square, ...	10 pounds.
1 lineal foot, 3 inches square,.........	30 pounds.
1 lineal foot, 6 inches square,.........	120 pounds, or 1.07 cwts.
1 lineal foot, 3 inches by 1 inch thick,	10 pounds.
1 lineal foot, ⅞ inch in diameter,....	2 pounds.
1 lineal foot, 2 inches in diameter,...	10.5 pounds.
1 lineal foot, 6½ in. in diameter, about	1 cwt.

Cast Iron.

1 cubic foot,.................................	450 pounds, or 4 cwts.
5 cubic feet, ...	1 ton.
1 square foot, 1 inch thick, ...	37.5 pounds.
1 square foot, 3 inches thick (¼ cub. ft.),	112.5 pounds, or 1 cwt.
3 square feet, 1 inch thick, ...	112.5 pounds, or 1 cwt.
1 cubic inch,.................................	0.26 pound.
3.84 cubic inches, ...	1 pound.

The Table No. 70 contains the weight of iron and other metals for the following volumes:—

1 cubic foot.
1 square foot, 1 inch thick, or $^1/_{12}$th of a cubic foot.
1 lineal foot, 1 inch square, or $^1/_{12}$th of a square foot.
1 cubic inch, or $^1/_{12}$th of a lineal foot.
A sphere, 1 foot in diameter.

The specific gravity due to the respective weights per cubic foot is also given, and likewise the specific weight or heaviness, taking the weight of wrought iron as 1, or unity.

The next Table, No. 71, contains the volumes of iron and other metals for the following weights:—

1 ton, in cubic feet.
1 cwt., in square feet, 1 inch thick.
1 cwt., in lineal feet, 1 inch square.
1 pound, in cubic inches.
1 ton, as a sphere, in feet of diameter.
1 ton, as a cube, in feet of lineal dimension.

The next Table, No. 72, contains the weight of 1 square foot of metals of various thickness, advancing by sixteenths and by twentieths of an inch, up to 1 inch in thickness.

The fourth Table, No. 73, contains the weight of prisms or bars of iron, and other metals, or metals of any other uniform section, for given sectional areas, varying from .1 square inch to 10 square inches of section, advancing by one-tenth of an inch, for 1 foot and 1 yard in length.

This table is useful in calculations of the weights of bars of every form, rails, joists, beams, girders, tubes, or pipes, &c., when the sectional area is given.

The table is available for finding the weight of a metal for any sectional area up to 100 square inches, by simply advancing the decimal points one place to the right; or, in round numbers, up to 1000 square inches, by advancing the decimal points two places. For example, to find the weight of wrought iron having a sectional area of 17 square inches:—

For 1.7 square inches, the weight per foot is 5.67 pounds.
For 17 square inches, the weight per foot is 56.7 pounds.
For 170 square inches, the weight per foot is 567 pounds.

Table No. 70.—Weight of Metals.

Metal.	Cubic Foot.	Square Foot, 1 Inch Thick.	Lineal Foot, 1 Inch Square.	Cubic Inch.	Sphere, 1 Foot Diameter.	Specific Gravity.	Specific Weight.
	lbs. or cwts.	lbs. or cwts.	lbs.	lb.	lbs.	Water =1.	Wro'ght Iron=1.
Wrought Iron.......	480 or 4.29	40 or .357	3.333	.278	251	7.698	1.000
Cast Iron............	450 or 4.02	37.5 or .335	3.125	.260	236	7.217	.9375
Steel..................	489.6 or 4.37	40.8 or .364	3.400	.283	257	7.852	1.020
Copper, Sheet	549 or 4.90	45.8 or .409	3.813	.318	287	8.805	1.144
Copper, Hammered	556 or 4.96	46.3 or .413	3.861	.322	291	8.917	1.158
Tin	462 or 4.13	38.5 or .344	3.208	.268	242	7.409	.962
Zinc	437 or 3.90	36.4 or .325	3.035	.253	229	7.008	.910
Lead..................	712 or 6.36	59.3 or .530	4.944	.412	373	11.418	1.483
Brass, Cast..........	505 or 4.51	42.1 or .375	3.507	.292	264	8.099	1.052
Brass, Wire.........	533 or 4.76	44.4 or .396	3.701	.308	279	8.548	1.110
Gun Metal...........	524 or 4.68	43.7 or .390	3.639	.304	274	8.404	1.092
Silver.....	655 or 5.85	54.6 or .488	4.549	.379	343	10.505	1.365
Gold	1200 or 10.72	100.0 or .893	8.333	.694	628	19.245	2.500
Platinum	1342 or 12.00	111.8 or 1.000	9.320	.777	703	21.522	2.796

Table No. 71.—Volume of Metals for given Weights.

Metal.	Cubic Feet to a Ton.	Square Feet, 1 Inch Thick, to a cwt.	Lineal Feet, 1 In. Square, to a cwt.	Cubic Inches to a lb.	Diameter of a Sphere of 1 Ton.	Side of a Cube of 1 Ton.
	cubic feet.	square feet.	feet.	cubic inches.	feet.	feet.
Wrought Iron.......	4.67	2.80	33.6	3.60	2.07	1.67
Cast Iron............	4.98	2.99	35.8	3.84	2.12	1.71
Steel..................	4.58	2.75	32.9	3.53	2.26	1.66
Copper, Sheet	4.08	2.44	29.4	3.15	1.98	1.60
Copper, Hammered	4.03	2.42	29.0	3.11	1.98	1.59
Tin	4.86	2.91	34.9	3.74	2.10	1.69
Zinc....................	5.13	3.08	36.8	3.95	2.14	1.73
Lead..................	3.15	1.89	22.7	2.43	1.81	1.47
Brass, Cast..........	4.44	2.67	31.9	3.42	2.04	1.64
Brass, Wire.........	4.20	2.30	30.3	3.24	2.00	1.61
Gun Metal...........	4.28	2.56	30.8	3.30	2.02	1.62
Silver..................	3.42	2.05	24.6	2.64	1.87	1.51
Gold	1.87	1.12	13.4	1.44	1.59	1.28
Platinum	1.67	1.00	12.0	1.29	1.47	1.19

Table No. 72.—Weight of 1 Square Foot of Metals.

Thickness advancing by Sixteenths of an Inch.

Thickness.	Wro't Iron. Specific wt.=1.	Cast Iron. Specific wt.=.9375.	Steel. Specific wt.=1.02.	Copper. Specific wt.=1.16.	Tin. Specific wt.=.962.	Zinc. Specific wt.=.910.	Brass. Specific wt.=1.052.	Gun Metal. Specific wt.=1.092.	Lead. Specific wt.=1.48.
inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1/16	2.50	2.34	2.55	2.89	2.41	2.28	2.63	2.73	3.71
⅛	5.00	4.69	5.10	5.79	4.81	4.55	5.26	5.46	7.41
3/16	7.50	7.03	7.65	8.68	7.22	6.83	7.89	8.19	11.1
¼	10.0	9.38	10.2	11.6	9.63	9.10	10.5	10.9	14.8
5/16	12.5	11.7	12.8	14.5	12.0	11.4	13.2	13.7	18.5
⅜	15.0	14.1	15.3	17.4	14.4	13.7	15.8	16.4	22.2
7/16	17.5	16.4	17.9	20.3	16.8	15.9	18.4	19.1	25.9
½	20.0	18.7	20.4	23.2	19.3	18.2	21.1	21.9	29.7
9/16	22.5	21.1	23.0	26.0	21.7	20.5	23.7	24.6	33.4
⅝	25.0	23.5	25.5	28.9	24.1	22.8	26.3	27.3	37.1
11/16	27.5	25.8	28.1	31.8	26.5	25.0	28.9	30.0	40.8
¾	30.0	28.1	30.6	34.7	28.9	27.3	31.6	32.8	44.5
13/16	32.5	30.5	33.2	37.6	31.3	29.6	34.2	35.0	48.2
⅞	35.0	32.8	35.7	40.5	33.7	31.9	36.8	38.2	51.9
15/16	37.5	35.2	38.3	43.4	36.1	34.1	39.5	41.0	55.6
1	40.0	37.5	40.8	46.3	38.5	36.4	42.1	43.7	59.3

Thickness advancing by Twentieths of an Inch.

Thickness.	Wro't Iron.	Cast Iron.	Steel.	Copper.	Tin.	Zinc.	Brass.	Gun Metal.	Lead.
inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
.05	2.00	1.88	2.04	2.32	1.93	1.82	2.11	2.19	2.96
.10	4.00	3.75	4.08	4.63	3.85	3.64	4.21	4.37	5.93
.15	6.00	5.63	6.12	6.95	5.78	5.46	6.32	6.56	8.90
.20	8.00	7.50	8.16	9.26	7.70	7.28	8.42	8.74	11.9
.25	10.0	9.38	10.2	11.6	9.63	9.10	10.5	10.9	14.8
.30	12.0	11.3	12.2	13.9	11.6	10.9	12.6	13.1	17.8
.35	14.0	13.1	14.3	16.2	13.5	12.7	14.7	15.3	20.8
.40	16.0	15.0	16.3	18.5	15.4	14.6	16.8	17.5	23.7
.45	18.0	16.9	18.4	20.8	17.3	16.4	18.9	19.7	26.7
.50	20.0	18.8	20.4	23.2	19.3	18.2	21.1	21.9	29.7
.55	22.0	20.6	22.4	25.5	21.2	20.0	23.2	24.0	32.7
.60	24.0	22.5	24.5	27.8	23.1	21.8	25.3	26.2	35.6
.65	26.0	24.4	26.5	30.1	25.0	23.7	27.4	28.4	38.6
.70	28.0	26.3	28.6	32.4	27.0	25.5	29.5	30.6	41.5
.75	30.0	28.1	30.6	34.7	28.9	27.3	31.6	32.8	44.5
.80	32.0	30.0	32.6	37.0	30.8	29.1	33.7	35.0	47.5
.85	34.0	31.9	34.7	39.4	32.7	30.9	35.8	37.2	50.4
.90	36.0	33.8	36.7	41.7	34.7	32.8	37.9	39.3	53.4
.95	38.0	35.6	38.8	44.0	36.6	34.6	40.0	41.5	56.3
1.00	40.0	37.5	40.8	46.3	38.5	36.4	42.1	43.7	59.3

Note to Table 73, next page.—To find the weight of 1 lineal foot or 1 lineal yard of hammered iron, copper, tin, zinc, or lead, multiply the tabular weight for rolled wrought iron of the given dimensions by the following multipliers, respectively:—

	Exact.	Approximate.			
Hammered Iron	1.008	1.01	equivalent to 1	per cent.	more.
Copper	1.158	1.16	,, 16	,,	more.
Tin	.962	.96	,, 4	,,	less.
Zinc	.91	.91	,, 9	,,	less.
Lead	1.483	1.48	,, 48	,,	more.

Table No. 73.—WEIGHT OF METALS, OF A GIVEN SECTIONAL AREA, PER LINEAL FOOT AND PER LINEAL YARD.

SECT. AREA.	ROLLED WROUGHT IRON. Sp. Weight=1.		CAST IRON. Sp. Weight=.9375.		STEEL. Sp. Weight=1.02.		BRASS. Sp. Weight=1.052.		GUN METAL. Sp. Weight=1.092.	
	1 Foot.	1 Yard.	1 Foot.	1 Yard.	1 Foot.	1 Yard.	1 Foot.	1 Yard.	1 Foot.	1 Yard.
sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
.1	.333	1.00	.313	.938	.340	1.02	.351	1.05	.364	1.09
.2	.667	2.00	.625	1.88	.680	2.04	.701	2.10	.728	2.18
.3	1.00	3.00	.938	2.81	1.02	3.06	1.05	3.16	1.09	3.28
.4	1.33	4.00	1.25	3.75	1.36	4.08	1.43	4.21	1.46	4.37
.5	1.67	5.00	1.56	4.69	1.70	5.10	1.75	5.26	1.82	5.46
.6	2.00	6.00	1.88	5.63	2.04	6.12	2.11	6.31	2.18	6.55
.7	2.33	7.00	2.19	6.56	2.38	7.14	2.46	7.36	2.55	7.64
.8	2.67	8.00	2.50	7.50	2.72	8.16	2.81	8.42	2.91	8.74
.9	3.00	9.00	2.81	8.44	3.06	9.18	3.16	9.47	3.28	9.83
1.0	3.33	10.0	3.15	9.38	3.40	10.2	3.51	10.5	3.64	10.9
1.1	3.67	11.0	3.44	10.3	3.74	11.2	3.86	11.6	4.00	12.0
1.2	4.00	12.0	3.75	11.3	4.08	12.2	4.21	12.6	4.37	13.1
1.3	4.33	13.0	4.06	12.2	4.42	13.3	4.56	13.7	4.73	14.2
1.4	4.67	14.0	4.38	13.1	4.76	14.3	4.91	14.7	5.10	15.3
1.5	5.00	15.0	4.69	14.1	5.10	15.3	5.26	15.8	5.46	16.4
1.6	5.33	16.0	5.00	15.0	5.44	16.3	5.61	16.8	5.82	17.5
1.7	5.67	17.0	5.31	15.9	5.78	17.3	5.96	17.9	6.19	18.6
1.8	6.00	18.0	5.63	16.9	6.12	18.4	6.31	18.9	6.55	19.7
1.9	6.33	19.0	5.94	17.8	6.46	19.4	6.66	20.0	6.92	20.8
2.0	6.67	20.0	6.25	18.8	6.80	20.4	7.01	21.0	7.28	21.8
2.1	7.00	21.0	6.56	19.7	7.14	21.4	7.36	22.1	7.64	22.9
2.2	7.33	22.0	6.88	20.6	7.48	22.4	7.72	23.1	8.01	24.0
2.3	7.67	23.0	7.19	21.6	7.82	23.5	8.07	24.2	8.37	25.1
2.4	8.00	24.0	7.50	22.5	8.16	24.5	8.42	25.3	8.74	26.2
2.5	8.33	25.0	7.81	23.4	8.50	25.5	8.77	26.3	9.10	27.3
2.6	8.67	26.0	8.13	24.4	8.84	26.5	9.12	27.4	9.46	28.4
2.7	9.00	27.0	8.44	25.3	9.18	27.5	9.47	28.4	9.83	29.5
2.8	9.33	28.0	8.75	26.3	9.52	28.6	9.82	29.5	10.2	30.6
2.9	9.67	29.0	9.06	27.2	9.86	29.6	10.2	30.5	10.6	31.7
3.0	10.0	30.0	9.38	28.1	10.2	30.6	10.5	31.6	10.9	32.8
3.1	10.3	31.0	9.69	29.1	10.5	31.6	10.9	32.6	11.3	33.9
3.2	10.7	32.0	10.0	30.0	10.9	32.6	11.2	33.7	11.7	34.9
3.3	11.0	33.0	10.3	30.9	11.2	33.7	11.6	34.7	12.0	36.0
3.4	11.3	34.0	10.6	31.9	11.6	34.7	11.9	35.8	12.4	37.1
3.5	11.7	35.0	10.9	32.8	11.9	35.7	12.3	36.8	12.7	38.2
3.6	12.0	36.0	11.3	33.8	12.2	36.7	12.6	37.9	13.1	39.3
3.7	12.3	37.0	11.6	34.7	12.6	37.7	13.0	38.9	13.5	40.4
3.8	12.7	38.0	11.9	35.6	12.9	38.8	13.3	40.0	13.8	41.5
3.9	13.0	39.0	12.2	36.6	13.3	39.8	13.7	41.0	14.2	42.6
4.0	13.3	40.0	12.5	37.5	13.6	40.8	14.0	42.1	14.6	43.7
4.1	13.7	41.0	12.8	38.4	13.9	41.8	14.4	43.1	14.9	44.8
4.2	14.0	42.0	13.1	39.4	14.3	42.8	14.7	44.2	15.3	45.9
4.3	14.3	43.0	13.4	40.3	14.6	43.9	15.1	45.2	15.7	46.9
4.4	14.7	44.0	13.8	41.3	15.0	44.9	15.4	46.3	16.0	48.0
4.5	15.0	45.0	14.1	42.2	15.3	45.9	15.8	47.3	16.4	49.1
4.6	15.3	46.0	14.4	43.1	15.6	46.9	16.1	48.4	16.7	50.2
4.7	15.7	47.0	14.7	44.1	16.0	47.9	16.5	49.4	17.1	51.3
4.8	16.0	48.0	15.0	45.0	16.3	49.0	16.8	50.5	17.5	52.4
4.9	16.3	49.0	15.3	45.9	16.7	50.0	17.2	51.6	17.8	53.5
5.0	16.7	50.0	15.6	46.9	17.0	51.0	17.5	52.6	18.2	54.6

Table No 73 (*continued*).

Sect. Area.	Rolled Wrought Iron. Sp. Weight=1.		Cast Iron. Sp.Weight=.9375.		Steel. Sp. Weight=1.02.		Brass. Sp.Weight=1.052.		Gun Metal. Sp.Weight=1.092.	
	1 Foot.	1 Yard.	1 Foot.	1 Yard.	1 Foot.	1 Yard.	1 Foot.	1 Yard.	1 Foot.	1 Yard.
sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
5.1	17.0	51.0	15.9	47.8	17.3	52.0	17.9	53.7	18.6	55.7
5.2	17.3	52.0	16.3	48.8	17.7	53.0	18.2	54.7	18.9	56.8
5.3	17.7	53.0	16.6	49.7	18.0	54.1	18.6	55.8	19.3	57.9
5.4	18.0	54.0	16.9	50.6	18.4	55.1	18.9	56.8	19.7	58.9
5.5	18.3	55.0	17.2	51.6	18.7	56.1	19.3	57.9	20.0	60.0
5.6	18.7	56.0	17.5	52.5	19.0	57.1	19.6	58.9	20.4	61.1
5.7	19.0	57.0	17.8	53.4	19.4	58.1	20.0	60.0	20.8	62.2
5.8	19.3	58.0	18.1	54.4	19.7	59.2	20.3	61.0	21.1	63.3
5.9	19.7	59.0	18.4	55.3	20.1	60.2	20.7	62.1	21.5	64.4
6.0	20.0	60.0	18.8	56.3	20.4	61.2	21.0	63.1	21.8	65.5
6.1	20.3	61.0	19.1	57.2	20.7	62.2	21.4	64.2	22.2	66.6
6.2	20.7	62.0	19.4	58.1	21.1	63.2	21.7	65.2	22.6	67.7
6.3	21.0	63.0	19.7	59.1	21.4	64.3	22.1	66.3	22.9	68.8
6.4	21.3	64.0	20.0	60.0	21.8	65.3	22.4	67.3	23.3	69.9
6.5	21.7	65.0	20.3	60.9	22.1	66.3	22.8	68.4	23.7	70.9
6.6	22.0	66.0	20.6	61.9	22.4	67.3	23.1	69.4	24.0	72.0
6.7	22.3	67.0	20.9	62.8	22.8	68.3	23.5	70.5	24.4	73.1
6.8	22.7	68.0	21.3	63.8	23.1	69.4	23.9	71.5	24.8	74.2
6.9	23.0	69.0	21.6	64.7	23.5	70.4	24.2	72.6	25.1	75.3
7.0	23.3	70.0	21.9	65.6	23.8	71.4	24.6	73.6	25.5	76.4
7.1	23.7	71.0	22.2	66.6	24.1	72.4	24.9	74.7	25.8	77.5
7.2	24.0	72.0	22.5	67.5	24.5	73.4	25.3	75.7	26.2	78.6
7.3	24.3	73.0	22.8	68.4	24.8	74.5	25.6	76.8	26.6	79.7
7.4	24.7	74.0	23.1	69.4	25.2	75.5	26.0	77.9	26.9	80.8
7.5	25.0	75.0	23.4	70.3	25.5	76.5	26.3	78.9	27.3	81.9
7.6	25.3	76.0	23.8	71.3	25.9	77.5	26.7	80.0	27.7	83.0
7.7	25.7	77.0	24.1	72.2	26.2	78.5	27.0	81.0	28.0	84.1
7.8	26.0	78.0	24.4	73.1	26.5	79.6	27.4	82.1	28.4	85.2
7.9	26.3	79.0	24.7	74.1	26.9	80.6	27.7	83.1	28.8	86.3
8.0	26.7	80.0	25.0	75.0	27.2	81.6	28.1	84.2	29.1	87.4
8.1	27.0	81.0	25.3	75.9	27.5	82.6	28.4	85.2	29.5	88.5
8.2	27.3	82.0	25.6	76.9	27.9	83.6	28.8	86.3	29.9	89.5
8.3	27.7	83.0	25.9	77.8	28.2	84.7	29.1	87.3	30.2	90.6
8.4	28.0	84.0	26.3	78.8	28.6	85.7	29.5	88.4	30.6	91.7
8.5	28.3	85.0	26.6	79.7	28.9	86.7	29.8	89.4	30.9	92.8
8.6	28.7	86.0	26.9	80.6	29.2	87.7	30.2	90.5	31.3	93.9
8.7	29.0	87.0	27.2	81.6	29.6	88.7	30.5	91.5	31.7	95.0
8.8	29.3	88.0	27.5	82.5	29.9	89.8	30.9	92.6	32.0	96.1
8.9	29.7	89.0	27.8	83.4	30.3	90.8	31.2	93.6	32.4	97.2
9.0	30.0	90.0	28.1	84.4	30.6	91.8	31.6	94.7	32.8	98.3
9.1	30.3	91.0	28.4	85.3	30.9	92.8	31.9	95.7	33.1	99.4
9.2	30.7	92.0	28.8	86.3	31.3	93.8	32.3	96.8	33.5	100.5
9.3	31.0	93.0	29.1	87.2	31.6	94.9	32.6	97.8	33.9	101.6
9.4	31.3	94.0	29.4	88.1	32.0	95.9	33.0	98.9	34.2	102.7
9.5	31.7	95.0	29.7	89.1	32.3	96.9	33.3	99.9	34.6	103.7
9.6	32.0	96.0	30.0	90.0	32.6	97.9	33.7	101.0	34.9	104.8
9.7	32.3	97.0	30.3	90.9	33.0	98.9	34.0	102.0	35.3	105.9
9.8	32.7	98.0	30.6	91.9	33.3	100.0	34.4	103.1	35.7	107.0
9.9	33.0	99.0	30.9	92.8	33.7	101.0	34.7	104.2	36.0	108.1
10.0	33.3	100.0	31.3	93.8	34.0	102.0	35.1	105.2	36.4	109.2

See note at foot of page 220.

RULES FOR THE WEIGHT OF IRON AND STEEL.

The following rules for finding the weight of wrought iron, cast iron, and steel, are based on the data contained in Tables No. 70 and 71.

RULE 1.—TO FIND THE WEIGHT OF IRON OR STEEL, *when the volume in cubic feet is given.* Multiply the volume by

4.29 for wrought iron,
4.02 for cast iron,
4.37 for steel.

The product is the weight in hundredweights.

RULE 2.—*When the volume in cubic inches is given*, multiply the volume by

.278 (or .28) for wrought iron,
.26 for cast iron,
.283 for steel.

The product is the weight in pounds.

RULE 3.—*When the quantity is reduced to square feet, one inch in thickness*, multiply the area by

40 for wrought iron,
37½ for cast iron,
40.8 (or 41) for steel.

The product is the weight in pounds.

Or, multiply the area by

.357 for wrought iron,
.335 for cast iron,
.364 for steel.

The product is the weight in hundredweights.

RULE 4.—*When the sectional area in square inches, and the length in feet, of a bar or prism are given*, multiply the sectional area by the length, and by

3 $^1/_3$ for wrought iron,
3⅛ for cast iron,
3.4 for steel.

The product is the weight in pounds.

For large masses, multiply the sectional area by the length, and divide the product by

672 for wrought iron,
717 for cast iron,
659 for steel.

The quotient is the weight in tons.

RULE 5.—*When the sectional area in square inches, and the length in yards, of a bar or prism, are given*, multiply the sectional area by the length, and by

10 for wrought iron,
9.375 for cast iron,
10.2 for steel.

The product is the weight in pounds.

RULE 6.—TO FIND THE SECTIONAL AREA OF A BAR OR PRISM OF IRON OR STEEL, *when the length and the total weight are given.* Divide the weight in pounds by the length in feet, and by

$3\,{}^1/_3$ for wrought iron,
$3\frac{1}{8}$ for cast iron,
3.4 for steel.

The quotient is the sectional area in square inches.

RULE 7.—TO FIND THE LENGTH OF A BAR, PRISM, OR OTHER PIECE OF UNIFORM SECTION OF IRON OR STEEL, *when the total weight and the sectional area are given.* Divide the weight in pounds by the sectional area in square inches, and by

$3\,{}^1/_3$ for wrought iron,
$3\frac{1}{8}$ for cast iron,
3.4 for steel.

The quotient is the length in feet.

In applying the last rule to calculate the length of wire of a given size, for a given weight, say 1 cwt. of wire, the sectional area of the wire is found, in the usual way, by multiplying the square of the thickness or diameter, d, by .7854. Then, by the rule, the length in feet of 1 cwt. of iron wire is equal to

$$\frac{112}{d^2 \times .7854 \times 3\,{}^1/_3} = \frac{42.78}{d^2}.$$

In the same way, the dividends of the fractions to express the length of 1 cwt. of other metals may be found, and the following is a special rule for wire:—

RULE 8.—TO FIND THE LENGTH OF ONE HUNDREDWEIGHT OF WIRE OF A GIVEN THICKNESS. Divide the following numbers by the square of the diameter or thickness, in parts of an inch:—

42.78 for wrought iron,
42 for steel,
37.43 for copper,
38.54 for brass,
31.34 for silver,
17.12 for gold,
15.28 for platinum.

The quotient is the length in feet.

Note.—This rule may be used for finding the weight of round bar iron.

2. It is known that the density of wire is not perfectly constant, but that there is some degree of variation, according to the size. It is generally understood that the density is reduced as the wire is drawn smaller, but it appears from the table of the weight of Warrington wire, that the density is greater for the smallest sizes. The same inference is to be drawn from tabular statements of the length of one kilogramme of wire according to the French gauge (Table No. 31, page 148). One of these statements is given on the next page, from which it is apparent that the length of iron

required to weigh a kilogramme decreases more rapidly than the sectional area increases. For example, the diameter being

6, 12, 24, 30 tenths of a millimetre,

the squares of which, or the relative volumes of a given length, are as

1, 4, 16, 25;

the lengths of a kilogramme are

405, 115, 30, 20 metres,

which are inversely as

1, 3.5, 13.5, 20.2.

Showing that a shorter length is required in proportion to the volume, as the diameter of the wire is reduced, and that the density of the smaller wire must therefore be the greater.

Table No. 73*a*.—Weight of Galvanized Iron Wire (French).

No. of Gauge.	Diameter.	Length of 1 Kilogramme.	No. of Gauge.	Diameter.	Length of 1 Kilogramme.
	millimetres.	metres.		millimetres.	metres.
1	0.6	405	13	2.0	40
2	0.7	370	14	2.2	35
3	0.8	260	15	2.4	30
4	0.9	215	16	2.7	25
5	1.0	175	17	3.0	20
6	1.1	140	18	3.4	15
7	1.2	115	19	3.9	10
8	1.3	103	20	4.4	9
9	1.4	82	21	4.9	6
10	1.5	70	22	5.4	5
11	1.6	65	23	5.9	4
12	1.8	50			

3. The densities of metals assumed in the foregoing rules are those which are tabulated in Table No. 65.

4. In estimating the weight of cast iron from plans, the weight is frequently calculated at the same rate as for wrought iron, which is heavier than cast iron, with the object of providing an allowance, by way of compensation, for occasional swellings or enlargements of castings in excess of the exact dimensions of patterns.

The following tables of the weight of metals in various forms have been calculated by means of the preceding rules. The sectional areas of bars and other pieces of uniform section are, in some tables, added for each scantling. The length of bar, and the area of plates and sheets, required to weigh 1 cwt., or 1 ton, are given.

LIST OF TABLES OF THE WEIGHT OF WROUGHT IRON,

In Bars, Plates, Sheets, Hoop-iron, Wire, and Tubes.

Table No. 74.—Weight of Flat Bar Iron; width, 1 to 11 inches; thickness, $^1/_{16}$ to 1 inch; length, 1 to 9 feet.

Table No. 75.—Weight of Square Iron; ⅛ to 6 inches square; length, 1 to 9 feet.

Table No. 76.—Weight of Round Iron, ⅛ to 24 inches in diameter; length, 1 to 9 feet.

Table No. 77.—Weight of Angle-Iron and Tee-Iron; sum of the width and depth, 1½ to 20 inches; thickness, ⅛ to 1 inch; length, 1 foot. In the composition of this table, it has been assumed that the base and the web or flange are of equal thicknesses; and that the reduction of area of section by rounding off the edges, is compensated by the filling in at the root of the flange.

Table No. 78.—Weight of Wrought-iron Plates; area, 1 to 9 square feet; thickness, ¼ to 15 inches.

Table No. 79.—Weight of Sheet Iron, according to wire-gauge used by South Staffordshire sheet-rollers; area, 1 to 9 square feet; thickness, No. 1 to No. 32 wire-gauge.

Table No. 80.—Weight of Black and Galvanized Iron Sheets (Morton's Table).

Table No. 81.—Weight of Hoop Iron; width, ⅝ to 3 inches; thickness, No. 4 to No. 21 wire-gauge; length, 1 foot.

Table No. 82.—Weight and Strength of Warrington Iron Wire.

Table No. 83.—Weight of Wrought-iron Tubes, by internal diameter; diameter, ⅝ to 36 inches; thickness, ⅝ inch to No. 18 wire-gauge; length, 1 foot.

Table No. 84.—Weight of Wrought-iron Tubes, by external diameter; diameter, 1 to 10 inches; thickness, No. 15 wire-gauge to $^5/_{16}$ inch; length, 1 foot.

Multipliers, derived from table No. 70, are subjoined, by which the tabulated weights of wrought iron may be multiplied, in order to find from these tables the weight of bars, plates, or sheets of other metal.—

	Multipliers.
Hammered Iron	1.01
Cast Iron	.94
Steel	1.02
Sheet Copper	1.14
Hammered Copper	1.16
Lead	1.48
Cast Brass	1.05
Brass Wire	1.11
Gun Metal	1.09

Table No. 74.—WEIGHT OF FLAT BAR IRON.

1 inch Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.250	.833	1.67	2.50	3.33	4.17	5.00	5.83	6.67	7.50	134.4
5/16	.313	1.04	2.08	3.12	4.16	5.20	6.24	7.28	8.32	9.36	117.5
⅜	.375	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.0	11.3	89.6
7/16	.438	1.46	2.92	4.38	5.84	7.29	8.76	10.2	11.7	13.1	76.8
½	.500	1.67	3.33	5.00	6.67	8.33	10.0	11.7	13.3	15.0	67.2
9/16	.563	1.88	3.75	5.62	7.50	9.37	11.3	13.1	15.0	16.9	59.7
⅝	.625	2.08	4.16	6.25	8.33	10.4	12.5	14.6	16.6	18.8	53.8
11/16	.688	2.29	4.58	6.87	9.17	11.4	13.8	16.0	18.3	20.6	48.9
¾	.750	2.50	5.00	7.50	10.0	12.5	15.0	17.5	20.0	22.5	44.8
13/16	.813	2.71	5.42	8.12	10.8	13.5	16.3	19.0	21.7	24.4	41.4
⅞	.875	2.92	5.84	8.76	11.7	14.6	17.5	20.4	23.4	26.3	38.4
15/16	.938	3.13	6.25	9.38	12.5	15.6	18.8	21.9	25.0	28.1	35.8
1	1.00	3.33	6.67	10.0	13.3	16.7	20.0	23.3	26.7	30.0	33.6

1⅛ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.281	.938	1.88	2.81	3.75	4.69	5.63	6.56	7.50	8.44	119.5
5/16	.352	1.17	2.34	3.52	4.68	5.86	7.03	8.20	9.37	10.6	95.6
⅜	.422	1.41	2.81	4.22	5.62	7.03	8.44	9.84	11.3	12.7	79.6
7/16	.492	1.64	3.28	4.92	6.56	8.20	9.84	11.5	13.1	14.8	68.3
½	.563	1.88	3.75	5.62	7.50	9.38	11.3	13.1	15.0	16.9	59.7
9/16	.633	2.11	4.22	6.33	8.44	10.6	12.7	14.8	16.9	19.0	53.1
⅝	.703	2.34	4.69	7.03	9.38	11.7	13.1	16.4	18.8	21.1	47.8
11/16	.773	2.58	5.16	7.73	10.3	12.9	15.5	18.0	20.6	23.2	43.4
¾	.844	2.81	5.63	8.44	11.3	14.0	16.9	19.7	22.5	25.3	39.8
13/16	.914	3.05	6.09	9.14	12.2	15.2	18.3	21.3	24.4	27.4	36.8
⅞	.984	3.28	6.56	9.84	13.1	16.4	19.7	23.0	26.3	29.5	34.1
15/16	1.06	3.52	7.03	10.6	14.1	17.6	21.1	24.6	28.1	31.6	31.9
1	1.13	3.75	7.50	11.3	15.0	18.8	22.5	26.3	30.0	33.8	29.9

1¼ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.313	1.04	2.08	3.12	4.17	5.21	6.25	7.29	8.33	9.37	107.5
5/16	.391	1.30	2.60	3.91	5.21	6.51	7.82	9.11	10.4	11.7	94.0
⅜	.469	1.56	3.13	4.69	6.25	7.81	9.38	10.9	12.5	14.1	71.7
7/16	.547	1.82	3.65	5.47	7.29	9.12	10.9	12.8	14.6	16.4	61.2
½	.625	2.08	4.17	6.25	8.33	10.4	12.5	14.6	16.7	18.8	53.8
9/16	.703	2.34	4.69	7.03	9.38	11.7	14.1	16.4	18.8	14.1	47.8
⅝	.781	2.60	5.21	7.81	10.4	13.0	15.6	18.2	20.8	23.4	43.0
11/16	.859	2.86	5.73	8.59	11.5	14.3	17.2	20.1	22.9	25.8	39.1
¾	.938	3.13	6.25	9.38	12.5	15.6	18.8	21.9	25.0	28.1	35.8
13/16	1.02	3.39	6.77	10.2	13.5	16.9	20.3	23.7	27.1	30.5	33.1
⅞	1.11	3.65	7.29	10.9	14.6	18.2	21.9	25.5	29.2	32.8	30.7
15/16	1.17	3.91	7.81	11.7	15.6	19.5	23.4	27.3	31.2	35.1	28.7
1	1.25	4.17	8.33	12.5	16.7	20.8	25.0	29.2	33.3	37.5	26.9

Weight of Flat Bar Iron.

1⅜ inches Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.344	1.15	2.29	3.44	4.58	5.73	6.87	8.02	9.17	10.3	97.7
5/16	.430	1.43	2.86	4.30	5.73	7.16	8.59	10.0	11.5	12.9	78.2
⅜	.516	1.72	3.44	5.16	6.87	8.59	10.3	12.0	13.7	15.5	65.6
7/16	.602	2.01	4.01	6.02	8.02	10.0	12.0	14.0	16.0	18.0	55.9
½	.688	2.29	4.58	6.87	9.17	11.5	13.8	16.0	18.3	20.6	48.9
9/16	.773	2.58	5.16	7.73	10.3	12.9	15.5	18.0	20.6	23.2	43.4
⅝	.859	2.86	5.73	8.59	11.5	14.3	17.2	20.1	22.9	25.8	39.1
11/16	.945	3.15	6.31	9.45	12.6	15.8	18.9	22.1	25.2	28.4	35.5
¾	1.03	3.44	6.88	10.3	13.8	17.2	20.6	24.1	27.5	30.9	32.6
13/16	1.12	3.72	7.45	11.2	14.9	18.6	22.3	26.1	29.8	33.5	30.1
⅞	1.20	4.01	8.02	12.0	16.0	20.0	24.1	28.1	32.1	36.1	27.9
15/16	1.29	4.30	8.59	12.9	17.2	21.5	25.8	30.1	34.4	38.7	26.1
1	1.38	4.58	9.17	13.8	18.3	22.9	27.5	32.1	36.7	41.3	24.4

1½ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.275	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.0	11.3	89.6
5/16	.469	1.56	3.13	4.69	6.25	7.82	9.38	10.9	12.5	14.1	78.3
⅜	.563	1.88	3.75	5.63	7.50	9.38	11.3	13.1	15.0	16.9	59.7
7/16	.656	2.19	4.38	6.56	8.75	10.9	13.1	15.3	17.5	19.7	51.2
½	.750	2.50	5.00	7.50	10.0	12.5	15.0	17.5	20.0	22.5	44.8
9/16	.844	2.81	5.63	8.44	11.3	14.1	16.9	19.7	22.5	25.3	39.8
⅝	.938	3.13	6.25	9.38	12.5	15.6	18.8	21.9	25.0	28.1	35.8
11/16	1.03	3.44	6.88	10.3	13.8	17.2	20.6	24.1	27.5	30.9	32.6
¾	1.13	3.75	7.50	11.3	15.0	18.8	22.5	26.3	30.0	33.8	29.9
13/16	1.22	4.06	8.13	12.2	16.3	20.3	24.4	28.4	32.5	36.6	27.6
⅞	1.31	4.38	8.75	13.1	17.5	21.9	26.3	30.6	35.0	39.4	25.6
15/16	1.41	4.69	9.38	14.1	18.8	23.4	28.1	32.8	37.5	42.2	23.9
1	1.50	5.00	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	22.4

1⅝ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.406	1.35	2.71	4.06	5.41	6.8	8.10	9.48	10.8	12.2	82.7
5/16	.508	1.69	3.39	5.07	6.77	8.5	10.2	11.8	13.5	15.2	66.2
⅜	.609	2.03	4.06	6.09	8.12	10.2	12.2	14.2	16.2	18.3	55.1
7/16	.711	2.37	4.74	7.11	9.48	11.8	14.2	16.6	19.0	21.3	47.3
½	.813	2.71	5.42	8.12	10.8	13.5	16.2	19.0	21.6	24.4	41.3
9/16	.914	3.05	6.09	9.14	12.2	15.2	18.3	21.3	24.4	27.4	36.8
⅝	1.02	3.39	6.77	10.2	13.5	16.9	20.3	23.7	27.1	30.5	33.1
11/16	1.12	3.72	7.45	11.2	14.9	18.6	22.3	26.1	29.8	33.5	30.1
¾	1.22	4.06	8.13	12.2	16.3	20.3	24.4	28.4	32.5	36.6	27.6
13/16	1.32	4.40	8.80	13.2	17.6	22.0	26.4	30.8	35.2	39.6	25.4
⅞	1.43	4.74	9.48	14.2	19.0	23.7	28.4	33.2	37.9	42.7	23.6
15/16	1.53	5.08	10.2	15.2	20.3	25.4	30.5	35.5	40.6	45.7	22.1
1	1.63	5.42	10.8	16.3	21.7	27.1	32.5	37.9	43.3	48.8	21.2

Weight of Flat Bar Iron.

1¾ inches Wide.

Thickness.	Sect. Area.	Length in Feet.									Length to weigh 1 cwt.
		1	2	3	4	5	6	7	8	9	
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.638	1.46	2.92	4.37	5.83	7.29	8.74	10.2	11.7	13.1	76.8
5/16	.547	1.82	3.65	5.47	7.29	9.11	10.9	12.8	14.6	16.4	61.4
⅜	.656	2.19	4.38	6.56	8.75	10.9	13.1	15.3	17.5	19.7	51.2
7/16	.766	2.55	5.10	7.66	10.2	12.8	15.3	17.9	20.4	23.0	43.9
½	.875	2.92	5.83	8.75	11.7	14.6	17.5	20.4	23.3	26.2	38.4
9/16	.984	3.28	6.56	9.84	13.1	16.4	19.7	23.0	26.2	29.5	34.1
⅝	1.09	3.65	7.29	10.9	14.6	19.2	21.9	25.5	29.2	32.8	30.7
1/16	1.20	4.01	8.02	12.0	16.0	20.0	24.1	28.1	32.1	36.1	27.9
¾	1.31	4.38	8.75	13.1	17.5	21.9	26.3	30.6	35.0	39.4	25.6
13/16	1.42	4.74	9.48	14.2	19.0	23.7	28.4	33.2	37.9	43.2	23.7
⅞	1.53	5.10	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9	21.9
15/16	1.64	5.47	10.9	16.4	21.9	27.3	32.8	38.3	43.7	49.2	20.5
1	1.75	5.83	11.7	17.5	23.3	29.2	35.0	40.8	46.7	52.5	19.2

1⅞ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.469	1.56	3.13	4.69	6.25	7.81	9.38	10.9	12.5	14.1	71.7
5/16	.586	1.95	3.91	5.86	7.81	9.66	11.7	13.7	15.6	17.6	57.3
⅜	.703	2.34	4.69	7.03	9.37	11.7	14.1	16.4	18.8	21.1	47.8
7/16	.820	2.73	5.47	8.20	10.9	13.7	16.4	19.1	21.9	24.6	41.0
½	.938	3.13	6.25	9.38	12.5	15.6	18.8	21.9	25.0	28.1	35.8
9/16	1.06	3.52	7.03	10.5	14.1	17.6	21.1	24.6	28.1	31.6	31.8
⅝	1.17	3.91	7.81	11.7	14.6	19.5	23.4	27.3	31.2	35.2	28.7
11/16	1.29	4.30	8.59	12.9	17.2	21.5	25.8	30.1	34.4	38.7	26.1
¾	1.41	4.69	9.38	14.1	18.8	23.4	28.1	32.8	37.5	42.2	23.9
13/16	1.52	5.08	10.2	15.2	20.3	25.4	30.5	35.5	40.6	45.7	22.1
⅞	1.64	5.47	10.9	16.4	21.9	27.3	32.8	38.3	43.9	49.4	20.5
15/16	1.76	5.86	11.7	17.6	23.4	29.3	35.1	41.0	46.9	52.7	19.1
1	1.88	6.25	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.2	17.9

2 inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.500	1.67	3.33	5.00	6.67	8.33	10.0	11.7	13.3	15.0	67.2
5/16	.625	2.08	4.17	6.25	8.33	10.4	12.5	14.6	16.7	18.8	53.8
⅜	.750	2.50	5.00	7.50	10.0	12.5	15.0	17.5	20.0	22.5	44.8
7/16	.875	2.92	5.83	8.75	11.7	14.6	17.5	20.4	23.3	26.3	38.4
½	1.00	3.33	6.67	10.0	13.3	16.7	20.0	23.3	26.7	30.0	33.6
9/16	1.13	3.75	7.50	11.3	15.0	18.8	22.5	26.3	30.0	33.8	29.9
⅝	1.25	4.17	8.33	12.5	16.7	20.8	25.0	29.2	33.3	37.5	26.9
11/16	1.38	4.58	9.16	13.8	18.3	22.9	27.5	32.1	36.7	41.2	24.4
¾	1.50	5.00	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	22.4
13/16	1.63	5.42	10.8	16.3	21.7	27.2	32.5	37.9	43.3	48.8	20.7
⅞	1.75	5.83	11.7	17.5	23.3	29.2	35.0	40.8	46.7	52.5	19.2
15/16	1.88	6.25	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.3	17.9
1	2.00	6.67	13.3	20.0	26.7	33.3	40.0	46.7	53.4	60.0	16.8

Weight of Flat Bar Iron.

2⅛ inches Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.531	1.77	3.54	5.31	7.08	8.85	10.6	12.4	14.2	15.9	63.2
5/16	.664	2.21	4.43	6.64	8.85	11.7	13.3	15.5	17.7	19.9	50.6
⅜	.797	2.66	5.31	7.97	10.6	13.3	15.9	18.6	21.2	23.9	42.2
7/16	.930	3.10	6.20	9.30	12.4	15.5	18.6	21.7	24.8	27.9	36.1
½	1.06	3.54	7.08	10.6	14.2	17.7	21.3	24.8	28.3	31.9	31.6
9/16	1.20	3.98	7.97	12.0	15.9	20.0	23.9	27.9	31.9	35.8	28.1
⅝	1.33	4.43	8.85	13.3	17.7	22.1	26.6	31.0	35.4	39.8	25.3
11/16	1.46	4.87	9.74	14.6	19.5	24.4	29.2	34.1	39.0	43.8	23.0
¾	1.59	5.31	10.6	15.9	21.2	26.6	31.9	37.2	42.5	47.8	21.1
13/16	1.74	5.76	11.5	17.3	23.0	28.8	34.5	40.3	46.0	51.8	19.8
⅞	1.86	6.20	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8	18.1
15/16	1.98	6.64	13.3	19.9	26.6	33.2	39.8	46.5	53.1	59.8	16.9
1	2.13	7.08	14.2	21.3	28.3	35.4	42.5	49.6	56.7	63.8	15.8

2¼ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.563	1.88	3.75	5.63	7.50	9.4	11.3	13.1	15.0	16.9	59.7
5/16	.703	2.34	4.69	7.03	9.38	11.7	14.1	16.4	18.8	21.1	47.8
⅜	.844	2.81	5.63	8.44	11.3	14.1	16.9	19.7	22.5	25.3	39.8
7/16	.984	3.28	6.56	9.84	13.1	16.4	19.7	23.0	26.3	29.5	34.1
½	1.13	3.75	7.50	11.3	15.0	18.8	22.5	26.3	30.0	33.8	29.9
9/16	1.27	4.22	8.44	12.7	16.9	21.1	25.3	29.5	33.8	38.0	26.5
⅝	1.41	4.69	9.38	14.1	18.8	23.4	28.1	32.8	37.5	42.2	23.9
11/16	1.55	5.16	10.3	15.5	20.6	25.8	30.9	36.1	41.3	46.4	21.7
¾	1.69	5.63	11.3	16.9	22.5	28.1	33.8	39.4	45.0	50.6	19.9
13/16	1.83	6.09	12.2	18.3	24.4	30.5	36.6	42.7	48.8	54.9	18.4
⅞	1.97	6.56	13.1	19.7	26.3	32.8	39.4	45.9	52.5	59.1	17.1
15/16	2.11	7.03	14.1	21.1	28.1	35.2	42.2	49.2	56.3	63.3	15.9
1	2.25	7.50	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	14.9

2⅜ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.594	1.98	3.96	5.94	7.92	9.90	11.9	13.9	15.8	17.8	56.6
5/16	.742	2.47	4.95	7.42	9.90	12.4	14.8	17.3	19.8	22.3	45.3
⅜	.891	2.97	5.94	8.91	11.9	14.8	17.8	20.8	23.8	26.7	37.7
7/16	1.04	3.46	6.93	10.4	13.9	17.3	20.8	24.2	27.7	31.2	32.3
½	1.19	3.96	7.92	11.9	15.8	19.8	23.8	27.7	31.7	35.6	28.3
9/16	1.34	4.45	8.91	13.4	17.8	22.3	26.7	31.2	35.6	40.1	25.2
⅝	1.48	4.95	9.90	14.8	19.8	24.7	29.7	34.6	39.6	44.5	22.6
11/16	1.67	5.44	10.9	16.3	21.8	27.2	32.7	38.1	43.5	49.0	20.6
¾	1.78	5.94	11.9	17.8	23.8	29.7	35.6	41.6	47.5	53.4	18.9
13/16	1.93	6.43	12.9	19.3	25.7	32.2	38.6	45.0	51.5	57.9	17.4
⅞	2.08	6.93	13.9	20.8	27.7	34.6	41.6	48.5	55.4	62.3	16.2
15/16	2.23	7.42	14.8	22.3	29.7	37.1	44.5	51.9	59.4	66.8	15.1
1	2.38	7.92	15.8	23.8	31.7	39.6	47.5	55.4	63.3	71.3	14.2

Weight of Flat Bar Iron.

2½ inches Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.625	2.08	4.17	6.25	8.33	10.4	12.5	14.6	16.7	18.8	53.8
5/16	.781	2.60	5.21	7.81	10.4	13.0	15.6	18.2	20.8	23.4	43.0
⅜	.938	3.13	6.25	9.38	12.5	15.6	18.8	21.9	25.0	28.1	35.8
7/16	1.09	3.65	7.29	10.9	14.6	18.2	21.9	25.5	29.2	32.8	30.7
½	1.25	4.17	8.33	12.5	16.7	20.8	25.0	29.2	33.3	37.5	26.9
9/16	1.41	4.69	9.38	14.1	18.8	23.4	28.1	32.8	37.5	42.2	23.9
⅝	1.56	5.21	10.4	15.6	20.8	26.0	31.3	36.5	41.7	46.9	21.5
11/16	1.72	5.73	11.5	17.2	22.9	28.6	34.4	40.1	45.8	51.6	19.6
¾	1.88	6.25	12.5	18.6	25.0	31.3	37.5	43.8	50.0	56.3	18.0
13/16	2.03	6.77	13.5	20.3	27.1	33.8	40.6	47.4	54.2	60.9	16.5
⅞	2.19	7.29	14.6	21.9	29.2	36.5	43.7	51.0	58.3	65.7	15.4
15/16	2.34	7.81	15.6	23.4	31.3	39.0	46.9	54.7	62.5	70.3	14.3
1	2.50	8.33	16.7	25.0	33.3	41.7	50.0	58.3	66.7	75.0	13.4

2⅝ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.656	2.19	4.38	6.56	8.75	10.9	13.1	15.3	17.5	19.7	51.2
5/16	.820	2.73	5.47	8.20	10.9	13.7	16.4	19.1	21.9	24.6	41.0
⅜	.984	3.28	6.56	9.84	13.1	16.4	19.7	23.0	26.2	29.5	34.2
7/16	1.15	3.83	7.66	11.5	15.3	19.1	23.0	26.8	30.6	34.4	29.3
½	1.31	4.38	8.75	13.1	17.5	21.9	26.3	30.6	35.0	39.4	25.6
9/16	1.48	4.92	9.84	14.8	19.7	24.6	29.5	34.5	39.4	44.3	22.8
⅝	1.64	5.47	10.9	16.4	21.9	27.3	32.8	38.3	43.8	49.2	20.5
11/16	1.81	6.02	12.0	18.1	24.1	30.2	36.1	42.1	48.1	54.1	18.6
¾	1.97	6.56	13.1	19.7	26.3	32.8	39.4	45.9	52.5	59.1	17.1
13/16	2.13	7.11	14.2	21.3	28.4	35.5	42.7	49.8	56.9	64.0	15.8
⅞	2.30	7.66	15.3	23.0	30.6	38.3	45.9	53.6	61.3	68.9	14.7
15/16	2.46	8.20	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8	13.7
1	2.63	8.75	17.5	26.3	35.0	43.8	52.5	61.3	70.0	78.8	12.8

2¾ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.688	2.29	4.58	6.87	9.17	11.5	13.8	16.1	18.3	20.6	48.9
5/16	.859	2.86	5.73	8.59	11.5	14.3	17.2	20.1	22.9	25.8	39.1
⅜	1.03	3.44	6.88	10.3	13.8	17.2	20.6	24.1	27.5	30.9	32.8
7/16	1.20	4.01	8.02	12.0	16.0	20.1	24.1	28.1	32.1	36.1	27.9
½	1.38	4.58	9.17	13.8	18.3	22.9	27.5	32.1	36.7	41.3	24.4
9/16	1.55	5.16	10.3	15.5	20.6	25.8	30.9	36.1	41.3	46.4	21.7
⅝	1.72	5.73	11.5	17.2	22.9	28.6	34.4	40.1	45.8	51.6	19.5
11/16	1.89	6.30	12.6	18.9	25.2	31.5	37.8	44.1	50.4	56.7	17.8
¾	2.06	6.88	13.8	20.6	27.5	34.4	41.3	48.1	55.0	61.9	16.3
13/16	2.23	7.45	14.9	22.3	29.8	37.2	44.7	52.1	59.6	67.0	15.0
⅞	2.41	8.02	16.0	24.1	32.1	40.1	48.1	56.1	64.2	72.2	14.0
15/16	2.58	8.59	17.2	25.8	34.4	43.0	51.6	60.1	68.8	77.3	13.0
1	2.75	9.17	18.3	27.5	36.7	45.8	55.0	64.2	73.3	82.5	12.2

Weight of Flat Bar Iron.

2⅞ inches Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.719	2.40	4.79	7.19	9.58	12.0	14.4	16.8	19.2	21.6	46.7
5/16	.898	3.00	6.00	9.00	12.0	15.0	18.0	21.0	24.0	27.0	37.4
⅜	1.08	3.59	7.19	10.8	14.4	18.0	21.6	25.2	28.8	32.3	31.2
7/16	1.26	4.19	8.39	12.6	16.8	21.0	25.2	29.4	33.5	37.7	26.7
½	1.44	4.79	9.58	14.4	19.2	24.0	28.8	33.5	38.3	43.1	23.4
9/16	1.62	5.39	10.8	16.2	21.6	27.0	32.3	37.7	43.1	48.5	20.8
⅝	1.80	6.00	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0	18.7
11/16	1.98	6.59	13.2	19.8	26.4	33.0	40.5	46.1	52.7	59.3	17.0
¾	2.16	7.19	14.4	21.6	28.8	36.0	43.1	50.3	57.5	64.7	15.6
13/16	2.34	7.79	15.6	23.4	31.1	39.0	46.7	54.5	62.3	70.1	14.4
⅞	2.52	8.39	16.8	25.2	33.5	42.0	50.3	58.7	67.1	75.5	13.4
15/16	2.70	8.98	18.0	27.0	35.9	45.0	53.9	62.9	71.9	80.9	12.4
1	2.88	9.58	19.2	28.8	38.3	48.0	57.5	67.1	76.7	86.3	11.7

3 inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.750	2.50	5.00	7.50	10.0	12.5	15.0	17.5	20.0	22.5	44.8
5/16	.938	3.13	6.25	9.38	12.5	16.7	18.8	21.9	25.0	28.1	35.8
⅜	1.13	3.75	7.50	11.3	15.0	18.8	22.5	26.3	30.0	33.8	29.9
7/16	1.31	4.38	8.75	13.1	17.5	21.9	26.3	30.6	35.0	39.4	25.6
½	1.50	5.00	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	22.4
9/16	1.69	5.63	11.3	16.9	22.5	28.2	33.8	39.4	45.0	50.6	19.9
⅝	1.88	6.25	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.3	17.9
11/16	2.06	6.88	13.8	20.6	27.5	34.4	41.3	48.1	55.0	61.9	16.3
¾	2.25	7.50	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	14.9
13/16	2.44	8.13	16.3	24.4	32.5	40.7	48.8	56.9	65.0	73.1	13.8
⅞	2.63	8.75	17.5	26.3	35.0	43.8	52.5	61.3	70.0	78.8	12.8
15/16	2.81	9.38	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	12.0
1	3.00	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	11.2

3¼ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.813	2.71	5.42	8.13	10.8	13.6	16.3	19.0	21.7	24.4	41.3
5/16	1.02	3.39	6.77	10.2	13.5	16.9	20.3	23.7	27.1	30.5	33.1
⅜	1.22	4.06	8.13	12.2	16.3	20.3	24.4	28.4	32.5	36.6	27.5
7/16	1.42	4.74	9.48	14.2	19.0	23.7	28.4	33.2	37.9	42.7	23.6
½	1.63	5.42	10.8	16.3	21.7	27.1	32.5	37.9	43.3	48.8	20.7
9/16	1.83	6.09	12.2	18.3	24.4	30.5	36.6	42.7	48.7	54.8	18.4
⅝	2.03	6.77	13.5	20.3	27.1	33.9	40.6	47.4	54.2	60.9	16.5
11/16	2.23	7.45	14.9	22.3	29.8	37.2	44.7	52.1	59.6	67.0	15.0
¾	2.44	8.13	16.3	24.4	32.5	40.6	48.8	56.9	65.0	73.1	13.7
13/16	2.64	8.80	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2	12.7
⅞	2.84	9.48	19.0	28.4	37.9	47.4	56.9	66.4	75.8	85.3	11.8
15/16	3.05	10.2	20.3	30.5	40.6	50.8	60.9	71.1	81.2	91.4	11.0
1	3.25	10.8	21.7	32.5	43.3	54.2	65.0	75.8	86.7	97.5	10.3

Weight of Flat Bar Iron.

3½ inches Wide.

Thickness.	Sect. Area.	Length in Feet.									Length to weigh 1 cwt.
		1	2	3	4	5	6	7	8	9	
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.875	2.92	5.83	8.75	11.7	14.6	17.5	26.4	23.3	26.3	38.4
5/16	1.09	3.65	7.29	10.9	14.6	18.2	21.9	25.5	29.2	32.8	30.7
⅜	1.31	4.38	8.75	13.1	17.5	21.9	26.3	30.6	35.0	39.4	25.6
7/16	1.53	5.10	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9	21.9
½	1.75	5.83	11.7	17.5	22.3	29.2	35.0	40.8	46.7	52.5	19.2
9/16	1.97	6.56	13.1	19.7	26.3	32.8	39.4	45.9	52.5	59.1	17.1
⅝	2.19	7.29	14.6	21.9	29.2	36.5	43.7	51.0	58.3	65.6	15.4
11/16	2.41	8.02	16.0	24.1	32.1	40.1	48.1	56.1	64.2	72.2	14.0
¾	2.63	8.75	17.5	26.3	35.0	43.8	52.5	61.3	70.0	78.8	12.8
13/16	2.84	9.48	19.0	28.4	37.9	47.4	56.9	66.4	75.8	85.3	11.9
⅞	3.06	10.2	20.4	30.6	40.8	51.0	61.2	71.5	81.6	91.9	11.0
15/16	3.28	10.9	21.9	32.8	43.8	54.7	65.6	76.6	87.5	98.4	10.2
1	3.50	11.7	23.3	35.0	46.7	58.3	70.0	81.7	93.3	105.0	9.60

3¾ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	.938	3.13	6.25	9.38	12.5	15.6	18.8	21.9	25.0	28.1	35.8
5/16	1.17	3.91	7.81	11.7	15.6	19.5	23.4	27.3	31.3	35.2	28.7
⅜	1.41	4.69	9.38	14.1	18.8	23.4	28.1	32.8	37.5	42.2	23.9
7/16	1.64	5.47	10.9	16.4	21.9	27.3	32.8	38.3	43.7	49.2	20.5
½	1.88	6.25	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.3	17.9
9/16	2.11	7.03	14.1	21.1	28.1	35.3	42.2	49.2	56.3	63.3	15.9
⅝	2.34	7.81	15.6	23.4	31.2	39.1	46.9	54.7	62.5	70.3	14.3
11/16	2.58	8.59	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.3	13.0
¾	2.81	9.38	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	12.0
13/16	3.05	10.2	20.3	30.5	40.6	50.8	60.9	71.1	81.3	91.4	11.0
⅞	3.28	10.9	21.9	32.8	43.8	54.7	65.6	76.6	87.5	98.4	10.2
15/16	3.52	11.7	23.4	35.2	46.9	58.6	70.3	82.0	93.7	105.5	9.56
1	3.75	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5	8.96

4 inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.00	3.33	6.67	10.0	13.3	16.7	20.0	23.3	26.7	30.0	33.6
5/16	1.25	4.17	8.33	12.5	16.7	20.8	25.0	29.2	33.3	37.5	26.9
⅜	1.50	5.00	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	22.4
7/16	1.75	5.83	11.7	17.5	23.3	29.2	35.0	40.8	46.7	52.5	19.2
½	2.00	6.67	13.3	20.0	26.7	33.3	40.0	46.7	53.3	60.0	16.8
9/16	2.25	7.50	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	14.9
⅝	2.50	8.33	16.7	25.0	33.3	41.7	50.0	58.3	66.7	75.0	13.4
11/16	2.75	9.17	18.3	27.5	36.7	45.8	55.0	64.2	73.3	82.5	12.2
¾	3.00	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	11.2
13/16	3.25	10.8	21.7	32.5	43.3	54.2	65.0	75.8	86.7	97.5	10.3
⅞	3.50	11.7	23.3	35.0	46.7	58.4	70.0	81.7	93.3	105.0	9.60
15/16	3.75	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5	8.96
1	4.00	13.3	26.7	40.0	53.3	66.7	80.0	93.3	106.7	120.0	8.40

Weight of Flat Bar Iron.

4¼ inches Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.06	3.54	7.08	10.6	14.2	17.7	21.3	24.8	28.3	31.9	31.6
5/16	1.33	4.43	8.85	13.3	17.7	22.1	26.6	31.0	35.4	39.8	25.3
⅜	1.59	5.31	10.6	15.9	21.3	26.6	31.9	37.2	42.5	47.8	21.1
7/16	1.85	6.20	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8	18.1
½	2.13	7.08	14.2	21.3	28.3	35.4	42.5	49.6	56.7	63.8	15.8
9/16	2.39	7.97	15.9	23.9	31.9	39.8	47.8	55.8	63.7	71.7	14.1
⅝	2.66	8.85	17.7	26.6	35.4	44.3	53.1	62.0	70.8	79.7	12.7
11/16	2.92	9.74	19.5	29.2	39.0	48.7	58.4	68.2	77.9	87.7	11.5
¾	3.19	10.6	21.3	31.9	42.5	53.1	63.8	74.4	85.0	95.6	10.5
13/16	3.45	11.5	23.0	34.5	46.0	57.6	69.1	80.6	92.1	103.6	9.9
⅞	3.72	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6	9.0
15/16	3.98	13.3	26.6	39.8	53.1	66.4	79.7	93.0	106.2	119.5	8.4
1	4.25	14.2	28.3	42.5	56.7	70.8	85.0	99.2	113.3	127.5	7.9

4½ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.13	3.75	7.5	11.3	15.0	18.8	22.5	26.3	30.0	33.8	29.9
5/16	1.41	4.69	9.38	14.1	18.8	23.4	28.1	32.8	37.5	42.2	23.9
⅜	1.69	5.63	11.3	16.9	22.5	28.1	33.8	39.4	45.0	50.6	19.9
7/16	1.97	6.56	13.1	19.7	26.3	32.8	39.4	45.9	52.5	59.1	17.1
½	2.25	7.50	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	14.9
9/16	2.53	8.44	16.9	25.3	33.8	42.2	50.6	59.1	67.5	75.9	13.3
⅝	2.81	9.38	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	12.0
11/16	3.09	10.3	20.6	30.9	41.3	51.6	61.9	72.2	82.5	92.8	10.9
¾	3.38	11.3	22.5	33.8	45.0	56.3	67.5	78.8	90.0	101.3	9.95
13/16	3.66	12.2	24.4	36.6	48.8	60.9	73.1	85.3	97.5	109.7	9.19
⅞	3.94	13.1	26.3	39.4	52.5	65.6	78.8	91.9	105.0	118.1	8.53
15/16	4.22	14.1	28.1	42.2	56.3	70.3	84.4	98.4	112.5	126.6	7.96
1	4.50	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0	7.46

4¾ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.19	3.96	7.92	11.9	15.8	19.8	23.8	27.7	31.7	35.6	28.3
5/16	1.48	4.95	9.90	14.8	19.8	24.8	29.7	34.6	39.6	44.4	22.6
⅜	1.78	5.94	11.9	17.8	23.8	29.7	35.6	41.6	47.5	53.4	18.9
7/16	2.08	6.93	13.9	20.8	27.7	34.7	41.6	48.5	55.4	62.3	16.2
½	2.38	7.92	15.8	23.8	31.7	39.6	47.5	55.4	63.3	71.3	14.2
9/16	2.67	8.91	17.8	26.7	35.6	44.6	53.4	62.3	71.3	80.2	12.6
⅝	2.97	9.90	19.8	29.7	39.6	49.5	59.4	69.3	79.2	89.1	11.3
11/16	3.27	10.9	21.8	32.7	43.5	54.5	65.3	76.2	87.1	98.0	10.3
¾	3.56	11.9	23.8	35.6	47.5	59.4	71.3	83.1	95.0	106.9	9.4
13/16	3.86	12.9	25.7	38.6	51.5	64.3	77.2	90.1	102.9	115.8	8.7
⅞	4.16	13.9	27.7	41.6	55.4	69.3	83.1	97.0	110.8	124.7	8.1
15/16	4.45	14.8	29.7	44.5	59.4	74.2	89.1	103.9	118.8	133.6	7.5
1	4.75	15.8	31.7	47.5	63.3	79.2	95.0	110.8	126.7	142.5	7.1

Weight of Flat Bar Iron.

5 inches Wide.

Thickness.	Sect. Area.	Length in Feet.									Length to weigh 1 cwt.
		1	2	3	4	5	6	7	8	9	
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.25	4.17	8.33	12.5	16.7	20.9	25.0	29.2	33.3	37.5	26.9
5/16	1.56	5.21	10.4	15.6	20.8	26.1	31.3	36.5	41.7	46.9	21.5
⅜	1.88	6.25	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.3	17.9
7/16	2.19	7.29	14.6	21.9	29.2	36.5	43.8	51.0	58.3	65.6	15.4
½	2.50	8.33	16.7	25.0	33.3	41.7	50.0	58.3	66.7	75.0	13.4
9/16	2.81	9.38	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	12.0
⅝	3.13	10.4	20.8	31.3	41.7	52.1	62.5	72.9	83.3	93.8	10.8
11/16	3.44	11.5	22.9	34.4	45.8	57.3	68.8	80.2	91.7	103.1	9.77
¾	3.75	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5	8.96
13/16	4.06	13.5	27.1	40.6	54.2	67.7	81.3	94.8	108.3	121.9	8.27
⅞	4.38	14.6	29.2	43.8	58.3	72.9	87.5	102.1	116.7	131.3	7.68
15/16	4.69	15.6	31.3	46.9	62.5	78.1	93.8	109.4	125.0	140.6	7.17
1	5.00	16.7	33.3	50.0	66.7	83.3	100.0	116.7	133.3	150.0	6.72

5¼ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.31	4.38	8.75	13.1	17.5	21.9	26.3	30.6	35.0	39.4	25.6
5/16	1.64	5.47	10.9	16.4	21.9	27.3	32.8	38.3	43.8	49.2	20.5
⅜	1.97	6.56	13.1	19.7	26.3	32.8	39.4	45.9	52.5	59.1	17.1
7/16	2.30	7.66	15.3	23.0	30.6	38.3	45.9	53.6	61.3	68.9	14.6
½	2.63	8.75	17.5	26.3	35.0	43.8	52.5	61.3	70.0	78.8	12.8
9/16	2.95	9.84	19.7	29.5	39.4	49.2	59.1	68.9	78.8	88.6	11.4
⅝	3.28	10.9	21.9	32.8	43.8	54.7	65.6	76.6	87.5	98.4	10.3
11/16	3.61	12.0	24.1	36.1	48.1	60.2	72.2	84.2	96.3	108.3	9.31
¾	3.94	13.1	26.3	39.4	52.5	65.6	78.8	91.9	105.0	118.1	8.55
13/16	4.27	14.2	28.4	42.7	56.9	71.1	85.3	99.5	113.7	128.0	7.88
⅞	4.59	15.3	30.6	45.9	61.3	76.6	91.9	107.2	122.5	137.8	7.31
15/16	4.92	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.3	147.7	6.83
1	5.25	17.5	35.0	52.5	70.0	87.5	105.0	122.5	140.0	157.5	6.40

5½ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.38	4.58	9.17	13.8	18.3	22.9	27.5	32.1	36.7	41.3	24.5
5/16	1.72	5.73	11.5	17.2	22.9	28.6	34.4	40.1	45.8	51.6	19.5
⅜	2.06	6.88	13.8	20.6	27.5	34.4	41.3	48.1	55.0	61.9	16.4
7/16	2.41	8.02	16.0	24.1	32.1	40.1	48.1	56.1	64.2	72.2	14.0
½	2.75	9.17	18.3	27.5	36.7	45.8	55.0	64.2	73.3	82.5	12.2
9/16	3.09	10.3	20.6	30.9	41.3	51.6	61.9	72.2	82.5	92.8	10.9
⅝	3.44	11.5	22.9	34.4	45.8	57.3	68.8	80.2	91.7	103.1	9.77
11/16	3.78	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4	8.88
¾	4.13	13.8	27.5	41.3	55.0	68.8	82.5	96.3	110.0	123.8	8.14
13/16	4.47	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134.1	7.52
⅞	4.81	16.0	32.1	48.1	64.2	80.2	96.3	112.3	128.3	144.4	6.98
15/16	5.16	17.2	34.4	51.6	68.8	85.9	103.1	120.3	137.5	154.7	6.52
1	5.50	18.3	36.7	55.0	73.3	91.6	110.0	128.4	146.7	165.0	6.11

Weight of Flat Bar Iron.

5¾ inches Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.44	4.79	9.58	14.4	19.2	24.0	28.8	33.5	38.3	43.1	23.4
5/16	1.80	5.99	12.0	18.0	24.0	30.0	35.9	41.9	47.9	53.9	18.7
⅜	2.16	7.19	14.4	21.6	28.8	35.9	43.1	50.3	57.5	64.7	15.6
7/16	2.52	8.39	16.8	25.2	33.5	41.9	50.3	58.7	67.1	75.5	13.4
½	2.88	9.58	19.2	28.8	38.3	47.9	57.5	67.1	76.7	86.3	11.7
9/16	3.23	10.8	21.6	32.3	43.1	53.9	64.7	75.5	86.2	97.0	10.4
⅝	3.59	12.0	24.0	36.0	48.0	60.0	71.9	83.9	95.8	107.8	9.35
11/16	3.95	13.2	26.4	39.5	52.7	65.9	79.1	92.2	105.4	118.6	8.50
¾	4.31	14.4	28.8	43.1	57.5	71.9	86.3	100.6	115.0	129.4	7.79
13/16	4.67	15.6	31.2	46.7	62.3	77.9	93.4	109.0	124.6	140.2	7.19
⅞	5.03	16.8	33.5	50.3	67.0	83.9	100.7	117.4	134.2	150.9	6.68
15/16	5.39	18.0	35.9	53.9	71.9	89.8	107.8	125.8	143.7	161.7	6.22
1	5.75	19.2	38.3	57.5	76.7	95.8	115.0	134.2	153.3	172.5	5.83

6 inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.50	5.00	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	22.4
5/16	1.88	6.25	12.5	18.8	25.0	31.8	37.5	43.8	50.0	56.3	17.9
⅜	2.25	7.50	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	14.9
7/16	2.63	8.75	17.5	26.3	35.0	43.8	52.5	61.3	70.0	78.8	12.8
½	3.00	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	11.2
9/16	3.38	11.3	22.5	33.8	45.0	56.3	67.5	78.8	90.0	101.3	10.0
⅝	3.75	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5	8.96
11/16	4.13	13.8	27.5	41.3	55.0	68.8	82.5	96.3	110.0	123.7	8.15
¾	4.50	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0	7.47
13/16	4.88	16.3	32.5	48.8	65.0	81.3	97.5	113.7	130.0	146.3	6.90
⅞	5.25	17.5	35.0	52.5	70.0	87.5	105.0	122.5	140.0	157.5	6.40
15/16	5.63	18.8	37.5	56.3	75.0	93.8	112.5	131.3	150.0	168.7	5.97
1	6.00	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0	5.60

6½ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.63	5.42	10.8	16.3	21.7	27.2	32.5	37.9	43.3	49.0	20.7
5/16	2.03	6.77	13.5	20.3	27.1	33.9	40.6	47.4	54.2	60.9	16.5
⅜	2.44	8.13	16.3	24.4	32.5	40.6	48.8	56.9	65.0	73.1	13.8
7/16	2.84	9.47	18.9	28.4	37.9	47.4	56.8	66.3	75.8	85.2	14.8
½	3.25	10.8	21.7	32.5	43.3	54.2	65.0	75.8	86.7	97.5	10.3
9/16	3.66	12.2	24.4	36.6	48.8	60.9	73.1	85.3	97.5	109.7	9.20
⅝	4.06	13.5	27.1	40.6	54.2	67.7	81.3	94.8	108.3	121.9	8.27
11/16	4.47	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134.1	7.52
¾	4.98	16.3	32.5	48.8	65.0	81.3	97.5	113.8	130.0	146.3	6.89
13/16	5.28	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4	6.36
⅞	5.68	19.0	37.9	56.9	75.8	94.8	113.8	132.7	151.7	170.6	5.91
15/16	6.09	20.3	40.6	60.9	81.3	101.6	121.9	142.8	162.5	182.8	5.51
1	6.50	21.7	43.3	65.0	86.7	108.3	130.0	151.7	173.3	195.0	5.29

Weight of Flat Bar Iron.

7 inches Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.75	5.83	11.7	17.5	23.3	29.2	35.0	40.8	46.7	52.5	19.2
5/16	2.19	7.29	14.6	21.9	29.2	36.5	43.8	51.0	58.3	65.6	15.4
⅜	2.63	8.75	17.5	26.3	35.0	43.8	52.5	61.3	70.0	78.8	12.8
7/16	3.06	10.2	20.4	30.6	40.8	51.0	61.3	71.5	81.7	91.9	11.0
½	3.50	11.7	23.3	35.0	46.7	58.3	70.0	81.7	93.3	105.0	9.60
9/16	3.94	13.1	26.3	39.4	52.5	65.6	78.8	91.9	105.0	118.1	8.53
⅝	4.38	14.6	29.2	43.8	58.3	72.9	87.5	102.1	116.7	131.3	7.68
11/16	4.81	16.0	32.1	48.1	64.2	80.2	96.3	112.3	128.3	144.4	6.98
¾	5.25	17.5	35.0	52.5	70.0	87.5	105.0	122.5	140.0	157.5	6.40
13/16	5.69	19.0	37.9	56.9	75.8	95.0	113.8	132.7	151.7	170.6	5.91
⅞	6.13	20.4	40.8	61.3	81.7	102.1	122.5	142.9	163.3	183.8	5.49
15/16	6.56	21.9	43.8	65.6	87.5	109.4	131.3	153.1	175.0	196.9	5.12
1	7.00	23.3	46.7	70.0	93.3	116.7	140.0	163.3	186.7	210.0	4.80

7½ inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	1.88	6.25	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.3	17.9
5/16	2.34	7.81	15.6	23.4	31.3	39.1	46.9	54.7	62.5	70.3	14.3
⅜	2.81	9.38	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	11.9
7/16	3.28	10.9	21.9	32.8	43.8	54.7	65.6	76.6	87.5	98.4	10.2
½	3.75	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5	8.96
9/16	4.22	14.1	28.1	42.2	56.3	70.3	84.4	98.4	112.5	126.6	7.96
⅝	4.69	15.6	31.3	46.9	62.5	78.1	93.8	109.4	125.0	140.6	7.17
11/16	5.16	17.2	34.4	51.6	68.8	85.9	103.1	120.3	137.5	154.7	6.52
¾	5.63	18.8	37.5	56.3	75.0	93.8	112.5	131.3	150.0	168.8	5.97
13/16	6.09	20.3	40.6	60.9	81.3	101.6	121.9	142.2	162.5	182.8	5.51
⅞	6.56	21.9	43.8	65.6	87.5	109.4	131.3	153.1	175.0	196.9	5.12
15/16	7.03	23.4	46.9	70.3	93.8	117.2	140.6	164.1	187.5	210.9	4.78
1	7.50	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0	4.48

8 inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	2.00	6.67	13.3	20.0	26.7	33.3	40.0	46.7	53.3	60.0	16.8
5/16	2.50	8.33	16.7	25.0	33.3	41.7	50.0	58.3	66.7	75.0	13.4
⅜	3.00	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	11.2
7/16	3.50	11.7	23.3	35.0	46.7	58.3	70.0	81.7	93.3	105.0	9.60
½	4.00	13.3	26.7	40.0	53.3	66.7	80.0	93.3	106.7	120.0	8.40
9/16	4.50	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0	7.47
⅝	5.00	16.7	33.3	50.0	66.7	83.3	100.0	116.7	133.3	150.0	6.72
11/16	5.50	18.3	36.7	55.0	73.3	91.7	110.0	128.3	146.7	165.0	6.11
¾	6.00	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0	5.60
13/16	6.50	21.7	43.3	65.0	86.7	108.3	130.0	151.7	173.3	195.0	5.17
⅞	7.00	23.3	46.7	70.0	93.3	116.7	140.0	163.3	186.7	210.0	4.80
15/16	7.50	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0	4.48
1	8.00	26.7	53.3	80.0	106.7	133.3	160.0	186.7	213.3	240.0	4.20

Weight of Flat Bar Iron.

9 inches Wide.

Thickness.	Sect. Area.	Length in Feet. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	2.25	7.50	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	14.9
5/16	2.81	9.38	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	11.9
⅜	3.38	11.3	22.5	33.8	45.0	56.3	67.5	78.8	90.0	101.3	10.0
7/16	3.94	13.1	26.3	39.4	52.5	65.6	78.8	91.9	105.0	118.1	8.53
½	4.50	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0	7.47
9/16	5.06	16.9	33.8	50.6	67.5	84.4	101.3	118.1	135.0	151.9	6.64
⅝	5.63	18.8	37.5	56.3	75.0	93.8	112.5	131.3	150.0	168.8	5.97
11/16	6.19	20.6	41.3	61.9	82.5	103.1	123.8	144.4	165.0	185.6	5.43
¾	6.75	22.5	45.0	67.5	90.0	112.5	135.0	157.5	180.0	202.5	4.98
13/16	7.31	24.4	48.8	73.1	97.5	121.9	146.3	170.6	195.0	219.4	4.59
⅞	7.88	26.3	52.5	78.8	105.0	131.3	104.5	183.8	210.0	236.3	4.26
15/16	8.44	28.1	56.3	84.4	112.5	140.6	168.8	196.9	225.0	253.1	3.98
1	9.00	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0	3.73

10 inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	2.50	8.33	16.7	25.0	33.3	41.7	50.0	58.3	66.7	75.0	13.4
5/16	3.13	10.4	20.8	31.3	41.7	52.1	62.5	72.9	83.3	93.8	10.7
⅜	3.75	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5	8.96
7/16	4.38	14.6	29.2	43.8	58.3	72.9	87.5	102.1	116.7	131.3	7.68
½	5.00	16.7	33.3	50.0	66.7	83.3	100.0	116.7	133.3	150.0	6.72
9/16	5.63	18.8	37.5	56.3	75.0	93.8	112.5	131.3	150.0	168.8	5.97
⅝	6.25	20.8	41.7	62.5	83.3	104.2	125.0	145.8	166.7	187.5	5.38
11/16	6.88	22.9	45.8	68.8	91.7	114.6	137.5	160.4	183.3	206.3	4.89
¾	7.50	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0	4.48
13/16	8.13	27.1	54.2	81.3	108.3	135.4	162.5	189.6	216.7	243.8	4.14
⅞	8.75	29.2	58.3	87.5	116.7	145.8	175.0	204.2	233.3	262.5	3.84
15/16	9.40	31.3	62.5	93.8	125.0	156.3	187.5	218.8	250.0	281.3	3.58
1	10.0	33.3	66.7	100.0	133.3	166.7	200.0	233.3	266.7	300.0	3.36

11 inches Wide.

inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
¼	2.75	9.17	18.3	27.5	36.7	45.8	55.0	64.2	73.3	82.5	12.2
5/16	3.44	11.5	22.9	34.4	45.8	57.3	68.8	80.2	91.7	103.1	9.77
⅜	4.13	13.8	27.5	41.3	55.0	68.8	82.5	96.3	110.0	123.8	8.15
7/16	4.81	16.0	32.1	48.1	64.2	80.2	96.3	112.3	128.3	144.4	6.98
½	5.50	18.3	36.7	55.0	73.3	91.7	110.0	128.3	146.7	165.0	6.11
9/16	6.19	20.6	41.3	61.9	82.5	103.1	123.8	144.4	165.0	185.6	5.43
⅝	6.88	22.9	45.8	68.8	91.8	114.6	137.5	160.4	183.3	206.3	4.89
11/16	7.56	25.2	50.4	75.6	100.8	126.0	151.3	176.5	201.7	226.9	4.44
¾	8.25	27.5	55.0	82.5	110.0	137.5	165.0	192.5	220.0	247.5	4.07
13/16	8.94	29.8	59.6	89.4	119.2	149.0	178.8	208.5	238.3	268.1	3.76
⅞	9.63	32.1	64.2	96.3	128.3	160.4	192.5	224.6	256.7	288.8	3.49
15/16	10.4	34.4	68.8	103.1	137.5	171.9	206.3	240.6	275.0	309.4	3.26
1	11.0	36.7	73.3	110.0	146.7	183.3	220.0	256.7	293.3	330.0	3.06

Table No. 75.—WEIGHT OF SQUARE IRON.

SIDE.	SECT. AREA.	LENGTH IN FEET. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
⅛	.0156	.052	.104	.156	.208	.260	.313	.365	.417	.469	2154
3/16	.0351	.117	.234	.351	.468	.584	.701	.818	.935	1.05	960.0
¼	.0625	.208	.417	.625	.833	1.04	1.25	1.46	1.67	1.88	537.6
5/16	.0977	.326	.651	.977	1.30	1.68	1.95	2.28	2.60	2.93	343.8
⅜	.141	.469	.938	1.41	1.88	2.34	2.81	3.28	3.75	4.22	238.3
7/16	.191	.638	1.28	1.91	2.55	3.19	3.83	4.46	5.10	5.74	176.0
½	.25	.833	1.67	2.50	3.33	4.17	5.00	5.83	6.67	7.50	134.4
9/16	.316	1.06	2.11	3.16	4.22	5.27	6.33	7.38	8.44	9.49	106.3
⅝	.391	1.30	2.60	3.91	5.21	6.51	7.81	9.11	10.4	11.7	85.9
11/16	.473	1.58	3.15	4.73	6.30	7.88	9.45	11.0	12.6	14.2	71.0
¾	.563	1.88	3.75	5.63	7.50	9.38	11.3	13.1	15.0	16.9	59.7
13/16	.661	2.20	4.40	6.61	8.80	11.0	13.2	15.4	16.6	19.8	50.8
⅞	.766	2.55	5.10	7.66	10.2	12.8	15.3	17.9	20.4	23.0	43.9
15/16	.879	2.93	5.86	8.79	11.7	14.7	17.6	20.5	23.4	26.4	38.2
1	1.00	3.33	6.67	10.0	13.3	16.7	20.0	23.3	26.7	30.0	33.6
1 1/16	1.13	3.76	7.53	11.3	15.1	18.8	22.6	26.3	30.1	33.9	29.7
1⅛	1.27	4.22	8.44	12.7	16.9	21.1	25.3	29.5	33.8	38.0	26.5
1 3/16	1.41	4.70	9.40	14.1	18.8	23.5	28.2	32.9	37.6	42.3	23.8
1¼	1.56	5.21	10.4	15.6	20.8	26.0	31.3	36.5	41.7	46.9	21.5
1 5/16	1.72	5.74	11.5	17.2	23.0	28.7	34.4	40.2	45.9	51.7	19.5
1⅜	1.89	6.30	12.6	18.9	25.2	31.5	37.8	44.1	50.4	56.7	17.8
1 7/16	2.07	6.89	13.8	20.7	27.6	34.5	41.3	48.2	55.1	62.0	16.2
1½	2.25	7.50	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	14.9
1 9/16	2.44	8.14	16.3	24.4	32.6	40.7	48.8	57.0	65.1	73.2	13.8
1⅝	2.64	8.80	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2	12.7
1 11/16	2.88	9.60	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4	11.7
1¾	3.06	10.2	20.4	30.6	40.8	51.0	61.3	71.4	81.6	91.9	11.0
1 13/16	3.29	11.0	21.9	32.9	43.8	54.8	65.7	76.7	87.6	98.6	10.2
1⅞	3.52	11.7	23.4	35.2	46.9	58.6	70.3	82.0	93.8	105.5	9.56
1 15/16	3.75	12.5	25.0	37.5	50.1	62.6	75.1	87.6	100.1	112.6	8.95
2	4.00	13.3	26.7	40.0	53.3	66.7	80.0	93.3	106.7	120.0	8.40
2⅛	4.52	15.1	30.1	45.2	60.2	75.3	90.3	105.4	120.0	135.5	7.43
2¼	5.06	16.9	33.8	50.6	67.1	84.4	101.3	118.1	135.0	151.9	6.64
2⅜	5.64	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2	5.96
2½	6.25	20.8	41.7	62.5	83.3	10.4	125.0	145.8	166.6	187.5	5.38
2⅝	6.89	23.0	45.9	68.9	91.9	114.9	137.8	160.8	183.9	206.7	4.99
2¾	7.56	25.2	50.4	75.6	100.8	126.1	151.3	176.5	201.7	226.9	4.44
2⅞	8.27	27.6	55.1	82.7	110.2	137.8	165.3	192.9	220.4	248.0	4.06
3	9.00	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0	3.73
3¼	10.6	35.2	70.4	105.6	140.8	176.0	211.3	246.5	281.7	316.9	3.17
3½	12.3	40.8	81.7	122.5	163.3	204.2	245.0	285.8	326.7	367.5	2.73
3¾	14.1	46.9	93.8	140.6	187.5	234.4	281.3	328.1	375.0	421.9	2.38
4	16.0	53.3	106.7	160.0	213.3	266.7	320.0	373.0	426.0	480.0	2.10
4¼	18.1	60.2	120.4	180.6	240.8	301.1	361.2	421.5	481.7	541.9	1.86
4½	20.3	67.5	135.0	202.5	270.0	337.5	405.0	472.5	540.0	607.5	1.66
4¾	22.6	75.2	150.4	225.6	300.8	376.1	451.3	526.5	601.7	676.9	1.49
5	25.0	83.3	166.7	250.0	333.3	416.7	500.0	583.3	666.7	750.0	1.34
5¼	27.6	91.9	183.8	275.6	367.5	459.4	551.3	643.1	735.0	826.9	1.21
5½	30.3	100.8	201.7	302.5	403.3	504.2	605.0	705.8	806.7	907.5	1.11
5¾	33.1	110.2	220.4	330.6	440.8	551.0	661.3	771.5	881.7	991.8	1.02
6	36.0	120.0	240.0	360.0	480.0	600.0	720.0	840.0	960.0	1080	.933

Table No. 76.—WEIGHT OF ROUND IRON.

DIAM.	SECT. AREA.	LENGTH IN FEET. 1	2	3	4	5	6	7	8	9	Length to weigh 1 cwt.
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	feet.
⅛	.0123	.041	.082	.123	.164	.205	.245	.286	.327	.368	2738
3/16	.0276	.092	.184	.276	.368	.460	.552	.644	.736	.828	1217
¼	.0491	.164	.327	.491	.655	.818	.982	1.15	1.31	1.47	684.4
5/16	.0767	.256	.511	.767	1.02	1.28	1.53	1.79	2.04	2.30	438.1
⅜	.110	.368	.736	1.10	1.47	1.84	2.21	2.58	2.94	3.31	305.4
7/16	.150	.501	1.00	1.50	2.00	2.51	3.01	3.51	4.01	4.51	224.0
½	.196	.654	1.21	1.96	2.62	3.27	3.93	4.58	5.23	5.89	171.4
9/16	.248	.828	1.66	2.49	3.31	4.14	4.97	5.80	6.63	7.46	135.5
⅝	.307	1.02	2.05	3.07	4.09	5.11	6.14	7.16	8.18	9.20	109.5
11/16	.371	1.24	2.48	3.71	4.95	6.19	7.42	8.66	9.90	11.1	90.6
¾	.442	1.47	2.94	4.42	5.89	7.36	8.83	10.3	11.8	13.3	76.0
13/16	.518	1.73	3.46	5.19	6.91	8.64	10.4	12.1	13.8	15.6	70.5
⅞	.601	2.00	4.01	6.01	8.02	10.0	12.0	14.0	16.0	18.0	55.9
15/16	.690	2.30	4.60	6.90	9.20	11.5	13.8	16.1	18.4	20.7	48.7
1	.785	2.62	5.24	7.85	10.5	13.1	15.7	18.3	20.9	23.6	42.8
1 1/16	.887	2.96	5.91	8.87	11.8	14.8	17.7	20.7	23.6	26.6	37.9
1⅛	.994	3.31	6.63	9.94	13.3	16.6	19.9	23.2	26.5	29.8	33.8
1 3/16	1.11	3.69	7.38	11.1	14.8	18.5	22.2	25.8	29.5	33.2	30.3
1¼	1.23	4.09	8.18	12.3	16.4	20.5	24.5	28.6	32.7	36.8	27.3
1 5/16	1.35	4.51	9.02	13.5	18.0	22.6	27.1	31.6	36.1	40.6	24.9
1⅜	1.48	4.95	9.90	14.9	19.8	24.8	29.7	34.6	39.6	46.6	22.7
1 7/16	1.62	5.08	10.2	16.2	20.3	25.9	32.5	35.5	40.6	48.7	20.7
1½	1.77	5.89	11.8	17.7	23.6	29.5	35.3	41.2	47.1	53.0	19.0
1 9/16	1.92	6.39	12.8	19.2	25.6	32.0	38.4	44.7	51.1	57.5	17.5
1⅝	2.07	6.91	13.8	20.7	27.7	34.6	41.5	48.4	55.3	62.9	16.2
1 11/16	2.24	7.46	14.9	22.4	29.8	37.3	44.7	52.2	59.6	67.1	15.0
1¾	2.41	8.02	16.0	24.1	32.1	40.1	48.1	56.1	64.1	72.2	13.9
1 13/16	2.58	8.60	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4	13.0
1⅞	2.76	9.20	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8	12.2
1 15/16	2.95	9.83	19.7	29.5	39.3	49.1	59.0	68.8	79.6	88.4	11.4
2	3.14	10.5	20.9	31.4	41.9	52.4	62.8	73.3	83.8	94.3	10.7
2⅛	3.55	11.8	23.6	35.5	47.3	59.1	70.9	82.8	94.6	106.4	9.47
2¼	3.98	13.3	26.5	39.8	53.0	66.3	79.5	92.8	106.0	119.3	8.44
2⅜	4.43	14.8	29.5	44.3	59.1	73.8	88.6	103.3	118.1	132.9	7.59
2½	4.91	16.4	32.7	49.1	65.5	81.8	98.2	114.5	130.9	147.3	6.84
2⅝	5.41	18.0	36.1	54.1	72.2	90.2	108.2	126.2	144.3	162.3	6.21
2¾	5.94	19.8	39.6	59.4	79.2	99.0	118.8	138.5	158.4	178.2	5.66
2⅞	6.49	21.6	43.3	64.9	86.6	108.2	129.8	151.5	173.1	194.8	5.18
3	7.07	23.6	47.1	70.7	94.3	117.8	141.4	164.9	188.5	212.1	4.75
3¼	8.30	27.7	55.3	83.0	110.4	138.3	165.9	193.6	221.2	248.9	4.05
3½	9.62	32.1	64.1	96.2	128.3	160.4	192.4	224.5	256.6	288.6	3.49
3¾	11.0	33.5	73.6	110.4	147.3	164.1	220.9	257.7	294.5	331.3	3.04
4	12.6	41.9	83.8	125.7	167.6	209.4	251.3	293.2	335.0	377.0	2.67
4¼	14.2	47.3	94.6	141.9	189.1	236.4	283.7	331.0	378.3	425.6	2.37
4½	15.9	53.0	106.0	159.0	212.1	265.1	319.1	371.1	424.1	477.1	2.11
4¾	17.7	59.1	118.1	177.2	236.3	295.3	354.4	413.5	472.5	531.6	1.90
5	19.6	65.5	130.9	196.4	261.8	327.3	392.7	458.2	523.6	589.1	1.71
5¼	21.7	72.2	144.3	216.5	288.6	360.8	432.9	505.1	577.3	649.4	1.55
5½	23.8	79.2	158.4	237.6	316.7	396.0	475.2	554.3	633.6	712.7	1.41
5¾	26.0	86.6	173.1	259.7	346.2	432.8	519.3	605.9	692.4	779.0	1.29
6	28.3	94.2	188.5	282.7	377.0	471.2	565.5	659.7	754.0	848.2	1.19

Weight of Round Iron.

Diam.	Sect. Area.	Length in Feet.									Length to weigh 1 ton.
		1	2	3	4	5	6	7	8	9	
inches.	sq. in.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	feet.
6½	33.2	.9876	1.975	2.963	3.950	4.938	5.926	6.613	7.901	8.888	20.2
7	38.5	1.145	2.291	3.436	4.582	5.727	6.872	8.018	9.163	10.31	17.5
7½	44.2	1.315	2.629	3.944	5.258	6.573	7.887	9.202	10.52	11.84	15.2
8	50.3	1.496	2.992	4.448	5.984	7.480	8.976	10.47	11.97	13.46	13.4
8½	56.7	1.689	3.378	5.067	6.756	8.444	10.13	11.82	13.50	15.20	11.8
9	63.6	1.893	3.786	5.680	7.572	9.46	11.36	13.25	15.14	17.04	10.6
9½	70.9	2.110	4.220	6.329	8.440	10.55	12.66	14.77	16.88	18.99	9.48
10	78.5	2.338	4.676	7.012	9.352	11.69	14.03	16.37	18.70	21.04	8.56
10½	86.6	2.577	4.754	7.731	10.31	12.89	15.46	18.04	19.02	23.19	7.76
11	95.0	2.828	5.656	8.485	11.31	14.14	16.97	19.80	22.62	25.46	7.07
11½	103.9	3.088	6.176	9.265	12.35	15.44	18.53	21.62	24.70	27.80	6.47
12	113.1	3.366	6.732	10.10	13.46	16.83	20.20	23.56	26.93	30.29	5.94
12½	122.7	3.656	7.312	10.96	14.62	18.28	21.91	25.59	29.25	32.90	5.48
13	132.7	3.950	7.900	11.85	15.80	19.75	23.70	27.65	31.60	35.15	5.06
13½	143.1	4.260	8.520	12.78	17.04	21.30	25.56	29.82	34.08	38.34	4.70
14	153.9	4.581	9.162	13.74	18.32	22.90	26.49	32.07	36.65	41.23	4.37
14½	165.1	4.915	9.830	14.74	19.66	24.58	28.49	34.41	39.32	44.24	4.07
15	176.7	5.259	10.52	15.78	21.04	26.30	31.46	36.81	42.08	47.33	3.80
15½	188.7	5.616	11.23	16.85	22.46	28.08	32.70	39.31	44.92	50.54	3.56
16	201.1	5.984	11.97	17.95	23.93	29.92	35.90	41.89	47.88	53.86	3.34
16½	213.8	6.364	12.73	19.09	25.46	31.82	38.18	44.55	50.92	57.28	3.14
17	227.0	6.755	13.51	20.27	27.02	33.78	40.53	47.29	54.04	60.80	2.96
17½	240.5	7.159	14.32	21.48	28.64	35.80	42.95	50.11	57.28	64.43	2.79
18	254.5	7.573	15.15	22.72	30.29	37.86	45.44	53.01	60.60	68.16	2.64
19	283.5	8.438	16.88	25.32	33.75	42.19	50.63	59.03	67.52	75.94	2.37
20	314.2	9.350	18.70	28.05	37.40	46.75	56.10	65.45	74.80	84.15	2.14
21	346.4	10.31	20.62	30.93	41.23	51.54	61.85	72.16	82.47	92.78	1.94
22	380.1	11.31	22.63	33.94	45.25	56.57	67.88	79.19	90.51	101.8	1.77
23	415.5	12.37	24.73	37.10	49.46	61.83	74.19	86.56	93.92	111.3	1.62
24	452.4	13.46	26.93	40.39	53.86	67.32	80.78	94.25	107.7	121.3	1.49

Table No. 77.—WEIGHT OF ANGLE-IRON AND TEE-IRON.

1 Foot in Length.

Note.—When the base or the web tapers in section, the mean thickness is to be measured.

Thickness.	Sum of the Width and Depth in Inches.										
	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾
inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
⅛	.57	.62	.68	.73	.78	.83	.88	.94	.99	1.04	1.09
3/16	.81	.89	.97	1.05	1.13	1.21	1.29	1.37	1.45	1.52	1.60
¼	1.04	1.15	1.25	1.36	1.46	1.56	1.67	1.77	1.88	1.98	2.08
5/16	1.24	1.37	1.50	1.63	1.76	1.89	2.02	2.15	2.28	2.41	2.54

	2⅞	3	3⅛	3¼	3⅜	3½	3⅝	3¾	3⅞	4	4¼
⅛	1.14	1.20	1.25	1.30	1.45	1.41	1.46	1.51	1.56	1.62	1.72
3/16	1.68	1.76	1.84	1.91	1.99	2.07	2.15	2.23	2.30	2.38	2.54
¼	2.19	2.29	2.40	2.50	2.60	2.71	2.81	2.92	3.02	3.13	3.33
5/16	2.67	2.80	2.93	3.06	3.19	3.32	3.45	3.58	3.71	3.84	4.10
⅜	3.13	3.28	3.44	3.59	3.75	3.91	4.06	4.22	4.38	4.53	4.84
7/16	3.57	3.75	3.93	4.11	4.29	4.48	4.66	4.84	5.02	5.20	5.56

	4½	4¾	5	5¼	5½	5¾	6	6¼	6½	6¾	7
3/16	2.70	2.85	3.01	3.16	3.32	3.48	3.63	3.79	3.95	4.10	4.26
¼	3.54	3.75	3.96	4.17	4.38	4.58	4.79	5.00	5.21	5.42	5.63
5/16	4.36	4.62	4.88	5.14	5.40	5.66	5.92	6.18	6.45	6.71	6.97
⅜	5.16	5.47	5.78	6.09	6.41	6.72	7.03	7.34	7.66	7.97	8.28
7/16	5.92	6.29	6.65	7.02	7.38	7.75	8.11	8.48	8.84	9.21	9.57
½	6.67	7.08	7.50	7.92	8.33	8.75	9.17	9.58	10.00	10.42	10.83
9/16	7.38	7.85	8.32	8.79	9.26	9.73	10.20	10.66	11.13	11.60	12.07

	7¼	7½	7¾	8	8¼	8½	8¾	9	9¼	9½	9¾
¼	5.83	6.04	6.25	6.46	6.67	6.88	7.08	7.29	7.50	7.71	7.92
5/16	7.23	7.49	7.75	8.01	8.27	8.53	8.79	9.05	9.31	9.57	9.83
⅜	8.59	8.91	9.22	9.53	9.84	10.16	10.47	10.78	11.09	11.41	11.72
7/16	9.93	10.30	10.66	11.03	11.39	11.76	12.12	12.49	12.85	13.22	13.58
½	11.25	11.67	12.08	12.50	12.92	13.33	13.75	14.17	14.58	15.00	15.42
9/16	12.54	13.01	13.48	13.94	14.41	14.88	15.35	15.82	16.29	16.76	17.23
⅝	13.80	14.32	14.84	15.36	15.89	16.41	16.93	17.45	17.97	18.49	19.01

	10	10½	11	11½	12	12½	13	13½	14	14½	15
⅜	12.03	12.66	13.28	13.91	14.53						
7/16	13.95	14.67	15.40	16.13	16.86	17.59	18.31	19.04	19.77	20.50	21.22
½	15.83	16.67	17.50	18.33	19.17	20.00	20.84	21.67	22.50	23.34	24.17
9/16	17.70	18.63	19.57	20.51	21.44	22.38	23.31	24.25	25.19	26.12	27.06
⅝	19.53	20.57	21.61	22.66	23.70	24.74	25.78	26.83	27.87	28.91	29.95
¾	23.13	24.38	25.63	26.88	28.13	29.37	30.63	31.88	33.13	34.38	35.63

	12	12½	13	13½	14	15	16	17	18	19	20
⅝	23.70	24.74	25.78	26.83	27.87	29.95	32.03	34.12	36.20	38.28	40.36
¾	28.13	29.37	30.63	31.88	33.13	35.63	38.13	40.63	41.13	43.63	46.13
⅞	32.45	33.91	35.36	36.82	38.28	41.19	44.12	47.02	49.95	52.87	55.78
1	36.67	38.33	40.00	41.67	43.33	46.67	50.00	53.33	56.67	60.00	63.33

Table No. 78.—WEIGHT OF WROUGHT-IRON PLATES.

Thickness.	Sect. Area, when 1 foot wide.	Area in Square Feet.									Number of sq. ft. in 1 ton.
		1	2	3	4	5	6	7	8	9	
inches.	sq. in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	sq. feet.
¼	3.00	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	224.0
5/16	3.75	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5	179.2
⅜	4.50	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0	149.3
7/16	5.20	17.5	35.0	52.5	70.0	87.5	105.0	122.5	140.0	157.5	128.0
½	6.00	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0	112.0
9/16	6.75	22.5	45.0	67.5	90.0	112.5	135.0	150.0	180.0	202.5	99.67
⅝	7.50	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0	89.60
11/16	8.25	27.5	55.0	82.5	110.0	137.5	165.0	192.5	220.0	247.5	81.45
¾	9.00	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0	74.67
13/16	9.75	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5	68.92
⅞	11.50	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0	64.00
15/16	11.25	37.5	75.0	112.5	150.0	187.5	225.0	262.5	300.0	337.5	59.73
1	12.00	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0	56.00
1 1/16	12.75	42.5	85.0	127.5	170.0	212.5	255.0	297.5	340.0	382.5	52.71
1⅛	13.50	45.0	90.0	135.0	180.0	225.0	270.0	315.0	360.0	405.0	49.78
1 3/16	14.25	47.5	95.0	142.5	190.0	237.5	285.0	332.5	380.0	427.5	47.16
1¼	15.0	50.0	100.0	150.0	200.0	250.0	300.0	350.0	400.0	450.0	44.80
1⅜	16.5	55.0	110.0	165.0	220.0	275.0	330.0	385.0	440.0	495.0	40.73
1½	18.0	60.0	120.0	180.0	240.0	300.0	360.0	420.0	480.0	540.0	37.33
1¾	21.0	70.0	140.0	210.0	280.0	350.0	420.0	490.0	560.0	630.0	32.00
2	24.0	80.0	160.0	240.0	320.0	400.0	480.0	560.0	640.0	720.0	28.00
		cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	
2½	30	.893	1.79	2.68	3.57	4.46	5.36	6.25	7.14	8.04	23.40
3	36	1.07	2.14	3.21	4.29	5.36	6.64	7.50	8.57	9.64	18.67
3½	42	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.00	11.25	16.00
4	48	1.43	2.86	4.29	5.71	7.14	8.57	10.00	11.43	12.86	14.00
4½	54	1.61	3.21	4.82	6.43	8.04	9.64	11.25	12.86	14.46	12.44
5	60	1.79	3.57	5.36	7.14	8.93	10.71	12.50	14.29	16.07	11.20
5½	66	1.96	3.93	5.89	7.86	9.82	11.79	13.75	15.71	17.68	10.18
6	72	2.14	4.29	6.43	8.57	10.71	12.86	15.00	17.14	19.29	9.33
7	84	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	8.00
8	96	2.86	5.71	8.57	11.43	10.29	17.14	20.00	22.86	25.71	7.00
9	108	3.21	6.43	9.64	12.86	16.07	19.29	22.50	25.71	28.93	6.22
10	120	3.57	7.14	10.71	14.29	12.86	21.43	25.00	28.56	32.14	5.60
11	132	3.93	7.86	11.79	15.71	19.64	23.57	27.50	31.43	35.36	5.09
12	144	4.29	8.57	12.86	17.14	21.43	25.71	30.00	34.29	38.57	4.67
13	156	4.64	9.29	13.93	18.57	23.21	27.86	32.50	37.14	41.79	4.31
14	168	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	4.00
15	180	5.36	10.71	16.07	21.43	26.79	32.14	37.50	42.86	48.21	3.73

Table No. 79.—WEIGHT OF SHEET IRON.

AT 480 LBS. PER CUBIC FOOT.

According to Wire-gauge used in South Staffordshire (Table No. 17).

Thickness.		Area in Square Feet.									Number of sq. ft. in 1 ton.
		1	2	3	4	5	6	7	8	9	
B.W.G.	inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	sq. ft.
32	.0125	.500	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	4480
31	.0141	.562	1.13	1.69	2.25	2.81	3.38	3.94	4.50	5.06	3986
30	.0156	.625	1.25	1.88	2.50	3.13	3.75	4.38	5.00	5.63	3584
29	.0172	.688	1.38	2.06	2.75	3.44	4.13	4.81	5.50	6.19	3256
28	.0188	.750	1.50	2.25	3.00	3.75	4.50	5.25	6.00	6.75	2987
27	.0203	.813	1.63	2.44	3.25	4.06	4.88	5.69	6.50	7.31	2755
26	.0219	.875	1.75	2.63	3.50	4.38	5.25	6.13	7.00	7.88	2560
25	.0234	.938	1.88	2.81	3.75	4.69	5.63	6.56	7.50	8.44	2388
24	.0250	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	2240
23	.0281	1.13	2.25	3.38	4.50	5.63	6.75	7.88	9.00	10.1	1982
22	.0313	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.0	11.3	1792
21	.0344	1.38	2.75	4.13	5.50	6.88	8.25	9.63	11.0	12.4	1623
20	.0375	1.50	3.00	4.50	6.00	7.50	9.00	10.5	12.0	13.5	1493
19	.0438	1.75	3.50	5.25	7.00	8.75	10.5	12.3	14.0	15.8	1280
18	.0500	2.00	4.00	6.00	8.00	10.0	12.0	14.0	16.0	18.0	1120
17	.0563	2.25	4.50	6.75	9.00	11.3	13.5	15.8	18.0	20.3	996
16	.0625	2.50	5.00	7.50	10.0	12.5	15.0	17.5	20.0	22.5	896
15	.0750	3.00	6.00	9.00	12.0	15.0	18.0	21.0	24.0	27.0	747
14	.0875	3.50	7.00	10.5	14.0	17.5	21.0	24.5	28.0	31.5	640
13	.1000	4.00	8.00	12.0	16.0	20.0	24.0	28.0	32.0	36.0	560
12	.1125	4.50	9.00	13.5	18.0	22.5	27.0	31.5	36.0	40.5	498
11	.1250	5.00	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	448
10	.1406	5.63	11.3	16.9	22.5	28.1	33.8	49.4	45.0	50.6	398
9	.1563	6.25	12.5	16.8	25.0	31.3	37.5	43.8	50.0	56.3	358
8	.1719	6.88	13.8	20.6	27.5	34.4	41.3	48.1	55.0	61.9	326
7	.1875	7.50	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	299
6	.2031	8.13	16.3	24.4	32.5	40.6	48.8	56.9	65.0	72.1	276
5	.2188	8.75	17.5	26.3	35.0	43.8	52.5	61.3	70.0	78.8	256
4	.2344	9.38	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	239
3	.2500	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	224
2	.2813	11.25	22.5	33.8	45.0	56.3	67.5	78.8	90.0	101.3	199
1	.3125	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5	179

Table No. 80.—WEIGHT OF BLACK AND GALVANIZED IRON SHEETS.

(Morton's Table, founded upon Sir Joseph Whitworth & Co.'s Standard Birmingham Wire-Gauge.)

Note.—The numbers on Holtzapffel's wire-gauge are applied to the thicknesses on Whitworth's gauge.

Gauge of Black Sheets.		Approximate number of square feet in 1 ton.		Gauge of Black Sheets.		Approximate number of square feet in 1 ton.	
Wire-Gauge.	Thickness.	Black Sheets.	Galvanized Sheets.	Wire-Gauge.	Thickness.	Black Sheets.	Galvanized Sheets.
No.	inch.	square feet.	square feet.	No.	inch.	square feet.	square feet.
1	.300	187	185	17	.060	933	876
2	.280	200	197	18	.050	1120	1038
3	.260	215	212	19	.040	1400	1274
4	.240	233	229	20	.036	1556	1403
5	.220	254	250	21	.032	1750	1558
6	.200	280	275	22	.028	2000	1753
7	.180	311	304	23	.024	2333	2004
8	.165	339	331	24	.022	2545	2159
9	.150	373	363	25	.020	2800	2339
10	.135	415	403	26	.018	3111	2553
11	.120	467	452	27	.016	3500	2808
12	.110	509	491	28	.014	4000	3122
13	.095	589	566	29	.013	4308	3306
14	.085	659	630	30	.012	4667	3513
15	.070	800	757	31	.010	5600	4017
16	.065	862	813	32	.009	6222	4327

Table No. 81.—WEIGHT OF HOOP IRON.

I FOOT IN LENGTH.

According to Wire-gauge used in South Staffordshire.

Thickness.		Width in Inches.							
		5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
B. W. G.	inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
21	.0344	.0716	.0861	.100	.115	.129	.144	.158	.172
20	.0375	.0781	.0938	.109	.125	.141	.156	.172	.188
19	.0438	.0911	.109	.128	.146	.164	.182	.200	.219
18	.0500	.104	.125	.146	.167	.188	.208	.229	.250
17	.0563	.117	.141	.164	.188	.211	.234	.258	.281
16	.0625	.130	.156	.182	.208	.234	.260	.286	.313
15	.0750	.156	.188	.219	.250	.281	.313	.344	.375
14	.0875	.183	.219	.256	.293	.329	.366	.402	.438
13	.1000	.208	.250	.292	.333	.375	.416	.458	.500
12	.1125	.234	.281	.328	.375	.422	.469	.516	.563
11	.1250	.260	.313	.365	.417	.469	.521	.573	.625
10	.1406	.293	.352	.410	.469	.527	.586	.645	.703
9	.1563	.326	.391	.456	.522	.587	.652	.717	.783
8	.1719	.358	.430	.501	.573	.644	.716	.788	.859
7	.1875	.391	.469	.547	.625	.703	.781	.859	.938
6	.2031	.423	.508	.593	.677	.762	.836	.931	1.02
5	.2188	.456	.547	.638	.729	.820	.912	10.0	1.09
4	.2344	.488	.586	.683	.781	.879	.977	10.7	1.17

Thickness.		Width in Inches.							
		1 5/8	1 3/4	1 7/8	2	2 1/4	2 1/2	2 3/4	3
B. W. G.	inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
21	.0344	.197	.201	.215	.229	.258	.287	.315	.344
20	.0375	.203	.219	.224	.250	.281	.313	.344	.375
19	.0438	.238	.257	.274	.292	.328	.365	.400	.437
18	.0500	.271	.292	.312	.333	.375	.417	.458	.500
17	.0563	.305	.328	.351	.375	.422	.469	.516	.563
16	.0625	.339	.365	.391	.417	.469	.521	.573	.625
15	.0750	.307	.438	.469	.500	.562	.625	.687	.750
14	.0875	.475	.512	.549	.585	.658	.731	.804	.875
13	.1000	.543	.584	.626	.667	.750	.833	.917	1.00
12	.1125	.609	.656	.703	.750	.842	.938	1.03	1.13
11	.1250	.677	.729	.781	.833	.937	1.04	1.15	1.25
10	.1406	.762	.820	.879	.938	1.06	1.17	1.29	1.16
9	.1563	.848	.913	.978	1.04	1.17	1.30	1.43	1.56
8	.1719	.931	1.00	1.07	1.15	1.29	1.43	1.58	1.72
7	.1875	1.02	1.09	1.17	1.25	1.41	1.56	1.72	1.88
6	.2031	1.10	1.19	1.27	1.35	1.52	1.69	1.86	2.03
5	.2188	1.19	1.28	1.37	1.46	1.64	1.82	2.00	2.19
4	.2344	1.27	1.37	1.46	1.56	1.76	1.95	2.15	2.35

Table No. 82.—WEIGHT AND STRENGTH OF WARRINGTON IRON WIRE.

TABLE OF WIRE MANUFACTURED BY RYLANDS BROTHERS.

NOTE.—The Wire-Gauge is that of Rylands Brothers.

Size on Wire-Gauge.	Diameter.		Weight of		Length of		Breaking Strain.		Specific Density, the average density of iron =1.
			100 Yds.	1 Mile.	1 Bundle of 63 lbs.	1 Cwt.	Annealed.	Bright.	
	inch.	millimetres.	lbs.	lbs.	yards.	yards.	lbs.	lbs.	average iron = 1.
7/0	½	12.7	193.4	3404	33	58	10470	15700	.9852
6/0	15/32	11.9	170.0	2991	37	66	9200	13810	
5/0	7/16	11.1	148.1	2606	43	76	8020	12000	
4/0	13/32	10.3	127.6	2247	49	88	6910	10370	
3/0	⅜	9.5	108.8	1915	58	103	5890	8835	.9852
2/0	11/32	8.7	91.4	1609	69	123	4960	7420	
0	.326	8.3	82.1	1447	77	136	4450	6678	
1	.300	7.6	69.6	1227	90	161	3770	5655	
2	.274	7.0	58.1	1022	108	193	3140	4717	
3	.250 (¼)	6.4	48.4	851	130	232	2618	3927	.9852
4	.229	5.8	40.6	714	155	276	2197	3295	
5	.209	5.3	33.8	595	186	332	1830	2740	
6	.191	4.9	28.2	495	223	397	1528	2290	
7	.174	4.4	23.4	412	269	479	1268	1900	
8	.159	4.0	19.6	344	322	573	1060	1558	
9	.146	3.7	16.5	290	382	680	893	1340	
10	.133	3.4	13.7	241	460	819	741	1110	
10½	.125 (⅛)	3.2	12.1	213	521	927	654	980	.9852
11	.117	3.0	10.6	186	595	1059	573	860	
12	.100 (1/10)	2.6	8.0	142	783	1393	436	650	
13	.090	2.3	6.3	110	1006	1790	339	509	
14	.079	2.0	4.8	85	1305	2322	261	390	
15	.069	1.8	3.7	65	1715	3052	199	299	
16	.0625 (1/16)	1.5	2.9	51	2188	3894	156	233	.9378
17	.053	1.3	2.2	38	2900	5160	118	176	
18	.047	1.2	1.7	30	3687	6560	93	138	
19	.041	1.0	1.3	23	4847	8620	70	105	
20	.036	.9	1.0	18	5985	11120	54	81	
21	.03125 (1/32)	.8	.8	14	7574	14152	43	64	1.0843
22	.028	.7	.6	11	9893	18486	33	49	

Mem. This Table of the weight and strength of Warrington wire is given by permission of Messrs. Rylands Brothers; and it is said to be based on very accurate measurements of sizes and weights. The last column is added by the author, to show that the density of the wire is stationary for diameters of from ½ inch to ⅛ inch, and probably somewhat smaller diameters; but that, contrary to current opinions of the density of wire, the density becomes greater when the diameter is reduced to 1/32 inch.

Table No. 83.—WEIGHT OF WROUGHT-IRON TUBES,

BY INTERNAL DIAMETER.

LENGTH, 1 FOOT. Thickness by Holtzapffel's Wire-Gauge.

THICKNESS. W. G.								4	5	6	7
INCH.	5/8	9/16	1/2	7/16	3/8	5/16	1/4	.238 15/64 f.	.220 7/32 f.	.203 13/64	.180 3/16 b.
INT. DIAM. inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1/8	4.91	4.05	3.27	2.58	1.96	1.43	.982	.905	.795	.698	.575
1/4	5.73	4.79	3.93	3.15	2.45	1.84	1.31	1.22	1.08	.963	.811
3/8	6.54	5.52	4.58	3.72	2.95	2.25	1.64	1.53	1.37	1.23	1.05
1/2	7.36	6.26	5.24	4.30	3.44	2.66	1.96	1.84	1.66	1.50	1.28
3/4	9.00	7.73	6.55	5.44	4.40	3.48	2.62	2.46	2.24	2.03	1.75
1	10.6	9.20	7.86	6.59	5.40	4.30	3.27	3.09	2.81	2.56	2.23
1 1/4	12.3	10.7	9.17	7.73	6.38	5.11	3.93	3.71	3.39	3.09	2.70
1 1/2	13.9	12.2	10.5	8.88	7.36	5.93	4.58	4.33	3.96	3.62	3.17
1 3/4	15.6	13.6	11.8	10.0	8.34	6.75	5.24	4.96	4.54	4.15	3.64
2	17.2	15.1	13.1	11.2	9.33	7.57	5.89	5.58	5.12	4.68	4.11
2 1/4	18.8	16.6	14.4	12.3	10.3	8.38	6.55	6.20	5.69	5.21	4.58
2 1/2	20.5	18.0	15.7	13.5	11.3	9.20	7.20	6.83	6.27	5.75	5.05
2 3/4	22.1	19.5	17.0	14.6	12.3	10.0	7.85	7.45	6.84	6.28	5.52
3	23.7	21.0	18.3	15.8	13.3	10.8	8.51	8.07	7.42	6.81	6.00
3 1/2	27.0	23.9	20.9	18.0	15.2	12.5	9.82	9.32	8.57	7.87	6.94
4	30.3	26.9	23.6	20.3	17.2	14.1	11.1	10.6	9.72	8.94	7.88
4 1/2	33.5	29.8	26.2	22.6	19.1	15.8	12.4	11.8	10.9	10.0	8.82
5	36.8	32.8	28.8	24.9	21.1	17.4	13.7	13.1	12.0	11.1	9.77
5 1/2	40.1	35.7	31.4	27.2	23.1	19.0	15.1	14.3	13.2	12.1	10.7
6	43.4	38.7	34.0	29.5	25.0	20.7	16.4	15.6	14.3	13.2	11.7
6 1/2	46.6	41.6	36.7	31.8	27.0	22.3	17.7	16.8	15.5	14.3	12.6
7	49.9	44.6	39.3	34.1	29.0	23.9	19.0	18.0	16.6	15.3	13.5
7 1/2	53.2	47.5	41.9	36.4	30.9	25.6	20.3	19.3	17.8	16.4	14.5
8	56.5	50.4	44.5	38.7	32.9	27.2	21.6	20.5	18.9	17.4	15.4
9	63.0	56.3	49.7	43.2	36.8	30.5	24.2	23.0	21.2	19.6	17.3
10	69.5	62.2	55.0	47.8	40.7	33.8	26.8	25.5	23.5	21.7	19.2
11	76.1	68.1	60.2	52.4	44.7	37.0	29.5	28.0	25.8	23.8	21.1
12	82.6	74.0	65.5	57.0	48.6	40.3	32.1	30.5	28.1	25.9	23.0
13	89.2	80.0	70.7	61.6	52.5	43.6	34.7	33.0	30.4	28.1	24.9
14	95.7	85.8	75.9	66.2	56.5	46.8	37.3	35.5	32.7	30.2	26.7
15	102.3	91.7	81.2	70.7	60.4	50.1	39.9	38.0	35.0	32.3	28.6
16	108.8	97.6	86.4	75.3	64.3	53.4	42.5	40.5	37.3	34.4	30.5
17	115.4	103.5	91.6	79.9	68.2	56.7	45.2	43.0	39.6	36.6	32.4
18	121.9	109.3	96.9	84.5	72.2	59.9	47.8	45.5	41.9	38.7	34.3
19	128.5	115.2	102.1	89.1	76.1	63.2	50.4	48.0	44.2	40.8	36.2
20	135.0	121.1	107.3	93.6	80.0	66.5	53.0	50.4	46.5	42.9	38.0
21	141.5	127.0	112.6	98.2	83.9	69.7	55.6	52.9	48.8	45.1	39.9
22	148.1	132.9	117.8	102.8	87.9	73.0	58.3	55.4	51.1	47.2	41.8
23	154.6	138.8	123.1	107.4	91.8	76.3	60.9	57.9	53.4	49.3	43.7
24	161.2	144.7	128.3	112.0	95.7	79.6	63.5	60.4	55.7	51.5	45.6
26	174.3	156.5	138.8	121.1	103.6	86.1	68.7	65.4	60.3	55.7	49.3
28	187.4	168.3	149.2	130.3	111.4	92.7	74.0	70.4	64.9	60.0	53.1
30	200.4	180.0	159.7	139.5	119.3	99.2	79.2	75.4	69.5	64.2	56.8
32	213.5	191.8	170.2	148.6	127.1	105.7	84.4	80.4	74.1	68.5	60.6
34	226.6	203.6	180.6	157.8	135.0	112.3	89.7	85.4	78.7	72.8	64.4
36	239.7	215.4	191.1	167.0	142.9	118.8	94.9	90.4	83.4	77.0	68.1

Table No. 83 (*continued*).

LENGTH, 1 FOOT. Thickness by Holtzapffel's Wire-Gauge.

THICKNESS. W. G.	8	9	10	11	12	13	14	15	16	17	18
INCH.	.165 11/64 *b.*	.148 9/64 *f.*	.134 9/64 *b.*	.120 ⅛ *b.*	.109 7/64	.095 3/32 *f.*	.083 5/64 *f.*	.072 5/64 *b.*	.065 1/16 *f.*	.058 1/16 *b.*	.049 3/64 *f.*
INT. DIAM. inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
⅛	.501	.423	.364	.318	.267	.219	.181	.149	.130	.111	.0895
¼	.717	.610	.539	.472	.410	.343	.290	.243	.215	.187	.154
⅜	.934	.797	.714	.625	.553	.468	.398	.337	.300	.263	.218
½	1.15	1.00	.890	.779	.695	.592	.507	.431	.385	.339	.282
¾	1.58	1.39	1.24	1.09	.981	.841	.718	.620	.555	.491	.410
1	2.01	1.78	1.59	1.41	1.27	1.09	.935	.808	.725	.643	.538
1¼	2.45	2.17	1.94	1.72	1.55	1.34	1.15	.997	.895	.795	.667
1½	2.88	2.55	2.29	2.04	1.84	1.59	1.37	1.19	1.07	.946	.795
1¾	3.31	2.94	2.64	2.35	2.12	1.84	1.59	1.37	1.24	1.10	.923
2	3.74	3.33	3.00	2.66	2.41	2.08	1.81	1.56	1.41	1.25	1.05
2¼	4.17	3.72	3.35	2.98	2.69	2.33	2.02	1.75	1.58	1.40	1.18
2½	4.61	4.10	3.70	3.29	2.98	2.58	2.24	1.94	1.75	1.55	1.31
2¾	5.04	4.49	4.05	3.61	3.26	2.83	2.46	2.13	1.92	1.71	1.44
3	5.47	4.88	4.40	3.92	3.55	3.08	2.68	2.31	2.09	1.86	1.57
3½	6.33	5.65	5.10	4.55	4.12	3.58	3.11	2.69	2.43	2.16	1.82
4	7.20	6.43	5.80	5.18	4.69	4.07	3.55	3.07	2.77	2.47	2.08
4½	8.06	7.20	6.50	5.81	5.26	4.57	3.98	3.45	3.11	2.77	2.34
5	8.93	7.98	7.21	6.44	5.83	5.07	4.42	3.83	3.45	3.07	2.59
5½	9.79	8.75	7.91	7.06	6.40	5.57	4.85	4.20	3.79	3.38	2.85
6	10.7	9.53	8.61	7.69	6.97	6.07	5.29	4.58	4.13	3.68	3.11
6½	11.5	10.3	9.31	8.32	7.55	6.56	5.72	4.96	4.47	3.98	3.36
7	12.4	11.1	10.0	8.95	8.12	7.06	6.16	5.33	4.81	4.29	3.62
7½	13.3	11.9	10.7	9.58	8.69	7.56	6.59	5.71	5.15	4.59	3.88
8	14.1	12.6	11.4	10.2	9.26	8.06	7.03	6.09	5.49	4.90	4.13
9	15.8	14.2	12.8	11.5	10.4	9.05	7.90	6.84	6.17	5.50	4.65
10	17.6	15.7	14.2	12.7	11.5	10.0	8.77	7.60	6.85	6.11	5.16
11	19.3	17.3	15.6	14.0	12.7	11.0	9.64	8.35	7.53	6.72	5.67
12	21.0	18.8	17.0	15.2	13.8	12.0	10.5	9.10	8.21	7.33	6.19
13	22.7	20.4	18.4	16.5	15.0	13.0	11.4	9.86	8.89	7.93	6.70
14	24.5	21.9	19.8	17.7	16.1	14.0	12.2	10.6	9.57	8.54	7.22
15	26.2	23.5	21.3	19.0	17.2	15.0	13.1	11.4	10.3	9.15	7.73
16	27.9	25.0	22.7	20.3	18.4	16.0	14.0	12.1	10.9	9.88	8.24
17	29.6	26.6	24.1	21.5	19.5	17.0	14.9	12.9	11.6	10.4	8.76
18	31.4	28.1	25.5	22.8	20.6	18.0	15.7	13.6	12.3	11.0	9.27
19	33.1	29.7	26.9	24.0	21.8	19.0	16.6	14.4	13.0	11.6	9.78
20	34.8	31.2	28.3	25.3	22.9	20.0	17.5	15.1	13.7	12.2	10.3
21	36.6	32.8	29.7	26.5	24.1	21.0	18.3	15.9	14.3	12.8	10.8
22	38.3	34.3	31.1	27.8	25.2	22.0	19.2	16.6	15.0	13.4	11.3
23	40.0	35.9	32.5	29.1	26.4	23.0	20.1	17.4	15.7	14.0	11.8
24	41.8	37.4	33.9	30.3	27.5	24.0	20.9	18.1	16.4	14.6	12.6
26	45.2	40.5	36.7	32.8	29.8	26.0	22.6	19.7	17.7	15.8	13.4
28	48.7	43.6	39.5	35.3	32.1	28.0	24.4	21.2	19.1	17.0	14.4
30	52.1	46.7	42.3	37.8	34.4	30.0	26.1	22.7	20.5	18.3	15.4
32	55.5	49.8	45.1	40.4	36.7	32.0	27.9	24.2	21.8	19.5	16.5
34	59.0	52.9	48.0	42.9	39.0	34.0	29.7	25.8	23.2	20.7	17.5
36	62.4	56.0	50.8	45.4	41.3	36.0	31.4	27.3	24.6	21.9	18.6

Table No. 84.—WEIGHT OF WROUGHT-IRON TUBES, BY EXTERNAL DIAMETER.

LENGTH, 1 FOOT. Thickness by Holtzapffel's Wire-Gauge.

THICKNESS. W. G.	7	8	9	10	11	12	13	14	15
INCH.	.180 3/16 *b.*	.165 11/64 *b.*	.148 9/64 *f.*	.134 9/64 *b.*	.120 1/8 *b.*	.109 7/64	.095 3/32 *f.*	.083 5/64 *f.*	.072 5/64 *b.*
EXT. DIAM.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1 inch.	1.55	1.44	1.32	1.22	1.11	1.02	.900	.797	.700
1⅛	1.78	1.66	1.51	1.39	1.26	1.16	1.03	.906	.794
1¼	2.02	1.88	1.71	1.57	1.42	1.30	1.15	1.01	.888
1⅜	2.25	2.09	1.90	1.74	1.58	1.45	1.27	1.12	.983
1½	2.49	2.31	2.10	1.92	1.73	1.59	1.40	1.23	1.08
1⅝	2.72	2.52	2.29	2.09	1.89	1.73	1.52	1.34	1.17
1¾	2.96	2.74	2.48	2.27	2.05	1.87	1.65	1.45	1.27
1⅞	3.19	2.96	2.68	2.45	2.21	2.02	1.77	1.56	1.36
2	3.43	3.17	2.87	2.62	2.36	2.16	1.90	1.67	1.45
2⅛	3.67	3.39	3.06	2.80	2.52	2.30	2.02	1.78	1.55
2¼	3.90	3.60	3.26	2.97	2.68	2.44	2.14	1.88	1.64
2⅜	4.14	3.82	3.45	3.15	2.83	2.59	2.27	1.99	1.74
2½	4.37	4.04	3.65	3.32	2.99	2.73	2.39	2.10	1.83
2⅝	4.61	4.25	3.84	3.50	3.15	2.87	2.52	2.21	1.93
2¾	4.84	4.47	4.03	3.67	3.31	3.02	2.64	2.32	2.02
2⅞	5.08	4.68	4.23	3.85	3.46	3.16	2.77	2.43	2.11
3	5.32	4.90	4.42	4.02	3.62	3.30	2.89	2.54	2.21
3¼	5.79	5.33	4.81	4.37	3.94	3.59	3.14	2.75	2.40
3½	6.26	5.76	5.20	4.72	4.25	3.87	3.39	2.97	2.59
3¾	6.73	6.19	5.58	5.07	4.57	4.16	3.64	3.19	2.77
4	7.20	6.63	5.97	5.43	4.88	4.44	3.89	3.40	2.96
4¼	7.67	7.06	6.36	5.78	5.20	4.73	4.13	3.62	3.15
4½	8.14	7.49	7.45	6.13	5.51	5.01	4.38	3.84	3.34
4¾	8.61	7.91	7.13	6.48	5.82	5.30	4.63	4.06	3.53
5	9.08	8.35	7.52	6.83	6.13	5.58	4.88	4.27	3.72
5¼	9.56	8.79	7.91	7.18	6.44	5.87	5.13	4.49	3.90
5½	10.0	9.22	8.30	7.53	6.76	6.15	5.38	4.71	4.09
5¾	10.5	9.65	8.68	7.88	7.07	6.44	5.63	4.93	4.28
6	11.0	10.1	9.07	8.23	7.39	6.73	5.87	5.14	4.47
6¼	11.4	10.5	9.46	8.58	7.70	7.01	6.12	5.36	4.66
6½	11.9	10.9	9.85	8.93	8.02	7.30	6.37	5.58	4.85
6¾	12.4	11.4	10.2	9.28	8.33	7.58	6.62	5.79	5.03
7	12.9	11.8	10.6	9.63	8.64	7.87	6.87	6.01	5.22
7¼	13.3	12.2	11.0	9.99	8.96	8.15	7.12	6.23	5.41
7½	13.8	12.7	11.4	10.3	9.27	8.44	7.37	6.45	5.60
7¾	14.3	13.1	11.8	10.7	9.59	8.72	7.62	6.66	5.79
8	14.7	13.5	12.2	11.0	9.90	9.01	7.86	6.88	5.98

THICKNESS. W. G.			4	5	6	7	8	9
INCH.	.3125 5/16	.281 9/32	.238 15/64 *f.*	.220 7/32	.203 13/64	.180 3/16 *b.*	.165 11/64 *b.*	.148 9/64 *f.*
EXT. DIAM.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
7 inch.	21.9	19.8	16.9	15.6	14.5	12.9	11.8	10.6
7½	23.5	21.3	18.1	16.8	15.5	13.8	12.7	11.4
8	25.2	22.7	19.3	17.9	16.6	14.7	13.5	12.2
8½	26.8	24.2	20.6	19.1	17.6	15.7	14.4	12.9
9	28.4	25.7	21.8	20.2	18.7	16.6	15.3	13.7
9½	30.1	27.1	23.1	21.4	19.8	17.6	16.1	14.5
10	31.7	28.6	24.3	22.5	20.8	18.5	17.0	15.3

LIST OF TABLES OF THE WEIGHT OF CAST IRON, STEEL, COPPER, BRASS, TIN, LEAD, AND ZINC.

The following Tables are devoted to the specialities of manufacture in Cast Iron, Steel, and other metals, embracing the utmost range of dimensions to which objects in the several metals are executed in the ordinary course of practice.

Thus, whilst it is customary for certain classes of Cylinders in Cast Iron—steam cylinders, for example—to be constructed according to given internal diameters, other classes are constructed according to diameters given externally, as the iron piers of railway bridges. Two distinct tables accordingly have been composed, showing the weights of Cylinders of various thicknesses, and of diameters as measured internally and externally.

The weights of Copper Pipes and Cylinders are only calculated for internal diameters, as it is not the practice to construct them to given external diameters. Brass Tubes, on the contrary, are calculated only for external diameters, as they are not ordinarily made to given internal diameters.

TABLE NO. 95.—Weight of Brass Tubes, 1 foot in length, advancing, by external measurement, from ⅛ inch to 6 inches in diameter, and from No. 3 to No. 25 wire-gauge in thickness.

TABLE NO. 96.—Weight of one square foot of Sheet Brass; from No. 3 to No. 25 wire-gauge in thickness.

TABLE NO. 97.—Size and weight of Tin Plates.

TABLE NO. 98.—Weight of Tin Pipes, as manufactured.

TABLE NO. 99.—Weight of Lead Pipes, as manufactured.

TABLE NO. 100.—Dimensions and weight of Sheet Zinc. *(Vielle-Montagne.)*

Table No. 85.—Weight of Cast-Iron Cylinders.

By Internal Diameter. 1 Foot Long.

Int. Diam.	Thickness in Inches.										
	¼	5/16	⅜	7/16	½	9/16	⅝	11/16	¾	⅞	1
inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1	3.07	4.03	5.06	6.17	7.36	8.63	9.97	11.4	12.9	16.1	19.6
1½	4.30	5.56	6.90	8.32	9.82	11.4	13.1	14.8	16.6	20.4	24.5
2	5.52	7.09	8.74	10.5	12.3	14.2	16.1	18.1	20.3	24.7	29.5
2½	6.75	8.63	10.6	12.6	14.7	16.9	19.2	21.5	23.9	29.0	34.4
3	7.98	10.2	12.4	14.8	17.2	19.7	22.2	24.9	27.6	33.3	39.3
3½	9.20	11.7	14.3	16.9	19.6	22.4	25.3	28.3	31.3	37.6	44.2
4	10.4	13.2	16.1	19.1	22.1	25.2	28.4	31.6	35.0	41.9	49.1
4½	11.7	14.8	18.0	22.1	24.5	28.0	31.5	35.0	38.7	46.2	54.0
5	12.9	16.3	19.8	23.4	27.0	30.7	34.5	38.4	42.3	50.5	58.9
5½	14.1	17.8	21.6	25.5	29.5	33.5	37.6	41.8	46.0	54.8	63.8
6	15.3	19.4	23.5	27.7	32.0	36.2	40.7	45.1	49.7	59.1	68.7

	Thickness in Inches.										
	⅜	7/16	½	9/16	⅝	11/16	¾	⅞	1	1⅛	1¼
inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
6	23.5	27.7	32.0	36.2	40.7	45.1	49.7	59.1	68.7	78.7	89.0
6½	25.3	29.8	34.4	39.0	43.7	48.5	53.4	63.4	73.6	84.2	95.1
7	27.2	32.0	36.8	41.8	46.8	51.9	57.1	67.7	78.5	89.7	101.2
7½	29.0	34.1	39.3	44.5	49.9	55.3	60.8	71.9	83.5	95.3	107.4
8	30.8	36.3	41.7	47.3	52.9	58.6	64.4	76.2	88.4	100.8	113.5
8½	32.7	38.4	44.2	50.0	55.9	62.0	68.1	80.5	93.3	106.3	119.7
9	34.5	40.5	46.6	52.8	59.0	65.4	71.8	84.8	98.2	111.8	125.8
9½	36.4	42.7	49.1	55.6	62.0	68.8	75.5	89.1	103.1	117.4	131.9
10	38.2	44.8	51.5	58.3	65.1	72.1	79.2	93.4	108.0	122.9	138.1
10½	40.0	47.0	54.0	61.1	68.2	75.5	82.8	97.7	112.9	128.4	144.2
11	41.9	49.1	56.5	63.9	71.2	78.9	86.5	102.0	117.8	133.9	150.3
11½	43.7	51.3	58.9	66.6	74.5	82.3	90.2	106.3	122.7	139.4	156.5
12	45.6	53.4	61.4	69.4	77.5	85.6	93.9	110.6	127.6	145.0	162.6
13	49.2	57.7	66.3	74.9	83.6	92.4	101.2	119.2	137.5	156.0	174.9
14	52.9	62.0	71.2	80.4	89.7	99.1	108.6	127.8	147.3	167.1	187.2
15	56.6	66.3	76.1	85.9	95.9	105.9	116.0	136.4	157.1	178.1	199.4
16	60.3	70.6	81.0	91.5	102.0	112.6	123.3	145.0	166.9	189.1	211.7
17	64.0	74.9	85.9	97.0	108.2	119.4	130.7	153.6	176.7	200.2	224.0
18	67.7	79.2	90.8	102.5	114.3	126.1	138.1	162.2	186.5	211.2	236.2

	Thickness in Inches.										
	⅜	7/16	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
inches.	cwt.	cwt.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.
18	.604	.707	.811	1.02	1.23	1.45	1.67	1.89	2.11	2.34	2.56
19	.637	.746	.855	1.08	1.30	1.52	1.75	1.99	2.22	2.46	2.70
20	.670	.784	.898	1.13	1.36	1.60	1.84	2.08	2.33	2.58	2.83
21	.703	.823	.942	1.19	1.43	1.68	1.93	2.18	2.44	2.70	2.96
22	.736	.861	.986	1.24	1.49	1.76	2.02	2.28	2.55	2.82	3.09
23	.769	.900	1.03	1.29	1.56	1.83	2.10	2.38	2.66	2.94	3.22
24	.802	.939	1.07	1.35	1.63	1.91	2.19	2.48	2.77	3.06	3.35
25	.835	.977	1.12	1.40	1.69	1.99	2.28	2.58	2.88	3.18	3.48

Table No. 85 (*continued*).

By Internal Diameter. 1 Foot Long.

Int. Diam.	Thickness in Inches.										
	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
inches.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.
26	.868	1.02	1.16	1.46	1.76	2.06	2.37	2.68	2.99	3.30	3.62
27	.901	1.05	1.21	1.51	1.82	2.14	2.45	2.77	3.09	3.42	3.75
28	.934	1.09	1.25	1.57	1.89	2.22	2.54	2.87	3.20	3.54	3.88
29	.967	1.13	1.29	1.62	1.96	2.29	2.63	2.97	3.31	3.66	4.01
30	.998	1.17	1.34	1.68	2.02	2.37	2.72	3.07	3.42	3.78	4.14
32	1.06	1.25	1.43	1.79	2.15	2.52	2.89	3.27	3.64	4.02	4.41
34	1.13	1.32	1.51	1.90	2.29	2.67	3.07	3.46	3.86	4.26	4.67
36	1.20	1.40	1.60	2.01	2.42	2.83	3.24	3.66	4.08	4.50	4.94
38	1.26	1.47	1.69	2.12	2.55	2.98	3.42	3.86	4.30	4.75	5.20
40	1.33	1.55	1.77	2.23	2.68	3.14	3.59	4.05	4.52	4.99	5.47
42	1.39	1.63	1.86	2.34	2.81	3.29	3.77	4.25	4.74	5.23	5.73
45	1.49	1.75	1.99	2.50	3.01	3.52	4.03	4.55	5.07	5.59	6.13
48	1.59	1.86	2.12	2.66	3.21	3.75	4.30	4.85	5.40	5.96	6.52

Int. Diam.	Thickness in Inches.										
	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2	2 1/4
inches.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.
48	2.66	3.21	3.75	4.30	4.85	5.40	5.96	6.52	7.63	8.77	9.91
51	2.82	3.40	3.98	4.56	5.14	5.73	6.32	6.91	8.09	9.29	10.5
54	2.99	3.60	4.21	4.82	5.44	6.06	6.69	7.31	8.55	9.82	11.1
57	3.15	3.80	4.44	5.09	5.73	6.38	7.05	7.70	9.01	10.4	11.7
60	3.32	4.00	4.67	5.35	6.03	6.71	7.41	8.10	9.47	10.9	12.3
63	3.48	4.19	4.90	5.61	6.33	7.04	7.78	8.49	9.93	11.4	12.9
66	3.64	4.39	5.13	5.88	6.62	7.37	8.14	8.89	10.4	11.9	13.5
69	3.81	4.59	5.36	6.14	6.92	7.70	8.51	9.28	10.9	12.5	14.1
72	3.97	4.78	5.59	6.40	7.21	8.03	8.87	9.67	11.3	13.0	14.7
75	4.14	4.98	5.82	6.66	7.51	8.36	9.24	10.1	11.8	13.5	15.2
78	4.30	5.18	6.05	6.93	7.81	8.69	9.60	10.5	12.2	14.0	15.8
81	4.46	5.38	6.28	7.19	8.10	9.02	9.97	10.9	12.7	14.6	16.4
84	4.63	5.57	6.51	7.45	8.40	9.35	10.3	11.3	13.2	15.1	17.0
87	4.79	5.77	6.74	7.72	8.69	9.67	10.7	11.6	13.6	15.6	17.6
90	4.96	5.97	6.97	7.98	8.99	10.0	11.1	12.0	14.1	16.1	18.2
93	5.12	6.17	7.20	8.24	9.29	10.3	11.4	12.4	14.5	16.7	18.8
96	5.28	6.36	7.43	8.51	9.58	10.7	11.8	12.8	15.0	17.2	19.4
99	5.45	6.56	7.66	8.77	9.88	11.0	12.2	13.2	15.5	17.7	20.0
102	5.61	6.76	7.89	9.03	10.2	11.3	12.5	13.6	15.9	18.2	20.6
105	5.78	6.95	8.12	9.29	10.5	11.7	12.9	14.0	16.4	18.8	21.2
108	5.94	7.15	8.36	9.56	10.8	12.0	13.3	14.4	16.8	19.3	21.8
111	6.10	7.35	8.59	9.82	11.1	12.3	13.6	14.8	17.3	19.8	22.3
114	6.27	7.55	8.82	10.1	11.4	12.6	14.0	15.2	17.8	20.3	22.9
117	6.43	7.74	9.05	10.4	11.7	13.0	14.3	15.6	18.2	20.9	23.5
120	6.59	7.94	9.28	10.6	12.0	13.3	14.7	16.0	18.7	21.4	24.1

Table No. 86.—Weight of Cast-Iron Cylinders.

By External Diameter. 1 Foot Long.

Ext. Diam.	Thickness in Inches.										
	3/16	¼	5/16	⅜	7/16	½	9/16	⅝	¾	⅞	1
inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
3	5.18	6.75	8.25	9.65	11.0	12.3	13.5	14.6	16.6	18.3	19.6
3½	6.10	7.98	9.78	11.5	13.2	14.7	16.2	17.6	20.3	22.6	24.5
4	7.02	9.20	11.3	13.3	15.3	17.2	19.0	20.7	24.0	26.9	29.5
4½	7.94	10.4	12.9	15.2	17.5	19.6	21.7	23.8	27.7	31.1	34.4
5	8.86	11.7	14.4	17.0	19.6	22.1	24.5	26.9	31.5	35.4	39.3
5½	9.78	12.9	15.9	18.9	21.8	24.5	27.3	29.9	35.2	39.7	44.2
6	10.7	14.1	17.5	20.7	23.9	27.0	30.0	33.0	38.9	44.0	49.1
6½	11.6	15.3	19.0	22.5	26.0	29.5	32.8	36.1	42.6	48.3	54.0
7	12.5	16.6	20.5	24.4	28.2	31.9	35.6	39.1	46.4	52.6	58.9
7½	13.5	17.8	22.1	26.2	30.3	34.4	38.3	42.2	50.1	56.9	63.8
8	14.4	19.0	23.6	28.1	32.5	36.8	41.1	45.3	53.8	61.2	68.7
8½	15.3	20.3	25.1	29.9	34.6	39.3	43.8	48.3	57.5	65.5	73.6
9	16.2	21.5	26.7	31.8	36.8	41.7	46.6	51.4	61.3	69.8	78.5
9½	17.2	22.7	28.2	33.6	38.9	44.2	49.4	54.5	65.0	74.1	83.5
10	18.1	23.9	29.7	35.4	41.1	46.6	52.1	57.5	68.7	78.4	88.4
11	19.9	26.4	32.8	39.1	45.4	51.5	57.6	63.7	76.0	87.0	98.2
12	21.8	28.8	35.9	42.8	49.7	56.5	63.2	69.8	83.4	95.6	108.0
13	23.6	31.3	38.9	46.5	54.0	61.4	68.7	75.9	90.7	104.2	117.8
14	25.5	33.8	42.0	50.2	58.3	66.3	74.2	82.1	98.0	112.8	127.6
15	27.3	36.2	45.1	53.8	62.6	71.2	79.7	88.2	105.4	121.3	137.4
16	29.1	38.7	48.1	57.5	66.9	76.1	85.3	94.3	112.7	129.9	147.3
17	31.0	41.1	51.2	61.2	71.1	81.0	90.8	100.5	120.0	138.5	157.1
18	32.8	43.6	54.3	64.9	75.4	85.9	96.3	106.6	127.4	147.1	166.9
19	34.6	46.0	57.3	68.6	79.7	90.8	101.8	112.8	134.7	155.7	176.7
20	36.5	48.5	60.4	72.3	84.0	95.7	107.3	118.9	142.0	164.3	186.5
21	38.3	50.9	63.5	75.9	88.3	100.6	112.9	125.0	149.4	172.9	196.4
22	40.2	53.4	66.5	79.6	92.6	105.5	118.4	131.2	156.7	181.5	206.2
23	42.0	55.8	69.6	83.3	96.9	110.5	123.9	137.3	164.0	190.1	215.0
24	43.8	58.3	72.7	87.0	101.2	115.4	129.4	143.4	171.4	198.7	225.8
25	45.7	60.8	75.7	90.7	105.5	120.3	135.0	149.6	178.7	207.2	235.6
26	47.5	63.2	78.8	94.3	109.8	125.2	140.5	155.7	186.1	215.8	245.4
27	49.4	65.7	81.9	98.0	114.1	130.1	146.0	161.8	193.4	224.4	255.3
28	51.2	68.1	85.0	101.7	118.4	135.0	151.5	168.0	200.7	233.0	265.1
29	53.0	70.6	88.0	105.4	122.7	139.9	157.0	174.1	208.1	241.6	274.9
30	54.9	73.0	91.1	109.1	127.0	144.8	162.6	180.2	215.4	250.2	284.7
31	56.7	75.5	94.2	112.8	131.3	149.7	168.1	186.4	222.7	258.8	294.5
32	58.6	77.9	97.2	116.4	135.6	154.6	173.6	192.5	230.1	267.4	304.3
33	60.4	80.4	100.3	120.1	139.9	159.5	179.1	198.7	237.5	276.0	314.2
34	62.2	82.8	103.4	123.8	144.2	164.5	184.7	204.8	244.8	284.6	324.0
35	64.1	85.3	106.4	127.5	148.5	169.4	190.2	210.9	252.2	293.1	333.8
36	65.9	87.8	109.5	131.2	152.7	174.3	195.7	217.1	259.5	301.7	343.6
38	69.6	92.7	115.6	138.5	161.3	184.1	206.8	229.3	274.3	318.9	363.2
40	73.3	97.6	121.8	145.9	169.9	193.9	217.8	241.6	289.0	336.1	382.9
42	77.0	102.5	127.9	153.3	178.5	203.7	228.8	253.9	303.7	353.3	402.5
45	82.5	109.8	137.1	164.3	191.2	218.5	245.4	272.3	325.8	379.1	432.0
48	88.0	117.2	146.3	175.4	203.8	233.2	262.0	290.7	347.9	404.8	461.4
51	93.6	124.6	155.5	186.4	216.5	247.9	278.6	309.1	370.0	430.6	490.9
54	99.1	131.9	164.7	197.5	229.2	262.6	295.1	327.5	392.1	456.4	520.3
57	104.6	139.3	173.9	208.5	241.8	277.4	311.7	345.9	414.2	482.1	549.8
60	110.1	146.6	183.1	219.6	254.5	292.1	328.3	364.3	436.3	507.9	579.3

Table No. 86 (*continued*).

By External Diameter. 1 Foot Long.

Ext. Diam.	Thickness in Inches.										
	3/16	¼	5/16	⅜	7/16	½	9/16	⅝	¾	⅞	1
ft. in.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.
5 3	1.03	1.44	1.71	2.06	2.39	2.74	3.08	3.42	4.09	4.77	5.43
5 6	1.08	1.50	1.80	2.16	2.50	2.87	3.22	3.58	4.29	5.00	5.70
5 9	1.13	1.55	1.88	2.26	2.62	3.00	3.37	3.75	4.49	5.23	5.96
6 0	1.18	1.61	1.96	2.36	2.74	3.14	3.52	3.91	4.69	5.46	6.22
6 3	1.23	1.67	2.05	2.45	2.85	3.27	3.66	4.08	4.88	5.69	6.49
6 6	1.28	1.73	2.13	2.55	2.97	3.40	3.81	4.24	5.08	5.92	6.75
6 9	1.33	1.78	2.21	2.65	3.09	3.53	3.96	4.41	5.28	6.15	7.01
7 0	1.38	1.84	2.29	2.75	3.20	3.66	4.10	4.57	5.47	6.38	7.28
7 6	1.48	1.95	2.46	2.95	3.43	3.92	4.39	4.90	5.87	6.84	7.80
8 0	1.58	2.07	2.62	3.15	3.67	4.19	4.69	5.23	6.26	7.30	8.33

	Thickness in Inches.										
	1⅛	1¼	1⅜	1½	1¾	2	2¼	2½	2¾	3	3½	4
inches.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.
6	.481	.520	.557	.592	.652	.701	.740	.761				
6½	.530	.575	.618	.657	.729	.789	.838	.872	.906			
7	.579	.630	.678	.723	.805	.876	.938	.982	1.03	1.05		
7½	.629	.685	.738	.789	.882	.964	1.04	1.09	1.15	1.18		
8	.678	.740	.799	.855	.959	1.05	1.14	1.20	1.27	1.32	1.38	
8½	.727	.794	.859	.921	1.04	1.14	1.23	1.31	1.39	1.45	1.53	
9	.777	.849	.919	.986	1.11	1.23	1.33	1.42	1.51	1.58	1.69	1.75
9½	.826	.904	.980	1.05	1.19	1.31	1.43	1.53	1.63	1.71	1.84	1.93
10	.875	.959	1.04	1.12	1.27	1.40	1.53	1.64	1.75	1.84	1.99	2.10
11	.974	1.07	1.16	1.25	1.42	1.58	1.73	1.86	1.99	2.10	2.30	2.46
12	1.07	1.18	1.28	1.38	1.57	1.75	1.92	2.08	2.23	2.37	2.61	2.81
13	1.17	1.29	1.40	1.51	1.73	1.93	2.12	2.30	2.47	2.63	2.92	3.16
14	1.27	1.40	1.52	1.64	1.88	2.10	2.32	2.52	2.71	2.89	3.22	3.51
15	1.37	1.51	1.65	1.78	2.03	2.28	2.52	2.74	2.95	3.16	3.53	3.86
16	1.47	1.62	1.77	1.91	2.19	2.45	2.71	2.96	3.19	3.42	3.84	4.21
17	1.57	1.73	1.89	2.04	2.34	2.63	2.91	3.18	3.44	3.68	4.14	4.56
18	1.66	1.84	2.01	2.17	2.49	2.81	3.11	3.40	3.68	3.95	4.45	4.91
20	1.86	2.06	2.25	2.43	2.80	3.16	3.50	3.83	4.16	4.47	5.06	5.61
22	2.06	2.27	2.49	2.70	3.11	3.51	3.90	4.27	4.64	5.00	5.68	6.32
24	2.26	2.49	2.73	2.96	3.41	3.86	4.29	4.71	5.12	5.52	6.29	7.01
27	2.55	2.82	3.09	3.35	3.87	4.38	4.88	5.37	5.85	6.31	7.21	8.06
30	2.85	3.15	3.46	3.75	4.33	4.91	5.47	6.03	6.57	7.10	8.13	9.12
33	3.14	3.48	3.82	4.14	4.79	5.44	6.06	6.68	7.29	7.89	9.05	10.2
36	3.44	3.81	4.18	4.54	5.25	5.96	6.66	7.34	8.01	8.68	9.97	11.2
39	3.74	4.14	4.54	4.93	5.72	6.49	7.25	8.00	8.74	9.47	10.9	12.3
42	4.03	4.47	4.90	5.33	6.18	7.01	7.84	8.66	9.46	10.3	11.8	13.3
45	4.33	4.79	5.26	5.72	6.64	7.54	8.43	9.31	10.2	11.1	12.7	14.4
48	4.62	5.12	5.62	6.12	7.10	8.07	9.02	9.98	10.9	11.8	13.7	15.4
51	4.92	5.45	5.98	6.51	7.56	8.59	9.61	10.6	11.6	12.6	14.6	16.5
54	5.22	5.78	6.35	6.91	8.02	9.12	10.2	11.3	12.4	13.4	15.5	17.5
57	5.51	6.11	6.71	7.30	8.48	9.64	10.8	11.9	13.1	14.2	16.4	18.6
60	5.81	6.44	7.07	7.70	8.94	10.2	11.4	12.6	13.8	15.0	17.3	19.6

Table No. 86 (*continued*).

BY EXTERNAL DIAMETER. 1 FOOT LONG.

EXT. DIAM.	THICKNESS IN INCHES.											
	1⅛	1¼	1⅜	1½	1¾	2	2¼	2½	2¾	3	3½	4
ft. in.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.
5 3	6.10	6.77	7.43	8.09	9.40	10.7	12.0	13.3	14.5	15.8	18.3	
5 6	6.40	7.09	7.79	8.48	9.86	11.2	12.6	13.9	15.2	16.6		
5 9	6.70	7.42	8.15	8.88	10.3	11.8	13.2	14.6	15.9			
6 0	7.00	7.75	8.51	9.27	10.8	12.3	13.8	15.2				
6 3	7.29	8.08	8.88	9.67	11.2	12.8	14.4	15.9				
6 6	7.58	8.41	9.24	10.1	11.7	13.3	14.9	16.6				
6 9	7.88	8.74	9.60	10.5	12.2	13.9	15.5	17.2				
7 0	8.17	9.07	9.96	10.9	12.6	14.4	16.1	17.9				
7 6	8.77	9.72	10.7	11.6	13.5	15.4	17.3	19.2				
8 0	9.36	10.4	11.4	12.4	14.5	16.5	18.5	20.5				
8 6	9.95	11.0	12.1	13.2	15.4	17.5	19.7	21.8				
9 0	10.5	11.7	12.9	14.0	16.3	18.6	20.8	23.1				
9 6	11.1	12.3	13.6	14.8	17.2	19.6	22.0	24.4				
10 0	11.7	13.0	14.3	15.6	18.1	20.7	23.2	25.7				
10 6	12.3	13.7	15.0	16.4	19.1	21.7	24.4	27.1				
11 0	12.9	14.3	15.7	17.2	20.0	22.8	25.6	28.4				
11 6	13.5	15.0	16.5	17.9	20.9	23.8	26.7	29.7				
12 0	14.1	15.6	17.2	18.7	21.8	24.9	27.9	31.0				
13 0	15.3	16.9	18.6	20.3	23.7	27.0	30.3	33.6				
14 0	16.5	18.3	20.1	21.9	25.5	29.1	32.7	36.3				
15 0	17.7	19.6	21.5	23.5	27.3	31.2	35.0	38.9				
16 0	18.8	20.9	23.0	25.0	29.2	33.3	37.4	41.5				
17 0	20.0	22.2	24.4	26.6	31.0	35.4	39.8	44.2				
18 0	21.2	23.5	25.9	28.2	32.9	37.5	42.2	46.8				
19 0	22.4	24.8	27.3	29.8	34.7	39.6	44.5	49.4				
20 0	23.6	26.1	28.8	31.4	36.5	41.7	46.9	52.0				

Table No. 87.—Volume and Weight of Cast-Iron Balls.

Given the Diameter.

Diameter.	Contents.	Weight.
inches.	cubic inches.	pounds.
1	.524	.136
1½	1.77	.460
2	4.19	1.09
2½	8.18	2.13
3	14.1	3.68
3½	22.5	5.85
4	33.5	8.73
4½	47.7	12.4
5	65.5	17.0
5½	87.1	22.7
6	113.1	29.5
6½	143.8	37.5
7	179.6	46.8
7½	220.9	57.5

Diameter.	Contents.	Weight.
inches.	cubic inches.	pounds.
8	268.1	69.8
8½	321.5	83.7
9	381.7	99.4
9½	448.9	116.9
10	523.6	136.4
inches.	cubic feet.	cwts.
11	.403	1.62
12	.524	2.10
13	.666	2.68
14	.832	3.34
15	1.023	4.11
16	1.241	4.99
17	1.489	5.98
18	1.767	7.10

Diameter.	Contents.	Weight.
inches.	cubic feet.	cwts.
19	2.078	8.35
20	2.424	9.74
21	2.806	11.28
22	3.227	12.97
23	3.688	14.82
24	4.188	16.83
25	4.736	19.03
26	5.327	21.40
27	5.963	23.96
28	6.651	26.72
29	7.390	29.69
30	8.181	32.87
31	9.027	36.27
32	9.930	39.90

Note.—To find the weight of balls of other metals, multiply the weight given in the table by the following multipliers:—

For Wrought Iron	1.067,	making about	7 per cent. more.
Steel	1.088	,,	9 ,,
Brass	1.12	,,	12 ,,
Gun Metal	1.165	,,	16½ ,,

Table No. 88.—Diameter of Cast-Iron Balls.

Given the Weight.

Weight.	Diameter.
pounds.	inches.
½	1.54
1	1.94
2	2.45
3	2.80
4	3.08
5	3.32
6	3.53
7	3.72
8	3.89
9	4.04
10	4.19
12	4.45

Weight.	Diameter.
pounds.	inches.
14	4.68
16	4.89
18	5.09
20	5.27
25	5.68
28	5.90
30	6.04
40	6.64
50	7.16
56	7.43
60	7.60
70	8.01

Weight.	Diameter.
pounds.	inches.
80	8.37
90	8.71
100	9.02
cwts.	inches.
1	9.37
1½	10.72
2	11.80
3	13.51
4	14.87
5	16.02
6	17.02
7	17.91

Weight.	Diameter.
cwts.	inches.
8	18.73
9	19.48
10	20.17
12	21.44
14	22.57
16	23.60
18	24.54
20	25.42
25	27.38
30	29.10
35	30.64
40	32.03

Table No. 89.—Weight of Flat Bar Steel.

1 Foot Long.

Thickness.	Width in Inches.							
	½	⅝	¾	⅞	1	1¼	1½	1¾
inches.	lb.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¼	.425	.533	.640	.743	.850	1.06	1.28	1.49
5/16	.531	.665	.800	.929	1.06	1.33	1.59	1.86
⅜	.638	.798	.960	1.11	1.28	1.59	1.91	2.23
7/16	.744	.931	1.12	1.30	1.49	1.86	2.23	2.60
½	.850	1.06	1.28	1.49	1.70	2.13	2.55	2.98
9/16	—	1.20	1.44	1.67	1.91	2.39	2.87	3.35
⅝	—	1.33	1.60	1.86	2.12	2.66	3.19	3.72
11/16	—	—	1.76	2.04	2.34	2.92	3.51	4.09
¾	—	—	1.92	2.23	2.55	3.19	3.83	4.46
13/16	—	—	—	2.41	2.76	3.45	4.14	4.83
⅞	—	—	—	2.60	2.98	3.72	4.46	5.21
15/16	—	—	—	—	3.19	3.98	4.78	5.58
1	—	—	—	—	3.40	4.25	5.10	5.95

Thickness.	Width in Inches.							
	2	2¼	2½	2¾	3	3¼	3½	4
inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¼	1.70	1.91	2.13	2.34	2.55	2.76	2.98	3.40
5/16	2.13	2.39	2.66	2.92	3.19	3.45	3.72	4.25
⅜	2.55	2.87	3.19	3.51	3.83	4.14	4.46	5.10
7/16	2.98	3.35	3.72	4.09	4.46	4.83	5.21	5.95
½	3.40	3.83	4.25	4.68	5.10	5.53	5.95	6.80
9/16	3.83	4.30	4.78	5.26	5.74	6.22	6.69	7.65
⅝	4.25	4.78	5.31	5.84	6.38	6.91	7.44	8.50
11/16	4.68	5.26	5.84	6.43	7.01	7.60	8.18	9.35
¾	5.10	5.74	6.38	7.01	7.65	8.29	8.93	10.2
13/16	5.53	6.22	6.91	7.60	8.29	8.98	9.67	11.1
⅞	5.95	6.69	7.44	8.18	8.93	9.67	10.4	11.9
15/16	6.38	7.17	7.97	8.77	9.56	10.4	11.2	12.8
1	6.80	7.65	8.50	9.35	10.2	11.1	11.9	13.6

Thickness.	Width in Inches.							
	4½	5	5½	6	6½	7	7½	8
inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¼	3.82	4.26	4.68	5.10	5.52	5.96	6.38	6.80
5/16	4.78	5.32	5.84	6.38	6.90	7.44	7.97	8.50
⅜	5.74	6.38	7.02	7.66	8.28	8.92	9.56	10.2
7/16	6.70	7.44	8.18	8.92	9.66	10.4	11.2	11.9
½	7.66	8.50	9.36	10.2	11.1	11.9	12.8	13.6
9/16	8.60	9.56	10.5	11.5	12.4	13.4	14.3	15.3
⅝	9.56	10.6	11.7	12.8	13.8	14.9	15.9	17.0
11/16	10.5	11.7	12.9	14.0	15.2	16.4	17.5	18.7
¾	11.5	12.8	14.0	15.3	16.6	17.9	19.1	20.4
13/16	12.4	13.8	15.2	16.6	18.0	19.3	20.7	22.2
⅞	13.4	14.9	16.4	17.9	19.4	20.8	22.3	23.8
15/16	14.3	15.9	17.5	19.1	20.8	22.4	23.9	25.6
1	15.3	17.0	18.7	20.4	22.1	23.8	25.5	27.2

Table No. 90.—Weight of Square Steel.

1 Foot in Length.

Size.	Weight.	Size.	Weight.	Size.	Weight.	Size.	Weight.
inches.	pounds.	inches.	pounds.	inches.	pounds.	inches.	pounds.
1/8	.053	15/16	3.06	1 3/4	10.4	3 1/4	35.9
3/16	.119	1	3.40	1 13/16	11.2	3 1/2	41.6
1/4	.212	1 1/16	3.83	1 7/8	11.9	3 3/4	47.8
5/16	.333	1 1/8	4.30	1 15/16	12.8	4	54.4
3/8	.478	1 3/16	4.79	2	13.6	4 1/4	61.4
7/16	.651	1 1/4	5.31	2 1/8	15.4	4 1/2	68.9
1/2	.850	1 5/16	5.86	2 1/4	17.2	4 3/4	76.7
9/16	1.08	1 3/8	6.43	2 3/8	19.2	5	85.0
5/8	1.33	1 7/16	7.03	2 1/2	21.2	5 1/4	93.7
11/16	1.61	1 1/2	7.65	2 5/8	23.5	5 1/2	102.8
3/4	1.92	1 9/16	8.30	2 3/4	25.7	5 3/4	112.4
13/16	2.24	1 5/8	8.98	2 7/8	28.2	6	122.4
7/8	2.60	1 11/16	9.79	3	30.6		

Table No. 91.—Weight of Round Steel.

1 Foot in Length.

Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.
inches.	pounds.	inches.	pounds.	inches.	pounds.	inches.	cwts.
1/8	.0417	1 7/16	5.18	4	42.7	12	3.433
3/16	.0939	1 1/2	6.01	4 1/4	48.3	12 1/2	3.729
1/4	.167	1 9/16	6.52	4 1/2	54.6	13	4.029
5/16	.260	1 5/8	7.05	4 3/4	60.3	13 1/2	4.345
3/8	.375	1 11/16	7.61	5	66.8	14	4.682
7/16	.511	1 3/4	8.18	5 1/4	73.6	14 1/2	5.013
1/2	.667	1 13/16	8.77	5 1/2	80.8	15	5.364
9/16	.845	1 7/8	9.38	5 3/4	88.3	15 1/2	5.728
5/8	1.04	1 15/16	10.0	6	96.1	16	6.103
11/16	1.27	2	10.7	inches.	cwts.	16 1/2	6.471
3/4	1.50	2 1/8	12.0	6 1/2	1.007	17	6.868
13/16	1.76	2 1/4	13.6	7	1.168	17 1/2	7.302
7/8	2.04	2 3/8	15.1	7 1/2	1.341	18	7.724
15/16	2.35	2 1/2	16.7	8	1.526	19	8.607
1	2.67	2 5/8	18.4	8 1/2	1.723	20	9.537
1 1/16	3.00	2 3/4	20.2	9	1.931	21	10.52
1 1/8	3.38	2 7/8	22.0	9 1/2	2.152	22	11.54
1 3/16	3.76	3	24.1	10	2.385	23	12.62
1 1/4	4.17	3 1/4	28.3	10 1/2	2.629	24	13.73
1 5/16	4.60	3 1/2	32.7	11	2.884		
1 3/8	5.05	3 3/4	34.2	11 1/2	3.150		

Table No. 92.—Weight of Chisel Steel—Hexagonal, Octagonal, and Oval-Flat.

1 Foot in Length.

Diameter across the Sides.	Hexagonal Section.			Octagonal Section.		
	Sectional Area.	Weight.	Length to weigh 1 cwt.	Sectional Area.	Weight.	Length to weigh 1 cwt.
inches.	square inches.	pounds.	feet.	square inches.	pounds.	feet.
3/8	.1217	.414	245	.1164	.396	268
1/2	.2165	.736	138	.2070	.704	151
5/8	.3383	1.15	88.3	.3236	1.10	96.5
3/4	.4871	1.66	61.3	.4659	1.58	67
7/8	.6631	2.25	45	.6342	2.16	49.3
1	.8661	2.94	34.5	.8284	2.82	37.7
1 1/8	1.096	3.73	27.3	1.048	3.56	30
1 1/4	1.353	4.60	22.5	1.294	4.40	24
1 3/8	1.637	5.57	18.3	1.566	5.32	20
1 1/2	1.949	6.63	15.3	1.864	6.34	16.8
	Oval-Flat Section.					
inch. inch.						
3/4 × 3/8	.2510	.853	119			
1 × 1/2	.4463	1.52	67			
1 1/4 × 5/8	.6974	2.37	43			

Table No. 93.—Weight of one Square Foot of Sheet Copper.

To Wire-Gauge employed by Williams, Foster, & Co.

Specific Weight taken as 1.16 (Specific Weight of Wrought Iron = 1).

Thickness.		Weight of 1 Square Foot.	Thickness.		Weight of 1 Square Foot.	Thickness.		Weight of 1 Square Foot.
Wire-Gauge. No.	Inch (approximate).	pounds.	Wire-Gauge. No.	Inch (approximate).	pounds.	Wire-Gauge. No.	Inch (approximate).	pounds.
1	.306	14.0	11	.123	5.65	21	.0338	1.55
2	.284	13.0	12	.109	5.00	22	.0295	1.35
3	.262	12.0	13	.0983	4.50	23	.0251	1.15
4	.240	11.0	14	.0882	4.00	24	.0218	1.00
5	.222	10.15	15	.0764	3.50	25	.0194	.89
6	.203	9.30	16	.0655	3.00	26	.0172	.79
7	.186	8.50	17	.0568	2.60	27	.0153	.70
8	.168	7.70	18	.0491	2.25	28	.0135	.62
9	.153	7.00	19	.0437	2.00	29	.0122	.56
10	.138	6.30	20	.0382	1.75	30	.0110	.50

Table No. 94.—Weight of Copper Pipes and Cylinders, by Internal Diameter.

Length, 1 Foot. Thickness by Holtzapffel's Wire-Gauge (Table No. 13).

Specific Weight = 1.16 (Specific Weight of Wrought-Iron = 1).

Thickness. W. G.	0000	000	00	0	1	2	3	4	5	6	7
Inch.	.454 29/64	.425 27/64 *f.*	.380 3/8 *f.*	.340 11/32	.300 19/64 *f.*	.284 9/32 *f.*	.259 1/4 *f.*	.238 15/64 *f.*	.220 7/32 *f.*	.203 13/64	.180 3/16 *b.*
Int. Diam. inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
⅛	3.14	2.84	2.33	1.92	1.53	1.41	1.21	1.05	.934	.809	.667
¼	3.84	3.49	2.91	2.44	1.99	1.84	1.60	1.41	1.27	1.12	.941
⅜	4.54	4.13	3.49	2.95	2.45	2.27	2.00	1.77	1.60	1.43	1.21
½	5.23	4.78	4.06	3.47	2.91	2.71	2.39	2.13	1.93	1.73	1.49
⅝	5.93	5.42	4.64	3.99	3.37	3.14	2.78	2.50	2.26	2.04	1.76
¾	6.63	6.07	5.22	4.50	3.83	3.57	3.17	2.86	2.60	2.35	2.03
⅞	7.32	6.71	5.79	5.02	4.29	4.00	3.57	3.22	2.93	2.66	2.31
1	8.02	7.36	6.37	5.53	4.74	4.43	3.96	3.57	3.26	2.97	2.58
1⅛	8.71	8.00	6.95	6.05	5.20	4.86	4.35	3.94	3.60	3.28	2.85
1¼	9.40	8.65	7.52	6.57	5.65	5.29	4.75	4.30	3.93	3.58	3.13
1⅜	10.1	9.30	8.10	7.08	6.11	5.72	5.14	4.66	4.26	3.89	3.40
1½	10.8	9.94	8.68	7.60	6.57	6.16	5.53	5.02	4.60	4.20	3.68
1⅝	11.5	10.6	9.26	8.12	7.02	6.59	5.93	5.39	4.93	4.51	3.95
1¾	12.1	11.2	9.83	8.63	7.48	7.02	6.32	5.75	5.27	4.82	4.22
1⅞	12.8	11.9	10.4	9.15	7.93	7.45	6.71	6.11	5.60	5.12	4.50
2	13.5	12.5	11.0	9.66	8.39	7.88	7.11	6.47	5.93	5.43	4.77
2⅛	14.2	13.2	11.6	10.2	8.84	8.31	7.50	6.83	6.27	5.74	5.04
2¼	14.9	13.8	12.1	10.7	9.30	8.74	7.89	7.19	6.60	6.05	5.32
2⅜	15.6	14.5	12.7	11.2	9.75	9.17	8.29	7.56	6.94	6.36	5.59
2½	16.3	15.1	13.3	11.7	10.2	9.60	8.68	7.92	7.27	6.67	5.86
2⅝	17.0	15.8	13.9	12.2	10.7	10.0	9.07	8.28	7.60	6.97	6.14
2¾	17.7	16.4	14.5	12.8	11.1	10.5	9.47	8.64	7.94	7.28	6.41
2⅞	18.4	17.1	15.0	13.3	11.57	10.9	9.86	9.00	8.27	7.59	6.68
3	19.1	17.7	15.6	13.8	12.0	11.3	10.3	9.36	8.61	7.90	6.95
3¼	20.4	19.0	16.8	14.8	12.9	12.2	11.1	10.1	9.27	8.52	7.50
3½	21.8	20.3	17.9	15.9	13.9	13.1	11.8	10.8	9.94	9.13	8.04
3¾	23.2	21.6	19.1	16.9	14.8	13.9	12.6	11.5	10.6	9.75	8.59
4	24.6	22.9	20.2	17.9	15.7	14.8	13.4	12.3	11.3	10.4	9.13
4¼	25.9	24.2	21.4	19.0	16.6	15.6	14.2	13.0	12.0	11.0	9.67
4½	27.3	25.4	22.5	20.0	17.5	16.5	15.0	13.7	12.7	11.6	10.2
4¾	28.7	26.7	23.7	21.0	18.4	17.4	15.8	14.4	13.3	12.2	10.8
5	30.1	28.0	24.8	22.1	19.3	18.2	16.6	15.1	14.0	12.8	11.3
5¼	31.5	29.3	26.0	23.1	20.2	19.1	17.3	15.9	14.6	13.5	11.9
5½	32.8	30.6	27.1	24.1	21.1	20.0	18.1	16.6	15.3	14.1	12.4
5¾	34.2	31.9	28.3	25.2	22.1	20.8	18.9	17.3	16.0	14.7	12.9
6	35.6	33.2	29.5	26.2	23.0	21.7	19.7	18.0	16.6	15.3	13.5

Table No 94 (*continued*).

LENGTH, 1 FOOT. Thickness by Holtzapffel's Wire-Gauge (Table No. 13).

Specific Weight = 1.16 (Specific Weight of Wrought Iron = 1).

THICKNESS. W. G.	8	9	10	11	12	13	14	15	16	17	18	19	20
INCH.	.165 11/64 b	.148 9/64 f	.134 9/64 b	.120 ⅛ b	.109 7/64	.095 3/32 f	.083 5/64 f	.072 5/64 b	.065 1/16 f	.058 1/16 b	.049 3/64 f	.042 3/64 b	.035 1/32 f
INT. DIAM. inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
⅛	.581	.491	.422	.357	.310	.254	.210	.173	.150	.129	.104	.086	.068
¼	.832	.716	.626	.540	.476	.398	.336	.282	.249	.217	.178	.149	.121
⅜	1.08	.941	.830	.722	.641	.543	.462	.391	.348	.305	.253	.213	.175
½	1.33	1.17	1.03	.904	.807	.687	.588	.500	.447	.393	.327	.277	.228
⅝	1.58	1.39	1.24	1.09	.972	.831	.714	.610	.545	.481	.402	.341	.281
¾	1.83	1.62	1.44	1.27	1.14	.975	.840	.719	.644	.570	.476	.404	.334
⅞	2.09	1.84	1.65	1.45	1.30	1.12	.966	.828	.743	.658	.550	.468	.387
1	2.34	2.05	1.85	1.63	1.47	1.26	1.09	.938	.842	.746	.625	.532	.440
1⅛	2.59	2.27	2.05	1.82	1.63	1.41	1.22	1.05	.940	.834	.699	.596	.493
1¼	2.84	2.49	2.26	2.00	1.80	1.55	1.34	1.16	1.04	.922	.774	.659	.547
1⅜	3.09	2.72	2.46	2.18	1.97	1.70	1.47	1.27	1.14	1.01	.848	.723	.600
1½	3.34	2.94	2.67	2.36	2.13	1.84	1.60	1.38	1.24	1.10	.922	.787	.653
1⅝	3.59	3.17	2.87	2.55	2.30	1.99	1.72	1.48	1.34	1.19	.997	.851	.706
1¾	3.84	3.39	3.07	2.73	2.46	2.13	1.85	1.59	1.43	1.27	1.07	.915	.759
1⅞	4.09	3.62	3.28	2.91	2.63	2.27	1.97	1.70	1.53	1.36	1.15	.978	.812
2	4.34	3.84	3.48	3.09	2.79	2.42	2.10	1.81	1.63	1.45	1.22	1.04	.865
2⅛	4.59	4.07	3.69	3.27	2.96	2.56	2.23	1.92	1.73	1.54	1.29	1.11	.919
2¼	4.84	4.29	3.89	3.45	3.12	2.71	2.35	2.03	1.83	1.63	1.38	1.17	.972
2⅜	5.09	4.52	4.09	3.64	3.29	2.85	2.48	2.14	1.93	1.71	1.45	1.23	1.03
2½	5.34	4.74	4.30	3.82	3.45	3.00	2.60	2.25	2.03	1.80	1.53	1.30	1.08
2⅝	5.59	4.97	4.50	4.00	3.62	3.14	2.73	2.36	2.13	1.89	1.60	1.36	1.13
2¾	5.84	5.19	4.71	4.18	3.79	3.28	2.86	2.47	2.22	1.98	1.68	1.43	1.18
2⅞	6.09	5.42	4.91	4.37	3.95	3.43	2.98	2.58	2.32	2.07	1.75	1.49	1.24
3	6.34	5.66	5.11	4.55	4.12	3.57	3.11	2.69	2.42	2.16	1.82	1.55	1.29
3¼	6.85	6.11	5.52	4.91	4.45	3.86	3.36	2.91	2.62	2.33	1.96	1.68	1.40
3½	7.35	6.56	5.93	5.28	4.78	4.15	3.61	3.12	2.82	2.51	2.11	1.81	1.51
3¾	7.85	7.01	6.33	5.64	5.11	4.44	3.87	3.34	3.01	2.68	2.26	1.94	1.62
4	8.35	7.46	6.74	6.01	5.44	4.73	4.12	3.56	3.21	2.86	2.41	2.06	1.73
4¼	8.85	7.91	7.14	6.37	5.77	5.02	4.37	3.78	3.41	3.04	2.56	2.19	1.84
4½	9.35	8.36	7.55	6.74	6.10	5.30	4.62	4.00	3.61	3.21	2.71	2.32	1.94
4¾	9.85	8.81	7.96	7.10	6.43	5.59	4.87	4.22	3.80	3.39	2.86	2.45	2.05
5	10.4	9.26	8.36	7.46	6.77	5.88	5.13	4.44	4.00	3.56	3.01	2.57	2.16
5¼	10.9	9.71	8.77	7.83	7.10	6.17	5.38	4.66	4.20	3.74	3.15	2.70	2.27
5½	11.4	10.2	9.18	8.19	7.43	6.46	5.63	4.88	4.40	3.92	3.30	2.83	2.38
5¾	11.9	10.6	9.58	8.56	7.76	6.75	5.88	5.09	4.59	4.09	3.45	2.96	2.48
6	12.4	11.1	9.99	8.92	8.09	7.04	6.14	5.31	4.79	4.27	3.60	3.09	2.58

Table No. 94 (*continued*).

LENGTH, 1 FOOT. Thickness by Holtzapffel's Wire-Gauge (Table No. 13).

Specific Weight = 1.16 (Specific Weight of Wrought-Iron = 1).

THICKNESS. W. G.	0000	000	00	0	1	2	3	4	5	6	7
INCH.	.454 26/64	.425 27/64 *f.*	.380 3/8 *f.*	.340 11/32	.300 19/64 *f.*	.284 9/32 *f.*	.259 1/4 *f.*	.238 15/64 *f.*	.220 7/32 *f.*	.203 13/64	.180 3/16 *b.*
INT. DIAM. inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
6½	38.4	35.8	31.8	28.3			21.3	19.5	18.0		14.6
7	41.1	38.3	34.1	30.3	26.6	25.1	22.8	20.9	19.3	17.8	15.7
7½	43.9	40.9	36.4	32.4	28.4			22.4	20.6		16.8
8	46.6	43.5	38.7	34.5	30.3	28.6	26.0	23.8	22.0	20.2	17.9
9	52.1	48.7	43.3	38.6	33.9	32.0	29.1	26.7	24.6	22.7	20.1
10	57.7	53.8	47.9	42.7	37.5	35.5	32.3	29.6	27.3	25.2	22.2
11	63.2	59.0	52.5	46.8	41.2	38.9	35.4	32.5	30.0	27.7	24.4
12	68.7	64.2	57.2	51.0	44.8	42.4	38.6	35.4	32.7	30.1	26.6
13	74.2	69.3	61.8	55.1	48.5	45.8	41.7	38.3	35.3	32.6	28.8
14	79.7	74.5	66.4	59.2	52.1	49.3	44.9	41.2	38.0	35.1	31.0
15	85.2	79.6	71.0	63.4	55.8	52.7	48.0	44.1	40.7	37.6	33.2
16	90.7	84.8	75.6	67.7	59.4	56.2	51.2	46.9	43.4	40.0	35.4
17	96.3	90.0	80.2	71.8	63.0	59.6	54.3	49.8	46.0	42.5	37.5
18	101.8	95.1	84.9	76.0	66.7	63.1	57.4	52.7	48.7	45.0	39.7
19	107.3	100.3	89.5	80.1	70.3	66.5	60.6	55.6	51.4	47.4	41.9
20	112.8	105.5	94.1	84.2	74.0	70.0	63.7	58.5	54.0	49.9	44.1
21	118.3	110.7	98.7	88.3	77.6	73.4	66.9	61.4	56.7	52.4	46.3
22	123.8	115.8	103.3	92.5	81.3	76.9	70.0	64.3	59.4	54.9	48.5
23	129.3	120.9	107.9	96.6	84.9	80.3	73.2	67.2	62.1	57.3	50.7
24	134.8	126.1	112.6	100.6	88.6	83.8	76.3	70.1	64.7	59.8	52.9
26	146.0	136.4	121.8	108.8	95.9	90.7	82.6	75.9	70.1	64.7	57.2
28	157.2	146.7	131.0	117.1	103.1	97.6	89.0	81.7	75.4	69.7	61.6
30	168.4	157.1	140.2	125.4	110.4	104.5	95.3	87.5	80.8	74.6	66.0
32	179.6	167.4	149.5	133.6	117.7	111.4	101.6	93.3	86.2	79.6	70.4
34	190.7	177.7	158.7	141.9	125.0	118.3	107.9	99.1	91.5	84.5	74.7
36	201.9	188.0	167.9	150.1	132.3	125.2	114.2	104.9	96.9	89.5	79.1

Table No. 94 (*continued*).

LENGTH, 1 FOOT. Thickness by Holtzapffel's Wire-Gauge (Table No. 13).

Specific Weight = 1.16 (Specific Weight of Wrought Iron = 1).

THICKNESS. W. G.	8	9	10	11	12	13	14	15	16	17	18	19	20
INCH.	.165 11/64 *b*	.148 9/64 *f*	.134 9/64 *b*	.120 1/8 *b*	.109 7/64	.095 3/32 *f*	.083 5/64 *f*	.072 5/64 *b*	.065 1/16 *f*	.058 1/16 *b*	.049 3/64 *f*	.042 3/64 *b*	.035 1/32 *f*
INT. DIAM. inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
6½	13.4	12.0	10.8	9.65	8.75	7.61	6.64	5.75	5.19	4.62	3.90	3.34	2.80
7	14.4	12.9	11.6	10.4	9.42	8.19	7.14	6.19	5.58	4.97	4.20	3.60	3.01
7½	15.4	13.8	12.47	11.1	10.1	8.77	7.65	6.63	5.98	5.33	4.49	3.85	3.23
8	16.4	14.7	13.2	11.8	10.74	9.34	8.15	7.06	6.37	5.68	4.79	4.10	3.43
9	18.4	16.5	14.9	13.3	12.1	10.5	9.16	7.94	7.16	6.38	5.39	4.61	3.86
10	20.4	18.2	16.5	14.8	13.4	11.7	10.2	8.81	7.95	7.08	5.98	5.12	4.28
11	22.4	20.0	18.1	16.2	14.7	12.8	11.2	9.69	8.74	7.79	6.58	5.63	4.70
12	24.4	21.8	19.8	17.7	16.0	14.0	12.2	10.6	9.53	8.49	7.18	6.14	5.13
13	26.4	23.6	21.4	19.1	17.4	15.1	13.2	11.4	10.3	9.20	7.77	6.65	5.55
14	28.4	25.4	23.0	20.6	18.7	16.3	14.2	12.3	11.1	9.90	8.37	7.16	5.98
15	30.4	27.2	24.6	22.1	20.0	17.4	15.2	13.2	11.9	10.6	8.96	7.67	6.40
16	32.4	29.0	26.3	23.5	21.3	18.6	16.2	14.1	12.7	11.3	9.56	8.18	6.82
17	34.4	30.8	27.9	25.0	22.7	19.7	17.2	14.9	13.5	12.1	10.2	8.69	7.27
18	36.4	32.6	29.5	26.4	24.0	20.9	18.2	15.8	14.3	12.7	10.7	9.20	7.69
19	38.4	34.4	31.2	27.9	25.3	22.0	19.2	16.7	15.1	13.4	11.3	9.71	8.12
20	40.4	36.2	32.8	29.3	26.6	23.2	20.2	17.6	15.9	14.1	11.9	10.2	8.54
21	42.4	38.0	34.4	30.8	27.9	24.3	21.3	18.4	16.6	14.8	12.5	10.7	8.96
22	44.4	39.8	36.0	32.3	29.3	25.5	22.3	19.3	17.4	15.5	13.1	11.2	9.39
23	46.4	41.6	37.7	33.7	30.6	26.7	23.3	20.2	18.2	16.2	13.7	11.8	9.81
24	48.5	43.4	39.3	35.2	31.9	27.8	24.3	21.1	19.0	16.9	14.3	12.3	10.2
26	52.5	47.0	42.6	38.1	34.6	30.1	26.3	22.8	20.6	18.4	15.5	13.3	11.1
28	56.5	50.6	45.8	41.0	37.2	32.4	28.3	24.6	22.2	19.8	16.7	14.3	11.9
30	60.5	54.2	49.1	43.9	39.9	34.7	30.3	26.3	23.7	21.2	17.9	15.3	12.8
32	64.5	57.8	52.3	46.8	42.5	37.0	32.3	28.1	25.3	22.6	19.1	16.3	13.6
34	68.5	61.4	55.6	49.8	45.1	39.4	34.4	29.8	26.9	24.0	20.3	17.4	14.5
36	72.5	65.0	58.8	52.7	47.8	41.7	36.4	31.6	28.5	25.4	21.5	18.4	15.3

Table No. 95.—Weight of Brass Tubes, by External Diameter.

Length, 1 Foot. Thickness by Holtzapffel's Wire-Gauge (Table No. 13).
Specific Weight = 1.11 (Specific Weight of Wrought Iron = 1).

Thickness. W. G.	15	16	17	18	19	20	21	22	23	24	25
Inch.	.072 5/64 b	.065 1/16 f.	.058 1/16 b.	.049 3/64 f.	.042 3/64 b.	.035 1/32 f.	.032 1/32	.028 1/32 b.	.025 1.6/64	.022 1.4/64	.020 1.3/64
Diam. inches.	lbs.	lbs.	lb.	lb.	lb.	lb.	lb.	lb.	lb.	lb.	lb.
1/8						.037	.035	.031	.029	.026	.024
3/16			.087	.079	.072	.062	.058	.052	.047	.042	.039
1/4			.130	.115	.102	.088	.081	.072	.065	.058	.053
5/16	.201	.187	.172	.150	.132	.113	.104	.092	.083	.074	.068
3/8	.255	.234	.214	.186	.163	.138	.128	.113	.102	.090	.082
7/16	.306	.281	.256	.221	.193	.164	.151	.133	.120	.106	.097
1/2	.358	.329	.298	.257	.224	.189	.174	.154	.138	.122	.111
9/16	.411	.376	.340	.293	.254	.215	.197	.174	.156	.138	.126
5/8	.463	.423	.382	.328	.285	.240	.221	.194	.174	.154	.141
11/16	.515	.470	.424	.364	.315	.265	.244	.215	.192	.170	.155
3/4	.567	.517	.467	.399	.346	.291	.267	.235	.211	.186	.170
13/16	.620	.564	.509	.435	.376	.316	.290	.255	.229	.202	.184
7/8	.672	.611	.551	.471	.407	.342	.314	.276	.247	.218	.199
15/16	.724	.658	.593	.506	.437	.367	.337	.296	.265	.234	.213
1	.777	.706	.635	.542	.468	.393	.360	.316	.283	.250	.228
1 1/8	.881	.801	.719	.613	.529	.443	.407	.357	.320	.282	.257
1 1/4	.986	.896	.804	.684	.590	.494	.453	.398	.356	.314	.286
1 3/8	1.09	.991	.888	.755	.651	.545	.500	.439	.392	.346	.315
1 1/2	1.20	1.09	.972	.827	.712	.596	.546	.479	.429	.378	.344

W. G.	9	10	11	12	13	14	15	16	17	18	19
Inch.	.148 9/64 f	.134 9/64 b.	.120 1/8 b.	.109 7/64	.095 3/32 f.	.083 5/64 f.	.072 5/64 b.	.065 1/16 f.	.058 1/16 b.	.049 3/64 f.	.042 3/64 b.
Diam. inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1 1/4	1.90	1.74	1.58	1.45	1.28	1.13	.986	.896	.804	.684	.590
1 3/8	2.11	1.93	1.76	1.60	1.41	1.25	.991	.991	.888	.755	.651
1 1/2	2.33	2.13	1.94	1.76	1.55	1.37	1.20	1.09	.972	.827	.712
1 5/8	2.54	2.32	2.12	1.92	1.69	1.49	1.30	1.18	1.06	.898	.773
1 3/4	2.76	2.52	2.30	2.08	1.83	1.61	1.40	1.28	1.14	.969	.834
1 7/8	2.97	2.71	2.47	2.24	1.97	1.73	1.51	1.37	1.23	1.04	.895
2	3.19	2.91	2.65	2.39	2.10	1.85	1.61	1.47	1.31	1.11	.956
2 1/8	3.40	3.10	2.83	2.55	2.24	1.97	1.72	1.56	1.39	1.18	1.02
2 1/4	3.62	3.30	3.01	2.71	2.38	2.09	1.82	1.66	1.48	1.25	1.08
2 3/8	3.83	3.49	3.19	2.86	2.52	2.21	1.93	1.75	1.56	1.33	1.14
2 1/2	4.04	3.69	3.37	3.02	2.66	2.33	2.03	1.85	1.65	1.40	1.20

Table No. 95 (*continued*).

LENGTH, 1 FOOT. Thickness by Holtzapffel's Wire-Gauge (Table No. 13).

Specific Weight = 1.11 (Specific Weight of Wrought Iron = 1).

THICKNESS. W. G.	3	4	5	6	7	8	9	10	11	12	13
INCH.	.259 ¼ *f.*	.238 15/64 *f.*	.220 7/32 *f.*	.203 13/64	.180 3/16 *b.*	.165 11/64 *b.*	.148 9/64 *f.*	.134 9/64 *b.*	.120 ⅛ *b.*	.109 7/64	.095 3/32
DIAM. inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
2	5.24	4.87	4.55	4.24	3.80	3.52	3.19	2.91	2.65	2.39	2.10
2⅛	5.62	5.22	4.87	4.54	4.07	3.76	3.40	3.10	2.83	2.55	2.24
2¼	5.99	5.57	5.19	4.83	4.33	4.00	3.62	3.30	3.01	2.71	2.38
2⅜	6.37	5.91	5.51	5.13	4.59	4.24	3.83	3.49	3.19	2.86	2.52
2½	6.75	6.26	5.83	5.42	4.85	4.48	4.04	3.69	3.37	3.02	2.66
2⅝	7.12	6.60	6.14	5.72	5.12	4.72	4.26	3.88	3.55	3.18	2.79
2¾	7.50	6.95	6.47	6.01	5.38	4.96	4.47	4.07	3.73	3.34	2.93
2⅞	7.88	7.30	6.79	6.31	5.64	5.20	4.69	4.27	3.91	3.50	3.07
3	8.25	7.64	7.11	6.60	5.90	5.44	4.90	4.46	4.09	3.66	3.21
3¼	9.01	8.33	7.75	7.19	6.43	5.92	5.49	4.85	4.43	3.98	3.48
3½	9.76	9.02	8.39	7.78	6.95	6.40	6.07	5.24	4.78	4.30	3.76
3¾	10.5	9.72	9.03	8.37	7.47	6.88	6.65	5.63	5.12	4.61	4.04
4	11.3	10.4	9.67	8.96	8.00	7.36	7.24	6.02	5.46	4.93	4.31
4¼	12.0	11.1	10.3	9.55	8.52	7.83	7.82	6.41	5.80	5.25	4.59
4½	12.8	11.8	10.9	10.1	9.04	8.31	8.41	6.80	6.15	5.56	4.87
4¾	13.5	12.5	11.6	10.7	9.56	8.79	8.99	7.19	6.49	5.88	5.14
5	14.3	13.2	12.2	11.3	10.1	9.27	9.57	7.58	6.83	6.20	5.42
5¼	15.0	13.9	12.9	11.9	10.6	9.75	10.2	7.97	7.17	6.51	5.69
5½	15.8	14.6	13.5	12.5	11.1	10.2	10.7	8.36	7.52	6.83	5.97
5¾	16.5	15.3	14.1	13.1	11.7	10.7	11.3	8.75	7.86	7.15	6.25
6	17.3	15.9	14.8	13.7	12.2	11.2	11.9	9.14	8.20	7.46	6.52

Table No. 96.—Weight of One Square Foot of Sheet Brass.

Thickness by Holtzapffell's Wire-Gauge (Table No. 13).

Thickness.		Weight of 1 Square Foot.	Thickness.		Weight of 1 Square Foot.	Thickness.		Weight of 1 Square Foot.
No. W.G.	inch.	pounds.	No. W.G.	inch.	pounds.	No. W.G.	inch.	pounds.
3	.259	10.9	11	.120	5.05	19	.042	1.77
4	.238	10.0	12	.109	4.59	20	.035	1.47
5	.220	9.26	13	.095	4.00	21	.032	1.35
6	.203	8.55	14	.083	3.49	22	.028	1.18
7	.180	7.58	15	.072	3.03	23	.025	1.05
8	.165	6.95	16	.065	2.74	24	.022	.926
9	.148	6.23	17	.058	2.44	25	.020	.842
10	.134	5.64	18	.049	2.06			

Table 97.—Size and Weight of Tin Plates.

Mark.	Size of Sheets.	Number of Sheets in a Box.	Weight per Box.
	inches. inches.	sheets.	pounds.
I C	14 × 10	225	112
IX	" "	"	140
IXX	" "	"	161
IXXX	" "	"	182
IXXXX	" "	"	203
S D C	15 × 11	200	168
S D X	" "	"	189
S D XX	" "	"	210
S D XXX	" "	"	231
S D XXXX	" "	"	252
D C	17 × 12½	100	98
D X	" "	"	126
D XX	" "	"	147
D XXX	" "	"	168
D XXXX	" "	"	169

Table No. 98.—Weight of Tin Pipes.

As manufactured.

1 FOOT IN LENGTH.

Diameter Externally.	Thickness.		Diameter Externally.	Thickness.
	3/32″ inch.	⅛ inch.		¼ inch.
inches.	lbs.	lbs.	inches.	lbs.
¼	.148	—	2¼	5.04
½	.384	.472	2½	5.67
¾	.620	.787	2¾	6.30
1	.856	1.103	3	6.93
1¼	1.095	1.417	3¼	7.56
1½	1.328	1.732	3½	8.19
1¾	1.564	2.047		
2	1.802	2.362		

Table No. 99.—Weight of Lead Pipes.

As manufactured.

Bore.	Length.	Weight and Thickness of Pipe.							
		Weight.	Calculated Thickness.	Weight.	Calculated Thickness.	Weight.	Calculated Thickness.	Weight.	Calculated Thickness.
inches.	feet.	lbs.	inch.	lbs.	inch.	lbs.	inch.	lbs.	inch.
½	15	14	.097	16	.112	18	.124	22	.146
⅝	„	—		17	.101	21	.121	30	.140
¾	„	24	.112	28	.147	32	.181	36	.215
1	„	36	.136	42	.156	56	.200	64	.225
1¼	12	36	.139	42	.160	48	.180	52	.193
1½	„	48	.156	56	.179	72	.224	84	.257
1¾	„	—		72	.199	84	.228	96	.256
2	„	72	.178	84	.204	96	.231	112	.266
2½	10	—		84	.200	96	.227	112	.261
3	„	—		—		112	.218	140	.275
3½	„	—		—		130	.225	160	.273
4	„	—		—		170	.257	220	.327
4½	„	—		—		170	.232	220	.295

4¼	5/16 inch thick. Weight per lineal foot.................. 22.04 lbs.
4½	„ „ .... 23.25 „
4¾	„ „ .... 24.45 „
5	„ „ .... 25.66 „

Table No. 100.—English Zinc Gauge. (*London Zinc Mills.*)

Gauge No.	Approximate Weight per Sq. Foot.	1,000ths of an Inch.	7 Ft. × 2 Ft. 8 Ins. Approximate Weight per Sheet.	7 Ft. × 2 Ft. 8 Ins. Approximate Number of Sheets in 10 Cwts.	7 Feet × 3 Feet. Approximate Weight per Sheet.	7 Feet × 3 Feet. Approximate Number of Sheets in 10 Cwts.	8 Feet × 3 Feet. Approximate Weight per Sheet.	8 Feet × 3 Feet. Approximate Number of Sheets in 10 Cwts.	Gauge No.	Nearest Birmingham Wire Gauge.
	ozs.		lbs. ozs.		lbs. ozs.		lbs. ozs.			
1	2¼	.004	2 10	427	—	—	—	—	1	41
2	3¼	.006	3 13	294	—	—	—	—	2	38
3	3¾	.007	—	—	4 15	227	—	—	3	37
4	4¾	.008	—	—	6 4	180	—	—	4	34
5	5¾	.010	—	—	7 9	148	—	—	5	31
6	6¾	.011	7 14	142	8 14	126	10 2	111	6	30
7	7¾	.013	9 1	124	10 3	110	11 10	96	7	29
8	9	.015	10 8	107	11 13	95	13 8	83	8	28
9	10	.017	11 11	96	13 2	85	15 0	75	9	27
10	11½	.019	13 7	83	15 2	74	17 4	65	10	25
11	13	.021	15 3	74	17 1	66	19 8	57	11	24
12	15	.025	17 8	64	19 11	57	22 8	50	12	23
13	17	.028	—	—	22 5	50	25 8	44	13	22
14	19	.031	—	—	24 15	45	28 8	39	14	21
15	22	.036	—	—	28 14	39	33 0	34	15	20
16	25	.041	—	—	32 13	34	37 8	30	16	19
17	28	.046	—	—	36 12	30	42 0	27	17	18
18	31	.051	—	—	40 11	28	46 8	24	18	—
19	35	.059	—	—	45 15	24	52 8	21	19	17
20	39	.065	—	—	51 3	22	58 8	19	20	16
21	43	.072	—	—	56 7	20	64 8	17	21	15

Sheets thicker than above are rolled to Birmingham Wire Gauge.

Table No. 100a.—"V M" Zinc Gauge. (*Vieille-Montagne*).

Gauge.	Approximate Thickness. Thousandths of an Inch.	Approximate Thickness. Metric Equivalent in Thousandths of a Millimètre.	Approximate Weight per Square Foot. Lbs.	Ozs.	Drms.	36 Ins. × 72 Ins. Approximate Weight of Sheets. Lbs.	Ozs.	Drms.	Sheets per 500 Kilos or 1102⅜ lbs. English about	36 Ins. × 84 Ins. Approximate Weight of Sheets. Lbs.	Ozs.	Drms.	Sheets per 500 Kilos or 1102⅜ lbs. English about	36 Ins. × 96 Ins. Approximate Weight of Sheets. Lbs.	Ozs.	Drs.	Sheets per 500 Kilos or 1102⅜ lbs. English about
1	0.004	0.100	—	2	5	Nos. 1 and 2 are only rolled to order and special dimensions.											
2	.006	.141	—	3	4												
3	.007	.171	—	3	15	4	6	14	249	5	2	11	213	5	14	8	187
4	.008	.209	—	4	13	5	6	10	204	6	5	1	175	7	3	8	153
5	.010	.247	—	5	11	6	6	6	172	7	7	7	147	8	8	8	129
6	.011	.291	—	6	11	7	8	6	146	8	12	7	126	10	—	8	110
7	.013	.337	—	7	12	8	11	8	126	10	2	12	108	11	10	—	95
8	.015	.386	—	8	14	9	15	12	110	11	10	6	95	13	5	—	83
9	.018	.450	—	10	5	11	9	10	95	13	8	9	81	15	7	8	71
10	.020	.500	—	11	7	12	13	14	86	15	—	3	73	17	2	8	64
11	.023	.580	—	13	5	14	15	10	74	17	7	9	63	19	15	8	55
12	.026	.660	—	15	2	17	0	4	65	19	13	10	56	22	11	—	49
13	.029	.740	1	—	15	19	0	14	57	22	3	11	50	25	6	8	43
14	.032	.820	1	2	12	21	1	8	52	24	9	12	45	28	2	—	39
15	.038	.950	1	5	12	24	7	8	45	28	8	12	39	32	10	—	34
16	.043	1.080	1	8	12	27	13	8	39	32	7	12	34	37	2	—	30
17	.048	1.210	1	11	11	31	2	6	35	36	5	7	30	41	8	8	27
18	.053	1.340	1	14	11	34	8	6	31	40	4	7	27	46	—	8	24
19	.058	1.470	2	1	11	37	14	6	29	44	3	7	25	50	8	8	22
20	.063	1.600	2	4	10	41	3	4	27	48	1	2	23	54	15	—	20
21	.070	1.780	2	8	12	45	13	8	24	53	7	12	21	61	2	—	18
22	.077	1.960	2	12	14	50	7	2	22	58	14	6	19	67	5	—	16
23	.084	2.140	3	1	1	55	3	2	20	64	6	5	17	73	9	8	15
24	.091	2.320	3	5	3	59	13	16	18	69	12	15	16	79	12	8	14
25	.098	2.500	3	9	5	64	7	10	17	75	3	9	15	85	15	8	13
26	.105	2.680	3	13	7	69	1	14	16	80	10	3	14	92	2	8	12

FUNDAMENTAL MECHANICAL PRINCIPLES.

FORCES IN EQUILIBRIUM.

Solid Bodies.

Parallelogram of Forces.—When a body remains at rest whilst being acted on by two or more forces, it is said to be in a state of equilibrium, and so also are the forces. Thus, if the forces P*p*, Q*q*, R*r*, Fig. 86, acting on the body *p q r*, keep it at rest, they are in equilibrium, and any two of them balance the third. The lines of force, if produced, meet at one point O within the body, and if a parallelogram be constructed having two adjacent sides proportional to and parallel to two of the forces respectively, to represent them in magnitude and direction, the diagonal of the parallelogram will represent the third force in magnitude and direction. Let the lines O P, O Q, Fig. 87, represent the forces P*p*, Q*q* in magnitude and direction, and complete the parallelogram by drawing the parallels P R, Q R, and draw O R. Then O R represents in magnitude and direction the resultant of the two forces; and R O taken in the opposite direction represents the third force R*r*, Fig. 86. If it be applied in this direction to the point O, as indicated by a dot line O R′, it would balance the other two. This construction is called the *parallelogram of forces*, and is applicable to any three forces in equilibrium.

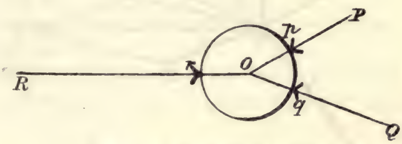


Fig. 86.—Equilibrium of Forces.

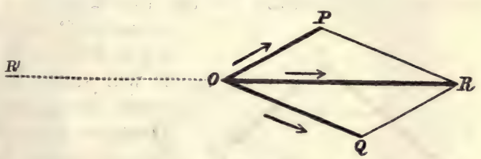


Fig. 87.—Parallelogram of Forces.

Three forces in equilibrium may also be represented by a triangle, or half a parallelogram. For example, the triangle O P R represents by its three sides the forces O P, O Q, O R, the side P R being substituted for O Q.

Three forces in equilibrium must be in the same plane.

When the directions of three forces holding a body at rest, and also the magnitude of one of them, are known, the magnitudes of the other two can be determined by constructing a parallelogram in the manner above exemplified, and measuring the lengths of the sides and the diagonal.

Polygon of Forces.—Equilibrium may subsist between more than three forces, which need not necessarily be in the same plane; and they can, like those already illustrated, be developed in direction and magnitude by diagram. Thus, let the point O, Fig. 88, representing a solid body, be kept at rest by a number of forces, O P, O Q, O R, O S, O T. Find the equivalent diagonal O *p* for the first two forces; then construct the parallelogram and diagonal O *r* for the resultant of O *p* and the third force O R; and again the parallelogram and diagonal O *s* for the resultant of O *r* and the fourth force O S. The last resultant O *s* represents in one the four distributed forces O P, O Q, O R, O S, and it balances the fifth force O T equal and opposite to it. As O *s* and O T are in the same straight line, their resultant is of course nothing.

The several forces thus dealt with may be combined into a *polygon of forces.* Draw O P, Fig. 89, parallel and equal to O P, Fig. 88, P Q parallel and equal to O Q, Q R parallel to O R, R S parallel to O S; then, finally, S O, completing the polygon, will be parallel and equal to O T, Fig. 88, the last of the series. Professor Mosely illustrates the polygon of forces by the united action of a number of bell-ringers, pulling by a number of ropes joined to a single rope. The polygon constructed as in Fig. 90, shows successively by corresponding letters, the individual contributions of the bell-ringers, combined into one vertical force.

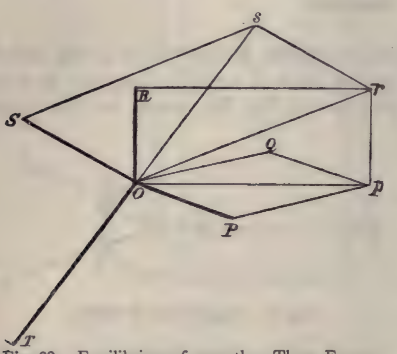


Fig. 88.—Equilibrium of more than Three Forces.

Again, equilibrium may be established between a number of forces acting in the same plane, applied to different points in a body, or system of bodies. For example, let the forces P O, Q O, R O, S O, and T O, be applied to several points, O, O, O, O, O, on a flat board A B C, Fig. 91, by means of cords and weights; it will settle into a position of equilibrium, when the opposing forces arrive at a balance between themselves. An axis or pivot may be established at any point, M, on the surface of the board, without disturbing the equilibrium, and it may be viewed as a centre of motion round which the forces tend to turn the board, some in one direction, the others the opposite way, balancing each other. The effect of each force to turn the body about the centre is represented by the product of its magnitude by the *leverage,* or perpendicular distance of its direction from the centre; draw these perpendiculars, and multiply each force by its perpendicular or leverage, then the resulting products will be divisible into two sets, tending to turn the board in opposite directions. The sum of the first set of products is equal to the sum of the second set, as is proved by the fact of equilibrium.

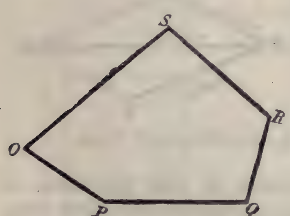


Fig. 89.—Polygon of Forces.

Moments of Forces.—The product of a force by the perpendicular distance of its direction from any given point, is called the *moment* of the

force about that point; and the equilibrium above discussed, in connection with Fig. 91, is the result of the *equality of moments.*

The law of the equality of moments may be thus set forth:—If any number of forces acting anywhere in the same plane, on the same body or connected system of bodies, be in equilibrium, then the sum of the forces tending to turn the system in one direction about any point in that plane, is equal to the sum of the moments of those forces tending to turn the system in the other direction.

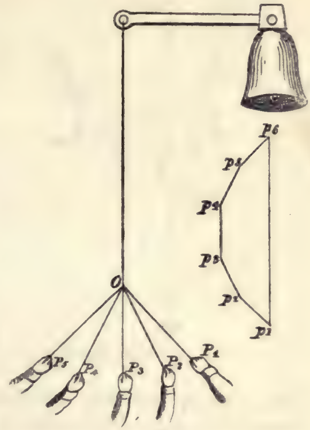


Fig. 90.—Bell-ringers. Polygon of Forces.

Such balanced forces may be transferred to a single point, and placed about it, as in Fig. 88, parallel to their directions as they stand; and they will continue in equilibrium, holding the point at rest. A polygon of the forces *p q r s t* within Fig. 90, may be constructed similarly to Fig. 89.

Though the principle of the polygon of forces be sufficient to test the equilibrium of a system of forces acting at one point, yet the principle of the equality of moments, in addition, is necessary to establish the equilibrium of a system applied to different points. The two principles conjointly are necessary, and they are sufficient, as conditions of equilibrium.

The Catenary.—When a chain, or a rope, or a flexible series of rods, is suspended by its extremities, supporting weights distributed along its length, in a state of rest, it forms a polygon of forces in equilibrium, as in Fig. 92. If all the forces except those which act on the extremities of the chain, be combined into a resultant, then the two extreme sides being produced, will meet the direction of the resultant at one point. Thus, in the polygon, Fig. 92, loaded with weights, W, W, &c., the vertical resultant of these weights W′ W″, passing through their common centre of gravity, intersects at W″ the two extreme sections P A, P′ B, when produced downwards.

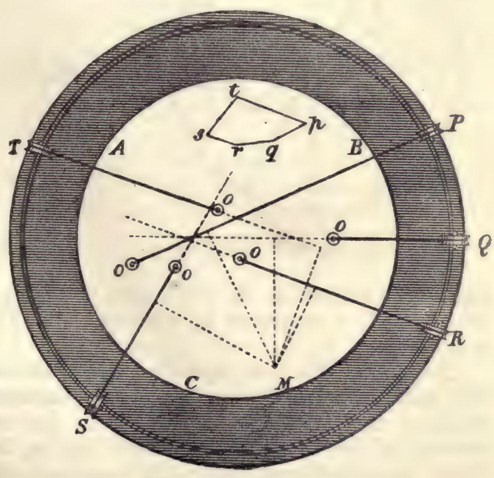


Fig. 91.—Equality of Moments.

Similarly, in the catenary, Fig. 93, which is the curve assumed by a rope or other flexible medium uniformly loaded and suspended by the two

extremities, if tangents be drawn to the extremities A, B, of the curve, meeting at W″, they represent the directions of the forces sustaining the curve at

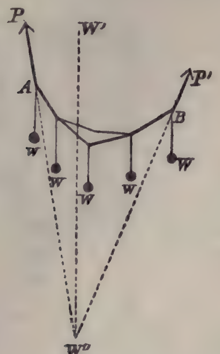


Fig. 92.—The Catenary.

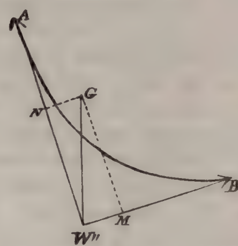


Fig. 93.—The Catenary.

those points, and they intersect at the same point W″, the vertical line G W″ passing through the centre of gravity of the curve. Let the weight of the

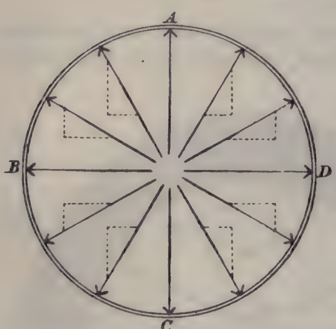


Fig. 94.—Centrifugal Forces in Equilibrium.

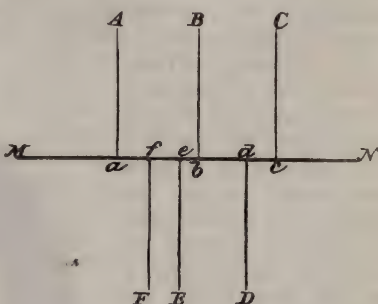


Fig. 95.—Parallel Forces in Equilibrium.

curve be represented by G W″, and complete the parallelogram M N, then W″ M and W″ N represent in force and direction the tension at the points B and A.

Centrifugal Forces in Equilibrium.—When a cylindrical vessel is exposed to a uniform internal pressure, as the pressure of steam within a boiler, for example, the pressure is balanced by the resistance of cohesion of the material of the boiler. Let A B C D, Fig. 94, be the section of a cylindrical boiler, the radial pressure of the steam may be represented by the arrows, which are equal and opposite in direction. The tension on the metal in resisting the internal pressure at any particular section B D, is equal to the sum of the pressures resolved into the direction at right angles to B D, or parallel to A C, according to the triangles, or half-parallelograms of force attached to each oblique arrow. The total vertical pressure thus obtained by the resolution of forces is equal to the total vertical pressure which

would be exerted on the sectional line B D if it be supposed to be a rigid diaphragm across the boiler, which is easily calculated. If the radial pressure be, for example, 100 lbs. on each square inch of surface, then the total pressure, or the tension on the two sides at B and D, would be $100 \times BD$ on each inch of length of the two sides; that is to say, if the diameter B D be equal to 60 inches, the tension on the two sides would be $60 \times 100 = 6000$ pounds for each inch of length.

A similar argument applies to the tension on the rim of a revolving fly-wheel.

Parallel Forces.—Systems of parallel forces are particular cases of the foregoing.—Let A, B, C, D, E, F, Fig. 95, be a system of parallel forces in equilibrium; and M N a line perpendicular to them in the same plane, and cut by them at the points *a, b, c, d, e, f.* They may act at any points in their lines of direction without disturbing the equilibrium, and they may be supposed to be applied at those points in the line M N. Then, the sum of the moments of the three forces A, B, C, acting in one direction, with respect to any point M as a centre, is equal to the sum of the moments of the forces D, E, F, opposed to them. Further, the sum of the simple forces A, B, C, irrespective of their moments, is equal to that of the forces D, E, F, so that the fact of their being in equilibrium resolves itself into a case of action and reaction, for the two equivalent forces representing the two opposing sums, act in the same straight line in opposite directions.

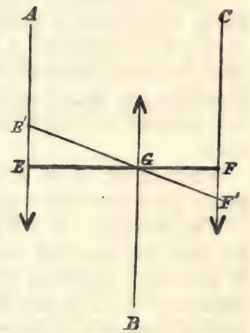


Fig. 96.—Three Parallel Forces in Equilibrium.

When three parallel forces balance each other, acting on a straight line, two of them must be opposed to the third; and the third must act between the other two, being equal and opposite to their resultant. Let A, B, C, Fig. 96, be three such forces applied to the line E G F, at the points E, G, F respectively; then, with respect to the point G, the moment of the force B is nothing, because it passes through that point and has no leverage on it. There remain the moments of the extreme forces, A and C, which are equal to each other, that is to say

$$A \times EG = C \times FG.$$

From this it follows, by proportion, that

$$A : C :: FG : EG,$$

and that the extreme forces are to each other inversely as their distances from the middle force.

In general, of three parallel forces acting in equilibrium on an inflexible line, the first in order is to the third as the distance of the third from the second, is to that of the first from the second.

The sum of the first and third is equal to the second; and when the distances or leverages are equal, the first and third forces are equal to each other.

If the position of the line E F be inclined to the direction of the three forces, and changed to E′ F′, Fig. 96, the forces A, B, C, continue in equilibrium;

for the perpendicular lines G E and G F continue, as before, to be the leverages of the extreme forces A and C, on the central point G.

When three forces not in the same plane act on one point, there cannot be equilibrium between them. Two of these may be reduced to their resultant by parallelogram, and this resultant reduced with the third force to a final resultant. For example, let the lines O P, O Q, O R, Fig. 97, represent in magnitude and direction three forces not in one plane acting on the point O. By parallelogram, O S is the resultant of the two forces O P, O Q, and O T is the final resultant of O S and the third force O R. That is to say, O T is the resultant of the three given forces.

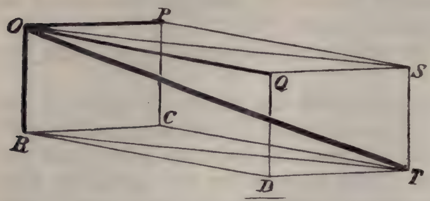


Fig. 97.—Parallelopiped of Forces.

If parallelograms be formed from each two of the three forces, they form, when duplicated, a parallelopiped of forces, of which the diagonal is found by the final resultant O T, and the principle of the parallelopiped of forces may be thus defined:—If three forces be represented in magnitude and direction by three adjacent edges of a parallelopiped, their resultant is represented in magnitude and direction by the adjacent diagonal of the solid.

There must be at least four forces to produce equilibrium about a point, when the forces are not in the same plane.

The triangle O S T, Fig. 97, comprises in its three sides the resultant of the first and second forces, the third force, and the resultant of the three. If the first resultant O S be replaced by the two lines O Q and Q S, which represent the first and second forces, they form the four-sided figure O Q S T, the polygon of the four equilibrating forces.

A greater number of forces than four acting on a point may be reduced in like manner.

Fluid Bodies.

The characteristic property of fluids is the capability of transmitting the pressure which is exerted upon a part of the surface of the fluid, in all directions, and of the same intensity:—the same pressure per square inch or per square foot.

The pressure of water in a vessel, caused by its own gravity, increases in proportion to the depth below the surface; and the pressure on a horizontal surface, say, the bottom, is equivalent to the weight of the superincumbent column of water, and the intensity of the pressure is independent of the form of the vessel. The same rule applies when the pressure is from below upwards.

The same rule also applies when the surface is either vertical or inclined, and the mean height of the superincumbent column of water is measured by the depth of the centre of gravity of the given surface below the surface of the water.

The water in open tubes communicating with each other, when in a state of equilibrium, stands at the same level in the tubes, whatever may be the relative diameters of the tubes.

The height of the superincumbent column of water is called the *head* of water.

The *buoyancy*, or the upward force with which water presses a body immersed in it, from below upwards, is equal to the weight of water displaced by the body, or of a quantity of water equal in volume to the submerged body, or submerged portion of a body. The resultant of the upward pressure passes through the centre of gravity of the water displaced; and also, when the floating body is at rest, through the centre of gravity of the body.

This resultant line is called the *axis of floatation*, and the horizontal section of the body at the surface of the water is the *plane of floatation.*

Bodies float either in an upright position or in an oblique position. A body floats with *stability*, when it strives to maintain the position of equilibrium, and when it can only be moved out of this position by force, and will return to it when the force is withdrawn. The *metacentre* is the point at which the axis of floatation intersects the axis of a symmetrical body, as a ship, when inclined. If the metacentre lies above the centre of gravity of the ship, the ship floats with stability; if below, the ship is unstable; if the centres coincide, which they must do in a cylinder or a sphere, for example, the body floats indifferently in any position.

For the weight, volume, and pressure of water and air, see *Water and Air as standards of measure*, page 124.

MOTION.

The motion of a body is *uniform*, when the body passes through equal spaces in equal intervals of time.

Velocity is the measure of motion, and is expressed by the number of feet or other unit of length moved through per second or other unit of time.

Motion is *accelerated*, when the body moves through continually increased spaces in equal intervals of time, like a railway train starting from a station. Motion is *retarded*, when the body moves through continually decreased spaces in equal intervals of time, like a railway train arriving at a station. The *acceleration* and *retardation* are *uniform*, when the spaces moved through increase or decrease by equal successive amounts, like a body falling by the action of gravity, or, on the contrary, projected upwards in opposition to gravity.

GRAVITY.

When bodies fall freely near the surface of the earth, the motion, as already said, is *uniformly accelerated;* equal additions of velocity being made to the motion of the body in equal intervals of time.

During the 1st second of time, the body, starting from a state of rest, falls through 16.095 feet, or, say 16.1 feet; during the 2d second, it falls through three times 16.1 feet; during the 3d second, it falls through five times 16.1 feet, and so on. The body having, in the 1st second, fallen through 16.1 feet, from a state of rest, with a motion uniformly accelerated, it would move through 32.2 feet in the next second, with the velocity thus acquired, without any additional stimulus from gravity; that is to say, the velocity acquired at the end of the 1st second is 32.2 feet per second. During the 2d second, it in fact acquires an additional velocity of 32.2 feet per second, making up, at the end of this second, a final velocity of 64.4

feet per second. In like manner the body acquires an additional velocity of 32.2 feet per second during the 3d second, making a final velocity of three times 32.2 feet, or 96.6 feet per second. And so on.

Each of these additional velocities is acquired in falling through 16.1 feet additional to the space fallen through in virtue of the velocity acquired at the beginning of each second.

The relations of *height fallen*, *velocity acquired*, and *time of fall*, are simply exhibited in the following manner:—

During the successive seconds the heights fallen through are consecutively as follow:—

time,	1,	1,	1,	1 second.
height of fall,	16.1,	16.1 × 3,	16.1 × 5,	16.1 × 7 feet.

And reckoning the totals from the commencement of the fall,

total times,	1,	2,	3,	4 seconds.
total height of fall,	16.1,	16.1 × 4,	16.1 × 9,	16.1 × 16 feet.
	or 16.1,	16.1×2^2,	16.1×3^2,	16.1×4^2 feet.
	or 16.1,	64.4,	144.9,	257.6 feet.

Showing that the *total height fallen* is as the square of the total time.

Again, during the successive seconds, the successive additional velocities acquired are:—

time,	1,	1,	1,	1 second.
velocities acquired,	32.2,	32.2,	32.2,	32.2 feet per second.

And the total or final velocities acquired, reckoning from the commencement of the fall, are:—

total times,	1,	2,	3,	4 seconds.
final velocities,	32.2,	32.2 × 2,	32.2 × 3,	32.2 × 4 feet per second.
	or 32.2,	64.4,	96.6,	128.8 feet per second.

Showing that the *velocity acquired* is in direct proportion to the time of the fall.

The above relations of time, height, and velocity are brought together for comparison, thus:—

time as,	1,	2,	3,	4,	&c.
velocity acquired as,	1,	2,	3,	4,	&c.
height of fall as,	1,	4,	9,	16,	&c.
or as	1,	2^2,	3^2,	4^2,	&c.

Showing that, whilst the *velocity* increases simply with the time, the *height of fall* increases as the square of the time, and as the square of the velocity.

The force of gravity is expressed by the velocity communicated by gravity to a body falling freely in a second, namely, 32.2 feet per second, and is symbolized by *g*.

The foregoing relations of *time*, *velocity*, and *height* of fall, are comprised in the six following propositions with their answers—applicable to the condition of a body falling freely. They are much used in mechanical calculations.

1 and 2. Given the *time*, to find the *velocity* and the *height*.
3 and 4. Given the *velocity*, to find the *time* and the *height*.
5 and 6. Given the *height*, to find the *time* and the *velocity*.

Rules for the Action of Gravity.

Putting t = the time of falling in seconds, v = the velocity in feet per second, h = the height of fall in feet, and g = gravity or 32.2; then,

Rule 1. Given the *time* of fall, to find the *velocity* acquired by a falling body. Multiply the time in seconds by 32.2, the product is the final velocity in feet per second. Or

$$v = 32.2\,t \qquad (1)$$

Rule 2. Given the *time* of fall, to find the *height* of the fall. Multiply the square of the time in seconds by 16.1. The product is the height of fall in feet. Or

$$h = 16.1\,t^2 \qquad (2)$$

Rule 3. Given the *velocity*, to find the *time* of falling. Divide the velocity in feet per second by 32.2. The quotient is the time in seconds. Or

$$t = \frac{v}{32.2} \qquad (3)$$

Rule 4. Given the *velocity*, to find the *height* of fall "due" to the velocity. Square the velocity in feet per second, and divide by 64.4. The quotient is the height of fall in feet. Or

$$h = \frac{v^2}{64.4} \qquad (4)$$

Rule 5. Given the *height* of fall, to find the *time* of falling. Divide the height in feet by 16.1, and find the square root of the quotient. The root is the time in seconds. Or

$$t = \sqrt{\frac{h}{16.1}}$$

$$\text{or } t = \tfrac{1}{4}\sqrt{h} \qquad (5)$$

Rule 6. Given the *height* of fall, to find the *velocity* due to the height. Multiply the height in feet by 64.4, and find the square root of the product. The root is the velocity in feet per second.

Or, multiply the square root of the height in feet by 8.025; the product is the velocity in feet per second.

Note.—It is usual to take the integer 8 only for the multiplier.

Symbolically, these operations are expressed as follows:—

$$v = 32.2\sqrt{\frac{2\,h}{32.2}} = \sqrt{64.4\,h} = 8.025\sqrt{h}$$

$$\text{or in a round number } v = 8\sqrt{h} \qquad (6)$$

The above rules are applicable, inversely, to the motion of bodies projected upwards and uniformly retarded by gravity. The height to which a body projected vertically upwards by an initial impulse, will ascend, is equal to the height through which the body must fall in order to acquire the initial velocity, and the same rule (4) applies in these two cases.

The following table, No. 101, gives the velocity acquired by a falling body in falling freely through a given height. Table No. 102 gives, conversely, the height of fall due to a given velocity. Table No. 103 gives the fall and the final velocity due to a given time of falling freely.

Table No. 101.—Velocity acquired by Falling Bodies, due to Given Heights of Fall.

$$v = 8.025 \sqrt{h}.$$

Height of Fall.	Velocity in Feet per Second.	Height of Fall.	Velocity in Feet per Second.	Height of Fall.	Velocity in Feet per Second.	Height of Fall.	Velocity in Feet per Second.
feet.	ft. per sec.	feet.	ft. per sec.	feet.	ft. per sec.	feet.	ft. per sec.
.01	.803	3.0	13.90	23	38.49	50	56.74
.02	1.14	3.5	15.01	24	39.31	100	80.25
.03	1.39	4.0	16.05	25	40.12	150	98.28
.04	1.61	4.5	17.03	26	40.92	200	113.5
.05	1.80	5.0	17.99	27	41.70	300	139.0
.06	1.97	5.5	18.82	28	42.47	400	160.5
.07	2.12	6.0	19.66	29	43.22	500	179.9
.08	2.27	6.5	20.46	30	43.95	600	196.6
.09	2.41	7.0	21.23	31	44.68	700	212.3
.1	2.54	7.5	21.97	32	45.39	800	226.9
.2	3.20	8.0	22.69	33	46.10	900	240.7
.3	4.40	8.5	23.40	34	46.79	1000	253.8
.4	5.07	9.0	24.07	35	47.47	1500	310.8
.5	5.68	9.5	24.73	36	48.15	2000	358.9
.6	6.22	10	25.38	37	48.81	2500	401.2
.7	6.71	11	26.62	38	49.47	3000	439.5
.8	7.18	12	27.80	39	50.11	3500	474.7
.9	7.61	13	28.93	40	50.75	4000	507.5
1.0	8.03	14	30.03	41	51.38	4500	538.3
1.2	8.79	15	31.08	42	52.01	5000	567.4
1.4	9.50	16	32.10	43	52.62	6000	621.6
1.6	10.15	17	33.09	44	53.23	7000	671.4
1.8	10.77	18	34.05	45	53.83	8000	717.8
2.0	11.35	19	34.98	46	54.43	9000	761.3
2.25	12.04	20	35.89	47	55.02	10000	802.5
2.50	12.69	21	36.77	48	55.60		
2.75	13.31	22	37.64	49	56.17		

Table No. 102.—Height of Fall due to Given Velocities.

$$h = \frac{v^2}{64.4}.$$

Velocity in Feet per Second.	Height of Fall.	Velocity in Feet per Second.	Height of Fall.	Velocity in Feet per Second.	Height of Fall.	Velocity in Feet per Second.	Height of Fall.
ft. per sec.	feet.	ft. per sec.	feet.	ft. per sec.	feet.	ft. per sec.	feet.
.25	.0010	19	5.61	46	32.9	73	82.7
.50	.0039	20	6.21	47	34.3	74	85.0
.75	.0087	21	6.85	48	35.8	75	87.4
1.00	.016	22	7.52	49	37.3	80	99.4
1.25	.024	23	8.21	50	38.8	85	112.2
1.50	.035	24	8.94	51	40.4	90	125.8
1.75	.048	25	9.71	52	42.0	95	140.1
2	.062	26	10.5	53	43.6	100	155.3
2.5	.097	27	11.3	54	45.3	105	171.2
3	.140	28	11.2	55	47.0	110	187.9
3.5	.190	29	13.1	56	48.7	115	205.4
4	.248	30	14.0	57	50.4	120	223.6
4.5	.314	31	14.9	58	52.2	130	262.4
5	.388	32	15.9	59	54.1	140	304.3
6	.559	33	16.9	60	55.9	150	349.4
7	.761	34	17.9	61	57.8	175	475.5
8	.994	35	19.0	62	59.7	200	621
9	1.26	36	20.1	63	61.6	300	1397
10	1.55	37	21.3	64	63.6	400	2484
11	1.88	38	22.4	65	65.6	500	3882
12	2.24	39	23.6	66	67.6	600	5590
13	2.62	40	24.9	67	69.7	700	7609
14	3.04	41	26.1	68	71.8	800	9938
15	3.49	42	27.4	69	73.9	900	12578
16	3.98	43	28.7	70	76.1	1000	15528
17	4.49	44	30.1	71	78.3		
18	5.03	45	31.4	72	80.5		

Table No. 103.—HEIGHT OF FALL AND VELOCITY ACQUIRED, FOR GIVEN TIME OF FALL.

$$h = 16.1\, t^2. \quad v = 32.2\, t.$$

Time of Fall.	Height of Fall.	Velocity acquired in Feet per Second.	Time of Fall.	Height of Fall.	Velocity acquired in Feet per Second.	Time of Fall.	Height of Fall.	Velocity acquired in Feet per Second.
seconds.	feet.	ft. per sec.	seconds.	feet.	ft. per sec.	seconds.	feet.	ft. per sec.
1	16.1	32.2	12	2318	386.4	23	8517	740.6
2	64.4	64.4	13	2721	418.6	24	9273	772.8
3	144.9	96.6	14	3156	450.8	25	10062	805.0
4	257.6	128.8	15	3623	483.0	26	10884	837.2
5	402.5	161.0	16	4122	515.2	27	11737	869.4
6	579.6	193.2	17	4653	547.4	28	12622	901.6
7	788.9	225.4	18	5217	579.6	29	13540	933.8
8	1030	257.6	19	5812	611.8	30	14490	966.0
9	1304	289.8	20	6440	644.0	31	15473	998.2
10	1610	322.0	21	7100	676.2	32	16487	1030
11	1948	354.2	22	7792	708.4			

ACCELERATED AND RETARDED MOTION IN GENERAL.

The same rules and formulas that have been applied to the action of gravity are applicable to the action of any other uniformly accelerating force on a body, the numerical constants being adapted to the force. If an accelerating or retarding force be greater or less than gravity; that is to say, than the weight of the body, the constants 16.1, 32.2, and 64.4 are to be varied in the same proportion.

To do this, multiply the constant by the accelerating force, and divide the product by the weight of the body. Let f be the accelerating force, and w the weight of the body, then the constant becomes

$$\frac{16.1\, f}{w}, \text{ or } \frac{32.2\, f}{w}, \text{ or } \frac{64.4\, f}{w}; \ldots\ldots\ldots\ldots\ldots (a)$$

and substituting this in the formulas (1) to (6) for gravity, the following general rules and formulas are arrived at for the action of uniformly accelerating or retarding forces. The rules are written for accelerating forces, but they apply by simple inversion to retarding forces also.

GENERAL RULES FOR ACCELERATING FORCES.

The accelerating force and the weight of the body are expressed in the same unit of weight; and the space in feet, the time in seconds, and the velocity in feet per second.

In the following rules the time during which a body is acted on by an accelerating force is called *the time;* the velocity acquired at the end of the

time is called *the final velocity;* the space traversed by the body during the time is called *the space;* the accelerating force is called *the force.*

t = the time.
v = the final velocity.
s = the space.
f = the force.
w = the weight.

RULE 7. Given the *weight*, the *force*, and the *time;* to find the final *velocity.* Multiply the force by the time and by 32.2, and divide by the weight. The quotient is the final velocity. Or

$$v = \frac{32.2 \, f \, t}{w} \qquad (7)$$

RULE 8. Given the *weight*, the *force*, and the *time;* to find the *space.* Multiply the force by the square of the time and by 16.1, and divide by the weight. Or

$$s = \frac{16.1 \, f \, t^2}{w} \qquad (8)$$

RULE 9. Given the *weight*, the *final velocity*, and the *force;* to find the *time.* Multiply the final velocity by the weight, and divide by the force, and by 32.2. The quotient is the time. Or

$$t = \frac{w \, v}{32.2 \, f} \qquad (9)$$

RULE 10. Given the *weight*, the *final velocity*, and the *force;* to find the *space.* Multiply the weight by the square of the velocity, and divide by the force, and by 64.4. The quotient is the space. Or

$$s = \frac{w \, v^2}{64.4 \, f} \qquad (10)$$

RULE 11. Given the *weight*, the *force*, and the *space;* to find the *time.* Multiply the weight by the space, and divide by the force; find the square root of the quotient, and divide it by 4. The last quotient is the time in seconds. Or

$$t = \tfrac{1}{4} \sqrt{\frac{w \, s}{f}} \qquad (11)$$

RULE 12. Given the *weight*, the *force*, and the *space;* to find the final *velocity.* Multiply the space by the force, and divide by the weight; find the square root of the quotient, and multiply by 8. The product is the final velocity. Or

$$v = 8 \sqrt{\frac{f \, s}{w}} \qquad (12)$$

RULE 13. Given the *weight*, the *space*, and the *final velocity;* to find the *force.* Multiply the weight by the square of the final velocity, and divide by the space, and by 64.4. The quotient is the force. Or

$$f = \frac{w \, v^2}{64.4 \, s} \qquad (13)$$

RULE 14. Given the *weight, time,* and *final velocity;* to find the *force.* Multiply the weight by the velocity, and divide by the time, and by 32.2. Or

$$f = \frac{w\,v}{32.2\,t} \quad \text{(14)}$$

Note 1. When the accelerating or retarding force bears a simple ratio to the weight of the body, the ratio may, for greater readiness in calculation, be substituted in the quantities (*a*) representing the modified constants, for the force and the weight. Suppose the accelerating force is a tenth part of the weight, then the ratio is 1 to 10, and

$$\frac{16.1}{10} = 1.61,$$

$$\frac{32.2}{10} = 3.22,$$

$$\frac{64.4}{10} = 6.44;$$

and these quotients may be substituted for 16.1, 32.2, and 64.4 respectively in the formulas for the action of gravity (1) to (6), to fit them for direct use in dealing with an accelerating force one-tenth of gravity, the height h in those formulas, of course, being taken to mean space traversed.

Note 2. The tables, Nos. 101–103, pages 280–282, for the relations of the velocity and height of falling bodies, may be employed in solving questions of accelerating force generally.

Example. A ball weighing 10 lbs. is projected with an initial velocity of 60 feet per second on a level bowling-green, and the frictional resistance to its motion over the green is 1 lb. What distance will it traverse before it comes to a state of rest? By rule 10,

$$\frac{10 \text{ lbs.} \times 60^2}{1 \text{ lb.} \times 64.4} = 559 \text{ feet},$$

the distance traversed.

Again, the same result may be arrived at, according to *Note* 1, by multiplying the constant 64.4, in rule 4, for gravity, by the ratio of the force and the weight, which in this case is $\frac{1}{10}$, and $64.4 \times \frac{1}{10} = 6.44$. Substituting 6.44 for 64.4 in that rule and formula, the formula becomes

$$h = \frac{v^2}{6.44} = \frac{60^2}{6.44} = 559 \text{ feet},$$

the distance traversed, as already found.

But the question may be answered more directly by the aid of the table for falling bodies (No. 102, page 281). The height due to a velocity of 60 feet per second, is 55.9 feet; and it is to be multiplied by the inverse ratio of the accelerating force and the weight of the body, or $\frac{10}{1}$, or 10; that is,

$$55.9 \times 10 = 559 \text{ feet},$$

the distance traversed.

If the question be put otherwise—What space will a ball move over before it comes to a state of rest, with an initial velocity of 60 feet per

second, allowing the friction to be 1-10th the weight of the ball? The answer may be given, that the friction, which is the retarding force, being 1-10th of the weight, that is of gravity, the space described will be 10 times as much as is necessary for gravity, supposing the ball to be projected vertically upwards to bring the ball to a state of rest. The height due to the velocity is 55.9 feet; then

$$55.9 \times 10 = 559 \text{ feet},$$

the space described by the ball.

Action of Gravity on Inclined Planes.

If a body freely descend an inclined plane by the force of gravity alone, the velocity acquired by the body when it arrives at the foot of the plane, is that which it would acquire by falling freely through the vertical height. Or, the velocity is that "due" to the height of the plane.

The time occupied in making the descent is greater than that due to the height, in the ratio of the length of the plane, or distance travelled, to the height. The time is therefore directly in proportion to the length of the plane, when the height is the same.

The impelling or accelerating force by gravitation acting in a direction parallel to the plane, is less than the weight of the body, in the ratio of the height of the plane to its length. It is, therefore, inversely in proportion to the length of the plane, when the height is the same.

The time of descent, under these conditions, is inversely in proportion to the accelerating force.

If, for instance, the length of the plane be five times the height, the time of making freely the descent on the plane by gravitation is five times that in which a body would freely fall vertically through the height; and the impelling force down the plane is $^1/_5$ th of the weight of the body.

Problems on the descent of bodies on inclined planes are soluble by the aid of the rules 7 to 14, for the relations of accelerating forces. But, as a preliminary step, the accelerating force is to be determined, by multiplying the weight of the descending body by the height of the plane, and dividing the product by the length of the plane. For example, let a body of 15 pounds weight gravitate freely down an inclined plane, the length of which is five times the height, the accelerating force is $15 \div 5 = 3$ pounds. If the length of the plane be 100 feet, the height is 20 feet, and the velocity acquired in falling freely from the top to the bottom of the plane would be, by rule 12,

$$v = 8\sqrt{\frac{3 \times 100}{15}} = 8\sqrt{20} = 35.776 \text{ feet per second.}$$

The time occupied in making the descent is, by rule 11,

$$t = \tfrac{1}{4}\sqrt{\frac{15 \times 100}{3}} = \tfrac{1}{4}\sqrt{500} = 5.59 \text{ seconds.}$$

Whereas, for a free vertical fall through the height, 20 feet, the time would be, by rule 5,

$$t = \tfrac{1}{4}\sqrt{20} = 1.118 \text{ seconds,}$$

which is $^1/_5$ th of the time of making the descent on the inclined plane.

Special Rules for the Descent on Inclined Planes.

The height and the length of an inclined plane may be substituted for the accelerating force and the weight respectively in the rule (11), to find the time. Putting h = the height of the plane, and l = the length of the plane, the formula (11)

$$t = \tfrac{1}{4}\sqrt{\frac{w\,s}{f}} \text{ becomes } t = \tfrac{1}{4}\sqrt{\frac{l\,s}{h}} = \tfrac{1}{4}\sqrt{\frac{l^2}{h}},$$

$$\text{or, } t = \frac{l}{4\sqrt{h}} \quad \ldots\ldots\ldots\ldots \quad (15)$$

Rule 15.—Given the *length* and the *height* of the inclined plane, to find the *time* in which a body would freely descend by gravitation. Divide the length by four times the square root of the height; the quotient is the time in seconds.

For example, the length of the plane is 100 feet, and the height is 20 feet, and the time is

$$t = \frac{100}{4\sqrt{20}} = 5.59 \text{ seconds,}$$

as was found before.

Again, by inversion of the formula (15),

$$l = 4t\sqrt{h}\text{ , and then}$$

$$h = \frac{l^2}{16\,t^2} \quad \ldots\ldots\ldots\ldots \quad (16)$$

Rule 16.—Given the *length* of the inclined plane, and the *time* of free descent by gravitation, to find the *height* through which the body descends. Divide the square of the length by the square of the time in seconds and by 16; the quotient is the length of the inclined plane.

For example, the length of the plane is 100 feet, and the time of descent is 5.59 seconds; then the vertical height of the descent is

$$h = \frac{100^2}{5.59^2 \times 16} = 20 \text{ feet, the height.}$$

Average Velocity of a Moving Body Uniformly Accelerated or Retarded.

The average velocity of a moving body uniformly accelerated or retarded, during a given time or in a given space, is equal to half the sum of the initial and final velocities; and if the body begin from a state of rest or arrive at a state of rest, the average speed is half the final or initial velocity, as the case may be. Thus, in the example of a ball rolling, the initial speed or velocity is, in either case, 60 feet per second, and the terminal speed is nothing; the average speed is therefore the same, namely, one-half of that, or 30 feet per second.

MASS.

Weight is not an essential property of a body; it is only the attraction of the earth exerted upon the body. Suppose the attractive force to be suspended, then the body would cease to have weight. What would remain? Mass, or substance, simply. But, though weight is not mass, it is a direct measure of mass, in the same locality, or wherever the force of gravitation is the same, for double the mass has twice the weight. Weight alone, however, is not sufficient as a universal measure of mass, since the weight of the same mass would vary according to the force of gravitation for different situations. The mass, therefore, varies inversely as the force of gravitation, when the weight remains the same. That force is measurable by the height through which a body falls in a given time, or by the velocity acquired at the end of that time, say, a second, expressed by g. In its most general form, then, the expression for the mass of a body comprises the weight directly and the force of gravitation inversely; thus

$$m = \frac{w}{g} \quad \text{.......................................} \quad (17)$$

in which m is the mass, w the weight, g the force of gravitation; that is to say, the mass of a body is equal to the weight of the body divided by the force of gravity. Since the weight and the force of gravity vary in the same ratio, the mass $\frac{w}{g}$ of a body is the same at all places. As the quantity of matter of the same body is also constant whatever place it occupies, the mass $\frac{w}{g}$ gives an exact idea of the quantity of matter, and is a measure of it.

MECHANICAL CENTRES.

There are four mechanical centres of force in bodies, namely, the centre of gravity, the centre of gyration, the centre of oscillation, and the centre of percussion.

CENTRE OF GRAVITY.

The centre of gravity is the physical centre of a body, or of a system of bodies in rigid connection with each other, about which the gravitation of the several particles of the body is self-balanced, and at which it can be freely suspended or supported in any position in a state of rest.

In various calculations, the whole weight or mass of a body is considered as placed at its centre of gravity.

To find the centre of gravity of any plane figure mechanically:—Suspend the figure by any point near its edge, and mark on it the direction of a plumb-line hung from that point; then suspend it from some other point, and again mark the direction of the plumb-line in like manner. Then the centre of gravity of the surface will be at the point of intersection of the two marks of the plumb-line.

The centre of gravity of parallel-sided objects may readily be found in this way. For instance, to find the centre of gravity of the arch of a bridge; draw the elevation upon paper to a scale, cut out the figure, and proceed with it as above directed, in order to find the position of the centre of

gravity in elevation for the model. In the actual arch, the centre of gravity will have the same relative position as in the paper model.

In regular figures or solids the centre of gravity is the same as their geometrical centres. Thus, the centre of gravity of a straight line, a parallelogram, a prism, a cylinder, a circle, the circumference of a circle, a ring, a sphere, and a regular polygon, is the geometrical centre of these figures and solids respectively.

To find the centre of gravity of a triangle; draw a straight line from one of its angles to the middle of the opposite side; the centre of gravity will be in this line at a distance of two-thirds of its length from the angle. Or, draw a straight line from two of the angles to the middle of the opposite sides respectively; the point of intersection of the two lines will be the centre of gravity.

For a trapezium, or irregular four-sided figure, draw the two diagonals, and find the centres of gravity of each of the four triangles thus formed; then join each opposite pair of these centres of gravity. The joining lines will cut each other in the centre of gravity of the figure.

For a cone and a pyramid, the centre of gravity is in the axis or centre line, at a distance of three-fourths of the length of the axis from the vertex, or one-fourth from the base.

For an arc of a circle, the centre of gravity lies in the radius bisecting the arc, and the distance of it from the centre of the arc is found by multiplying the radius by the chord of the arc, and dividing by the length of the arc; the quotient is the distance of the centre of gravity from the centre of the circle.

For a sector of a circle, the centre of gravity is two-thirds of the distance of that of an arc, from the centre of the circle. It is found independently by multiplying the radius by twice the chord of the arc, and dividing by three times the length of the arc; the quotient is the distance of the centre of gravity from the centre of the circle.

For a parabolic space, the centre of gravity is in the axis, or centre line, and its distance from the vertex is three-fifths of the centre line or axis.

For a paraboloid, the centre of gravity is in the axis, at a distance from the vertex of two-thirds of the axis.

For two bodies, fixed or suspended one at each end of a straight bar, the common centre of gravity is in the bar, at that point which divides the distance between their individual centres of gravity, in the inverse ratio of the weights respectively. For example, if two weights of 20 lbs. and 10 lbs. be suspended on a bar at a distance of 9 feet apart between their centres of gravity, the common centre of gravity will divide the distance in the ratio of 1 to 2, being 3 feet from the heavier weight, and 6 feet from the lighter. In this example, the weight of the bar is neglected; but it may be allowed for according to the following direction.

For more than two bodies connected in one system, the common centre of gravity may be found by finding, in the first place, the common centre of gravity of two of them, and then finding that of these two jointly with a third, and so on to the last body in the group.

Centre of Gyration.—Radius of Gyration.—Moment of Inertia.

The centre of gyration, or revolution, is that point in a revolving body, or system of bodies, at a certain distance from the axis of motion, in which the whole of the matter in revolution may, as an equivalent condition, be

conceived to be concentrated, just as if a pound of platinum were substituted for a pound of revolving feathers, whilst the moment of inertia remains the same. The work accumulated in the body, of which the moment of inertia is a measure, remains in such a case the same, at the same angular velocity; and, as a necessary consequence, if the same accelerating force be applied to the body at the centre of gyration, as would actually be expended over the distributed matter of the body to communicate to it its angular velocity, the same angular velocity would be generated.

The distance of the centre of gyration from the axis of motion is called the radius of gyration; and the moment of inertia is equal to the product of the square of the radius of gyration by the mass or weight of the body.

The moment of inertia of a revolving body is found exactly by ascertaining the moments of inertia of every particle separately, and adding them together; or, approximately, by adding together the moments of the small parts arrived at by the subdivision of the body.

RULE 1. To find the moment of inertia of a revolving body. Divide the body into small parts of regular figure. Multiply the mass, or the weight, of each part by the square of the distance of its centre of gravity from the axis of revolution. The sum of the products is the moment of inertia of the body.

Note.—The value of the moment of inertia obtained by this process will be more nearly exact, the smaller and more numerous the parts into which the body is divided.

RULE 2. To find the length of the radius of gyration of a body about a given axis of revolution. Divide the moment of inertia of the body by its mass, or its weight, and find the square root of the quotient. The square root is the length of the radius of gyration; or

$$r = \sqrt{\frac{m}{w}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which m is the moment of inertia, and w is the weight of the body.

Note.—When the parts into which the body is divided are equal, the radius of gyration may be determined by taking the mean of all the squares of the distances of the parts from the axis of revolution, and finding the square root of the mean square.

The following are useful examples of the radius of gyration of bodies:—

In a straight bar, or a thin rectangular plate, revolving about one of its ends, the radius of gyration is equal to the length of the rod, multiplied by

$$\sqrt{1/3}, \text{ or } 0.5773.$$

In a straight bar, or a thin rectangular plate, revolving about its centre, the radius of gyration is equal to half the length, multiplied by

$$\sqrt{1/3}, \text{ or } 0.5773.$$

The general expression for the radius of gyration in a straight bar revolving on any point of its length, is

$$\sqrt{\left(\frac{a^3 + b^3}{3\ (a + b)}\right)};$$

in which a and b are the lengths of the two parts of the bar; that is to say,

divide the sum of the cubes of the two parts by three times the length of the bar, and extract the square root of the quotient. The root thus found is equal to the radius of gyration.

In a circular plate, a solid wheel of uniform thickness, or a solid cylinder of any length, revolving on its axis, the radius of gyration is equal to the radius of the object, multiplied by

$$\sqrt{\tfrac{1}{2}}, \text{ or } 0.7071.$$

In a plane ring, like the rim of a fly-wheel, revolving on its axis, the radius of gyration is equal to the square root of half the sum of the squares of the inside and outside radius of the rim.

In a thin circular plate, put in motion round one of its diameters, the radius of gyration is equal to half the radius of the circle.

For the circumference of a circle, revolving about a diameter, the radius of gyration is equal to the radius multiplied by 0.7071.

In the circumference of a circle revolving about its own axis, the radius of gyration is equal to the radius of the circle.

In a solid sphere revolving about a diameter, the radius of gyration is equal to the radius multiplied by

$$\sqrt{2/5}, \text{ or } 0.6325.$$

In the surface of a sphere, or an insensibly thin spherical shell, the radius of gyration is equal to the radius multiplied by

$$\sqrt{\tfrac{2}{3}}, \text{ or } 0.8165.$$

In a cone revolving about its axis, the radius of gyration is equal to the radius multiplied by $\sqrt{.3}$, or .5477.

Centre of Oscillation.

The centre of oscillation of a body vibrating about a fixed axis or centre of suspension, by the action of gravity, is that point in which, if, as an equivalent condition, the whole matter of the vibrating body were concentrated, the body would continue to vibrate in the same time. It is the resultant point of the whole vibrating energy, or of the action of gravity in causing oscillation. As the particles of the body further from the centre of suspension have greater velocity of vibration than those nearer to it, it is apparent that the centre of oscillation is more distant than the centre of gravity is from the axis of suspension, but it is situated in the centre line which passes from the axis through the centre of gravity. It differs also from the centre of gyration in this, that whilst the motion of oscillation is produced by the gravity of the body, that of gyration is caused by some other force acting at one place only.

The radius of oscillation, or the distance of the centre of oscillation from the axis of suspension, is a third proportional to the distance of the centre of gravity from the axis of suspension and the radius of gyration. Hence the following rule for finding the radius of oscillation:—

RULE 3. To find the radius of oscillation in a body vibrating on an axis. Square the radius of gyration of the body, and divide by the distance of the centre of gravity from the axis of suspension. The quotient is the radius of oscillation. Or,

$$\text{Radius of oscillation} = \frac{\text{radius}^2 \text{ of gyration.}}{\text{distance of centre of gravity from axis.}} \ldots\ldots (3)$$

If the axis of suspension be in the vertex or uppermost point of a plane figure, and the motion be edgewise, then,

In a right line, or straight rod, the radius of oscillation is two-thirds of the length of the rod.

In a circle suspended at the circumference, the radius of oscillation is three-fourths of the diameter.

In a rectangle suspended by one of its angles, it is two-thirds of the diagonal.

In a parabola suspended by the vertex, it is five-sevenths of the axis plus one-third of the parameter.

In a parabola suspended by the middle of its base, it is four-sevenths of the axis plus half the parameter.

But, if the oscillation of the plane figure be sidewise, then,

In an isosceles, or equal-sided triangle, it is three-fourths of the height of the triangle.

In a circle it is five-eighths of the diameter.

In a parabola it is five-sevenths of the axis.

In a sector of a circle suspended by the centre, it is three-fourths of the radius multiplied by the length of the arc, and divided by the length of the chord.

In a cone it is four-fifths of the axis, plus the quotient obtained by dividing the square of the radius of the base by five times the axis.

In a sphere it is two-fifths of the square of the radius divided by the sum of the radius and the length of the cord by which the sphere is suspended, plus the radius and the length of the cord. For example, in a sphere 16 inches in diameter, suspended by a cord 25 inches long, the radius of oscillation is

$$\frac{2 \times 8^2}{5\,(8 + 25)} + 8 + 25 = 0.78 + 33 = 33.78 \text{ inches,}$$

or 0.78 inch below the centre of the sphere.

It may be noted that the depression of the centre of oscillation below the centre of the sphere, namely, 0.78 inch, is signified in the first quantity in this equation.

The Pendulum.

A "simple pendulum" is the most elementary form of oscillating body,—consisting theoretically of a heavy particle attached to one end of a cord, or an inflexible rod, without weight, and caused to vibrate on an axis at the other end, or the centre of suspension.

If an ordinary pendulum be inverted, so that the centre of oscillation shall become the centre of suspension, then the first centre of suspension will become the new centre of oscillation, and the pendulum will vibrate in the

same time as before. This reciprocal action of the pendulum is a property of all pendulous bodies, and it is known as the reciprocity of the pendulum.

The time of vibration of an ordinary pendulum depends on the angle or the arc of vibration, and is greater when the arc of vibration is greater, but in a very much smaller proportion; and if this arc do not exceed 4° or 5°, that is to say, from 2° to 2½° on each side of the vertical line, the time of vibration is sensibly the same, however the length of the arc may vary within that limit. This property of a pendulum, of equal times of vibration, is known as *isochronism*.

To construct a pendulum such that the time of vibration shall be the same whatever the magnitude of the angle of vibration may be, it is necessary to cause the pendulum to vibrate, not in a circular arc, but in a cycloidal curve. For this object the pendulum is suspended by a flexible thread or rod, which oscillates between two cycloidal surfaces, on which it alternately laps and unlaps itself; these are generated by a circle of which the diameter is equal to half the length of the pendulum. By means of the circle O B, Fig. 98, for example, of which the diameter is half the length of the pendulum, describe the right and left cycloidal curves O C A, O C′ A′, on the horizontal line A A′; and draw the tangent C B C′, touching the cycloids at the middle of their lengths. The half-lengths O C, O C′, are equal to twice the diameter of the generating circle O B, and consequently equal to the length of the pendulum, which will vibrate in equal times, on the centre of suspension O, between the entire half-lengths O C, O C′, or in any shorter path. The curve C P C′ thus described by the pendulum, is itself a cycloidal curve, and is a duplicate of the other cycloids. Though a cycloidal motion of the pendulum is necessary to render it isochronous for all angles of vibration, yet taking very small arcs of the cycloidal path on either side of the vertical line, they do not sensibly differ from the circular arcs which would be described by an ordinary pendulum of the same length (O P) swinging freely. Hence the reason that the ordinary pendulum vibrates in equal times when its vibrations do not exceed 4° or 5° in extent.

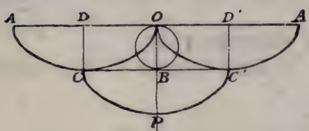


Fig. 98.—Cycloidal Pendulum.

The length of the pendulum vibrating seconds at the level of the sea in the latitude of London is 39.1393 inches, nearly a metre; at Paris it is 39.1279; at Edinburgh it is 39.1555 inches; at New York, 39.10153 inches; at the equator it is 39.027 inches, and at the pole it is 39.197 inches. Generally, if the force of gravity, or the length of the seconds pendulum at the equator be represented by 1, the gravity, or the length of pendulum at other latitudes will be as follows:—

Length of Seconds Pendulum.

At the equator	1.00000
„ 30° latitude	1.00141
„ 45 „	1.00283
„ 52 „	1.00357
„ 60 „	1.00423
„ 90 „ (the pole)	1.00567

According to these ratios, the force of gravity, and the length of the seconds pendulum, at the pole, are $^1/_{176}$th greater than at the equator; there being a difference of length of between a fourth and a fifth of an inch.

The following are the relations of the lengths of pendulums and the times of their vibrations, that is to say, of such as vibrate through equal angles, or of which the total angle of vibration does not exceed 4° or 5°:—

The times of vibration of pendulums are proportional to the square root of the lengths of the pendulums.

Conversely, the lengths of pendulums are to each other as the squares of the times of one vibration, or inversely as the squares of the numbers of vibrations in a given time.

The length of the seconds pendulum at London, 39.1393 inches, may be taken as a datum for calculation applicable to pendulums of different lengths, and to different times of vibration.

RULE 4. To find the time of vibration of a pendulum of a given length. Divide the square root of the given length in inches by the square root of 39.1393, or 6.2561. The quotient is the time of a vibration in seconds. Or

$$t=\sqrt{\frac{l}{39.1393}}=\frac{\sqrt{l}}{6.2561};\qquad (4)$$

in which l is the given length of pendulum in inches, and t the time of vibration in seconds.

RULE 5. To find the number of vibrations per second of a pendulum of given length. Divide 6.2561 by the square root of the length in inches. The quotient is the number of vibrations per second.

For the number of vibrations per minute. Divide 375.366 by the square root of the length in inches. The quotient is the number of vibrations per minute. Or

$$n=\frac{6.2561}{\sqrt{l}}\text{ (per second)};\qquad (5)$$

$$n=\frac{375.366}{\sqrt{l}}\text{ (per minute)};\qquad (5)$$

in which n is the number of vibrations.

RULE 6. To find the length of a pendulum when the time of a vibration is given. Multiply the square of the time of one vibration in seconds by 39.1393. The product is the length of the pendulum in inches. Or

$$l=t^2\times 39.1393\qquad (6)$$

RULE 7. To find the length of a pendulum when the number of vibrations per second is given. Divide 39.1393 by the square of the number of vibrations in a second. The quotient is the length of the pendulum in inches.

When the number of vibrations per minute is given. Divide 140,900 by the square of the number of vibrations in a minute. The quotient is the length of the pendulum in inches. Or

$$l=\frac{39.1393}{n^2\text{ (per second)}};\qquad (7)$$

$$l=\frac{140{,}900}{n^2\text{ (per minute)}}\qquad (7)$$

A pendulum may be shortened and yet vibrate in the same time as before, by the action of a second weight fixed on the pendulum rod above the centre of suspension. Here the upper weight counteracts the lower, and there is only the balance of gravitating force due to the preponderance of the lower weight available for vibrating both masses. The mass being thus increased while the gravitating force is diminished, a longer time is required for each vibration when the length of pendulum remains unaltered, or the pendulum may be shortened so that the time of the vibrations continues the same. By varying the height of the upper weight above the centre of suspension, and thus varying the level of the common centre of gravity, the period of vibration is varied in proportion.

Rule 8. To find the weight of the upper bob of a compound pendulum necessary to vibrate seconds, when the weight of the lower bob is given, and the respective distances of the bobs from the centre of suspension. Multiply the distance in inches of the lower bob from the centre of suspension by 39.1393, and from the product subtract the square of that distance (1); again, multiply the distance in inches of the upper bob from the centre of suspension by 39.1393, and add the square of that distance (2); multiply the lower weight by the remainder (1), and divide by the sum (2). The quotient is the weight of the upper bob. Or

$$w = \mathrm{W}\frac{(39.1393 \times \mathrm{D}) - \mathrm{D}^2}{(39.1393 \times d) + d^2}; \qquad (8)$$

in which D and d are the respective distances of the lower and upper bobs from the centre of suspension, and W, w, their respective weights.

Thus, by means of a second bob, pendulums of small dimensions may be made to vibrate as slowly as may be desired. The metronome, an instrument for marking the time of music, is constructed on this principle, the upper weight being slid and adjusted on a graduated rod to measure fast or slow movements.

The Centre of Percussion.

If a blow is struck by an oscillating or revolving body moving about a fixed centre, the percussive action is the same as if the whole mass of the body were concentrated at the centre of oscillation. That is to say, the centre of percussion is identical with the centre of oscillation, and its position is found by the same rules as for the centre of oscillation. If an external body is so struck that the mean line of resistance passes through the centre of percussion, then the whole force of percussion is transmitted directly to the external body; on the contrary, if the revolving body be struck at the centre of percussion, the motion of the revolving body will be absolutely destroyed, so that the body shall not incline either way, just as if every particle separately had been struck.

CENTRAL FORCES.

When a body revolves on an axis, every particle moves in a circle of revolution, but would, if freed, move off in a straight line, forming a tangent to the circle. The force required to prevent the body or particle flying from the centre is called *centripetal* force, and the tendency to fly from the centre is *centrifugal* force. These forces are equal and opposite—examples of action and reaction—and are classed as *central forces.*

Centrifugal force varies as the square of the speed of revolution.

It varies as the radius of the circle of revolution.

It varies as the mass or the weight of the revolving body.

Let c be the centrifugal force in pounds, w the weight of the revolving body in pounds, r the radius of revolution or gyration in inches, m the mass of the body $= \frac{w}{g}$, in which $g = 32.2$ or gravity; and v the linear or circumferential velocity in feet per second; then

$$c = \frac{m\,v^2}{r} = \frac{w\,v^2}{32.2\,r}.$$

That is to say, the centrifugal force of a revolving body is equal to the weight of the body multiplied by the square of the linear velocity, divided by 32.2 times the radius of gyration.

If the height due to the velocity be substituted for the velocity in the above equation, the height h being equal to $\frac{v^2}{64.4}$, then

$$c = \frac{2\,w\,v^2}{64.4\,r} = \frac{2\,w\,h}{r},$$

and

$$c : w :: 2\,h : r.$$

That is to say, the centrifugal force is to the weight of the body as twice the height due to the velocity is to the radius of gyration.

From the first equation the following rules for revolving bodies are deduced, for finding one of the four elements when the other three are given:—namely, the centrifugal force, the radius of gyration, the linear velocity, and the weight.

RULE 1. For the centrifugal force. Multiply the weight by the square of the speed, and divide by 32.2 times the radius of gyration. The quotient is the centrifugal force. Or

$$c = \frac{w\,v^2}{32.2\,r} \quad \text{(1)}$$

RULE 2. For the linear velocity. Multiply the centrifugal force by the radius of gyration, and by 32.2, and divide by the weight; and find the square root of the quotient. The root is the velocity. Or

$$v = \sqrt{\frac{32.2\,c\,r}{w}} \quad \text{(2)}$$

RULE 3. For the weight. Multiply the centrifugal force by the radius of gyration, and by 32.2, and divide by the square of the velocity. The quotient is the weight. Or

$$w = \frac{32.2\,c\,r}{v^2} \quad \text{(3)}$$

RULE 4. For the radius of gyration. Multiply the weight by the square of the velocity, and divide by the centrifugal force, and by 32.2. The quotient is the radius of gyration. Or

$$r = \frac{w\,v^2}{32.2\,c} \quad \text{(4)}$$

Note.—When the velocity is expressed as angular velocity, in revolutions per unit of time, it is to be reduced to linear or circumferential velocity by multiplying it by the radius of gyration and by 6.2832; or

$$v = 6.2832\ v'\ r,$$

in which v' is the angular velocity.

By substitution and reduction in equation (1), the following equation in terms of the angular velocity is arrived at:—

$$0.8156\ c = w\ r\ v'^2, \qquad (5)$$

from which is found

$$c = \frac{w\ r\ v'^2}{0.8156} = 1.226\ w\ r\ v'^2. \qquad (6)$$

That is to say, the centrifugal force is equal to the weight multiplied by the radius of gyration and by the square of the angular velocity, and by 1.226.

MECHANICAL ELEMENTS.

The function of mechanism is to receive, concentrate, diffuse, and apply *power* to overcome *resistance.* The combinations of mechanism are numberless; but the primary elements are only two, namely, the *lever* and the *inclined plane.* By the lever, power is transmitted by circular or angular action; that is to say, by action about an axis; by the inclined plane, it is transmitted by rectilineal action. The principle of the lever is the basis of the *pulley* and the *wheel and axle;* that of the inclined plane is the basis of the *wedge* and the *screw.*

For the present, frictional resistance and the weight of the mechanism are not considered; the terminal resistance is called the *weight;* and the elemental mechanisms are to be treated as in a state of equilibrium, in which the power exactly balances the weight without actual movement. The action, or work done, will be subsequently treated.

The Lever.

The elementary lever is an inflexible straight bar, turning on an axis or fixed point, called the fulcrum; the force being transmitted by angular motion about the fulcrum, from the point where the power is applied to the point where the weight is raised, or other resistance overcome. There are three varieties of the lever, according as the fulcrum, the weight, or the power is placed between the other two, but the action is, in every case, reducible to that of three parallel forces in equilibrium (page 275).

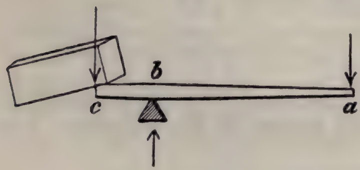


Fig. 99.—Lever.

First. The power is applied at one end a, of the lever $a\ b\ c$, Fig. 99, and transmitted through the fulcrum, b, to the weight at the other end c. The moments of the power and the weight about the fulcrum are equal, or

$$\text{power} \times a\ b = \text{weight} \times b\ c.$$

That is, the product of the power by its distance from the fulcrum is equal

to the product of the weight by its distance from the fulcrum. Consequently

$$\text{power} : \text{weight} :: b\,c : a\,b,$$

that is, the power and the weight are to each other inversely as their respective distances from the fulcrum.

The ratio of the length of the power end of the lever to the length of the weight end is called the leverage of the power. The respective lengths, Fig. 99, being 7 feet and 1 foot, the leverage is 7 to 1, or 7.

The three varieties of the lever are grouped together in Figs. 100, 101, and 102. In each case, the lever is supposed to be 8 feet long and divided into feet. The leverage, in the first, is 7 to 1, or 7; in the second, 8 to 1, or 8; in the third, $\frac{1}{8}$ to 1, or $\frac{1}{8}$: showing that, in the first case, the power balances seven times its own amount; in the second case, eigh times its amount; in the third case, only

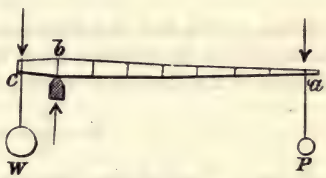


Fig. 100.—Lever, 1st kind.

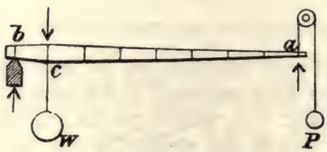


Fig. 101.—Lever, 2d kind.

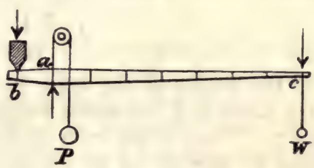


Fig. 102.—Lever, 3d kind.

one-eighth of itself, because it is nearer to the fulcrum than the weight.

In each case the moments of the power and the weight about the fulcrum are equal, for, in each case,

$$P \times a\,b = W \times b\,c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

The pressures exerted at the extremities of the lever act in the same direction, and the sum of them is equal and opposite to the intermediate pressure, whether it be that of the fulcrum, the weight, or the power (—). From this the pressure on the fulcrum may be found. If it be in the middle, the pressure is equal to the sum of the power and the weight, that is, $60 + 420 = 480$ lbs. in the example above; if at one end, it is equal to the difference of them, that is, it is $480 - 60 = 420$ lbs. when the weight is in the middle, and it is $60 - 7\frac{1}{2} = 52\frac{1}{2}$ lbs. when the power is in the middle.

From the equation for the equality of moments,

$$\begin{aligned} P \times a\,b &= W \times b\,c, \\ \text{or } P \times L \ &= W \times l, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b) \end{aligned}$$

in which L and l are the respective distances of the power and the weight from the fulcrum, rules may be formed for finding any one of the four quantities, when the other three are given.

RULE 1. To find the power. Multiply the weight by its distance from the fulcrum, and divide by the distance of the power from the fulcrum. The quotient is the power.

Or, divide the weight by the leverage. The quotient is the power. Or

$$P = \frac{W\,l}{L} \quad \dots\dots\dots\dots\dots\dots \quad (1)$$

RULE 2. To find the weight. Multiply the power by its distance from the fulcrum, and divide by the distance of the weight from the fulcrum. The quotient is the weight.

Or, multiply the power by the leverage. The product is the weight. Or

$$W = \frac{P\,L}{l} \quad \dots\dots\dots\dots\dots\dots \quad (2)$$

RULE 3. To find the distance of the power from the fulcrum. Multiply the weight by its distance from the fulcrum, and divide by the power. The quotient is the distance of the power from the fulcrum. Or

$$L = \frac{W\,l}{L} \quad \dots\dots\dots\dots\dots\dots \quad (3)$$

RULE 4. To find the distance of the weight from the fulcrum. Multiply the power by its distance from the fulcrum, and divide by the weight. The quotient is the distance of the weight from the fulcrum. Or

$$l = \frac{P\,L}{W} \quad \dots\dots\dots\dots\dots\dots \quad (4)$$

If the weight of the lever be included in such calculations, its influence is the same as if its whole weight or its mass were collected at its centre of gravity. Thus, if the lever of the first kind, Fig. 100, weighs 30 lbs., and its centre of gravity be at the middle of its length, the weight of the lever co-operates with the power, at a mean distance of 3 feet from the fulcrum. By equality of moments

$$(P \times 7) \times (30 \times 3) = W \times 1 = 420 \text{ lbs.} \times 1,$$
$$\text{and } P \times 7 = 420 - 90 = 330 \text{ lbs.};$$

therefore P, the power at the end of the lever required to balance the

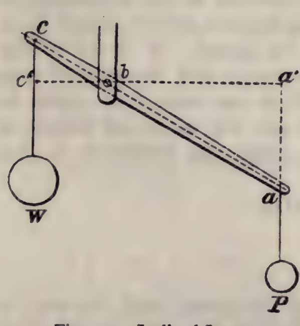


Fig. 103.—Inclined Lever.

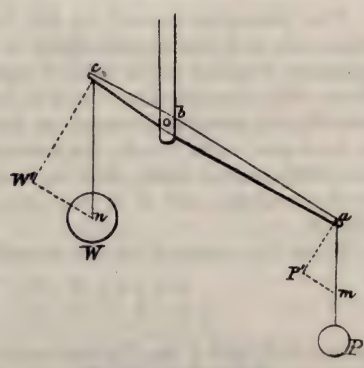


Fig. 104.—Inclined Lever.

weight, is only $330 \div 7 = 47.1$ lbs. in co-operation with the weight of the lever, as compared with 60 lbs., without reckoning the aid from this source.

When the lever is inclined to the direction of the forces, as in Fig. 103,

equilibrium, or the equality of moments, may nevertheless be maintained. Drawing the horizontal line $a'\ b\ c'$ through the fulcrum, to meet the verticals through the power and the weight at a' and c', the moments of the power and the weight are to be estimated on the horizontal lengths $a'\ b$, $b\ c'$; and

$$\text{the moment P} \times a'\ b = \text{the moment W} \times b\ c'.$$

The equality of moments may be proved in another way. Let the power and the weight be resolved, in order to find the pressures on the ends of the lever, at right angles to it, and thus to arrive at the moments as estimated on the actual length of the lever. Let the verticals through the ends of the lever, *a m* and *c n*, Fig. 104, represent the power and the weight respectively, and draw *a* P′ and *c* W′ perpendicular to the lever, and *m* P′ and *n* W parallel to it, completing the triangles *a m* P′, *c n* W′. Then *a* P′ and *c* W′ are the components of the power and the weight respectively tending to turn the lever; and, it may be added, they bear the same ratio to each other as the power and the weight. Consequently, if these components be multiplied by the respective lengths of the lever, the products will be the moments of the components, and the moments will be equal; or

$$\text{the moment } a\ \text{P}' \times a\ b = \text{the moment } c\ \text{W}' \times b\ c.$$

These two methods of analyzing and finding the moments of the forces acting on an inclined lever—one, combining a reduced length of lever with the whole power and weight; the other, combining the whole length of lever with a reduced power and weight—lead to one conclusion, that a lever, if balanced in one position, is balanced in every other position, when the forces continue to act in parallel lines.

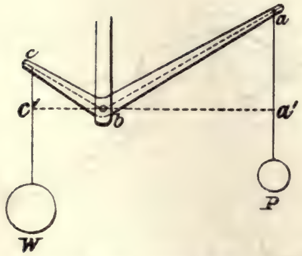


Fig. 105.—Bent Lever.

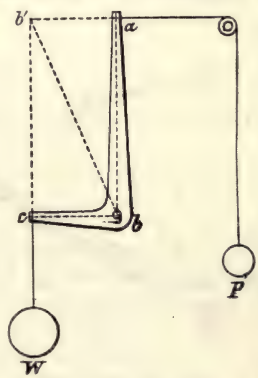


Fig. 106.—Bent Lever.

The conditions of equilibrium in a bent lever may be defined similarly. Let the lever *a b c*, Fig. 105, be bent at the fulcrum *b;* draw the horizontal line $a'\ b\ c'$, then the moments of the power and the weight are reckoned on the lines $a'\ b$, $b\ c'$, and they are equal to each other; or

$$\text{P} \times a'\ b = \text{W} \times b\ c'.$$

Again, let the forces acting on a lever, whether straight or bent, be otherwise than vertical or parallel. When the arms of the lever are at right angles, and the power and the weight applied at right angles to the arms, as in Fig. 106, the moments are reckoned directly on the arms, $a\,b$, $b\,c$, as in a straight lever; and

$$\text{the moment } P \times a\,b = \text{the moment } W \times b\,c.$$

The thrust, or pressure on the fulcrum, is in this case less than the sum of the power and the weight; and it may be determined by constructing a parallelogram upon the two arms of the lever, the arms representing inversely the respective forces. That is, $a\,b$ represents the magnitude and direction of the weight W, and $b\,c$ those of the power P. The diagonal $b\,b'$, of the parallelogram represents the magnitude and direction of the third force acting at the fulcrum to oppose the other two and maintain equilibrium.

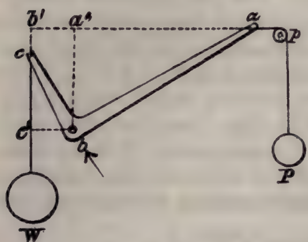


Fig. 107.—Bent Lever.

When the same lever is tilted into an oblique position, the power continuing to act horizontally on the lever, Fig. 107, draw the vertical $b'\,c'$ through the end c of the lever, and produce the power line $a\,p$ to meet it at b'. Complete the parallelogram $a'\,b'\,c'\,b$; then the sides $a'\,b$ and $b\,c'$ are the perpendiculars to the directions to the power and weight, on which the moments are reckoned, so that

$$\text{the moment } P \times a'\,b = \text{the moment } W \times b\,c'.$$

The diagonal $b\,b'$ is the resultant force at the fulcrum.

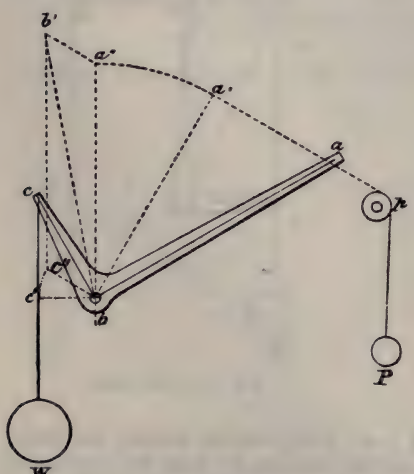


Fig. 108.—Bent Lever.

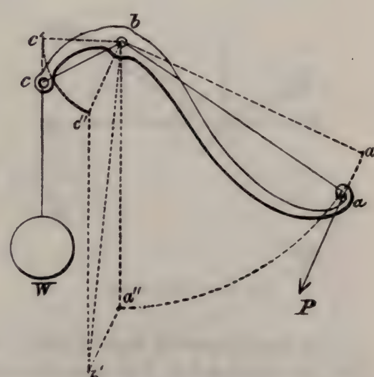


Fig. 109.—Serpentine Lever.

If the power do not act horizontally, but in some other direction, $a\,p$, Fig. 108, produce the power-line $p\,a$ and draw $b\,a'$ perpendicular to it.

Draw $b\ c'$ as before; then the moments are reckoned on the perpendiculars $b\ a'$, $b\ c'$, and, as before,

$$P \times a'\ b = W \times b\ c'.$$

To find the resultant force at the fulcrum. On the fulcrum b as a centre describe arcs of circles with the radii $b\ a'$ and $b\ c'$, and draw $b\ a''$, $b\ c''$ respectively parallel to the directions of the weight and the power, to cut the arcs at a'' and c''. Complete the parallelogram, and the diagonal $b\ b'$ represents in magnitude and direction the resultant force at the fulcrum.

In this solution the power and the weight are assumed to act exactly, or sensibly, in the same plane.

Again, in the serpentine lever $a\ b\ c$, Fig. 109, supposed to be a pump-handle, the power P is applied obliquely in the direction a P. Produce P a and W c, and draw the perpendiculars $b\ a'$, $b\ c'$ from the fulcrum for the lengths of the moments, then

$$P \times a'\ b = W \times b\ c'.$$

Construct the parallelogram, as in the foregoing figure, and the diagonal $b\ b''$ represents the resultant force at the fulcrum.

By similar treatment the action of the forces in levers of the second and third kinds may be analyzed. The lever of the second kind, $a\ c\ b$, Fig. 110, in an oblique position, is acted on horizontally by the power and the weight at a and c; draw the vertical $b\ c'\ a'$, then $b\ c'$ and $b\ a'$ are the distances at which the forces act from the fulcrum, or the lengths of the moments, and

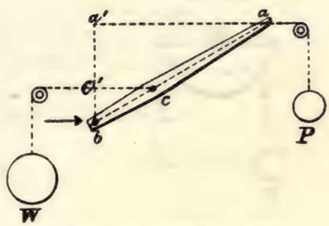


Fig. 110.—Lever of the 2d kind.

$$P \times b\ a' = W \times b\ c';$$

and the horizontal resultant force at the fulcrum is equal to the difference of the weight and the power.

If more than two forces be applied to a lever in a state of equilibrium, the sum of the moments tending to turn the lever in one direction is equal to the sum of those tending in the opposite direction.

If two or more levers are connected consecutively one to the other, so that they act as one system, with the power and the weight at the extremities, then, in equilibrium, the ratio of the power to the weight is the product of the separate inverse ratios of all the levers. For example, in a connected series of three levers, having each their arms in the ratio of 2 to 1, the combined inverse ratio is found by multiplying 2 by 2 and by 2; thus

first lever	2 to 1 ratio,
second lever	2 to 1 ratio,
third lever	2 to 1 ratio,
compound ratio	8 to 1.

That is, the power is to the weight as 1 to 8.

The Pulley.

The pulley is a wheel over which a cord, or chain, or band is passed, in order to transmit the force applied to the cord in another direction. It is equivalent to a continuous series of levers, with equal arms on one fulcrum or axis, and affords a continuous circular motion instead of the intermittent circular motion of one lever. The weight W, Fig. 111, is sustained by the power P, by means of a cord passed over the pulley A, in fixed supports, and the centre line *a b c* represents the element of the lever, from the ends of which the power and the weight may be conceived to depend, turning on the fulcrum *b*. By equality of moments, $P \times ab = W \times bc$; and the arms or radii *a b*, *b c* being equal, the power is equal to the weight, and the counter-pressure at the fulcrum is equal to twice the weight.

When the power and weight act in directions at an angle with each other, as in Fig. 112, the acting radii *a b*, *b c*, representing the element of a bent

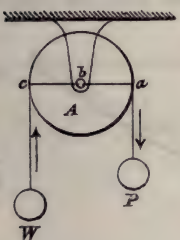


Fig. 111.—Pulley.

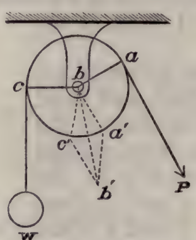


Fig. 112.—Pulley.

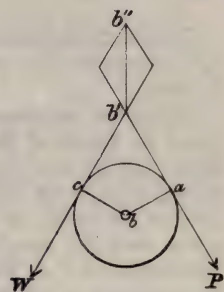


Fig. 113.—Pulley.

lever, are lines drawn from the centre perpendicular to the directions of the power and weight. The power is equal to the weight, but the counter-pressure on the fulcrum is less than twice the weight, and is represented by the diagonal *b b′* of the parallelogram formed by the radii *b a′*, *b c′*, drawn from the fulcrum parallel to the directions of the power and the weight respectively.

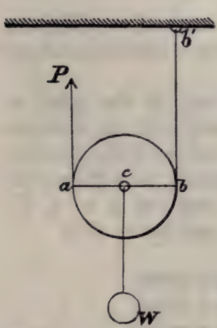


Fig. 114.—Movable Pulley, as a lever of the 2d kind.

Another construction for the parallelogram of forces in the action of the pulley is obtained by producing the directions of the power and the weight beyond the pulley, Fig. 113, intersecting each other at *b′*, then forming the parallelogram, and drawing the diagonal *b′ b″* as the resultant pressure on the fulcrum.

Thus the single fixed pulley acts like a lever of the first kind, and simply changes the direction of force, without modifying the intensity of the power.

But the pulley may be employed as a lever of the second kind by suspending the weight to the axis of the pulley, and fixing one end of the cord to a point as a fulcrum point. Thus, in Fig. 114, the weight W is suspended from the axis *c*,

the cord is fixed to the point b', and the power P acts through the diameter $a\,c\,b$, in which b is the fulcrum. By equality of moments,

$$P \times a\,b = W \times b\,c;$$

that is, the product of the power by the diameter of the pulley is equal to the product of the weight by the radius of the pulley, and the leverage being as 2 to 1, the power is only half the weight.

In acting as a lever of the third kind, the power is applied to the axis a, Fig. 115, one end of the cord being fixed at b', and the weight attached at the other end, c. In this case, by equality of moments the product of the power by the radius of the pulley is equal to that of the weight by the diameter, and the leverage being as 1 to 2, the power is twice the weight.

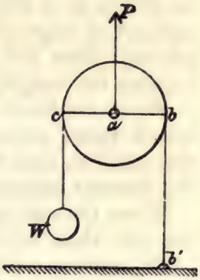


Fig. 115.—Movable Pulley, as a lever of the 3d kind.

These demonstrations with respect to movable pulleys only apply to cases of parallel cords; that is to say, when the direction of the power is parallel to that of the weight. If, on the contrary, they be inclined to each other, as in Fig. 116, in which the weight is suspended by the axis, the power becomes greater than half the weight, as is shown by the parallelogram of which the diagonal $c'\,c''$ represents the weight, and the sides $c'\,b''$, $c'\,a''$, the pull on the fulcrum, and the power exerted to sustain the weight. Each of these sides is greater than half the diagonal.

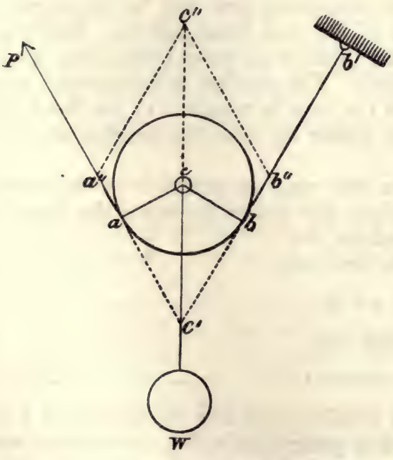


Fig. 116.—Movable Pulley.

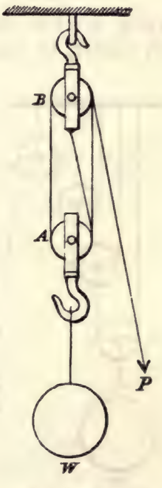


Fig. 117.—Pulley-Blocks.

Combinations of Pulleys.—Fast and Loose Pulleys.—In these last two applications of the pulley, it becomes movable when in action, and by combining two or more movable pulleys on the same or different axles in one block, with one cord, the gain of power may be increased in the same proportion. The movable block A, Fig. 117, carrying the weight, is used

with a fixed counterpart B, the rope is attached by one end to the fixed block, and is passed over the movable and fixed pulleys, from one to the other in succession, the power being applied to the other end, as at P. This system is known as fast and loose pulley-blocks.

The fixed end of the rope is sometimes attached to the movable block.

RULE 1. To find the power necessary to balance a weight or resistance by means of a system of fast and loose pulleys. Divide the weight by the number of ropes by which it is carried; that is, the number of ropes which proceed from the movable block. The quotient is the power required to balance the weight.

When the fixed end of the rope is attached to the fixed block, the number of ropes proceeding from the loose block is twice the number of movable pulleys, and the power may be found by dividing the weight by twice the number of movable pulleys.

When the end of the rope is attached to the movable block, the divisor may be taken at twice the number of movable pulleys plus 1.

Or, putting n for the number of movable pulleys; if the fixed end of the rope is attached to the fixed block,

$$P = \frac{W}{2\,n}; \qquad (1)$$

and if the fixed end of the rope be attached to the movable block,

$$P = \frac{W}{2\,n+1} \qquad (1a)$$

RULE 2. To find the weight or resistance that will be balanced by a given power, by means of a system of fast and loose pulleys. Multiply the power by the number of ropes proceeding from the movable block. The product is the required weight.

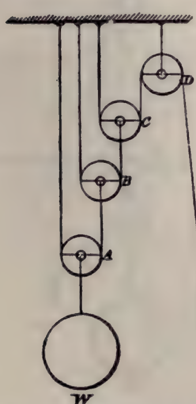


Fig. 118.—Movable Pulleys.

Or, when the rope is attached to the fixed block, multiply the power by twice the number of movable pulleys.

Or, when the rope is attached to the movable block, multiply the power by twice the number of movable pulleys plus 1.

Or, in the first case,

$$W = 2\,n\,P; \qquad (2)$$

in the second case,

$$W = (2\,n+1)\,P \qquad (2a)$$

Again, a combination may be formed of a number of movable pulleys, as in Fig. 118, each of which, A, B, C, is suspended by a cord, with one end fixed to the roof and the other end fixed to the axis of the next pulley. The weight W is hung to the axis of the first pulley A, which delivers half the weight to the second pulley B, which delivers half of the weight hanging to it, or one-fourth of the first weight W, to the third pulley C; from which only one-eighth of the first weight passes over the guide or neutral pulley D to the power P. In

general the divisor for the power is 2^n, or the nth power of 2, n being the number of movable pulleys.

Rule 3. To find the power necessary to balance a weight by means of a system of separate movable pulleys, with separate cords consecutively connected as above described. Divide the weight by that power of 2 of which the index is the number of movable pulleys. The quotient is the power or force required to balance the weight.

Or, divide and subdivide the weight successively by 2 as many times as there are movable pulleys to find the power required. Or

$$P = \frac{W}{2^n} \quad \text{.......................................} \quad (3)$$

Rule 4. To find the weight that can be balanced by a given power, by means of a system of separate movable pulleys as above described. Multiply the power by that power of 2 of which the index is the number of movable pulleys. The product is the weight required.

Or, multiply the power successively by 2 as many times as there are pulleys. Or

$$W = P \times 2^n \quad \text{.......................................} \quad (4)$$

Note.—It is necessary that the cords should be parallel to each other, as in the illustration, in order that the above rules, 3 and 4, may apply.

Wheel and Axle.

The wheel and axle may be likened to a couple of pulleys of different diameters united together on one axis, of which the larger, a, Fig. 119, is the wheel, and the smaller, c, the axle, with a common fulcrum, b; the centre line $a\,b\,c$ representing the elements of a lever. The weight W on the axle at c is balanced by the power P, on the wheel at a. The moments are equal, or

$$P \times a\,b = W \times b\,c;$$

and the power is to the weight inversely as their distances from the centre; or

$$P : W :: b\,c : a\,b.$$

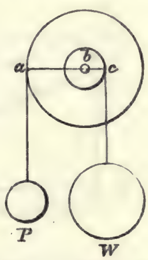


Fig. 119.—Wheel and Axle.

If a crank handle be substituted for the wheel, making a windlass, the radius of the crank is substituted for that of the wheel in estimating the ratio of the forces.

Of the four elements, namely, the radius of the wheel or crank, the radius of the axle or roller, the power, and the weight, if three be given, the fourth can be found as follows, putting R and r for the respective radii.

Rule 1. To find the power. Multiply the weight by the radius of the axle, and divide by the radius of the wheel. The quotient is the power. Or

$$P = W \times \frac{r}{R} \quad \text{.......................................} \quad (1)$$

Rule 2. To find the weight. Multiply the power by the radius of the

wheel, and divide by the radius of the axle. The quotient is the weight. Or

$$W = P \times \frac{R}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

RULE 3. To find the radius of the wheel. Multiply the weight by the radius of the axle, and divide by the power. The quotient is the radius of the wheel. Or

$$R = \frac{W\,r}{P} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

RULE 4. To find the radius of the axle. Multiply the power by the radius of the wheel, and divide by the weight. The quotient is the radius of the axle. Or

$$r = \frac{P\,R}{W} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Note.—The diameters of the wheel and the axle or roller may be employed in the calculations instead of the radii.

INCLINED PLANE.

The inclined plane is a slope, or a flat surface inclined to the horizon, on which weights may be raised. By such substitution of a sloping path for a direct vertical line of ascent, a given weight can be raised by a power which is less than the weight itself.

There are three elements of calculation in the inclined plane:—the plane itself, A B, Fig. 120; the base, or horizontal length, A C; and the height or vertical rise B C; together forming a right-angled triangle. The weight W is to be raised through a height equal to C B, and for that object is drawn up the slope from A to B. It is partly supported during the ascent, and it is in virtue of this degree of support given to the weight that such a "dead pull" as that of a direct vertical lift is avoided, and that it can be raised by a power much less than its own weight. Let the weight W be kept at rest on the incline by the power P, acting in the line b P′, parallel to the plane. Draw the vertical line $b\,a$ to represent the weight; also $b\,b'$ perpendicular to the plane, and complete the parallelogram $b'\,c$. Then the vertical weight $b\,a$ is equivalent to $b\,b'$, which is the measure of support given by the plane to the weight, and $b\,c$, which is the force of gravity tending to draw the weight down the plane. The power required to maintain the weight in equilibrium is represented by this force $b\,c$. Thus, the power and the weight are in the ratio of $b\,c$ to $b\,a$.

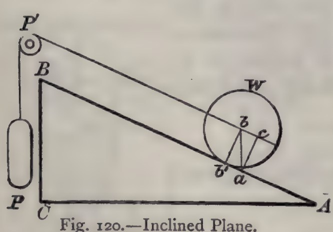


Fig. 120.—Inclined Plane.

Since the triangle of forces $a\,b\,c$ is similar to the triangle of the incline A B C, the latter may be substituted for the former in determining the relative magnitude of the forces, and

$$P : W :: bc : ab :: BC : AB,$$

that is, the power, acting parallel to the inclined plane, is to the weight, as the height of the plane to its length. Then, by equality of moments,

$$P \times A\,B = W \times B\,C,$$

or P × length of inclined plane = W × height of inclined plane..... (*a*)

For example, take the length of the inclined plane, 24 feet; the height, 2 feet; and the weight to be raised, 360 lbs. The power required to balance the weight is equal to $360 \times 2 \div 24 = 30$ lbs.

Again, the base, A C, of the inclined plane, represents the element of the pressure of the weight on the inclined plane.

It is thus seen that the sides of the triangle formed by an inclined plane, its base, and its height, are respectively proportional as follows:—

The inclined plane	to the weight at rest on the plane.
The base	to the pressure of the weight on the plane.
The height	to the power acting parallel to the plane.

When the power acts in a direction parallel to the base, as in Fig. 121, in which the power P supports the weight W in the direction *b* P′, parallel to the base; draw the vertical *b a* to represent the weight, and the line *b b′* perpendicular to the incline, and complete the parallelogram *b′ c*. The weight *b a*, decomposed, is equivalent to *b b′*, the perpendicular to the incline, representing the pressure of the weight upon the plane, and *b c*, the force of traction, or the power P. Here the pressure *b b′* on the plane is greater than the weight, and the power *b c* is greater than when the line of traction is parallel to the incline.

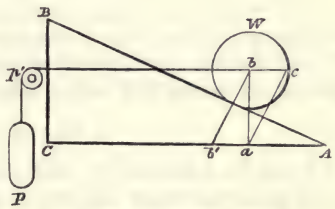


Fig. 121.—Inclined Plane.

The triangles *a b c*, A B C, being similar, the ratios of the power and the weight are as follows:—

P : W : : *b c* : *a b* : : B C : A C;........... (*b*)

that is, they are to each other as the height of the plane to its base; and the inclined plane, the base, and the height, are respectively proportional as follows:—

The inclined plane	to the pressure of the weight on the plane.
The base	to the weight at rest on the plane.
The height	to the power acting parallel to the base.

If the power be applied in any direction above that which is parallel to the incline, though the pressure of the weight on the plane will be less than the weight itself, yet, as in the previous case, the power is greater than is necessary when it acts in a direction parallel to the plane. Thus, in Fig. 122, in which the power P acts at a divergent angle in the direction *b* P′, draw the vertical *b a*, the perpendicular *b b′*, to the plane, and the extension of the power line to *c*, and complete the parallelogram. Then, the weight is represented by *b a*, the pressure on the incline by *b b′*, and the power by *a b′* or *b c*.

For comparison, the parallelogram that would represent the relative forces arising from a power acting parallel to the plane, is added on the figure in dotted lines extending to the angles b'' and c'. It shows that the pressure on the plane is greater than when the power is divergent, but that the power is less.

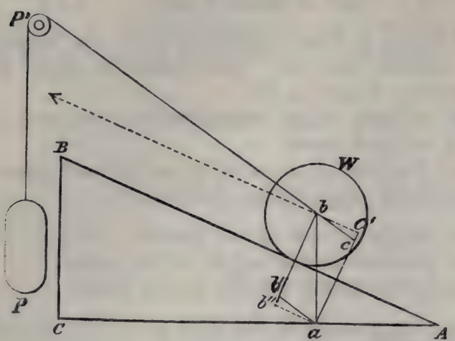


Fig. 122.—Inclined Plane.

It follows that the longer the inclined plane, when the height is the same, the less is the power required to balance the weight; in fact, the power simply varies in the inverse ratio of the length of the plane.

If two inclines, A B and B D, of unequal lengths and the same height, be united back to back on the line B C, then two weights, W and W′, on the respective planes, connected by a cord over a pulley at the summit B, will balance each other, when they are in the ratio of the lengths of the planes on which they rest. That is,

$$W : W' :: A B : B D.$$

From the formula (a), rules may be formed for finding one of the following four elements when the other three are given, namely, the length of the inclined plane, the height of it, the weight, and the power to balance the weight when acting in a direction parallel to the incline.

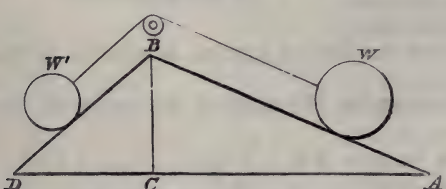


Fig. 123.—Double Inclined Plane.

RULE 1. To find the power. Multiply the weight by the height of the plane, and divide by the length. The quotient is the power.

RULE 2. To find the weight. Multiply the power by the length of the plane, and divide by the height. The quotient is the weight.

RULE 3. To find the height of the inclined plane. Multiply the power by the length, and divide by the weight. The quotient is the height.

RULE 4. To find the length of the inclined plane. Multiply the weight by the height of the plane, and divide by the power. The quotient is the length.

Identity of the Inclined Plane and the Lever.

Though the inclined plane is distinguished from the lever in the mode of operation, inasmuch as there is no motion about a mechanical centre, as in the lever, yet the conditions of equilibrium on the inclined plane may be established on the principle of the lever. Suppose a round weight W kept at rest on the incline A B by a power P parallel to the incline, passing

through the centre *a*. Draw *a b* perpendicular to the incline; the point *b* is the point of contact of the weight with the incline. Draw the vertical line *a d*, and the perpendicular *b c* to it. Then the lines *a b*, *b c* form a bent lever *a b c*, of which *b* is the fulcrum, and *a b*, *b c* the arms. The weight acts at the extremity *c* of the short arm, and the power at the extremity *a* of the long arm; and the power and the weight are to each other inversely as the relative arms of the lever, *a b*, *b c*. Now, as *a b c* and A B C are similar triangles, the arms *a b*, *b c* are to each other as the length and the height A B, B C, of the incline, and

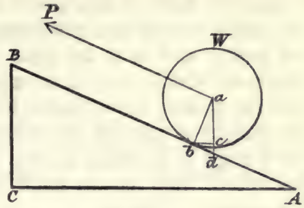


Fig. 124.—Leverage on an Inclined Plane.

$$P : W :: bc : ab :: BC : AB;$$

that is, the power is to the weight as the height of the inclined plane to its length, which is the proportion already established (*a*) page 307).

The ratio of the length of an inclined plane to its height may be called the leverage of the plane, and the products of the power into the length of the plane, and of the weight into the height of the plane, may represent the moments of the power and the weight.

Suppose, again, that the power is applied at P, Fig. 125, through a cord *a* P, passed round and over the weight parallel to the incline; then the

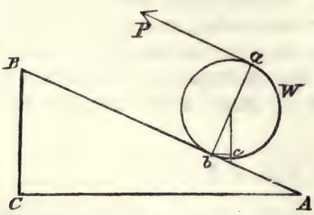


Fig. 125.—Leverage on an Inclined Plane.

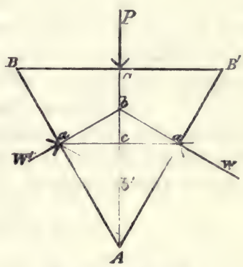


Fig. 126.—Wedge.

diameter of the weight *a b* becomes the long arm of the lever *a b c*, through which the power acts, being double the length of the arm *a b*, Fig. 124, where the power is applied at the centre of the weight. By thus doubling the leverage, the power is halved, and the ratio of the power to the weight is as half the height of the plane to its length.

In this case there is the action of a movable pulley combined with an inclined plane; the rolling weight moved by a cord lapped round it, representing a movable pulley with the weight attached to the axle. Thus the leverage of the power on the inclined plane can be doubled.

The Wedge.

The wedge is a pair of inclined planes united by their bases, or "back to back," as A B C B′, Fig. 126. Whereas inclined planes are fixed, wedges are moved, and in the direction of the centre line C A, against a resistance equally acted on by both planes of the wedge. The function of the wedge

is to separate two bodies by force, or divide into two a single body. In some cases the wedge is moved by blows, as in splitting timber; in others it is moved by pressure. The action by simple pressure is now to be considered.

The pressure P is applied to a wedge at the head B B′ at right angles to the surface, and the resistance or "weight" to be overcome is opposed to the wedge and acts at right angles to the faces A B, A B′, at the middle points of which, *a*, *a*, it is supposed to be concentrated. Whilst the wedge and the power move through a space equal to the length of the wedge A C, the weight is moved or overcome through a space equal to the breadth of the wedge B B′; and, as the power is to the weight inversely as the spaces described, they are to each other directly as the breadth to the length of the wedge. That is,

$$P : W :: B B' : A C,$$

and the product of the power by the length of the wedge is equal to the product of the weight by the breadth of the wedge; or

$$P \times A\ C = W \times B\ B',$$

$$\text{or } P \times \text{length} = W \times \text{breadth of wedge} \quad \ldots\ldots\ldots\ldots \quad (c)$$

By the aid of the parallelogram the same conclusions are arrived at. Thus, in Fig. 126, produce the directions of the two resistances, W *a*, W *a*, to meet in the middle of the wedge at *b*, complete the parallelogram, and draw the diagonals *a c a* and *b b′*. The diagonal *b b′* is the resultant of the two forces *a b*, *a b*, and represents the pressure on the head of the wedge. Again, in the triangle *a b c*, whilst *a b* represents, in magnitude and direction, the perpendicular pressure of the resistance on the wedge, *a c*, which is perpendicular to the centre line of the wedge, represents, in magnitude and direction, the force applied in overcoming the resistance. The ratio of the power to the weight is therefore as *b b′* to *a c*. And, as the triangle *a b b′* is similar to the triangle A B B′,

$$P : W :: bb' : ac :: B B' : A C;$$

that is, the power is to the weight as the breadth of the wedge to its length.

From the formula (*c*), the following rules for wedges acting under pressure, as distinct from impact, are deduced:—

RULE 1. To find the weight or transverse resistance. Multiply the power by the length of the wedge, and divide by the breadth of the head. The quotient is the weight.

RULE 2. To find the power. Multiply the weight or transverse resistance by the breadth of the head, and divide by the length of the wedge. The quotient is the power.

RULE 3. To find the length of the wedge. Multiply the weight by the breadth of the wedge, and divide by the power. The quotient is the length of the wedge.

RULE 4. To find the breadth of the wedge. Multiply the power by the length of the wedge, and divide by the weight. The quotient is the breadth of the wedge.

Note.—1. The length of the wedge is taken as the distance from the head to the point of intersection of the sides.

2. The power may be applied at the point of the wedge by drawing, instead of at the head by pressing.

3. The power may be applied in a direction parallel to one of the sides of the wedge, and the relation of the power to the weight may be found by construction, in the same manner as for the inclined plane, when the power is applied in a direction parallel to the base. See proportion (*b*), page 307.

The Screw.

The screw is an inclined plane lapped round a cylinder. Take, for example, an inclined plane A B C, Fig. 127, and bend it into a circular form, resting on its base, Fig. 128, so that the ends meet. The incline may be

Fig. 127.

continued winding upwards round the same axis, and thus winding or helical inclined planes of any required length and height may be constructed. The helix thus arrived at being placed upon a solid cylinder, and the dead parts of the helix removed, the product is an ordinary screw. The inclined fillet is the "thread" of the screw, and the screw is called "external." But the thread may also be applied within a hollow cylinder, and then it is "internal," such as an ordinary "nut" is.

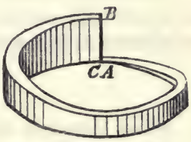


Fig. 128.

The distance of two consecutive coils apart, measured from centre to centre, or from upper side to upper side,—literally the height of the inclined plane,—for one revolution, is the "pitch" of the screw.

The effect of a screw is estimated in terms of the pitch and the radius of the handle employed to turn either it or the nut, one on the other; and the leverage of the power is the ratio of the circumference of the circle described by the power end of the handle to the pitch. The radius is to be measured to the central point where the power is applied.

The circumference being equal to the radius multiplied by twice 3.1416, or 6.28,

$$P : W :: p : r \times 6.28,$$

in which p is the pitch and r the radius; also

$$6.28\ P\ r = W \times p; \ldots\ldots\ldots\ldots \quad (d)$$

that is, 6.28 times the product of the power by the radius of the handle is equal to the product of the weight by the pitch. Whence the following rules relative to the power of a screw, for finding any one of those four quantities when the other three are given:—

Rule 1. To find the power. Multiply the weight by the pitch, and

divide by the radius of the handle and by 6.28. The quotient is the power. Or

$$P = \frac{W p}{6.28\ r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

RULE 2. To find the weight. Multiply the power by the radius and by 6.28, and divide by the pitch. The quotient is the weight. Or

$$W = \frac{6.28\ P r}{p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

RULE 3. To find the pitch. Multiply the power by the radius of the handle and by 6.28, and divide by the weight. The quotient is the pitch. Or

$$p = \frac{6.28\ P r}{W} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

RULE 4. To find the radial length of the handle. Multiply the weight by the pitch, and divide by the power and by 6.28. The quotient is the length of the handle. Or

$$r = \frac{W p}{6.28\ P} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Note.—When the power is applied through a wheel fixed to the screw, the acting diameter of the wheel may be substituted for the radius in the above rules and formulas, and the constant becomes 3.14.

Similarly, should the power-wheel be fixed to the nut so as to turn it upon the screw, instead of the screw within the nut, the same substitutions may be made.

WORK.

Work consists of the sustained exertion of pressure through space.

The English unit of work is one foot-pound; that is, a pressure of one pound exerted through a space of one foot.

The French unit of work is one kilogrammetre; that is, a pressure of one kilogramme exerted through a space of one metre.

One kilogrammetre is equal to 7.233 foot-pounds.

In the performance of work by means of mechanism, the work done upon the weight is equal to the work done by the power. This principle of the equality of work is deducible from the principle of the equality of moments, and is expressed generally by the equation

$$P \times H = W \times h, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (a)$$

in which H is the height or space moved through by the power, and h the height or space moved through by the weight at the same time. It signifies that the product of the power by the space through which it has acted is equal to the product of the weight by the space through which it has acted.

Again,

$$P : W :: h : H,$$

signifying that the power is to the weight inversely as the respective heights or spaces moved through by them in the same time.

Work done with the Lever.

On the principle of the equality of moments, the power and the weight in the lever, neglecting frictional resistance, are to each other inversely as the lengths of the arms upon which they act, that is, of their radii of motion; and inversely as the arcs or spaces passed through or described by the ends of the arms. If the weighted lever, Fig. 99, page 296, be moved by the power, the power descends through an arc at *a*, and the weight is raised through an arc at *c*. These arcs may be taken as the heights moved through, and are proportional to the lengths of the respective arms, *a b*, *b c*. In this example, these are as 7 to 1, and if the power descend 7 inches the weight is raised only 1 inch; but the weight raised is seven times the power applied, and "what is gained in power is lost in speed," or, more correctly, in space moved through. The equality of work thus developed from the equality of moments is thus expressed

$$\text{power} \times \text{arc } a = \text{weight} \times \text{arc } c \ldots\ldots\ldots\ldots \quad (a)$$

To show this arithmetically, let the weight be raised through 1 foot; then, with a leverage of 7 to 1, the power descends 7 feet, and taking it, as before, at 60 lbs., the weight it raises will be 60 lbs. × 7 = 420 lbs., and the equation of work is

$$\begin{array}{ccc} 60 \text{ lbs.} \times 7 \text{ feet} & = & 420 \text{ lbs.} \times 1 \text{ foot.} \\ \text{(or 420 foot-pounds)} & & \text{(or 420 foot-pounds).} \end{array}$$

Again,

$$\text{power} : \text{weight} :: \text{arc } c : \text{arc } a,$$

expressing the principle of virtual velocities, the relative velocities being indicated by the arcs *a*, *c*.

Work done with the Pulley.

In using the single fixed pulley, Fig. 111, page 302, the power is equal to the weight, and the spaces through which they move in the same time are equal.

With the movable pulley, Fig. 114, the weight is suspended at the axle, and in raising the weight 1 foot, the power at the circumference, with a leverage of 2, passes through 2 feet and is only half the weight. If P and W be 20 lbs. and 40 lbs. respectively, the equality of work is thus expressed—

$$\text{(P) } 20 \text{ lbs.} \times 2 \text{ feet} = \text{(W) } 40 \text{ lbs.} \times 1 \text{ foot} = 40 \text{ foot-pounds};$$

and by means of this pulley a weight double the power is raised half the height through which the power is applied.

Conversely, when the weight is suspended at the circumference of the movable pulley, Fig. 115, and the power applied at the axle, the leverage is ½; the power is therefore double the weight, and moves through 1 foot whilst the weight moves through 2 feet. Thus

$$\text{(P) } 40 \text{ lbs.} \times 1 \text{ foot} = \text{(W) } 20 \text{ lbs.} \times 2 \text{ feet} = 40 \text{ foot-pounds}.$$

In a system of fast and loose pulley blocks, Fig. 117, page 303, the power being equal to the weight divided by the number of ropes, then, by

equality of work, the space through which the power is moved is equal to the height through which the weight is raised, multiplied by the number of ropes. Suppose that there are three movable pulleys and six ropes; if the weight, 120 lbs., be raised 1 foot, each rope is shortened 1 foot and the power is moved 6 feet. And

$$(P)\ 20 \text{ lbs.} \times 6 \text{ feet} = (W)\ 120 \text{ lbs.} \times 1 \text{ foot} = 120 \text{ foot-pounds.}$$

Work done with the Wheel and Axle.

While the wheel, Fig. 119, page 305, makes one revolution, the axle also makes one. The power descends or traverses a space equal to the circumference of the wheel $= 2\ (a\ b) \times 3.1416$, whilst the weight is raised through a space equal to the circumference of the axle $= 2\ (b\ c) \times 3.1416$. If the radius of the wheel be 1 foot 6 inches, and that of the axle 3 inches, the circumferences are 9.42 feet and 1.57 feet, being as 6 to 1; and the power and the weight, conversely, are as 1 to 6. If the power be 20 lbs., then

$$\begin{array}{cc} (P)\ 20 \text{ lbs.} \times 9.42 \text{ feet} = & (W)\ 120 \text{ lbs.} \times 1.57 \text{ feet.} \\ (188.4 \text{ foot-pounds}) & (188.4 \text{ foot-pounds}). \end{array}$$

Work done with the Inclined Plane.

The weight is raised in opposition to gravity, and the work done on it is expressed by the product of the weight into the vertical height of the inclined plane. The work done by the power is expressed by the product of the power into the length of the plane. These two products express equal quantities of work, and

$$P \times l = W \times h,$$

as before intimated at (*a*), page 307, to express equality of moments.

For example, the length of the plane is 24 feet and the height 2 feet; the weight is 120 lbs., the power 10 lbs. Then, the work done in raising the weight up the whole of the incline is 240 lbs., thus

$$\begin{array}{cc} (P)\ 10 \text{ lbs.} \times 24 \text{ feet} = & (W)\ 120 \text{ lbs.} \times 2 \text{ feet.} \\ (240 \text{ foot-pounds}) & (240 \text{ foot-pounds}). \end{array}$$

The power is here supposed to be applied in a direction parallel to the plane. If applied in a direction at an angle to the plane, as in Fig. 122, page 308, it is to be resolved into its components, parallel and perpendicular to the plane. Draw the line $b\ c'$ parallel to the incline; then the power applied, $b\ c$, is equivalent to the force actually expended $b\ c'$, and to the pressure without motion $c\ c'$. The consumption of power is expressed by the product of its parallel equivalent, $b\ c'$, into the length of the plane. Taking, for example, as above, the weight, 120 lbs., and the active power, 10 lbs., represented by the parallel force $b\ c'$; then the amount of the horizontal force, or the power applied, $b\ c$, is found by proportion, thus

$$A\,C : A\,B :: b\,c' : b\,c;$$

that is, the parallel and horizontal forces are to each other as the base to the length of the incline.

Work done with the Wedge.

Supposing the wedge driven by a constant pressure through a distance equal to its length, the work done by the power is expressed by the power into the length, and the work done on the weight is expressed by the product of the weight into the breadth of the wedge. By equality of work,

$$P \times L = W \times B,$$

as before stated, in expressing equality of moments.

If the wedge be driven for only a part of its length, the work done by the power is in the proportion of the part of the length driven; and the work done on the weight is similarly in the proportion of the part of the breadth by which the resisting surfaces are separated.

Work done with the Screw.

In one revolution of the screw, the weight is raised through a height equal to the pitch of the thread, whilst the power acts through the circumference of the circle described by the point at which it is applied to a lever. The products of the power and the weight by the spaces described by them are equal, or

$$P \times 6.28\, r = W \times p,$$

as before stated (page 311) to express equality of moments.

Work done by Gravity.

The work done by gravity on a falling body is equal to the weight of the body multiplied by the height through which it falls.

Work accumulated in Moving Bodies.

The quantity of work stored in a body in motion is the same as that which would be accumulated in it by gravity if it fell from such a height as would be sufficient to give it the same velocity; in short, from the height due to the velocity. (See Gravity, page 277). The accumulated work expressed in foot-pounds, is equal to the height so found in feet, multiplied by the weight of the body in pounds. The height due to the velocity is equal to the square of the velocity divided by 64.4, and the work and the velocity may be found directly from each other, according to the following rules:—

Rule 1. Given the weight and velocity of a moving body, to find the work accumulated in it. Multiply the weight in pounds by the square of the velocity in feet per second, and divide by 64.4. The quotient is the accumulated work in foot-pounds.

Or, putting U for the work, v for the velocity, and w for the weight,

$$U = \frac{v^2 \times w}{64.4} \quad \ldots\ldots\ldots\ldots (1)$$

Or, secondly:—Multiply the weight in pounds by the height in feet due to the velocity. The product is the accumulated work in foot-pounds. Or, putting h for the height,

$$U = w \times h \quad \ldots\ldots\ldots\ldots (1a)$$

Work done by Percussive Force.

If a wedge be driven by blows or strokes of a hammer or other heavy mass, the effect of the percussive force is measured by the quantity of work accumulated in the striking body. This work is calculated by the preceding rules, from the weight of the body and the velocity with which the blow is delivered, or directly from the height of the fall, if gravity be the motive power.

The useful work done through the wedge is equal to the work delivered upon the wedge, supposing that there is no elastic or vibrating reaction from the blow, just as if the work had been delivered by a constant pressure equal to the weight of the striking body, exerted through a space equal to the height of the fall, or the height due to its final velocity.

Of course, in order to give effect to the constant pressure on the wedge, now imagined to be brought into action, the pressure would require to be applied to the resisting medium through some combination of the mechanical elements.

But where elastic action intervenes, a portion of the work delivered is uselessly absorbed in elastically straining the resisting body; and the elastic action may be, in some situations, so excessive as to absorb the whole of the work delivered. In this case, there would not be any useful work done.

These remarks, applied to the action of a blow on a wedge, are applicable equally to the action of a blow of the monkey of a pile-driver upon a pile. If there be no elastic action, the work delivered being the product of the weight of the monkey by the height of its fall, is equal to the work done in sinking the pile: that is, to the product of the frictional and other resistance to its descent by the depth through which it descends for one blow of the monkey.

Supposing that the pile rests upon and is absolutely resisted by a hard unyielding obstacle, the work done becomes wholly useless, and consists of elastic or vibrating action; or it may be that the head of the pile is split open.

HEAT.

THERMOMETERS.

The action of Thermometers is based on the change of volume to which bodies are subject with a change of temperature, and they serve, as their name implies, to measure temperature. Thermometers are filled with air, water, or mercury. Mercurial thermometers are the most convenient, because the most compact. They consist of a stem or tube of glass, formed with a bulbous expansion at the foot to contain the mercury, which expands into the tube. The stem being uniform in bore, and the apparent expansion of mercury in the tube being equal for equal increments of temperature, it follows that if the scale be graduated with equal intervals, these will indicate equal increments of temperature. A sufficient quantity of mercury having been introduced, it is boiled to expel air and moisture, and the tube is hermetically sealed. The freezing and the boiling points on the scale are then determined respectively by immersing the thermometer in melting ice and afterwards in the steam of water boiling under the mean atmospheric pressure, 14.7 lbs. per square inch, and marking the two heights of the column of mercury in the tube. The interval between these two points is divided into 180 degrees for Fahrenheit's scale, or 100 degrees for the Centigrade scale, and degrees of the same interval are continued above and below the standard points as far as may be necessary. It is to be noted that any inequalities in the bore of the glass must be allowed for by an adaptation of the lengths of the graduations. The rate of expansion of mercury is not strictly constant, but increases with the temperature, though, as already referred to, this irregularity is more or less nearly compensated by the varying rates of expansion of glass.

In the *Fahrenheit Thermometer*, used in Britain and America, the number 0° on the scale corresponds to the greatest degree of cold that could be artificially produced when the thermometer was originally introduced. 32° ("the freezing-point") corresponds to the temperature of melting ice, and 212° to the temperature of pure boiling water—in both cases under the ordinary atmospheric pressure of 14.7 lbs. per square inch. Each division of the thermometer represents 1° Fahrenheit, and between 32° and 212° there are 180°.

In the *Centigrade Thermometer*, used in France and in most other countries in Europe, 0° corresponds to melting ice, and 100° to boiling water. From the freezing to the boiling point there are 100°.

In the *Réaumur Thermometer*, used in Russia, Sweden, Turkey, and Egypt, 0° corresponds to melting ice, and 80° to boiling water. From the freezing to the boiling point there are 80°.

Each degree Fahrenheit is $\frac{5}{9}$ of a degree Centigrade, and $\frac{4}{9}$ of a degree Réaumur, and the relations between the temperatures indicated by the different thermometers are as follows:—

$$C. = \tfrac{5}{9}\,(F. - 32). \quad R. = \tfrac{4}{9}\,(F. - 32). \quad C. = \tfrac{5}{4}\,R.$$

C. being the temperature in degrees Centigrade.			
R.	do.	do.	Réaumur.
F.	do.	do.	Fahrenheit.

That is to say, that Centigrade temperatures are converted into Fahrenheit temperatures by multiplying the former by 9 and dividing by 5, and adding 32° to the quotient; and conversely, Fahrenheit temperatures are converted into Centigrade by deducting 32°, and taking $\frac{5}{9}$ths of the remainder.

Réaumur degrees are multiplied by $\frac{5}{4}$ to convert them into the equivalent Centigrade degrees; conversely, $\frac{4}{5}$ths of the number of Centigrade degrees give their equivalent in Réaumur degrees.

Fahrenheit is converted into Réaumur by deducting 32° and taking $\frac{4}{9}$ths of the remainder, and Réaumur into Fahrenheit by multiplying by $\frac{9}{4}$, and adding 32° to the product.

Tables No. 104, 105 contain equivalent temperatures in degrees Centigrade for given degrees Fahrenheit, from 0° F., or zero on the Fahrenheit scale, to 608° F.; and conversely, the temperature in degrees Fahrenheit corresponding to degrees Centigrade, from 0° C., or zero on the Centigrade scale, to 320° C.

Table No. 104.—Equivalent Temperatures by the Fahrenheit and Centigrade Thermometers.

Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.
0	− 17.78	+ 38	+ 3.34	+ 76	+ 24.45	+ 114	+ 45.56
+ 1	17.23	39	3.90	77	25.00	115	46.11
2	16.67	40	4.45	78	25.56	116	46.67
3	16.11	41	5.00	79	26.12	117	47.23
4	15.56	42	5.56	80	26.67	118	47.78
5	15.00	43	6.11	81	27.23	119	48.34
6	14.45	44	6.67	82	27.78	120	48.90
7	13.90	45	7.23	83	28.34	121	49.45
8	13.34	46	7.78	84	28.89	122	50.00
9	12.78	47	8.34	85	29.45	123	50.56
10	12.23	48	8.89	86	30.00	124	51.11
11	11.67	49	9.45	87	30.55	125	51.67
12	11.11	50	10.00	88	31.11	126	52.23
13	10.56	51	10.56	89	31.67	127	52.78
14	10.00	52	11.11	90	32.22	128	53.34
15	9.45	53	11.67	91	32.78	129	53.90
16	8.89	54	12.23	92	33.33	130	54.45
17	8.34	55	12.78	93	33.89	131	55.00
18	7.78	56	13.34	94	34.45	132	55.56
19	7.23	57	13.90	95	35.00	133	56.11
20	6.67	58	14.45	96	35.56	134	56.67
21	6.11	59	15.00	97	36.11	135	57.23
22	5.56	60	15.56	98	36.67	136	57.78
23	5.00	61	16.11	99	37.23	137	58.34
24	4.45	62	16.67	100	37.78	138	58.90
25	3.90	63	17.23	101	38.34	139	59.45
26	3.34	64	17.78	102	38.90	140	60.00
27	2.78	65	18.34	103	39.45	141	60.56
28	2.23	66	18.89	104	40.00	142	61.11
29	1.67	67	19.45	105	40.56	143	61.67
30	1.11	68	20.00	106	41.11	144	62.23
31	0.56	69	20.56	107	41.67	145	62.78
32	0.00	70	21.11	108	42.23	146	63.34
33	+ 0.56	71	21.67	109	42.78	147	63.90
34	1.11	72	22.23	110	43.34	148	64.45
35	1.67	73	22.78	111	43.90	149	65.00
36	2.23	74	23.34	112	44.45	150	65.56
37	2.78	75	23.90	113	45.00	151	66.11

Table No. 104 (*continued*).

FAHRENHEIT AND CENTIGRADE.

Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.
+ 152	+ 66.67	+ 193	+ 89.45	+ 234	+ 112.23	+ 275	+ 135.00
153	67.23	194	90.00	235	112.78	276	135.56
154	67.78	195	90.56	236	113.34	277	136.11
155	68.34	196	91.11	237	113.90	278	136.67
156	68.90	197	91.67	238	114.45	279	137.23
157	69.45	198	92.23	239	115.00	280	137.78
158	70.00	199	92.78	240	115.56	281	138.34
159	70.56	200	93.34	241	116.11	282	138.90
160	71.11	201	93.90	242	116.67	283	139.45
161	71.67	202	94.45	243	117.23	284	140.00
162	72.23	203	95.00	244	117.78	285	140.56
163	72.78	204	95.56	245	118.34	286	141.11
164	73.34	205	96.11	246	118.90	287	141.67
165	73.90	206	96.27	247	119.45	288	142.23
166	74.45	207	97.23	248	120.00	289	142.78
167	75.00	208	97.78	249	120.56	290	143.34
168	75.56	209	98.34	250	121.11	291	133.90
169	76.11	210	98.90	251	121.67	292	144.45
170	76.67	211	99.45	252	122.23	293	145.00
171	77.23	212	100.00	253	122.78	294	145.56
172	77.78	213	100.56	254	123.34	295	146.11
173	78.34	214	101.11	255	123.90	296	146.67
174	78.90	215	101.67	256	124.45	297	147.23
175	79.45	216	102.23	257	125.00	298	147.78
176	80.00	217	102.78	258	125.56	299	148.34
177	80.56	218	103.34	259	126.11	300	148.90
178	81.11	219	103.90	260	126.67	301	149.45
179	81.67	220	104.45	261	127.23	302	150.00
180	82.23	221	105.00	262	127.78	303	150.56
181	82.78	222	105.56	263	128.34	304	151.11
182	83.34	223	106.11	264	128.90	305	151.67
183	83.90	224	106.67	265	129.45	306	152.23
184	84.45	225	107.23	266	130.00	307	152.78
185	85.00	226	107.78	267	130.56	308	153.34
186	85.56	227	108.83	268	131.11	309	153.90
187	86.11	228	108.90	269	131.67	310	154.45
188	86.67	229	109.45	270	132.23	311	155.00
189	87.23	230	110.00	271	132.78	312	155.56
190	87.78	231	110.56	272	133.34	313	156.11
191	88.34	232	111.11	273	133.90	314	156.67
192	88.90	233	111.67	274	134.45	315	157.23

Table No. 104 (*continued*).

FAHRENHEIT AND CENTIGRADE.

Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.
+ 316	+ 157.78	+ 357	+ 180.56	+ 398	+ 203.34	+ 439	+ 226.11
317	158.34	358	181.11	399	203.90	440	226.67
318	158.90	359	181.67	400	204.45	441	227.23
319	159.45	360	182.23	401	205.00	442	227.78
320	160.00	361	182.78	402	205.56	443	228.34
321	160.56	362	183.34	403	206.11	444	228.90
322	161.11	363	183.90	404	206.67	445	229.45
323	161.67	364	184.45	405	207.23	446	230.00
324	162.23	365	185.00	406	207.78	447	230.56
325	162.78	366	185.56	407	208.34	448	231.11
326	163.34	367	186.11	408	208.90	449	231.67
327	163.90	368	186.67	409	209.45	450	232.23
328	164.45	369	187.23	410	210.00	451	232.78
329	165.00	370	187.78	411	210.56	452	233.34
330	165.56	371	188.34	412	211.11	453	233.90
331	166.11	372	188.90	413	211.67	454	234.45
332	166.67	373	189.45	414	212.23	455	235.00
333	167.23	374	190.00	415	212.78	456	235.56
334	167.78	375	190.56	416	213.34	457	236.11
335	168.34	376	191.11	417	213.90	458	236.67
336	168.90	377	191.67	418	214.45	459	237.23
337	169.45	378	192.23	419	215.00	460	237.78
338	170.00	379	192.78	420	215.56	461	238.34
339	170.56	380	193.34	421	216.11	462	238.90
340	171.11	381	193.90	422	216.67	463	239.45
341	171.67	382	194.45	423	217.23	464	240.00
342	172.23	383	195.00	424	217.78	465	240.56
343	172.78	384	195.56	425	218.34	466	241.11
344	173.34	385	196.11	426	218.90	467	241.67
345	173.90	386	196.67	427	219.45	468	242.23
346	174.45	387	197.23	428	220.00	469	242.78
347	175.00	388	197.78	429	220.56	470	243.34
348	175.56	389	198.34	430	221.11	471	243.90
349	176.11	390	198.90	431	221.67	472	244.45
350	176.67	391	199.45	432	222.23	473	245.00
351	177.23	392	200.00	433	222.78	474	245.56
352	177.78	393	200.56	434	223.34	475	246.11
353	178.34	394	201.11	435	223.90	476	246.67
354	178.90	395	201.67	436	224.45	477	247.23
355	179.45	396	202.23	437	225.00	478	247.78
356	180.00	397	202.78	438	225.56	479	248.34

Table No. 104 (*continued*).

FAHRENHEIT AND CENTIGRADE.

Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.	Degrees Fahr.	Degrees Centigrade.
+ 480	+ 248.90	+ 513	+ 267.23	+ 546	+ 285.56	+ 579	+ 303.90
481	249.45	514	267.78	547	286.11	580	304.45
482	250.00	515	268.34	548	286.67	581	305.00
483	250.56	516	268.90	549	287.23	582	305.56
484	251.11	517	269.45	550	287.78	583	306.11
485	251.67	518	270.00	551	288.34	584	306.67
486	252.23	519	270.56	552	288.90	585	307.23
487	252.78	520	271.11	553	289.45	586	307.78
488	253.34	521	271.67	554	290.00	587	308.34
489	253.90	522	272.23	555	290.56	588	308.90
490	254.45	523	272.78	556	291.11	589	309.45
491	255.00	524	273.34	557	291.67	590	310.00
492	255.56	525	273.90	558	292.23	591	310.56
493	256.11	526	274.45	559	292.78	592	311.11
494	256.67	527	275.00	560	293.34	593	311.67
495	257.23	528	275.56	561	293.90	594	312.23
496	257.78	529	276.11	562	294.45	595	312.78
497	258.34	530	276.67	563	295.00	596	313.34
498	258.90	531	277.23	564	295.56	597	313.90
499	259.45	532	277.78	565	296.11	598	314.45
500	260.00	533	278.34	566	296.67	599	315.00
501	260.56	534	278.90	567	297.23	600	315.56
502	261.11	535	279.45	568	297.78	601	316.11
503	261.67	536	280.00	569	298.34	602	316.67
504	262.23	537	280.56	570	298.90	603	317.23
505	262.78	538	281.11	571	299.45	604	317.78
506	263.34	539	281.67	572	300.00	605	318.34
507	263.90	540	282.23	573	300.56	606	318.90
508	264.45	541	282.78	574	301.11	607	319.45
509	265.00	542	283.34	575	301.67	608	320.00
510	265.56	543	283.90	576	302.23		
511	266.11	544	284.45	577	302.78		
512	266.67	545	285.00	578	303.34		

Table No. 105.—Equivalent Temperatures by the Centigrade and Fahrenheit Thermometers.

Degrees Cent.	Degrees Fahr.	Degrees Cent.	Degrees Fahr.	Degrees Cent.	Degrees Fahr.	Degrees Cent.	Degrees Fahr.
−20	−4.0	+21	+69.8	+62	+143.6	+103	+217.4
19	2.2	22	71.6	63	145.4	104	219.2
18	0.4	23	73.4	64	147.2	105	221.0
17	+1.4	24	75.2	65	149.0	106	222.8
16	3.2	25	77.0	66	150.8	107	224.6
15	5.0	26	78.8	67	152.6	108	226.4
14	6.8	27	80.6	68	154.4	109	228.2
13	8.6	28	82.4	69	156.2	110	230.0
12	10.4	29	84.2	70	158.0	111	231.8
11	12.2	30	86.0	71	159.8	112	233.6
10	14.0	31	87.8	72	161.6	113	235.4
9	15.8	32	89.6	73	163.4	114	237.2
8	17.6	33	91.4	74	165.2	115	239.0
7	19.4	34	93.2	75	167.0	116	240.8
6	21.2	35	95.0	76	168.8	117	242.6
5	23.0	36	96.8	77	170.6	118	244.4
4	24.8	37	98.6	78	172.4	119	246.2
3	26.6	38	100.4	79	174.2	120	248.0
2	28.4	39	102.2	80	176.0	121	249.8
1	30.2	40	104.0	81	177.8	122	251.6
0	32.0	41	105.8	82	179.6	123	253.4
+1	33.8	42	107.6	83	181.4	124	255.2
2	35.6	43	109.4	84	183.2	125	257.0
3	37.4	44	111.2	85	185.0	126	258.8
4	39.2	45	113.0	86	186.8	127	260.6
5	41.0	46	114.8	87	188.6	128	262.4
6	42.8	47	116.6	88	190.4	129	264.2
7	44.6	48	118.4	89	192.2	130	266.0
8	46.4	49	120.2	90	194.0	131	267.8
9	48.2	50	122.0	91	195.8	132	269.6
10	50.0	51	123.8	92	197.6	133	271.4
11	51.8	52	125.6	93	199.4	134	273.2
12	53.6	53	127.4	94	201.2	135	275.0
13	55.4	54	129.2	95	203.0	136	276.8
14	57.2	55	131.0	96	204.8	137	278.6
15	59.0	56	132.8	97	206.6	138	280.4
16	60.8	57	134.6	98	208.4	139	282.2
17	62.6	58	136.4	99	210.2	140	284.0
18	64.4	59	138.2	100	212.0	141	285.8
19	66.2	60	140.0	101	213.8	142	287.6
20	68.0	61	141.8	102	215.6	143	289.4

Table No. 105 (*continued*).

CENTIGRADE AND FAHRENHEIT.

Degrees Cent.	Degrees Fahr.	Degrees Cent.	Degrees Fahr.	Degrees Cent.	Degrees. Fahr.	Degrees Cent.	Degrees Fahr.
+144	+291.2	+189	+372.2	+234	+453.2	+279	+534.2
145	293.0	190	374.0	235	455.0	280	536.0
146	294.8	191	375.8	236	456.8	281	537.8
147	296.6	192	377.6	237	458.6	282	539.6
148	298.4	193	379.4	238	460.4	283	541.4
149	300.2	194	381.2	239	462.2	284	543.2
150	302.0	195	383.0	240	464.0	285	545.0
151	303.8	196	384.8	241	465.8	286	546.8
152	305.6	197	386.6	242	467.6	287	548.6
153	307.4	198	388.4	243	469.4	288	550.4
154	309.2	199	390.2	244	471.2	289	552.2
155	311.0	200	392.0	245	473.0	290	554.0
156	312.8	201	393.8	246	474.8	291	555.8
157	314.6	202	395.6	247	476.6	292	557.6
158	316.4	203	397.4	248	478.4	293	559.4
159	318.2	204	399.2	249	480.2	294	561.2
160	320.0	205	401.0	250	482.0	295	563.0
161	321.8	206	402.8	251	483.8	296	564.8
162	323.6	207	404.6	252	485.6	297	566.6
163	325.4	208	406.4	253	487.4	298	568.4
164	327.2	209	408.2	254	489.2	299	570.2
165	329.0	210	410.0	255	491.0	300	572.0
166	330.8	211	411.8	256	492.8	301	573.8
167	332.6	212	413.6	257	494.6	302	575.6
168	334.4	213	415.4	258	496.4	303	577.4
169	336.2	214	417.2	259	498.2	304	579.2
170	338.0	215	419.0	260	500.0	305	581.0
171	339.8	216	420.8	261	501.8	306	582.8
172	341.6	217	422.6	262	503.6	307	584.6
173	343.4	218	424.4	263	505.4	308	586.4
174	345.2	219	426.2	264	507.2	309	588.2
175	347.0	220	428.0	265	509.0	310	590.0
176	348.8	221	429.8	266	510.8	311	591.8
177	350.6	222	431.6	267	512.6	312	593.6
178	352.4	223	433.4	268	514.4	313	595.4
179	354.2	224	435.2	269	516.2	314	597.2
180	356.0	225	437.0	270	518.0	315	599.0
181	357.8	226	438.8	271	519.8	316	600.8
182	359.6	227	440.6	272	521.6	317	602.6
183	361.4	228	442.4	273	523.4	318	604.4
184	363.2	229	444.2	274	525.2	319	606.2
185	365.0	230	446.0	275	527.0	320	608.0
186	366.8	231	447.8	276	528.8		
187	368.6	232	449.6	277	530.6		
188	370.4	233	451.4	278	532.4		

AIR-THERMOMETERS.

Air-thermometers, or gas-thermometers, though inconvenient because bulky, are, by reason of the great expansiveness of air, superior to such as depend upon the expansion of liquids or solids, in point of delicacy and exactness. In any thermometer, whether liquid or gas, the indications depend jointly upon the expansion by heat of the fluid substance, and that of the tube which holds it. The expansion of mercury is scarcely seven times that of the glass tube within which it expands, and the exactness of its indications are interfered with by the variation in the expansiveness of glass of different qualities. In the gas-thermometer, on the contrary, the expansiveness of the gas is 160 times that of the glass, and the inequalities of the glass do not sensibly affect the indications of the instrument.

Gas-thermometers, or, as they are commonly called, air-thermometers, are designed either to maintain a constant pressure with a varying volume of air, or to maintain a constant volume of air while the pressure varies. In the first case, Fig. 119, the thermometer consists of a reservoir A, to be placed in the substance of which the temperature is to be ascertained; a tube *df*, connected at a suitable distance by a small tube *ab* to the reservoir; a tube *cd*, open above, through which mercury is introduced into the instrument; a stop-cock *r* to open or close a communication—1st, between the tube *df* and the atmosphere; 2d, between the base of the tube *cd* and the atmosphere; 3d, between the two tubes *df*, *cd*; 4th, between both these tubes and the atmosphere. The tube *df*, which is carefully gauged, answers the purpose of the graduated tube of the mercury-thermometer, and receives the air driven over by expansion from the reservoir, at the same time that it is maintained at or near the temperature of the surrounding atmosphere. Thus the air is divided between the reservoir A and the tube *df*, of which the air in the former is at the temperature of the substance under observation, and that in the latter is at the temperature of the atmosphere. These two portions of air support the same pressure, which can at all times be approximated to that of the atmosphere by means of the cock *r*, through which the mercury is allowed to escape until it arrives at the same level in the two tubes. By means of a formula embracing the respective volumes of the two portions of air and the temperature of the atmosphere, the temperature of the substance under observation is determined. But it is apparent that, when applied as a pyrometer to the measurement of high temperatures—higher, that is to say, than the boiling point of mercury (676° F.)—the greater part of the air passes by expansion into the tube *df*, leaving but a small remainder in the reservoir A. A serious objection to this is that the proportion of air which passes over into the tube *df* for a new increase of temperature is very small, and is with difficulty measured with sufficient precision.

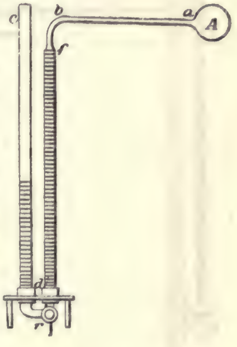


Fig. 119.—Air-Thermometer.

The second form of air thermometer, in which the pressure varies whilst the volume remains the same, was used by M. Regnault in his researches.

The temperature is measured by means of the increased elastic force of the inclosed air, and the instrument is both more convenient and more precise than that in which the volume varies, for at all temperatures the sensibility of the instrument is the same. At high temperatures the apparatus is liable to distortion under the pressure of the inclosed air; but this may be prevented, if needful, by introducing air of a lower than atmospheric pressure at an ordinary temperature, even so low as one-fourth of an atmosphere; for, although the apparatus is less sensitive in proportion as the first supply of air is of less density and pressure, yet withal it is sufficiently sensitive. The thermometer, as employed by M. Regnault, is shown in Fig. 120. Two glass tubes, *df*, *cd*, about half-an-inch bore, are united at the base by a stop-cock *r*. The tube *cd* is open above, and *df* is connected to the reservoir A by a small tube *ab*. The cover of the boiler in which the reservoir is inclosed is shown at B, and the tubes are protected from the heat of the boiler by the partition C D. By means of a three-way connection, *g*, and tube *h*, the connecting tube *ab* communicates with an air pump, by means of which the apparatus may be dried, and air or other gas supplied to it. The first thing to be done is to completely dry the apparatus, and for this object, a little mercury is passed into the tube *bd*, and the cock *r* is closed against it. The exhausting pump is then set to work to exhaust the tube, which is done several times, the air being slowly re-admitted after each exhaustion, after having been passed through a filter of pumice-stone in connection with the pump, saturated with concentrated sulphuric acid to absorb moisture, and thus desiccate the air. During this part of the process, the reservoir is maintained at a temperature of 130° F., or 140° F., to insure complete desiccation. Next, the reservoir is plunged into melting ice, the two vertical tubes *bd*, *cd*, are put into communication, and filled with mercury up to a suitable level *f*, marked on the tube *bd*. If it is desired to establish an internal pressure less than that of the atmosphere, the air is partially exhausted by means of the pump, the degree of exhaustion being recorded by the difference of level in the two tubes. The exhausting tube *h* is then hermetically sealed, and the mercury adjusted to the level *f* in the tube *bd*.

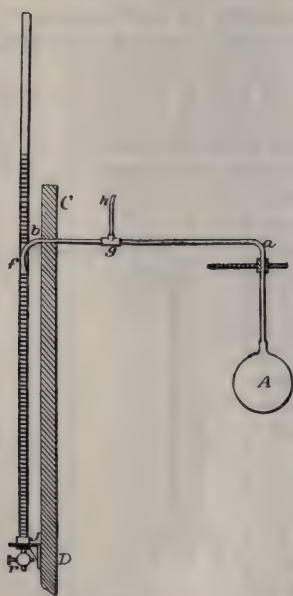


Fig. 120.

Pyrometers.

Pyrometers are employed to measure temperatures above the boiling point of mercury, about 676° F. They depend upon the change of form of either solid or gaseous bodies, liquids being necessarily inadmissible. Pyrometric estimations are of three classes:—First, those of which the

indications are based upon the change of dimensions of a particular body, solid or gaseous—the pyrometer; second, those based on the heat imparted to water by a heated body; third, those which are based upon the melting points of metals and metallic alloys.

Wedgwood's pyrometer, invented in 1782, was founded on the property possessed by clay of contracting at high temperatures, an effect which is due partly to the dissipation of the water in clay, and subsequently to partial vitrification. The apparatus consists of a metallic groove, 24 inches long, the sides of which converge, being half-an-inch wide above and three-tenths below. The clay is made up into little cylinders or truncated cones, which fit the commencement of the groove after having been heated to low redness; their subsequent contraction by heat is determined by allowing them to slide from the top of the groove downwards till they arrive at a part of it through which they cannot pass. The zero point is fixed at the temperature of low redness, 1077° F. The whole length of the groove or scale is divided into 240 degrees, each of which was supposed by Wedgwood equivalent to 130° F., the other end of the scale being assumed to represent 32,277° F. Wedgwood also assumed that the contraction of the clay was proportional to the degree of heat to which it might be exposed; but this assumption is not correct, for a long-continued moderate heat is found to cause the same amount of contraction as a more violent heat for a shorter period. Wedgwood's pyrometer is not employed by scientific men, because its indications cannot be relied upon for the reason just given, and also because the contraction of different clays under great heat is not always the same.

In Daniell's pyrometer the temperature is measured by the expansion of a metal bar inclosed in a black-lead earthenware case, which is drilled out longitudinally to $\frac{3}{10}$ inch in diameter and $7\frac{1}{2}$ inches deep. A bar of platinum or soft iron, a little less in diameter, and an inch shorter than the bore, is placed in it and surmounted by a porcelain index $1\frac{1}{2}$ inches long, kept in its place by a strap of platinum and an earthenware wedge. When the instrument is heated, the bar, by its greater rate of expansion compared with the black-lead, presses forward the index, which is kept in its new situation by the strap and wedge until the instrument cools, when the observation can be taken by means of a scale.

The *air-pyrometer*. The principle and construction of the air-thermometer are directly applicable for pyrometric purposes, substituting a platinum globe for the glass reservoir already described, for resisting great heat, and as large as possible. The chief cause of uncertainty is the expansion of the metal at high temperatures.

The second means of estimation is best represented by the "pyrometer" of Mr. Wilson, of St. Helen's. He heats a given weight of platinum in the fire of which the temperature is to be measured, and plunges it into a vessel containing twice the weight of water of a known temperature. Observing the rise of temperature in the water, he calculates the temperature to which the platinum was subjected, in terms of the rise of temperature of the water, the relative weights of the platinum and the water, and their specific heats. In fact, the elevation of the temperature of the water is to that of the platinum above the original temperature of the water in the compound ratio of the weights and specific heats inversely; that is to say, that the weights of the platinum and the water being as 1 to 2, and

their specific heats as .0314 to 1, the rise of temperature of the water is to that of the platinum as 1 × .0314 to 2 × 1, or as 1 to 63.7, and the rule for finding the temperature of the fire is to multiply the rise of temperature of the water by 63.7, and add its original temperature to the product. The sum is the temperature of the fire, subject to correction for the heat absorbed by the thermometer in the water, and by the iron vessel containing the water, and the heat retained by the platinum. The correction is estimated by Mr. Wilson at $\frac{1}{17}$th, taking the weight of water at 2000 grains, and that of the platinum 1000 grains, and it may be allowed for by increasing the above-named multiplier by $\frac{1}{17}$th, to 67.45.

Mr. Wilson proposed that for general practical purposes a small piece of Stourbridge clay be substituted for platinum, to lessen the cost of the apparatus. With a piece of such clay, weighing 200 grains, and 2000 grains of water, he found that the correct multiplier was 46.

The third means of estimation, based on the melting points of metals and metallic alloys, is applied simply by suspending in the heated medium a piece of metal or alloy of which the melting point is known, and, if necessary, two or more pieces of different melting points, so as to ascertain, according to the pieces which are melted and those which continue in the solid state, within certain limits of temperature, the heat of the furnace. A list of melting points of metals and metallic alloys is given in a subsequent chapter.

Luminosity at High Temperatures.

The luminosity or shades of temperature have been observed by M. Pouillet by means of an air-pyrometer to be as follows:—

Shade.	Temperature, Centigrade.	Temperature, Fahrenheit.
Nascent Red	525°	977°
Dark Red	700	1292
Nascent Cherry Red	800	1472
Cherry Red	900	1652
Bright Cherry Red	1000	1832
Very Deep Orange	1100	2012
Bright Orange	1200	2192
White	1300	2372
"Sweating" White	1400	2552
Dazzling White	1500	2732

A bright bar of iron, slowly heated in contact with air, assumes the following tints at annexed temperatures (Claudel):—

	Centigrade.		Fahrenheit.
1. Cold iron at about	12°	or	54°
2. Yellow at	225		437
3. Orange at	243		473
4. Red at	265		509
5. Violet at	277		531
6. Indigo at	288		550
7. Blue at	293		559
8. Green at	332		630
9. Oxide Gray *(gris d'oxyde)* at	400		752

MOVEMENTS OF HEAT.

When two bodies in the neighbourhood of each other have unequal temperatures, there exists between them a transfer of heat from the hotter of the two to the other. The tendency to an equalization, or towards an equilibrium, of temperatures in this way is universal, and the passage of heat takes place in three ways: by radiation, by conduction, and by convection or carriage from one place to another by heated currents.

Radiation of Heat from Combustibles.

It is a common assumption that the quantity of heat radiated from combustibles is very small in comparison with the total quantity of heat evolved. Holding the hand near the flame of a candle, laterally, the radiant heat, which is the only heat thus experienced, is much less than the heat experienced by the hand when held above the flame, which is the heat by convection of the hot current of air which rises from the flame. But it is to be noted that, whilst the radiant heat is dissipated all round the flame, the diameter of the upward current is little more than that of the flame, and the conveyed heat is therefore concentrated in a narrow compass.

M. Peclet, by means of a simple apparatus, consisting of a cage suspending the combustible within a hollow cylinder filled with water in an annular space, ascertained that the proportion of the total heat radiated from different combustibles was as follows:—

Radiant heat	from	wood	nearly	$\frac{1}{4}$.
Do.	do.	wood charcoal	„	$\frac{1}{2}$.
Do.	do.	oil	„	$\frac{1}{5}$.

These values serve to show that radiation of heat is considerable, and that flameless carbon radiates much more than flame, though the proportion of heat radiated from fuels depends very much upon the disposition of the material and the extent of radiating surface.

With respect to heated bodies, apart from combustibles as such, the radiation or emission of heat implies the reverse process of absorption, and the best radiators are likewise the best absorbents of heat. All bodies possess the property of radiating heat. The heat rays proceed in straight lines, and the intensity of the heat radiated from any one source of heat becomes less as the distance from the source of heat increases, in the inverse ratio of the square of the distance. That is to say, for example, that at any given distance from the source of radiation, the intensity of the radiant heat is four times as great as it is at twice the distance, and nine times as great as it is at three times the distance.

The quantity of heat emitted by radiation increases in some proportion with the difference of temperatures of the radiating body and the surrounding medium, but more rapidly than the simple proportion for the greater differences; and the quantity of heat, greater or less, emitted by bodies by radiation under the same circumstances is the measure of their *radiating power*.

Radiant heat traverses air without heating it.

When a polished body is struck by a ray of heat, it absorbs a part of the heat and reflects the rest. The greater or less proportion of heat absorbed by the body is the measure of its *absorbing power*, and the reflected heat is the measure of its *reflecting power*.

When the temperature of a body remains constant it indicates that the quantity of heat emitted is equal to the quantity of heat absorbed by the body. The reflecting power of a body is the complement of its absorbing power; that is to say, that the sum of the absorbing and reflecting powers of all bodies is the same, which amounts to this, that a ray of heat striking a body is disposed of by absorption and reflection together, that which is not absorbed being necessarily reflected.

For example, the radiating power of a body being represented by 90, the reflecting power is also 90, and the absorbing power is 10, supposing that

Table No. 106.—Comparative Radiating or Absorbent and Reflecting Powers of Substances.

Substance.	Powers.	
	Radiating or Absorbing.	Reflecting.
Lamp Black	100	0
Water	100	0
Carbonate of Lead	100	0
Writing Paper	98	2
Ivory, Jet, Marble	93 to 98	7 to 2
Isinglass	91	9
Ordinary Glass	90	10
China Ink	85	15
Ice	85	15
Gum Lac	72	28
Silver Leaf on Glass	27	73
Cast Iron, brightly polished	25	75
Mercury, about	23	77
Wrought Iron, polished	23	77
Zinc, polished	19	81
Steel, polished	17	83
Platinum, a little polished	24	76
Do. deposited on Copper	17	83
Do. in Sheet	17	83
Tin	15	85
Brass, cast, dead polished	11	89
Do. hammered, dead polished	9	91
Do. cast, bright polished	7	93
Do. hammered, bright polished	7	93
Copper, varnished	14	86
Do. deposited on iron	7	93
Do. hammered or cast	7	93
Gold, plated	5	95
Do. deposited on polished Steel	3	97
Silver, hammered, polished bright	3	97
Do. cast, polished bright	3	97

the total quantity of heat which strikes the body is represented by 100. The reflecting power of soot is sensibly *nil*, and its absorbing and radiating powers are 100.

The absorbing power varies with the nature of the source of heat, with the condition of the substance, and with the inclination of the direction of the heat radiated upon the body. That of a metallic surface is so much the less, and consequently the reflecting power is so much the more, in proportion as the surface is better polished.

The reflecting power of metals, according to MM. de la Provostaye and Desains, is practically the same, when the angle of incidence, that is the angle at which the rays of heat strike the surface, is less than 70° of inclination with the surface; but for greater angles, approaching more nearly to 90°, perpendicular to the surface, it sensibly diminishes.

For example, at angles of from 75 to 80 degrees, the reflecting power is only 94 per cent. of what it is under the smaller angles of incidence.

The table No. 106 contains the radiating and absorbing powers and the reflecting powers of various substances. *(Leslie, De la Provostaye and Desains, and Melloni.)*

The reflecting power of glass has been found to be the same for heat and for light.

Conduction of Heat.—Conduction is the movement of heat through substances, or from one substance to another in contact with it. The table No. 107 contains the relative internal conducting power of metals and earths, according to M. Despretz. A body which conducts heat well is called a good conductor of heat; if it conducts heat slowly, it is a bad conductor of heat. Bodies which are finely fibrous, as cotton, wool, eider-down, wadding, finely divided charcoal, are the worst conductors of heat. Liquids and gases are bad conductors; but if suitable provision be made for the free circulation of fluids they may abstract heat very quickly by contact with heated surfaces, acting by convection.

Convection of Heat.—Convected or carried heat is that which is transferred from one place to another by a current of liquid or gas: for example, by the products of combustion in a furnace towards the heating surface in the flues of a boiler.

Table No. 107.—Relative Internal Conducting Power of Bodies.

Substance.	Relative conducting power.	Substance.	Relative conducting power.
Gold	1000	Zinc	363
Platinum	981	Tin	304
Silver	973	Lead	180
Copper	892	Marble	24
Brass	749	Porcelain	12
Cast Iron	562	Terra Cotta	11
Wrought Iron	374		

THE MECHANICAL THEORY OF HEAT.

Heat and mechanical force are identical and convertible. Independently of the medium through which heat may be developed into mechanical action, the same quantity of heat is resolved into the same total quantity of work. The English unit of heat is that which is required to raise the temperature of 1 lb. of water at 39°.1, 1 degree Fahr. If 2 lbs. of water be raised 1 degree, or 1 lb. be raised 2 degrees in temperature, the expenditure of heat is the same in amount, namely, two units of heat; and to express the mechanical equivalent of heat, the comparison lies between the unit of heat on the one part, and the unit of work, or the foot-pound, on the other part. The most precise determination yet made of the numerical relation subsisting between heat and mechanical work was obtained by the following experiment of Dr. Joule. He constructed an agitator, Fig. 121, consisting of a vertical shaft carrying a brass paddle-wheel, of which the paddles revolved between stationary vanes, which served to prevent the liquid in the vessel from being bodily whirled in the direction of rotation. The vessel was filled with water, and the agitator was made to revolve by means of a cord wound round the upper part of the shaft, and attached to a weight which descended in front of a scale, by which the work done was measured. When all corrections had been applied, it was found that the heat communicated to the water by the agitation amounted to one pound-degree Fahrenheit for every 772 foot-pounds of work expended in producing it. It was deduced, inversely, that one unit of heat was capable of raising 772 lbs. weight 1 foot in height. The mechanical equivalent of heat, known as "Joule's equivalent," is therefore taken as 772 foot-pounds for 1 unit of heat. Sperm oil was tried as the fluid medium, and it yielded the same results as water.

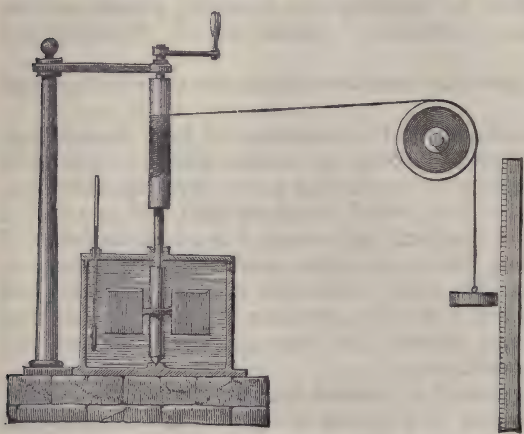


Fig. 121.—Dr. Joule's Agitator.

The following are the values of Joule's equivalent for different thermometric scales, and in English and French units:—

1 English thermal unit, or 1 degree Fahrenheit in 1 pound of water...... } 772 foot-pounds.

1 French thermal unit, or 1 degree centigrade in 1 kilogramme of water...... } 423.55 (say 424) kilogrammetres.

1 degree Centigrade in 1 pound of water—1389.60 (say 1390) foot-pounds.

1 French thermal unit is equal to 3.968 English thermal units—about 4 English units.

According to the mechanical theory of heat, in its general form, heat, mechanical force, electricity, chemical affinity, light, and sound, are but different manifestations of motion. Dulong and Gay Lussac proved by their experiments on sound, that the greater the specific heat of a gas, the more rapid are its atomic vibrations. Elevation of temperature does not alter the rapidity but increases the length of their vibrations, and in consequence produces "expansion" of the body. All gases and vapours are assumed to consist of numerous small atoms, moving or vibrating in all directions with great rapidity; but the average velocity of these vibrations can be estimated when the pressure and weight of any given volume of the gas is known, pressure being, as explained by Joule, the impact of those numerous small atoms striking in all directions, and against the sides of the vessel containing the gas. The greater the number of these atoms, or the greater their aggregate weight, in a given space, and the higher the velocity, the greater is the pressure. A double weight of a perfect gas, when confined in the same space, and vibrating with the same velocity—that is, having the same temperature—gives a double pressure; but the same weight of gas, confined in the same space, will, when the atoms vibrate with a double velocity, give a quadruple pressure. An increase or decrease of temperature is simply an increase or decrease of molecular motion. When the piston in the cylinder yields to the pressure of steam, the atoms will not rebound from it with the same velocity with which they strike, but will return after each succeeding blow, with a velocity continually decreasing as the piston continues to recede, and the length of the vibrations will be diminished. The motion gained by the piston will be precisely equivalent to the energy, heat, or molecular motion lost by the atoms of the gas; and it would be as reasonable to expect one billiard ball to strike and give motion to another without losing any of its own motion, as to suppose that the piston of a steam-engine can be set in motion without a corresponding quantity of energy being lost by some other body.

In expanding air spontaneously to a double volume, delivering it, say, into a vacuous space, it has been proved repeatedly that the air does not appreciably fall in temperature, no external work being performed; but that, on the contrary, if the air at a temperature, say, of 230° F., be expanded against an opposing pressure or resistance, as against the piston of a cylinder, giving motion to it and raising a weight or otherwise doing work, the temperature will fall nearly 170° F. when the volume is doubled, that is from 230° F. to about 60° F., and, taking the initial pressure at 40 lbs., the final pressure would be 15 lbs. per square inch.

When a pound weight of air, in expanding, at any temperature or pressure, raises 130 lbs. one foot high, it loses 1° F. in temperature; in other words, this pound of air would lose as much molecular energy as would equal the energy acquired by a weight of one pound falling through a height of 130 feet. It must, however, be remarked that but a small portion of this work—130 foot-pounds—can be had as available work, as the heat which disappears does not depend on the amount of work or duty realized, but upon the total of the opposing forces, including all resistance from any external source whatever. When air is compressed the atmosphere descends and follows the piston, assisting in the operation with its whole weight; and when air is expanded the motion of the piston is, on the contrary, opposed by the whole weight of the atmosphere, which is again raised. Although,

therefore, in expanding air, the heat which disappears is in proportion to the total opposing force, it is much in excess of what can be rendered available; and, commonly, where air is compressed the heat generated is much greater than that which is due to the work which is required to be expended in compressing it, the atmosphere assisting in the operation.

Let a pound of water, at a temperature of 212° F., be injected into a vacuous space or vessel, having 26.36 cubic feet of capacity—the volume of one pound of saturated steam at that temperature—and let it be evaporated into such steam, then 893.8 units of heat would be expended in the process. But if a second pound of water, at 212°, be injected and evaporated at the same temperature, under a uniform pressure of 14.7 lbs. per square inch, being the pressure due to the temperature, the second pound must dislodge the first, supposing the vessel to be expansible, by repelling that pressure; and this involves an amount of labour equal to 55,800 foot-pounds (that is, 14.7 lbs. $\times$ 144 square inches $\times$ 26.36 cubic feet), and an additional expenditure of 72.3 units of heat (that is, 55,800 $\div$ 772), making a total, for the second pound, of 965.1 units.

Similarly, when 1408 units of heat are expended in raising the temperature of air under a constant pressure, 1000 of these units increase the velocity of the molecules, or produce a sensible increase of temperature; while the remaining 408 units, which disappear as the air expands, are directly consumed in repelling the external pressure for the expansion of volume.

Again, if steam be permitted to flow from a boiler into a comparatively vacuous space without giving motion to another body, the temperature of the steam entering this space would rise higher than that of the steam in the boiler. Or, suppose two vessels, side by side, one of them vacuous and the other filled with air at, say, two atmospheres; if a communication be opened between them, the pressure becomes the same in both. But the temperature would fall in one vessel and rise in the other; and although the air is expanded in this manner to double its first volume, there would not, on the whole, be any appreciable loss of heat, for if the separate portions of air be mixed together, the resulting average temperature of the whole would be very nearly the same as at first. It has been proved experimentally, corroborative of this statement, that the quantity of heat required to raise the temperature of a given weight of air, to a given extent, is the same, irrespective of the density or the volume of the air. Regnault and Joule found that to raise the temperature of a pound of air, whether 1 cubic foot or 10 cubic feet in volume, the same quantity of heat was expended.

In rising against the force of gravity steam becomes colder, and it partially condenses while ascending, in the effort of overcoming the resistance of gravity. For instance, a column of steam weighing, on a square inch of base, 250.3 lbs., that is to say, having a pressure of 250.3 lbs. per square inch, would, at a height of 275,000 feet, be reduced to a pressure of 1 lb. per square inch, and, in ascending to this height, the temperature would fall from 401° to 102° F., while, at the same time, nearly 25 per cent. of the whole vapour would be precipitated in the form of water, unless it were supplied with additional heat while ascending.

If a body of compressed air be allowed to rush freely into the atmosphere, the temperature falls in the rapid part of the current, by the conversion of

heat into motion, but the heat is almost all reproduced when the motion has quite subsided. From recent experiments, it appears that nearly similar results are obtained from the emission of steam under pressure.

When water falls through a gaseous atmosphere, its motion is constantly retarded as it is brought into collision with the particles of that atmosphere, and by this collision it is partly heated and partly converted into vapour.

If a body of water descends freely through a height of 772 feet, it acquires from gravity a velocity of 223 feet per second; and, if suddenly brought to rest when moving with this velocity, it would be violently agitated, and would be raised one degree of temperature. But suppose a water-wheel, 772 feet in diameter, into the buckets of which the water is quietly dropped; when the water descends to the foot of the fall, and is delivered gently into the tail-race, it is not sensibly heated. The greatest amount of work it is possible to obtain from water falling from a given level to a lower level is expressible by the weight of water multiplied by the height of the fall.

These illustrative exhibitions of the nature and reciprocal action of heat and motive power, show that the nature and extent of the change of temperature of a gas while expanding depend nearly altogether upon the circumstances under which the change of volume takes place.

EXPANSION BY HEAT.

All bodies are expanded by the application of heat, but in different degrees. Expansion is measurable in three directions:—Length, breadth, and thickness; and it may be measured as linear expansion, in one direction; as superficial expansion, in two directions; or as cubical expansion, in three directions. Linear expansion, or the expansion of length, is that which will be exposed in the following tables for solids and liquids. The expansion of gases is measured cubically, by volume.

Superficial expansion, it may be added, is twice the linear expansion, and cubical expansion is three times the linear expansion. That is to say, the additional volume by expansion in two directions, as in length and breadth, is twice the additional volume in one direction; and the additional volume in three directions is three times that in one direction. For example, take a solid cube *a b c d e f g*; the expansion in one direction *e a*, on the face *a b c d*, is, say, equal to that indicated by the dot lines projected from that face, and the volume by expansion is equal to the extension of the surface *a b c d* thus projected. In each of the two other directions, *d a*, upwards, and *a b*, laterally, the volume by expansion is the same as that of the expansion on the face *a b c d*. Consequently, the total increase of volume by expansion, as measured cubically, in the three directions of length, breadth, and thickness, is three times the increase of volume in one direction singly; and, as measured superficially, in two of these directions, it is twice the increase of volume in one direction.

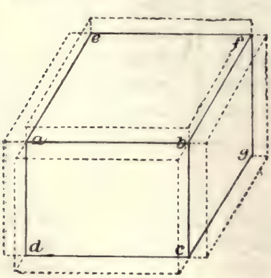


Fig. 122.

Table No. 108.—LINEAR EXPANSION OF SOLIDS BY HEAT, BETWEEN 32° AND 212° F.

METALS.	Expansion between 32° and 212° F. in common fractions.	Expansion between 32° and 212° F. in a length =100.	Expansion between 32° and 212° F. in a length of 10 feet.	Expansion for 1° F. in a length of 100 feet.
		length = 100.	inch.	inch.
Zinc, sheet	1/340	.29416	.353	.0196
Do., forged	1/321	.31083	.374	.0207
Lead	1/351	.28484	.342	.0190
Zinc 8 + 1 tin, slightly hammered,	1/372	.26917	.322	.0179
White Solder:—tin 1 + 2 lead.	1/399	.25053	.301	.0167
Tin, grain	1/403	.24833	.298	.0166
Tin	1/462	.21730	.260	.0145
Silver	1/524	.19075	.229	.0127
Speculum metal	1/517	.19333	.232	.0130
Brass	1/532	.18782	.225	.0125
Copper	1/581	.17220	.207	.0115
Gun Metal:—16 copper + 1 tin	1/524	.19083	.229	.0127
8 copper + 1 tin	1/550	.18167	.218	.0121
Yellow Brass:—Rod	1/528	.18930	.227	.0126
Do. Trough form..	1/528	.18945	.227	.0126
Gold:—				
Paris standard, annealed	1/661	.15153	.181	.0101
Do. unannealed	1/645	.15516	.186	.0103
Bismuth	1/719	.13917	.167	.00928
Iron, forged	1/819	.12204	.146	.00814
Do. wire	1/812	.12350	.148	.00823
Steel, rod, 5 feet long	1/874	.11450	.137	.00763
Do. tempered	1/807	.12396	.149	.00826
Do. not tempered	1/926	.10792	.130	.00719
Cast Iron, rod, 5 feet long	1/901	.11100	.133	.00740
Antimony	1/923	.10833	.130	.00722
Palladium	1/1000	.10000	.120	.00667
Platinum	1/1167	.08570	.103	.00571
From 0° to 300° C. (32° F. to 572° F.)				
Copper, 0° to 100° C.	1/582	.17182	.206	.0115
Copper, 0° to 300° C.	1/531	.18832	.226	.00418
Iron, 0° to 100° C.	1/846	.11821	.142	.00788
Iron, 0° to 300° C.	1/681	.14684	.176	.00326
Platinum, 0° to 100° C.	1/1103	.08842	.106	.00589
Platinum, 0° to 300° C.	1/1089	.09183	.111	.00204

Table No. 108 (*continued*).

GLASS.	Expansion between 32° and 212° F. in common fractions.	Expansion between 32° and 212° F. in a length =100.	Expansion between 32° and 212° F. in a length of 10 feet.	Expansion for 1° F. in a length of 100 feet.
			inch.	inch.
Flint Glass	1/1248	.08117	.0974	.00541
French Glass, with lead	1/1147	.08720	.105	.00581
Glass tube, without lead	1/1090	.09175	.110	.00612
Glass of St. Gobain	1/1122	.08909	.107	.00594
Barometer tubes (Smeaton)	1/1175	.08333	.100	.00555
Glass tube (Roy)	1/1289	.07755	.0931	.00517
Glass rod, solid (Roy)	1/1237	.08083	.0970	.00539
Glass (Dulong and Petit)	1/1161	.08613	.103	.00574
Do. (0° to 200° C.)	1/1032	.09484	.114	.00632
Do. (0° to 300° C.)	1/987	.10108	.121	.00674
Ice				.0333

STONES.	Initial Temperature.	Final Temperature.	Expansion in a length =100.	Expansion for 1° F. in a length of 100 feet.
			length = 100.	inch.
Granite	45° F.	220° F.	.2916	.0200
Do.	45	100	.0416	.00908
Clay-slate	46	87	.0416	.0122
Do.	46	104	.0693	.0143
York paving	46	95	.1695	.0415
Micaceous sandstone	52	200	.1736	.0141
Do. do.	52	200	.1041	.00844
Do. do.	52	150	.0832	.0102
Do. do.	52	100	.0520	.01300
Do. do.	45	100	.0416	.00908
Do. do.	45	260	.1458	.00814
Carrara marble	32	212	.0849	.00566
Sost do.	32	212	.0568	.00380
Stock Brick	52	260	.2500	.00144

Speaking exactly, the cubical expansion is rather less than three times, and the superficial expansion rather less than twice, the linear expansion; for, in fact, the expanded corners of the body are carried out to the full square figure, and have not the entering angles shown in the figure, and there is, in this way, a certain overlapping of the strata of expansion at the ends, sides, and top.

The same kind of demonstration applies to bodies of any other than a cubical shape.

A hollow body expands by heat to the same extent as if it were a solid body having the same exterior dimensions.

The rate of expansion of solids from the freezing point to the boiling point of water, 32° to 212° F., is sensibly uniform.

The table, No. 108, gives the linear expansion of a number of metals, and of glass, between the freezing and boiling points; and of ice for one degree, and of stones for various intervals of temperature. Authorities:—Laplace and Lavoisier, Smeaton, Roy, Troughton, Wollaston, Dulong and Petit, Froment, Rennie.

Zinc is the most expansible of the metals; it expands fully one-third per cent., or as much as $^1/_{321}$st part of its length, when heated from 32° F. to 212° F. Iron expands about one-seventh to one-eighth per cent.; and cast-iron and platinum about one-tenth per cent. The expansion of metals proceeds at a less rate above the boiling point than below it. Ice expands at the rate of $^1/_{36,000}$th of its length for one degree Fahrenheit; which, for 180°, would be $^1/_{200}$th of its length,—greatly more than that of any metal.

Expansion of Liquids.

The measurement of the expansion of liquids by the application of heat cannot well be taken lineally; that is, as linear expansion, in the sense in which the expansion of solids is observed. For liquids must be contained in vessels, which only admit of expansion in one direction, seeing that the liquid is limited by the bottom and sides of the vessel, which throw the whole of the expansion or enlargement of volume upwards. The observations on the expansion of liquids, therefore, though measured in one direction only, necessarily indicate the cubical expansion or total enlargement of volume. But, of course, it is easy to reduce the expansion of a liquid for comparison with the linear expansion of a solid by taking one-third of the observed measurement.

When the temperature of water at the freezing point, 32° F., is raised, the water does not at first expand, but, on the contrary, contracts in volume until the temperature is raised to 39°.1 F., which is 7.1 degrees above the freezing point. This is called "the temperature of maximum density." From this point water expands as the temperature rises, until, at 46° F., it regains its initial volume, that is, the volume at 32° F. Thence, it continues to expand until it reaches the boiling point, 212° F., under one atmosphere. Passing this point upwards, if the pressure be suitably increased, water continues to expand with a rise of temperature.

The cubical expansion of water when heated from 32° to 212° F. is .0466; that is, the volume is increased from 1 at 32° F. to 1.0466 at 212° F. This expansion is rather more than 4⅝ per cent., or between $^1/_{21}$st and $^1/_{22}$d part of the volume at 32°. The expansion of water increases in a

Table No. 109.—Expansion and Density of Pure Water, from 32° to 390° F.

(Calculated by means of Rankine's approximate formula.)

Temperature.	Comparative Volume.	Comparative Density.	Density, or weight of 1 cubic foot.	Weight of 1 gallon.	Remarkable Temperatures.
Fahr.	Water at 32° = 1.	Water at 32° = 1.	Pounds.	Pounds.	
32°	1.00000	1.00000	62.418	10.0101	Freezing point.
35	0.99993	1.00007	62.422	10.0103	
39.1	0.99989	1.00011	62.425	10.0112	Point of maximum density.
40	0.99989	1.00011	62.425	10.0112	
45	0.99993	1.00007	62.422	10.0103	
46	1.00000	1.00000	62.418	10.0101	Same volume and density as at the freezing point.
50	1.00015	0.99985	62.409	10.0087	
52.3	1.00029	0.99971	62.400	10.0072	Weight taken for ordinary calculations.
55	1.00038	0.99961	62.394	10.0063	
60	1.00074	0.99926	62.372	10.0053	
62	1.00101	0.99899	62.355	10.0000	Mean temperature.
65	1.00119	0.99881	62.344	9.9982	
70	1.00160	0.99832	62.313	9.9933	
75	1.00239	0.99771	62.275	9.9871	
80	1.00299	0.99702	62.232	9.980	
85	1.00379	0.99622	62.182	9.972	
90	1.00459	0.99543	62.133	9.964	
95	1.00554	0.99449	62.074	9.955	
100	1.00639	0.99365	62.022	9.947	Temperature of condenser water.
105	1.00739	0.99260	61.960	9.937	
110	1.00889	0.99119	61.868	9.922	
115	1.00989	0.99021	61.807	9.913	
120	1.01139	0.98874	61.715	9.897	
125	1.01239	0.98808	61.654	9.887	
130	1.01390	0.98630	61.563	9.873	
135	1.01539	0.98484	61.472	9.859	
140	1.01690	0.98339	61.381	9.844	
145	1.01839	0.98194	61.291	9.829	
150	1.01989	0.98050	61.201	9.815	
155	1.02164	0.97882	61.096	9.799	
160	1.02340	0.97714	60.991	9.781	
165	1.02589	0.97477	60.843	9.757	
170	1.02690	0.97380	60.783	9.748	
175	1.02906	0.97193	60.665	9.728	
180	1.03100	0.97006	60.548	9.711	
185	1.03300	0.96828	60.430	9.691	

Table No. 109 (*continued*).

(Calculated by means of Rankine's approximate formula.)

Temperature.	Comparative Volume.	Comparative Density.	Density, or weight of 1 cubic foot.	Weight of 1 gallon.	Remarkable Temperatures.
Fahr.	Water at 32° = 1.	Water at 32° = 1.	Pounds.	Pounds.	
190	1.03500	0.96632	60.314	9.672	
195	1.03700	0.96440	60.198	9.654	
200	1.03889	0.96256	60.081	9.635	
205	1.0414	0.9602	59.93	9.611	
210	1.0434	0.9584	59.82	9.594	
212	1.0444	0.9575	59.76	9.584	Boiling point; by formula.
212	1.0466	0.9555	59.64	9.565	Boiling point; by direct measurement.
230	1.0529	0.9499	59.36	9.520	
250	1.0628	0.9411	58.75	9.422	
270	1.0727	0.9323	58.18	9.331	
290	1.0838	0.9227	57.59	9.236	
298	1.0899	0.9175	57.27	9.185	Temperature of steam of 50 lbs. effective pressure per square inch.
338	1.1118	0.8994	56.14	9.004	Temperature of steam of 100 lbs. effective pressure per square inch.
366	1.1301	0.8850	55.29	8.867	Temperature of steam of 150 lbs. effective pressure per square inch.
390	1.1444	0.8738	54.54	8.747	Temperature of steam of 205 lbs. effective pressure per square inch.

greater ratio than the temperature. The annexed table No. 109 shows approximately the cubical expansion, comparative density, and comparative volume of water for temperatures between 32° and 212° F., calculated by means of an approximate formula constructed by Professor Rankine as follows:—

$$D_1 \text{ nearly} = \frac{2\,D_0}{\frac{t+461}{500} + \frac{500}{t+461}} \quad \cdots\cdots\cdots\cdots \quad (1)$$

in which $D_0 = 62.425$ lbs. per cubic foot, the maximum density of water, and D_1 = its density at a given temperature t F.

RULE.—*To find approximately the density of water at a given temperature, the maximum density being* 62.425 *lbs. per cubic foot.* To the given temperature in Fahrenheit degrees, add 461, and divide the sum by 500. Again, divide 500 by that sum. Add together the two quotients, and divide 124.85 by the sum. The final quotient is the density nearly.

The results given by this rule are very nearly exact for the lower temperatures, but for the higher temperatures they are too great. For 212° F. the density of water by the rule is 59.76 lbs. per cubic foot, but it is actually only 59.64 lbs., showing an error of about $^1/_{500}$th part in excess.

From the table it appears that the density of water at 46° F., or about 8° C., is the same as at the freezing point, 32° F., and that the temperature of maximum density, 39°.1 F., or 4° C., lies midway between those temperatures. The expansion of water towards and down to the freezing point is $^1/_{9000}$th part of the volume at the temperature of maximum density. It would appear that in thus expanding from 39°.1 F. downwards, the particles of water enter on a preparatory stage of separation, anticipating the still further separation which ensues on the conversion of water into the solid state; for ice is considerably lighter than water and floats on it, and its density is little more than nine-tenths that of water.

In passing upwards from the freezing point towards higher temperatures, the increase of volume of water by expansion, in parts of the volume at the freezing point, is as follows:—

		Expansion in parts of the volume at 32° F.
		per cent.
at 52°.3 F.	corresponding to the weight per cubic foot (62.4 lbs.) usually taken for ordinary calculations	.03
at 62°	the mean temperature	.10
at 100°	the temperature of condenser water	.64
at 212°	the boiling point	4.66
at 298°	the temperature of steam of 50 lbs. effective pressure per square inch	9.0
at 338°	the temperature of steam of 100 lbs. effective pressure per square inch	11.2
at 366°	the temperature of steam of 150 lbs. effective pressure per square inch	13.0
at 390°	the temperature of steam of 205 lbs. effective pressure per square inch	14.4

The expanded volume of some liquids from 32° to 212° F. is given in table No. 110; that is, the apparent expansion as seen through glass. It is shown that alcohol and nitric acid are the most expansible, and water and mercury the least; the former expand one-ninth of their initial volume, and of the latter, water, as already stated, expands $^1/_{22}$d part, and mercury $^1/_{65}$th part of their initial volumes respectively. Observations on the absolute expansion of mercury are added, and they show that whereas the apparent expansion in glass is $^1/_{65}$th part, the real expansion is $^1/_{55}$th part of the initial volume.

No other liquid besides water has a point of maximum density; that is, a point higher than the freezing point of the liquid.

Table No. 110.—EXPANSION OF LIQUIDS BY HEAT, FROM 32° to 212° F.

Apparent Expansion, in Glass.

LIQUID.	Volume at 212° F.	Expansion in Vulgar Fractions.
	volume at 32° F.=1.	volume at 32° F.=1.
Alcohol	1.1100	1/9
Nitric Acid	1.1100	1/9
Olive Oil	1.0800	1/12
Linseed Oil	1.0800	1/12
Turpentine	1.0700	1/14
Sulphuric Ether	1.0700	1/14
Hydrochloric Acid (density 1.137)	1.0600	1/17
Sulphuric Acid (density 1.850)	1.0600	1/17
Water saturated with Sea Salt	1.0500	1/20
Water	1.0466	1/22
Mercury	1.0154	1/65

Absolute Expansion of Mercury.

			Volume at 212° F.	Expansion.
Mercury,	from 32° to 212° F. (0° to 100° C.),	Dulong and Petit,	1.0180180	1/55.5
Do.	from 212° to 392° F. (100° to 200° C.),	do.	1.0184331	1/54.25
Do.	from 392° to 572° F. (200° to 300° C.),	do.	1.0188679	1/53
Do.	from 32° to 212° F. (0° to 100° C.),	Regnault,	1.0181530	1/55.12

EXPANSION OF GASES BY HEAT.

Gases are divisible into two classes—permanent gases and vapours. Gases for which great pressure and extremely low temperatures are necessary to reduce them to the liquid form, are called *permanent gases*, and those which exist in the fluid state under ordinary temperatures, are called *vapours*.

The influence of heat in expanding a permanent gas maintained under a constant pressure, is such that, for equal increments of temperature, the increments of volume by expansion are also equal or very nearly equal; in other words, the gas expands uniformly, or very nearly uniformly, in proportion to the rise of temperature.

Again, it has been observed that when the volume of permanent gases is maintained constant, the pressure increases uniformly, or nearly uniformly, with an increase of temperature.

A perfect or ideal *gas* is one which, under a constant pressure, expands with perfect uniformity in proportion to the rise of temperature; and of which, also, when confined to a constant volume, the pressure increases with perfect uniformity in proportion to the rise of temperature.

When the temperature of atmospheric air is raised from 32° to 212° F., the following are the total increments of volume or of pressure, according to the treatment, as determined by Regnault, when the volume at 32° is taken as 1:—

AIR.	TEMPERATURE.	INCREMENT.
Pressure constant......	32° to 212° F.....	Volume increased from 1 to 1.3670.
Volume constant......	32° to 212° F.....	Pressure increased from 1 to 1.3665.

Showing that the increase of pressure, .3665, with a constant volume, is sensibly the same as, though less than, the expansion or increase of volume, .3670, when the pressure is constant.

The table No. 111 gives the expansion and the increase of pressure, for several gases, when raised from 32° to 212° F.:—

Table No. 111.—EXPANSION AND PRESSURE OF GASES RAISED FROM 32° to 212° F.

(Regnault.)

GASES.	Expansion of Gases under 1 Atmosphere.		Increase of Pressure of Gases under a Contant Volume.
	Final Volume at 212° F.	Expansion at 212° F., in Common Fractions.	Final Pressure at 212°.
	Initial volume at 32°=1.	Initial volume at 32°=1.	Initial pressure at 32°=1.
Atmospheric Air.....	1.3670	$^1/_{2.76}$	1.3665
Hydrogen............	1.3661	$^1/_{2.73}$	1.3667
Nitrogen............	—	—	1.3668
Carbonic Oxide......	1.3669	$^1/_{2.73}$	1.3667
Carbonic Acid.......	1.3710	$^1/_{2.71}$	1.3688
Nitrous Oxide.......	1.3719	$^1/_{2.72}$	1.3676
Cyanogen............	1.3877	$^1/_{2.61}$	1.3829
Sulphurous Acid.....	1.3903	$^1/_{2.60}$	1.3843

Table No. 112.—EXPANSION OF GASES RAISED FROM 32° TO 212° F., UNDER DIFFERENT PRESSURES, THESE PRESSURES REMAINING CONSTANT FOR EACH OBSERVATION.

(Regnault.)

Gas.	Pressure.		Volume at 212°.
	Millimetres.	Atmospheres.	Volume at 32° F. = 1.
Air	760	1.00	1.36706
	2525	3.32	1.36944
	2620	3.45	1.36964
Hydrogen	760	1.00	1.36613
	2545	3.35	1.36616
Carbonic Acid	760	1.00	1.37099
	2520	3.32	1.38455
Sulphurous Acid	760	1.00	1.3903
	980	1.16	1.3980

The first part of the table, No. 111, on the expansion of gases by heat, shows that the expansion, which is a little more than a third of the initial volume, is nearly the same for air, hydrogen, and carbonic oxide, which are sensibly perfect gases, and have never been liquefied. On the contrary, carbonic acid, cyanogen, and sulphurous acid have a greater enlargement of volume than those gases, and they are gases which may easily be liquefied.

The second part of the table, column 4, shows that, when the volume is constant, the pressure is increased nearly in the same proportion as the volume is increased, when the pressure is constant. This nearness of the proportions is particularly close in the cases of the three sensibly perfect gases,—air, hydrogen, and carbonic oxide.

The next table, No. 112, contains the results of Regnault's experiments on the expansion of gases from 32° to 212° F., under various constant pressures of from 1 to 3½ atmospheres. It is shown that the expansions of air and of hydrogen are sensibly the same, whether the constant pressure be 1 atmosphere or between 3 and 4 atmospheres; whilst the expansions of carbonic acid and sulphurous acid are higher at the higher pressure.

The deductions of Regnault, from his experiments, comprised the following principles:—

That for air, and all other gases except hydrogen, the coefficient of dilatation, or the increment of expansion for one degree rise of temperature, increases to some extent with their density.

That all gases possess the same coefficient of dilatation when in a state of extreme tenuity; but that this law is departed from as gases become dense.

Adopting, nevertheless, the mean of the results of the experiments of M. Regnault and of M. Rudberg, the expansion of one volume of air measured at 32° F., when heated to 212° F., under a constant pressure, will, for future calculation, be taken as equal to 0.365; the ratio of the initial to the expanded volume being as 1 to 1.365. As the expansion is uniform with the rise of the temperature through 180°, the expansion for each degree Fahr. is—

$$.365 \div 180 = \frac{1}{493.2},$$

the volume at 32° F. being = 1. The same uniform rate of expansion holds sensibly for temperatures higher than 212°; it has been verified experimentally up to 700° F., under one atmosphere. It is inferred that, conversely, air would contract uniformly under uniform reductions of temperature below 32° F., until, on arriving at 493°.2 below the freezing point, or 461°.2 F. below zero, the air would be reduced to a state of collapse, without elasticity. This point in the Fahrenheit scale has thus been adopted as that of absolute zero, standing at the foot of the natural scale of temperature; and the temperature, measured from absolute zero, or −461°.2 F., is called the *absolute temperature*.

Accordingly, if a given weight of air at 0° F. be raised in temperature to +461° F., under a constant pressure, it is expanded to twice its original volume; and if heated from 0° F. to twice 461°, or 922°, its original volume is trebled.

In brief, it follows that, sensibly,

1st. The pressure of air varies inversely as the volume when the temperature is constant.

2d. The pressure varies directly as the absolute temperature when the volume is constant.

3d. The volume varies as the absolute temperature when the pressure is constant.

4th. The product of the pressure and volume is proportional to the absolute temperature.

The absolute zero-point by different thermometrical scales is as follows:—

Reaumur	$-219°.2$
Centigrade	$-274°$
Fahrenheit	$-461°.2$

To simplify calculation, the decimal is usually dropped from the Fahrenheit temperature, which is taken as $-461°$.

The foregoing laws do not apply exactly to the expansion and contraction of the more easily condensable gases, for these, as they approach the point of liquefaction, become sensibly more compressible than air. Oxygen, nitrogen, hydrogen, nitric oxide, and carbonic oxide follow the same ratio of compression as that of air, being incondensable gases, at least as far as 100 atmospheres of pressure. Sulphurous acid, ammoniacal gas, carbonic acid, and protoxide of nitrogen, which have been proved, on the contrary, to be condensable, become sensibly more compressible than air when they are reduced to one-third or one-fourth of their original volume at atmospheric pressure. Carbonic acid, under five atmospheres, occupies only 97 per cent. of the volume which air occupies under the same pressure; and under forty atmospheres, near the condensing point, it occupies only 74 per cent., or barely three-fourths of the volume of air at the same pressure. It has, nevertheless, been established that all gases, at some distance from the point of maximum density for the pressure, beyond which point they must condense, sensibly follow the first law above recited, according to which the pressure and the density vary directly as each other, when the temperature is constant. With such limitations, they rank as perfect gases.

The table No. 113 contains examples of the progressive pressures required to compress air, nitrogen, carbonic acid, and hydrogen, into one-twentieth of their original volumes, founded on experiments made by M. Regnault. The pressures are expressed in metres of mercury, the pressure of a column of mercury one metre high being equal to 19.34 lbs. per square inch. The table shows that hydrogen is the most perfect type of gaseity. When compressed to a twentieth of its original volume, it supports something more than twenty times the original pressure. Air, on the contrary, requires a quarter of a metre less than 20 metres of pressure; nitrogen requires a fifth of a metre less; and carbonic acid, like an overloaded spring, 3¼ metres less.

Table No. 113.—Compression of Gases by Pressure under a Constant Temperature.

Ratio of the original volume to the reduced volume.	Pressure in Metres of Mercury for			
	Air.	Nitrogen.	Carbonic Acid.	Hydrogen.
	Metres.	Metres.	Metres.	Metres.
1	1.000	1.000	1.000	1.000
2	1.998	1.997	1.983	2.001
4	3.987	3.992	3.897	4.007
6	5.970	5.980	5.743	6.018
8	7.946	7.964	7.519	8.034
10	9.916	9.944	9.226	10.056
12	11.882	11.919	10.863	12.084
14	13.845	13.891	12.430	14.119
16	15.804	15.860	13.926	16.162
18	17.763	17.825	15.351	18.211
20	19.720	19.789	16.705	20.269

Note.—20 metres of mercury are equal to a pressure of 386.8 lbs. per square inch, or 26.3 atmospheres.

Relations of the Pressure, Volume, and Temperature of Air and other Gases.

In accordance with the relations of pressure, volume, and temperature above stated, it is found that air and other perfect gases, and, within practical limits, the permanent gases generally, are expanded by heat at the rate of $^1/_{461}$ part of their volume at 0° F. for each degree of temperature, under a constant pressure. If the volume at the freezing point, 32° F., be taken as the point of departure, the denominator of the fraction is $(461° + 32° =)$ 493°, and the expansion is at the rate of $^1/_{493}$ part of the volume at 32° F. for each degree of temperature. In general, for any other initial temperature the denominator of the fraction showing the rate of expansion for each degree is found by adding 461° to the initial temperature. But, for convenience of calculation, the initial temperature is usually taken at 0° F.

Similarly, the pressure of air having a given constant volume, is increased by heat at the rate of $^1/_{461}$ part of the pressure at 0° F.

The fraction of expansion when the pressure is constant, and the fraction of pressure when the volume is constant, for each degree of temperature by Fahrenheit's scale above 0°, is, then,

$$\frac{1}{461};$$

and the same fraction expresses the rate of contraction of volume for each degree of temperature below 0° F.

A number of proportions and rules for the relations of the pressure, volume, and temperature of a constant weight of a gas are readily deduced from the above defined ratios.

1. When the pressure is constant, the volume varies as the absolute temperature; or,

$$V : V' : : t + 461 : t' + 461, \text{ and}$$

$$V' = V\frac{t' + 461}{t + 461}; \qquad (1)$$

in which V is the volume of the air or other gas at the temperature t, and V′ is the volume at the temperature t'. Whence the rule—

RULE 1. *To find the volume of a constant weight of air or other permanent gas,* at any other temperature, when the volume at a given temperature is known, the pressure being constant. Multiply the given volume by the new absolute temperature, and divide by the given absolute temperature. The quotient is the new volume.

Note.—The absolute temperature is found by adding 461° to the temperature indicated by the Fahrenheit thermometer.

As a common case of the above rule, air may be taken at the mean temperature, 62° F. The increased volume, by expansion by heat, taking the initial volume = 1, is found by substitution and reduction to be as follows:—

$$V' = \frac{t' + 461}{523} \qquad (2)$$

RULE 2. *To find the increased volume of a constant weight of air,* of which the initial volume = 1, taken at 62° F., heated under a constant pressure, to a given temperature. To the given temperature add 461, and divide the sum by 523. The quotient is the increased volume by expansion.

2. When the temperature of a constant weight of air, or other gas, is constant, the volume varies inversely as the pressure; or,

$$V : V' : : p' : p, \text{ and}$$

$$V' = V\frac{p}{p'}; \qquad (3)$$

in which V and V′ are the volumes respectively at the pressures p and p'.

RULE 3. *To find the volume of a constant weight of air or other permanent gas,* for any pressure, when the volume at a given pressure is known, the temperature remaining constant. Multiply the given volume by the given pressure, and divide by the new pressure. The quotient is the new volume.

3. When the pressure and temperature of a constant weight of air or other gas both change, the volume varies in the compound ratio of the absolute temperature directly, and the pressure inversely; or,

$$V : V' : : p'(t + 461) : p(t' + 461);$$

$$\text{or } V' p'(t + 461) = V p(t' + 461), \text{ and}$$

$$V' = V\frac{p(t' + 461)}{p'(t + 461)} \qquad (4)$$

Rule 4. *To find the volume of a constant weight of air or other permanent gas* for any other pressure and temperature, when the volume is known at a given pressure and temperature. Multiply the given volume by the given pressure, and by the new absolute temperature, and divide by the new pressure, and by the given absolute temperature. The quotient is the new volume.

4. When the volume and temperature of a constant weight of air or other gas both change, the pressure varies in the compound ratio of the absolute temperature directly, and the volume inversely.

$$p : p' :: V' (t+461) : V (t'+461);$$

or $V' p' (t+461) = V p (t'+461)$, and

$$p' = p \frac{V (t'+461)}{V'(t+461)} \dots\dots (5)$$

Rule 5. *To find the pressure of a constant weight of air or other permanent gas* for any other volume and temperature, when the pressure is known for a given volume and temperature. Multiply the given pressure by the given volume, and by the new absolute temperature, and divide by the new volume, and by the given absolute temperature. The quotient is the new pressure.

For the common case, when the initial temperature is 62° F., and the initial pressure is 14.7 lbs. per square inch, the formula (5) becomes, by substitution and reduction,

$$p' = \frac{V (t'+461)}{35.58 V'} \dots\dots (6)$$

Rule 6. *To find the pressure of a constant weight of air or other gas taken at* 62° F., *and at* 14.7 *lbs. pressure per square inch*, with a given volume, for any other volume and temperature. Multiply the initial volume by the final temperature plus 461, and divide the product by the final volume, and by 35.58. The quotient is the new pressure in lbs. per square inch.

When the volume is constant, with an initial temperature of 62° F., and an initial pressure of 14.7 lbs. per square inch, the above formula (6) is simplified thus:—

$$p' = \frac{t'+461}{35.58} \dots\dots (7)$$

Rule 7. *To find the pressure of a constant weight of air or other gas taken at* 62° F., *and at* 14.7 *lbs. pressure per square inch*, with a constant volume, for a given temperature. Add 461 to the given temperature, and divide the sum by 35.58. The quotient is the pressure in lbs. per square inch.

5. The mutual relations of pressure, volume, and temperature are condensed in the following formula:—

$$V p \doteqdot t+461, \dots\dots (a)$$

the product of the volume and pressure of a constant weight of air being proportional to the absolute temperature. And, as that product bears always the same ratio to the absolute temperature, an equation may be

formed between them by multiplying the absolute temperature by a coefficient, which may be put $=a$. Then—

$$V p = a\ (t + 461); \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (b)$$

that is, the product of the volume and pressure of a constant weight of air or other permanent gas, is equal to the absolute temperature multiplied by a constant coefficient, which is to be determined for each gas according to its density.

Special Rules for One Pound Weight of a Gas.

The application of formula (b) to a particular constant weight of gas, will suffice for many purposes. Let the constant weight be one pound of gas. To settle the coefficients for the different gases, take, for example, the temperature 32° F., giving an absolute temperature of 493°, and the pressure one atmosphere, or 14.7 lbs. per square inch. The volume of one pound of air at this temperature and this pressure is as before stated, 12.387 cubic feet. Substitute these values for V, t, p, in the formula (b), then—

$$12.387 \times 14.7 = a \times 493,$$

whence the coefficient, a, for air is—

$$a = .36935, \text{ or } \frac{1}{2.7074};$$

and the formula (b) becomes, for air,

$$V p = \frac{t + 461}{2.7074}; \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (c)$$

Table No. 114.—Of Coefficients or Constants, a, in the Equation (b) for the Relations of the Volume, Pressure, and Temperature of Gases; namely, $V p = a\ (t + 461)$.

Name of gas.	Volume of one pound of gas, at 32° F., under one atmosphere.	Value of coefficient a.
	cubic feet.	
Hydrogen	178.83	5.33200, or $^1/_{0.1875}$
Gaseous steam	19.913	0.59372, or $^1/_{1.6842}$
Nitrogen	12.723	0.37937, or $^1/_{2.6359}$
Olefiant gas	12.580	0.37506, or $^1/_{2.6662}$
Air	12.387	0.36935, or $^1/_{2.7074}$
Oxygen	11.205	0.33406, or $^1/_{2.9935}$
Carbonic acid (ideal)*	8.157	0.24322, or $^1/_{4.1114}$
Do. do. (actual)	8.101	0.24155, or $^1/_{4.1399}$
Ether vapour*	4.777	0.14246, or $^1/_{7.0195}$
Vapour of mercury*	1.776	0.05296, or $^1/_{18.878}$

* The densities are computed by Rankine for the ideal condition of perfect gas.

that is to say, the volume of one pound of air, multiplied by the pressure per square inch, is equal to the absolute temperature divided by the constant 2.7074.

To adapt the formula (*b*) for other gases, the respective coefficients, or constants, are found in the same manner, in terms of the volume of one pound of each gas, at 32° F., under one atmosphere of 14.7 lbs. per square inch. They are given in table No. 114.

6. The volume of one pound of air at any pressure and any temperature is deduced as follows:—

$$V = \frac{t + 461}{2.7074\, p} \quad \text{.......... (8)}$$

Rule 8.—*To find the volume of one pound of air, of a given temperature and pressure.* Divide the absolute temperature by the pressure in lbs. per square inch, and by 2.7074. The quotient is the volume in cubic feet.

For the ordinary case when the pressure is constant at 14.7 lbs. per square inch, the formula (8) becomes, by substituting and reducing,

$$V = \frac{t + 461}{39.80} \quad \text{.......... (9)}$$

Rule 9.—*To find the volume of one pound of air under* 14.7 *lbs. pressure per square inch, at a given temperature.* Add 461 to the temperature, and divide the sum by 39.80. The quotient is the volume in cubic feet.

7. The pressure of one pound of air of any volume, and at any temperature, is found as follows:—

$$p = \frac{t + 461}{2.7074\, V} \quad \text{.......... (10)}$$

Rule 10.—*To find the pressure of one pound of air, of a given temperature and volume.* Divide the absolute temperature by the volume and by 2.7074. The quotient is the pressure in lbs. per square inch.

8. The temperature of one pound of air of any volume and pressure is found as follows:—

$$t = 2.7074\, V p - 461 \quad \text{.......... (11)}$$

Rule 11.—*To find the temperature of one pound of air, of a given volume and pressure.* Multiply the volume by the pressure in pounds per square inch, and also by 2.7074; subtract 461 from the product. The remainder is the temperature.

9. The density of air is inversely as the volume, and is expressed by an inversion of the formula (8), for the volume; thus, putting D for the density, or the weight in pounds of one cubic foot of air—

$$D = 2.7074 \frac{p}{t + 461} \quad \text{.......... (12)}$$

Rule 12.—*To find the density of air, at a given temperature and pressure.* Multiply the pressure in pounds per square inch by 2.7074, and divide by the absolute temperature. The quotient is the density, or weight in pounds of one cubic foot.

Table No. 115.—Volume, Density, and Pressure of Air at various Temperatures.

Temperature.	Volume of one pound of air at constant atmospheric pressure, 14.7 lbs. per square inch. *Datum*—Volume at 62° F. = 1.		Density, or weight of one cubic foot of air at atmospheric pressure.	Pressure of a given weight of air having a constant volume. *Datum*—Atmospheric pressure at 62° F. = 1.	
Fahrenheit.	cubic feet.	comparative volume.	pounds.	pounds per square inch.	comparative pressure.
0°	11.583	.881	.086331	12.96	.881
32	12.387	.943	.080728	13.86	.943
40	12.586	.958	.079439	14.08	.958
50	12.840	.977	.077884	14.36	.977
62	13.141	1.000	.076097	14.70	1.000
70	13.342	1.015	.074950	14.92	1.015
80	13.593	1.034	.073565	15.21	1.034
90	13.845	1.054	.072230	15.49	1.054
100	14.096	1.073	.070942	15.77	1.073
120	14.592	1.111	.068500	16.33	1.111
140	15.100	1.149	.066221	16.89	1.149
160	15.603	1.187	.064088	17.50	1.187
180	16.106	1.226	.062090	18.02	1.226
200	16.606	1.264	.060210	18.58	1.264
210	16.860	1.283	.059313	18.86	1.283
212	16.910	1.287	.059135	18.92	1.287
220	17.111	1.302	.058442	19.14	1.302
230	17.362	1.321	.057596	19.42	1.321
240	17.612	1.340	.056774	19.70	1.340
250	17.865	1.359	.055975	19.98	1.359
260	18.116	1.379	.055200	20.27	1.379
270	18.367	1.398	.054444	20.55	1.398
280	18.621	1.417	.053710	20.83	1.417
290	18.870	1.436	.052994	21.11	1.436
300	19.121	1.455	.052297	21.39	1.455
320	19.624	1.493	.050959	21.95	1.493
340	20.126	1.532	.049686	22.51	1.532
360	20.630	1.570	.048476	23.08	1.570
380	21.131	1.608	.047323	23.64	1.608
400	21.634	1.646	.046223	24.20	1.646
425	22.262	1.694	.044920	24.90	1.694
450	22.890	1.742	.043686	25.61	1.742
475	23.518	1.789	.042520	26.31	1.789
500	24.146	1.837	.041414	27.01	1.837
525	24.775	1.885	.040364	27.71	1.885
550	25.403	1.933	.039365	28.42	1.933
575	26.031	1.981	.038415	29.12	1.981
600	26.659	2.029	.037510	29.82	2.029

Table No. 115 (*continued*).

Temperature.	Volume of one pound of air at constant atmospheric pressure, 14.7 lbs. per square inch. *Datum*—Volume at 62° F. = 1.		Density, or weight of one cubic foot of air at atmospheric pressure.	Pressure of a given weight of air having a constant volume. *Datum*—Atmospheric pressure at 62° F. = 1.	
Fahrenheit.	cubic feet.	comparative volume.	pounds.	pounds per square inch.	comparative pressure.
650	27.915	2.124	.035822	31.23	2.124
700	29.172	2.220	.034280	32.63	2.220
750	30.428	2.315	.032865	34.04	2.315
800	31.685	2.411	.031561	35.44	2.411
850	32.941	2.507	.030358	36.85	2.507
900	34.197	2.602	.029242	38.25	2.602
950	35.453	2.698	.028206	39.66	2.698
1000	36.710	2.793	.027241	41.06	2.793
1500	49.274	3.749	.020295	55.12	3.749
2000	61.836	4.705	.016172	69.17	4.705
2500	74.400	5.661	.013441	83.22	5.661
3000	86.962	6.618	.011499	97.28	6.618

Note to Rules 8, 10, 11, 12.—The coefficients or constants for other gases, in the application of the preceding five formulas and rules, are given in table No. 114.

The table No. 115 contains the volume, density, and pressure of air at various temperatures from 0° to 3000° F., starting from 62° F. and 14.7 lbs. per square inch respectively as unity for the proportional volumes and pressures. The second column of the table, containing the volumes of one pound of air at different temperatures, was calculated by means of the formula (9), page 350. The third column, of comparative volumes, the volume at 62° F. being = 1, was calculated by means of formula (2), page 347. The fourth column, of density, contains the reciprocals of the volumes in column 2, but it is calculable independently by means of formula (12), page 350. The fifth column, of pressures, due to the temperatures, was calculated by means of formula (7), p. 348. The sixth column contains these pressures expressed comparatively, the atmospheric pressure, 14.7 lbs. per square inch, being taken as 1.

SPECIFIC HEAT.

The specific heat of a body signifies its capacity for heat, or the quantity of heat required to raise the temperature of the body one degree Fahrenheit, compared with that required to raise the temperature of a quantity of water of equal weight one degree. The British unit* of heat is that which is required to raise the temperature of one pound of water one degree, from 32° F. to 33° F.; and the specific heat of any other body is expressed by the quantity of heat, in units, necessary to raise the temperature of one pound weight of such body one degree.

The specific heat of water at 32° F. is represented by 1, or unity, and there are very few bodies of which the specific heat equals or exceeds that of water. Specific heats are, therefore, almost universally expressible by fractions of a unit.

* See page 332

It is necessary to fix a standard of temperature, such as the freezing point, for the datum of specific heat, as the specific heat of water is not *exactly* the same at different parts of the scale of temperatures, but increases in an appreciable degree, as well as in an increasing ratio, as the temperature rises. For temperatures not higher than 80° or 90° F., the quantity of heat required to raise the temperature of water one degree is sensibly constant; at 86° F., it is not above one-fifth per cent. in excess of that at the freezing-point. At 212° F., it is about 1⅓ per cent. in excess of that at 32° F. Above 212° F., it increases more rapidly; at 302°, it is 2⅝ per cent. more than at 32°, and at 402°, it is 4½ per cent. more.

The average specific heat of water between the freezing and the boiling points is 1.005, or one-half per cent. more than the specific heat at the freezing point.

It follows from the increasing specific heat of water, as the temperature rises, that the consumption of heat in raising the temperature is slightly greater expressed in units than in degrees of temperature. To raise, for example, one pound of water from 0° to 100° C., or from 32° to 212° F., there are required 100.5 C. units, or 180.9 F. units, of heat.

The specific heats of water in the solid, liquid, and gaseous state are grouped as follows:—

Ice	0.504
Water	1.000
Gaseous Steam	0.622

showing that in the solid state, as ice, the specific heat of water is only half that of liquid water; and that, in the gaseous state, it is a little more than that of ice, or barely five-eighths of that of liquid water.

The specific heat of all liquid and solid substances is variable, increasing sensibly as the temperature rises, and the specific heats of such bodies, as tabulated, are not to be taken as exact for all temperatures, but rather as approximate average values, sufficiently near for practical purposes. The specific heat of the same body is, however, nearly constant for temperatures under 212° F.

The specific heats of such gases, on the contrary, as are perfectly gaseous, or nearly so, do not sensibly vary with density or with temperature.

For the same body, the specific heat is greater in the liquid than in the solid state. For example:—

	Liquid.	Solid.
Water (specific heat)	1.000	0.504
Bromine „	0.111	0.084
Mercury „	0.0333	0.0319

M. Regnault has verified, by numerous experiments, the conclusion arrived at by previous experimentalists, that, for metals, the specific heats are in the inverse ratio of their chemical equivalents. Consequently the products of the specific heats of metals, by their respective chemical equivalents, are a constant quantity. The same rule holds good for other groups of bodies of the same composition, and of similar chemical constitution. The specific heat of alloys is sensibly equal to the mean of those of the alloyed metals.

The following are the specific heats of water for various tempera-

tures from 0° to 230° C., or 32° to 446° F., by the air-thermometer, calculated by means of Regnault's formula:—

$$c = 1 + 0.00004\,t + 0.0000009\,t^2; \quad \ldots\ldots\ldots\ldots \quad (1)$$

in which c is the specific heat of water at any temperature t, the specific heat at the freezing point being = 1.

Table No. 116.—Specific Heat of Water.

Temperature.		Units of Heat required to raise the temperature from the freezing point to the given temperature.		Specific Heat at the given temperature.	Mean Specific Heat between the freezing point and the given temperature.
Centigrade.	Fahrenheit.	Cent. units.	Fahr. units.	Freezing point = 1.	
0°	32°	0.000	0.000	1.0000	
10	50	10.002	18.004	1.0005	1.0002
20	68	20.010	36.018	1.0012	1.0005
30	86	30.026	54.047	1.0020	1.0009
40	104	40.051	72.090	1.0030	1.0013
50	122	50.087	90.157	1.0042	1.0017
60	140	60.137	108.247	1.0056	1.0023
70	158	70.210	126.378	1.0072	1.0030
80	176	80.282	144.508	1.0089	1.0035
90	194	90.381	162.686	1.0109	1.0042
100	212	100.500	180.900	1.0130	1.0050
110	230	110.641	199.152	1.0153	1.0058
120	248	120.806	217.449	1.0177	1.0067
130	266	130.997	235.791	1.0204	1.0076
140	284	141.215	254.187	1.0232	1.0087
150	302	151.462	272.628	1.0262	1.0097
160	320	161.741	291.132	1.0294	1.0109
170	338	172.052	309.690	1.0328	1.0121
180	356	182.398	328.320	1.0364	1.0133
190	374	192.779	347.004	1.0401	1.0146
200	392	203.200	365.760	1.0440	1.0160
210	410	213.660	384.588	1.0481	1.0174
220	428	224.162	403.488	1.0524	1.0189
230	446	234.708	422.478	1.0568	1.0204

The Specific Heat of Air and other Gases.

The specific heat, or capacity for heat, of permanent gases is sensibly constant for all temperatures, and for all densities. That is to say, the capacity for heat of each gas is the same for each degree of temperature. For air, M. Regnault proved that the capacity for heat was uniform for temperatures varying from −30° C. to + 225° C. (−22° to 437° F.); thus the specific heat for equal weights of air, at constant pressure, were as follows:—

Air between	$-30°$ and $+10°$ C.	Specific heat,	0.2377
Do.	$10°$ and $+100°$ C.	Do.	0.2379
Do	$100°$ and $+225°$ C.	Do.	0.2376
	Average		0.2377

The temperature is then without any sensible influence on the specific heat of air; neither has the pressure, so far as it has been subjected to experiment—from one to ten atmospheres—any influence on the magnitude of the specific heat.

The specific heat of gases is to be observed from two points of view:—1st, When the pressure remains the same, and the gas expands by heat. 2d, When the volume remains the same, and the pressure increases with the temperature. There is a striking difference in the specific heat, or capacity for heat, according as it is measured under an increasing volume, or an increasing pressure. When the temperature is raised one degree, under constant pressure, with increasing volume, the gas not only becomes hotter to the same extent as when the volume remains the same and the pressure alone is increased, but it also expands $^1/_{493}$d part of its volume at 32° F., and thus absorbs an additional quantity of heat in proportion to the work done by expansion against the pressure. It follows that the specific heat of a gas at constant pressure is greater than that of the same gas under a constant volume; and though the former alone has been made the subject of direct experiment, the latter being of a difficult nature for experimenters, yet the latter, which is properly the specific heat, is easily deducible from the former on the principle of the mechanical theory of heat.

When the volume of a gas is enlarged by expansion against pressure, the work thus done in expanding the gas may be expressed in foot-pounds by multiplying the enlargement of volume in cubic feet by the resistance to expansion in pounds per square foot. Having thus found the work done in foot-pounds, it may be divided by Joule's equivalent, 772, and the quotient will be the expression of that work in units of heat. It becomes latent, or insensible to the thermometer, and is called the latent heat of expansion. It constitutes an expenditure of heat in addition to the heat that is sensible to the thermometer, and that raises the temperature. The sum of these two quantities of heat is that which has been observed in the gross by experimentalists, and which gives the specific heat at constant pressure.

It follows that, when the specific heat at constant pressure is known, the specific heat at constant volume may be arrived at by subtracting the proportion of heat devoted to the enlargement of the volume from the total heat absorbed at constant pressure. The remainder is the proportion of heat necessary and sufficient to elevate the temperature when the volume remains unaltered, from which the specific heat at constant volume is deduced by simple proportion; thus—

As the total heat absorbed at constant pressure,
Is to the proportion of heat absorbed at constant volume,
So is the specific heat at constant pressure
To the specific heat at constant volume.

For example, the specific heat of air at constant pressure and with in-

creasing volume has been observed to be .2377, that of water being 1. Let one pound of air at atmospheric pressure, and at 32° F., having a volume equal to 12.387 cubic feet, be expanded by heat to twice its initial volume, the pressure remaining the same. The absolute temperature, which is 32° + 461 = 493° F., will be doubled, and the indicated temperature will be 32 + 493 = 525° F. Thus, 493 degrees of heat are appropriated, and if the capacity for heat of the air were the same as that of water, 493 units of heat would be expended in the process of doubling the volume. But, as the specific heat is only .2377, or less than a fourth of that of water, the expenditure of heat is just 493 × .2377 = 117.18 units, and this quantity comprises the fraction of heat consumed in displacing the atmosphere and overcoming its resistance through a space of 12.387 cubic feet additional to the original or initial volume of the same amount. Now, the work thus done is equal to—

12.387 cubic feet × 2116.4 lbs. pressure per sq. foot = 26,216 foot-pounds;

and dividing this by 772, Joule's equivalent, the work of enlarging or doubling the volume is found to be equivalent to 33.96 units of heat. Deducting these 33.96 units from the gross expenditure, which is 117.18 units, the remainder, 83.22 units, is the proportion of heat required to raise the temperature through 493 degrees, under an increasing pressure simply, without increasing the volume; and this remainder is the datum from which the proper specific heat of air is to be deduced.

The distribution of heat thus detailed may be concisely exhibited thus:—

	Units.
To double the temperature without adding to the volume....	83.22
To double the volume, in addition..............................	33.96
To double the temperature and double the volume at constant pressure	117.18

Now, as before stated, the specific heat at constant volume bears the same ratio to that at constant pressure, as the respective quantities, or units of heat, absorbed, do to each other, or as 83.22 and 117.18; and it is found by simple proportion to be .1688; thus—

$$117.18 : 83.22 :: .2377 : .1688.$$

The proper specific heat of air is then .1688, in raising the temperature without enlarging the volume, and it bears to the so-called specific heat of air, at constant pressure and with expanding volume, the ratio of 1 to 1.408.

This ratio, 1 to 1.408, deduced by means of the mechanical theory of heat, is practically identical with the ratio experimentally arrived at by M. Masson from the fall of temperature of a quantity of compressed air, which was liberated and allowed to expand back until it regained its initial pressure. The ratio he deduced from his inverse experiment was 1 to 1.41; which is the ratio of

$$1 \text{ to } \sqrt{2}.$$

It may be added, by way of explanation, and to enforce the distinction, that though the pressure of a gas under constant volume rises with the temperature,—a phenomenon which is analogous, at first sight, to that of the volume of a gas at constant pressure increasing with the temperature,—yet there is no expenditure of work in simply raising the pressure in the former case, while the volume remains unaltered; whereas, in the latter case, there is an expenditure in increasing the volume, as has already been shown.

To generalize the foregoing process, by which the specific heat of air at constant volume has been deduced from the specific heat for constant pressure; and to show its applicability for finding the specific heat of all gases at constant volume:—

Given t = the initial temperature of the gas, in degrees Fahrenheit.
,, t' = the final temperature to which the gas is raised.
,, V = the initial volume of the gas, under one atmosphere of pressure, in cubic feet.
,, v = the final volume of the gas, heated under constant pressure.
,, h = the specific heat of the gas under constant pressure.
Put h' = the specific heat of the gas under constant volume.
,, H = the total heat expended at constant pressure, in units of heat.
,, H′ = the total heat expended at constant volume.
,, H″ = the fractional quantity of heat expended in increasing the volume, at constant pressure; or the latent heat of expansion.

To find the value of h'; then by proportion,

$$\mathrm{H} : \mathrm{H}' :: h : h',$$

$$\text{And } h' = \frac{\mathrm{H}' h}{\mathrm{H}}.$$

$$\text{Now } \mathrm{H}' = \mathrm{H} - \mathrm{H}'',$$

$$\text{And } \frac{\mathrm{H}'}{\mathrm{H}} = \frac{\mathrm{H} - \mathrm{H}''}{\mathrm{H}}, \text{ and, by substitution,}$$

$$h' = \frac{(\mathrm{H} - \mathrm{H}'')\, h}{\mathrm{H}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (a)$$

$$\text{Again, } \mathrm{H} = (t' - t) \times h,$$

$$\text{And } \mathrm{H}'' = (\mathrm{V} - v) \times 14.7 \times 144 \div 772$$

$$= (\mathrm{V} - v) \times 2.742;$$

$$\text{And } \frac{\mathrm{H} - \mathrm{H}''}{\mathrm{H}} = \frac{h\,(t' - t) - 2.742\,(\mathrm{V} - v)}{h\,(t' - t)}.$$

Substituting this value in equation (a) above,

$$h' = \frac{h\,(h\,(t' - t) - 2.742\,(\mathrm{V} - v))}{h\,(t' - t)};$$

$$\text{or } h' = \frac{h\,(t' - t) - 2.742\,(\mathrm{V} - v)}{(t' - t)} \quad \ldots\ldots\ldots\ldots \quad (b)$$

Whence the following rule:—

RULE 1. *To find the specific heat of a gas at constant volume,* when the specific heat at constant pressure is given together with the initial and final temperatures due to given initial and final volumes under an atmosphere of

pressure. Multiply the difference of the initial and final temperatures by the specific heat at constant pressure. Also, multiply the difference of the initial and final volumes by 2.742. Find the difference of these two products, and divide it by the difference of the temperatures. The quotient is the specific heat of the gas at constant volume.

Applying the rule to the example of one pound of air at atmospheric pressure, and at 32° F., doubled in volume by heat; $h = .2377$, $t' - t = 493°$, and $V - v = 12.387$ cubic feet. Then

$$h' = \frac{(.2377 \times 493) - (2.742 \times 12.387)}{493} = .1688,$$

the specific heat of air at constant volume, as already found.

The comparative volumes of other gases are given in table No. 69, page 216, of the Weight and Specific Gravity of Gases and Vapours.

The Specific Heat of Gases for Equal Volumes.

The specific heats of equal volumes of gases are deducible from their specific heats proper,—which are for equal weights. The greater the density, the less is the volume, and the greater the weight of gas that is necessary to equal in volume a lighter gas; it is greater, in fact, in proportion to the density.

Hence the following rule:—

Rule 2. *To find the specific heat of a gas for equal volumes of the gas and of air.* Multiply the specific heat of the gas, that is, the specific heat for equal weights of the gas and air, by the specific gravity of the gas. The product is the specific heat for equal volumes.

Note.—The specific heat for equal volumes may be found for constant pressure, and for constant volume, in terms respectively of the specific heat of equal weights at constant pressure and constant volume.

Tables of the Specific Heat of Solids, Liquids, and Gases.

The annexed table, No. 117, contains the specific heats of a number of solids, classified for convenience of reference, into

Metals,
Stones,
Precious Stones,
Sundry Mineral Substances,
Woods.

It appears from the tables that the metals, generally speaking, have the least specific heat: ranging from bismuth, having a specific heat of .031, to iron, which has a specific heat of from .11 to .13, and iridium, which has the greatest specific heat, namely, .189.

Stones show a specific heat of about .20, or a fifth of that of water. Precious stones average less than that.

Of the sundry mineral substances, glass, sulphur, and phosphorus average about a fifth of the specific heat of water, and coal and coke a fourth.

Woods average a half of the specific heat of water.

It is a useful practical conclusion, as Dr. Rankine remarks, that the average specific heat of the non-metallic materials and contents of a furnace, whether bricks, stones, or fuel, does not greatly differ from one-fifth of that of water.

Of the liquids specified in the table No. 118, it appears that all, with the exception of bromine, which has a specific heat of 1.111, have less specific heat than water. Olive oil has the lowest,—only .31; alcohol averages .65, and vinegar, .92.

The table No. 119 of the specific heat of gases, contains, in the second column, their specific heat, for equal weights, at constant pressure, as determined by M. Regnault. The third column contains the specific heat, for equal weights, at constant volume, calculated by means of the Rule 1, above. The fourth and fifth columns contain the specific heat of gases, for equal volumes, at constant pressure, and at constant volume, arrived at by means of the Rule 2, above.

There is a remarkable nearness to equality in the specific heat for equal volumes of air, oxygen, hydrogen, carbonic oxide, and nitrogen. It may be noted, in particular, that hydrogen, though it has fourteen times the specific heat of air for equal weights, and has barely a fourteenth of the density of air, has no more specific heat than air, for equal volumes.

Table No. 117.—Specific Heat of Solids.

(Authority, *Regnault*, when not otherwise stated.)

METALS, from 32° to 212° F.	Water at 32° = 1.
Bismuth	.03084
Lead	.0314
Platinum, sheet	.03243
Do. spongy	.03293
Do. 32° F. to 212° F. (*Petit and Dulong*)	.0335
Do. 32° F. to 572° F. (300° C.) "	.0355
Do. at 212° F. (100° C.) (*Pouillet*)	.0335
Do. at 572° F. (300° C.) "	.03434
Do. at 932° F. (500° C.) "	.03518
Do. at 1292° F. (700° C.) "	.036
Do. at 1832° F. (1000° C.) "	.03718
Do. at 2192° F. (1200° C.) "	.03818
Gold	.03244
Mercury, solid	.0319
Do. liquid	.03332
Do. 59° to 68° F. (15° to 20° C.)	.029
Do. 32° to 212° F. (*Petit and Dulong*)	.033
Do. 32° to 572° F. (300° C.) "	.035
Tungsten	.03636
Antimony	.05077
Do. 32° to 212° F. (*Petit and Dulong*)	.0507
Do. 32° to 572° F. (300° C.) "	.0547
Tin, English	.05695
Do. Indian	.05623

METALS (*continued*).	Water at 32° = 1.
Cadmium	.05669
Silver	.05701
Do. 32° to 212° F. (*Petit and Dulong*)	.0557
Do. 32° to 572° F. (300° C.) ,,	.0611
Palladium	.05927
Uranium	.0619
Molybdenum	.07218
Brass	.09391
Cymbal metal	.086
Copper	.09515
Do. 32° to 212° F. (*Petit and Dulong*)	.094
Do. 32° to 572° F. (300° C.) ,,	.1013
Zinc	.09555
Do. 32° to 212° F. (*Petit and Dulong*)	.0927
Do. 32° to 572° F. (300° C.) ,,	.1015
Cobalt	.10696
Do. carburetted	.11714
Nickel	.10863
Do. carburetted	.11192
Wrought iron	.11379
Do. 32° to 212° F. (*Petit and Dulong*)	.1098
Do. 32° to 392° F. (200° C.) ,,	.115
Do. 32° to 572° F. (300° C.) ,,	.1218
Do. 32° to 662° F. (350° C.) ,,	.1255
Steel, soft	.1165
Do. tempered	.1175
Do. Haussman	.11848
Cast iron, white	.12983
Manganese, highly carburetted	.14411
Iridium	.1887
STONES.	
Brickwork and masonry (*Rankine*) about	.20
Marble, gray	.20989
Do. white	.21585
Chalk, white	.21485
Quicklime	.2169
Dolomite (Magnesian limestone)	.21743
PRECIOUS STONES.	
Sapphire	.21732
Zircon	.14558
Diamond	.14687
SUNDRY MINERAL SUBSTANCES.	
Tellurium	.05155
Iodine	.05412
Selenium	.0837
Bromine	.0840
Phosphorus, 50° to 86° F.	.1887

Sundry Mineral Substances (*continued*).	Water at 32° = 1.
Phosphorus, 32° to 212° F.	.25034
Glass	.19768
Do. flint	.19
Do. 32° to 212° F. (*Petit and Dulong*)	.177
Do. 32° to 572° F.	.19
Sulphur	.20259
Do. crystallized, natural	.1776
Do. cast for two years	.1764
Do. cast for two months	.1803
Do. cast recently	.1844
Chloride of lead	.06641
Do. zinc	.13618
Do. manganese	.14255
Do. tin	.14759
Do. calcium	.16420
Do. potassium	.17295
Do. magnesium	.19460
Do. sodium	.214 to .230
Perchloride of tin	.10161
Protochloride of mercury	.06889
Nitrate of silver	.14352
Do. barytes	.15228
Do. potass	.23875
Do. soda	.27821
Sulphate of lead	.08723
Do. barytes	.11285
Do. potash	.1901
Carbonaceous:—	
Coal	.24111
Charcoal	.2415
Coke of cannel coal	.20307
Do. pit coal	.20085
Coal and coke, average (*Rankine*)	.20
Anthracite, Welsh	.20172
Do. American	.201
Graphite, natural	.20187
Do. of blast furnaces	.497
Animal black	.26085
Sulphate of lime	.19659
Magnesia	.22159
Soda	.23115
Ice	.504
WOODS.	
Turpentine	.467
Pear tree	.500
Oak	.570
Fir	.650

Table No. 118.—Specific Heat of Liquids.

	Water at 32° = 1.
Mercury	... .0333
Olive oil (*Laplace and Lavoisier*)	... .3096
Sulphuric acid, density 1.87 "	... .3346
Do. do. 1.30 "	... .6614
Benzine, 59° to 68° F.	... .3932
Turpentine,	... .4160
Do. density .872 (*Despretz*)	... .4720
Ether, oxalic	... .4554
Do., sulphuric, density 0.76 (*Dalton*)	... .6600
Do. do. do. 0.715 (*Despretz*)	... .5200
Essence of juniper	... .4770
Do. lemon	... .4879
Do. orange	... .4886
Hydrochloric acid	... .6000
Wood spirit, 59° to 68° F.	... .6009
Chloride of calcium, solution	... .6448
Acetic acid, concentrated	... .6581
Alcohol	... .6588
Do. density 0.793 (*Dalton*)	... .6220
Do. do. 0.81 "	... .7000
Vinegar	... .9200
Water, at 32° F.	... 1.0000
Do. at 212° F.	... 1.0130
Do. from 32° to 212° F.	... 1.0050
Bromine	... 1.1110

Table No. 119.—SPECIFIC HEAT OF GASES.

Water at 32° F. = 1.

Gas.	Specific Heat for Equal Weights.		Specific Heat for Equal Volumes.	
	At constant pressure.	At constant volume. (Real specific heat.)	At constant pressure.	At constant volume.
	water = 1.	water = 1.	air = .2377, as in col. 2	air = .1688, as in col. 3.
Sulphurous acid	0.1553	0.1246	0.3489	0.2799
Vapour of chloroform	0.1568	0.1438	0.8310	0.7621
Carbonic acid	0.2164	0.1714	0.3308	0.2620
Oxygen	0.2182	0.1559	0.2412	0.1723
Air	0.2377	0.1688	0.2377	0.1688
Nitrogen	0.2440	0.1740	0.2370	0.1690
Carbonic oxide	0.2479	0.1768	0.2399	0.1711
Olefiant gas	0.3694	0.2992	0.3572	0.2893
Hydrogen	3.4046	2.4096	0.2356	0.1667
Vapour of Benzine	0.3754	0.3499	1.0114	0.9427
Acetic ether	0.4008	0.3781	1.2184	1.1490
Vapour of alcohol	0.4513	0.4124	0.7171	0.6553
Gaseous steam	0.4750	0.3643	0.2950	0.2262
Vapour of turpentine	0.5061	0.4915	2.3776	2.3090
Ammoniacal gas	0.5080	0.3911	0.2994	0.2305
Light carburetted hydrogen	0.5929	0.4683	0.3277	0.2588

FUSIBILITY OR MELTING POINTS OF SOLIDS.

The metals are solid at ordinary temperatures, with the exception of mercury, which is liquid down to − 39° F. Hydrogen, it is believed, is a metal in a gaseous form.

All the metals are liquid at temperatures more or less elevated, and they probably vaporize at very high temperatures. Their melting points range from 39 degrees below zero of Fahrenheit's scale, the melting, or rather the freezing, point of mercury, up to more than 3000 degrees, beyond the limits of measurement by any known pyrometer. Certain of the metals, as potassium, sodium, iron, platinum, become pasty and adhesive at temperatures much below their melting points. Potassium and sodium, which melt at temperatures between 130° and 200° F., can be moulded like wax at 62° F. Two pieces of iron raised to a welding heat, are softened, and readily unite under the hammer; and pieces of platinum unite at a white heat.

The melting points of alloys do not follow the ratios of those of their constituent metals, so that it is impossible to infer their melting points from these data. A remarkable instance of the absence of this relation is afforded in the fusible metal consisting of five parts of lead, three of tin, and eight of bismuth, which melts at 212° F., the heat of boiling water, though the

melting point, if it were an average of those of the component metals, would be about 520° F. The addition of bismuth to mixtures of lead and tin has the effect of lowering the melting points.

According to Professor Rankine, the melting point of ice is lowered by pressure, at the rate of 0.0000063° F. for each pound of pressure on the square foot. An atmosphere of pressure being 2116 lbs. per square foot, the lowering of the melting point per atmosphere of pressure, is—

$$0^\circ.0000063 \times 2116 = 0^\circ.0133 \text{ Fahrenheit.}$$

To lower the melting point one degree, a pressure of 75 atmospheres would be required.

In the case of water, antimony, and cast iron, and probably other substances, the bulk of the substance in the solid state exceeds that in the liquid state, as is evidenced by the floating of ice on water, and of solid iron on molten iron. The volume of water is to that of ice at 32° F., as 1 to 1.088; that is to say, that water, in freezing at 32° F., expands nearly 9 per cent.

The following table, No. 120, contains the melting points of metals, metallic alloys, and other substances:—

Table No. 120.—Melting Points of Solids.

VARIOUS SUBSTANCES (*Pouillet, Claudel,* &c.)	Melting Points.
Sulphurous acid	– 148° F.
Carbonic acid	– 108
Bromine	+ 9.5
Turpentine	14
Hyponitric acid	16
Ice	32
Nitro-glycerine	45
Tallow	92
Phosphorus	112
Acetic acid	113
Stearine	109 to 120
Spermaceti	120
Margaric acid	131 to 140
Wax, rough	142
„ bleached	154
Stearic acid	158
Iodine	225
Sulphur	239

Table No. 120 (*continued*).

METALS.	MELTING POINTS.	
	Pouillet, Claudel.	Wilson.
	Fahrenheit degrees.	Fahrenheit degrees.
Mercury	$-39°$	—
Rubidium	—	101°
Potassium	+136	144
Sodium	194	208
Lithium	—	356
Tin	446	442
Cadmium	—	442
Bismuth	504	507
Thallium	—	561
Lead	608	617
Zinc	680	773
Antimony	810	1150
Bronze	1692	—
Aluminium	—	full red heat.
Calcium	—	full red heat.
Silver	(very pure) 1832	1873
Copper	—	1996
Gold, standard	2156	—
Gold	(very pure) 2282	2016
Cast Iron, white	1922 to 2012	—
„ „ gray	2012	2786
„ „ „ 2d melting	2192	—
„ „ with manganese	2282	—
Steel	2372 to 2552	—
Wrought Iron, French	2732	—
Hammered Iron, English	2912	—
Malleable Iron	—	Fusible in highest heat of forge.
Cobalt	—	
Nickel	—	
Manganese	—	
Palladium	—	Not fusible in forge fire, but soften and agglomerate.
Molybdenum	—	
Tungsten	—	
Chromium	—	
Platinum	—	Only fusible before the oxyhydrogen blow-pipe.
Rhodium	—	
Iridium	—	
Vanadium	—	
Ruthenium	—	
Osmium	—	

Table No. 120 (*continued*).

ALLOYS OF LEAD, TIN, AND BISMUTH.	MELTING POINTS.	
	Holtzapffel.	Claudel.
No. 1. 1 Tin, 25 Lead........................	558°	
2. 1 ,, 10 ,, ...	541	
3. 1 ,, 5 ,, ...	511	
4. 1 ,, 3 ,, ...	482	
5. 1 ,, 2 ,, ...	441	
6. 1 ,, 1 ,, ...	370	466°
7. 1½ ,, 1 ,, ...	334	
8. 2 ,, 1 ,, ...	340	385
9. 3 ,, 1 ,, ...	356	367
10. 4 ,, 1 ,, ...	365	372
11. 5 ,, 1 ,, ...	378	381
12. 6 ,, 1 ,, ...	381	
13. 4 Lead, 4 Tin, 1 Bismuth	320	
14. 3 ,, 3 ,, 1 ,, ...	310	
15. 2 ,, 2 ,, 1 ,, ...	292	
16. 1 ,, 1 ,, 1 ,, ...	254	
17. 2 ,, 1 ,, 2 ,, ...	236	
18. 3 ,, 5 ,, 8 ,, ...	202	

SUNDRY ALLOYS OF TIN, LEAD, AND BISMUTH.	MELTING POINTS.
3 Lead, 2 Tin, 5 Bismuth.............................. *Ure*	199°
1 ,, 1 ,, 2 ,, ... ,,	201
1 ,, 1 ,, 4 ,, ... *Claudel*	201
5 ,, 3 ,, 8 ,, ... *Ure*	212
2 ,, 3 ,, 5 ,, ... *Claudel*	212
1 ,, 4 ,, 5 ,, ... ,,	246
1 ,, 1 ,, ... ,,	286
1 ,, 3 ,, ... ,,	334
2 ,, 1 ,, ... ,,	334
1 ,, 2 ,, ... *Holtzapffel*	360
Claudel	385
3 ,, 1 ,, ... ,,	392
3 ,, 1 ,, ... ,,	552

ALLOYS FOR FUSIBLE PLUGS.	Softens at	Melts at
2 Tin, 2 Lead..............................	365° F.	372° F.
2 ,, 6 ,, ...	372	383
2 ,, 7 ,, ...	377½	388
2 ,, 8 ,, ...	395½	406 to 410

Latent Heat of Fusion of Solid Bodies.

When a solid body is exposed to heat, and ultimately passes into the liquid state under the influence of the heat, the temperature of the body rises until it attains the point of fusion, or melting point. The temperature of the body remains stationary at this point until the whole of it is melted; and the heat meantime absorbed, without affecting the temperature, is said to become latent, as it is not sensible to the thermometer. It is, in fact, *the latent heat of fusion*, or *the latent heat of liquidity*, and its function is to separate the particles of the body, hitherto solid, and change their condition into that of a liquid. When, on the contrary, the liquid is solidified, the latent heat is disengaged.

M. Person gave the following law as the result of his experiments on the latent heat of fusion of non-metallic substances:—Let c be the specific heat of the substance in the solid state, and c' its specific heat in the liquid state; t the temperature of fusion, or melting point, by Fahrenheit's scale, and l the latent heat. Then the latent heat of fusion of one pound, in British thermal units, is

$$l = c' - c\,(t + 256°) \qquad (1)$$

Rule.—*To find the latent heat of fusion of a non-metallic substance.* Subtract the specific heat of the substance in the solid state from its specific heat in the liquid state, and multiply the remainder by the temperature of fusion or melting point by the Fahrenheit scale, plus 256. The product is the latent heat of fusion in heat-units.

Table No. 121.—Latent Heat of Fusion of Solid Bodies.

Person.

Non-metallic.	Melting Point.	Specific Heat. Liquid.	Specific Heat. Solid.	Latent Heat in heat-units, of 1 pound.
Ice	32° F.	1.0000	.5040	142.6
Chloride of calcium	83	.5550	.3450	73
Phosphate of soda	97	.7467	.4077	120
Phosphorus	112	.2045	.1788	9
Spermaceti	120	—	—	148
Wax	142	—	—	175
Sulphur	239	.2340	.2026	17
Nitrate of soda	591	.4130	.2782	113
Nitrate of potass	642	.3319	.2388	85
Metallic.				
Tin	442	.0637	.0562	25.6
Cadmium	442	.0642	.0567	25.6
Bismuth	507	.0363	.0308	22.7
Lead	617	.0402	.0314	9.86
Zinc	773	—	.0956	50.6
Silver	1873	—	.0570	37.9

EXAMPLE.—To find the latent heat of fusion of ice, the specific heat of ice, $c = 0.504$, and that of water $c' = 1$; $t = 32°$ F. Then

the latent heat $= (1 - 0.504)\ (32° + 256) = 0.496 \times 288 =$	142.86
Do., by M. Person's experiment.......................... $=$	142.65
Difference................................	0.21

showing that the latent heat of fusion of one pound of ice is 142.86 units.

The table No. 121 contains the latent heat of fusion of several metals and other bodies, according to M. Person. On an inspection of the table, it appears generally that the latent heat of fusion of non-metallic bodies is greater for those which have the lower melting points, and that, for metals, the proportion lies rather the other way. The greatest latent heat of fusion belongs to wax, which has 175 units per pound, and the least to phosphorus and lead, which have only 9 and 9.86 units respectively per pound weight.

BOILING POINTS OF LIQUIDS.

When a cold liquid, contained in a vessel open to the air, is subjected to heat, the temperature of the liquid is raised, and a quantity of vapour is emitted from the surface of the fluid, the pressure of which gradually increases until it becomes equal to the pressure of the atmosphere. When this pressure is reached, the aggregation of the vapour becomes visible in the interior of the liquid, and the vapour rises to the surface and escapes. This is *ebullition*, or *evaporation*, or *vaporization*. When the liquid has attained to the state of ebullition, the temperature ceases to rise, and remains stationary, and it so remains until the whole of the liquid is evaporated. This phenomenon of stationary temperature is analogous to that which attends the fusion of solids into liquids.

The proper boiling point of water, under one atmosphere of pressure, is 100° C., or 212° F. It is affected to some extent by the nature of the vessel which contains it and the presence of other objects in the vessel. In a glass retort, for example, water boils with jolts and small explosions, and the temperature of the water at which ebullition takes place is from two to three degrees higher than when it is evaporated in an iron vessel. Sulphuric acid behaves similarly under ebullition, and the explosions are as much more violent as the liquid has greater cohesiveness, and as it acts chemically upon the matter of the vessel in which it boils. A few pieces of metal thrown into the glass vessel arrest the explosive ebullition, and the temperature of the liquid falls to the same level as in the metallic vessel.

The boiling point of liquids is not altered by the presence of foreign bodies mechanically in mixture with them, such as sand, sulphate of lime, and carbonate of lime. But it is always greater when matters are present in chemical combination with the liquids. All the soluble salts have this effect when dissolved in water; but, on the contrary, it has been proved experimentally,—

That the vapour produced at the surface of saline solutions is the steam of pure water,

And that at atmospheric pressure the temperature of the steam formed is invariably 212° F., whatever be the nature of the dissolved salt, or of the vessel containing the solution. It further appears that at higher pressures

Table No. 122.—BOILING POINTS OF LIQUIDS UNDER ONE ATMOSPHERE OF PRESSURE.

	Fahrenheit.
Sulphuric ether	100°
Sulphuret of carbon	118.4
Ammonia	140
Chloroform	140
Bromine	145
Wood spirit	150
Alcohol	173
Benzine	176
Water	212
Average sea-water	213.2
Saturated brine	226
Nitric acid	248
Oil of turpentine	315
Phosphorus	554
Sulphur	570
Sulphuric acid	590
Linseed oil	597
Mercury	648

Table No. 123.—BOILING POINTS OF SATURATED SOLUTIONS OF SALTS UNDER ONE ATMOSPHERE.

NAME OF SALT.	BOILING POINT.		Quantity of salt which saturates 100 parts of water.
	Centigrade.	Fahrenheit.	per cent.
Chlorate of potash	104°.2	219°.6	61.5
Chloride of barium	104.4	220.0	60.1
Carbonate of soda	104.6	220.3	48.5
Phosphate of soda	105.5	222.0	113.2
Chloride of potassium	108.3	227.0	59.4
Chloride of sodium (common salt)	108.4	227.2	41.2
Hydrochlorate of ammonia	114.2	237.6	88.9
Neutral tartrate of potash	114.67	238.4	296.2
Nitrate of potash	115.9	240.6	335.1
Chloride of strontium	117.9	244.2	117.5
Nitrate of soda	121.0	250.0	224.8
Acetate of soda	124.37	255.8	209.0
Carbonate of potash	135.0	275.0	205.0
Nitrate of lime	151.0	304.0	362.2
Acetate of potash	169.0	336.0	798.2
Chloride of calcium	179.5	355.1	325.0
Nitrate of ammonia	180.0	356.0	unlimited.

the temperatures of steam formed from sea-water are the same as those of steam generated from fresh-water under equal pressures.

Table No. 122 contains the boiling points of liquids under one atmosphere of pressure. It shows that the boiling points vary from 100° F. for sulphuric ether to 648° F. for mercury. Linseed oil boils at 597° F., and the great elevation of this temperature, as representing more or less approximately the boiling points of oils and fats generally, explains the capacity of these substances when heated for cooking meat immersed in them.

It is shown that sea-water boils at 213°.2 F. under one atmosphere, and that saturated brine does not boil until the temperature rises to 226° F. The boiling point of sea-water is raised in proportion to its concentration as brine.

Table No. 123 contains the boiling points of saturated solutions of various salts in water, under one atmosphere, according to the experiments of M. Legrand. They vary from 220° to 356° F. They present a striking diversity, even among salts having the same base.

Boiling Points of Liquids at various Pressures.

The boiling points of liquids rise as the pressure increases under which they are evaporated, and they contrast strikingly in this respect with the melting points of solids, which are practically constant under all pressures. It has already been stated that, to lower the melting point of ice only one degree Fahrenheit, 75 atmospheres of pressure were required. On the contrary the boiling point of water is raised 75 degrees by an augmentation of less than three atmospheres above the atmospheric pressure.

Table No. 124 contains a comparative statement of the pressures of the vapours of water and other liquids, at temperatures varying from 0° C., or 32° F., to 222° C., or 432° F.—in fact, their boiling points for various pressures—the results of experiments by Regnault.

The table No. 124 shows a great diversity of pressure of saturated vapours for given temperatures. At the temperature of 212° F., for example, at which water boils under one atmosphere of pressure; in other words, at which the pressure of the vapour of boiling water is 14.7 lbs. per square inch, the pressures of the saturated vapours of the several liquids are as follows:—

				per square inch.
Vapour of	water at 212° F.	....	pressure,	14.7 lbs.
Do.	alcohol ,,	....	,,	32.6 lbs.
Do.	ether ,,	....	,,	95.17 lbs.
Do.	chloroform ,,	....	,,	45.54 lbs.
Do.	turpentine ,,	....	,,	2.61 lbs.

The relations of the vapour of water or steam are fully considered in a subsequent section.

Latent Heat and Total Heat of Evaporation of Liquids.

Liquids, in the course of being transformed into vapour on the application of heat, absorb a certain quantity of heat which remains latent in the vapour, and is, on the contrary, restored to sensibility, and communicated to other bodies when the vapours are condensed into liquids. The following

Table No. 124.—Boiling Points of Saturated Vapours under various Pressures, or their corresponding Temperatures and Pressures.

Regnault.

Temperature.		Pressure per square inch of the vapour of the following liquids:—				
		Water.	Alcohol.	Ether.	Chloroform.	Turpentine.
Centigrade.	Fahrenheit.	lbs.	lbs.	lbs.	lbs.	lbs.
0°	32°	.089	.246	3.53	—	.041
10	50	.178	.466	5.54	2.52	.045
20	68	.337	.851	8.60	3.68	.083
30	86	.609	1.52	12.32	5.34	.135
40	104	1.06	2.59	17.67	7.04	.217
50	122	1.78	4.26	24.53	10.14	.333
60	140	2.88	6.77	33.47	14.27	.520
70	158	4.51	10.43	44.67	18.88	.810
80	176	6.86	15.72	57.01	26.46	1.18
90	194	10.16	23.02	75.41	35.03	1.74
100	212	14.70	32.60	95.17	45.54	2.61
110	230	20.80	45.50	120.9	58.42	3.62
116	240.8	25.37	—	137.0	—	—
120	248	29.88	62.05	—	—	4.97
130	266	39.27	83.80	—	—	6.71
136	276.8	46.87	—	—	—	—
140	284	52.56	109.1	—	—	8.94
150	302	69.27	140.4	—	—	11.70
152	305.6	73.07	147.3	—	—	—
160	320	89.97	147.3	—	—	13.10
170	338	115.3	—	—	—	19.13
180	356	146.0	—	—	—	23.70
190	374	182.6	—	—	—	29.30
200	392	226.1	—	—	—	36.09
210	410	277.1	—	—	—	43.54
220	428	336.4	—	—	—	52.04
222	431.6	349.3	—	—	—	53.74

Boiling Points under One Atmosphere.

1st. According to table No. 122.

Water.	Alcohol.	Sulphuric ether.	Chloroform.	Turpentine.
212° F.	173° F.	100° F.	140° F.	315° F.

2d. By interpolation in the above table.

212°	...... 173°	...... 94°	...... 142°	...... 335°

table, No. 125, gives, on the authority of Despretz, Favre and Silbermann, and Regnault, the latent heat of evaporation of several vapours under one atmosphere. The total heat of evaporation, reckoned from 32° F., is added in the last column. It is calculated for each liquid by multiplying its

boiling point less 32°, by its specific heat, to find the quantity of heat in units required to raise it to the boiling point, and adding this product to the latent heat.

Table No. 125.—Latent Heat and Total Heat of Evaporation of Liquids under One Atmosphere.

Liquid.	Boiling Point.	Latent Heat of Evaporation.	Total Heat, reckoned from 32° F.
	Fahrenheit.	Units of heat.	Units of heat.
Sulphuric ether	100°	175	210.4
Wood spirit	150	475	545.9
Acetic ether	165	191	—
Alcohol, pure	173	374	461.7
Water	212	965.2	1146.1
Essence of lemon	297	126	255.3
Oil of turpentine	315	124	256.6

LIQUEFACTION AND SOLIDIFICATION OF GASES.

Professor Faraday succeeded in liquefying, and even solidifying, many gases, and it is probable that all gases are susceptible of being solidified, and that they might be so condensed if sufficiently low temperatures and sufficiently strong vessels could be produced.

At – 112° F., and under a pressure less than one atmosphere, Faraday reduced the following gases to the liquid or the solid state:—Chlorine, cyanogen, ammonia, hydrosulphuric acid, arseniated hydrogen, hydriodic acid, hydrobromic acid, and carbonic acid.

The following gases were solidified at the annexed temperatures:—

Cyanogen	– 31° F.
Hydriodic acid	– 60
Carbonic acid	– 72
Oxide of chlorine	– 76
Ammonia	– 103
Sulphurous acid	– 105
Sulphuretted hydrogen	– 123
Hydrobromic acid	– 126
Protoxide of nitrogen	– 148

The following gases were not solidified, even at a temperature of – 166° F.:—

Olefiant gas.
Fluosilicic acid.
Protophosphuretted hydrogen.
Fluoboric acid.
Hydrochloric acid.
Arseniated hydrogen.

The following gases gave no sign of even approaching liquefaction, even at – 166° F., and with many atmospheres of pressure:—

Hydrogen	at	$-166°$ F.,	and 27	atmospheres.
Oxygen	„	-166	and 27	„
Do.	„	-140	and 58	„
Nitrogen	„	-166	and 50	„
Nitric oxide	„	-166	and 50	„
Carbonic oxide	„	-166	and 40	„
Coal gas	„	-166	and 32	„

The greatest known degree of cold, $-166°$ F., or $-110°$ C., was produced for these experiments by Professor Faraday. As to the method of production, see SOURCES OF COLD.

According to the results of more recent experiments, hydrogen has been subjected to a pressure of 8000 atmospheres without making any sign of condensation.

SOURCES OF COLD.

The production of cold—the abstraction of heat—is a curious subject of inquiry. When a salt is dissolved in water, cold is produced. When a liquid vaporizes, the heat, latent and sensible, necessary for the production of the vapour, is abstracted from some other body in contact with the liquid, and cold is produced. When spirits of wine or aromatic vinegar, for example, is thrown on the body, a sense of cold immediately results from the vaporization of the liquid which draws heat from the body. If air is allowed to expand, there is a reduction of temperature, and a translation of heat from neighbouring bodies. Again, in hot climates, water is successfully cooled in porous vessels, through the pores of which the water passes to the exterior, and is vaporized, and the cooling process is accelerated by a current of air directed upon the vessel, which quickens the vaporization.

Siebe's ice-making machine, invented originally by Jacob Perkins in 1834, is based on the principle of producing cold by the evaporation of a volatile fluid—ether by preference. The fluid is placed in an air-tight vessel, and evaporated in vacuo, the vacuum being formed by means of a pump, which, in its continued efforts to reduce the pressure, promotes the evaporation of the fluid at a low temperature. A temperature 50° below the freezing point may be effected; but in place of an unprofitably low temperature, the cooling action is distributed through the mass of salt water employed as the freezing medium, the salt water retaining its fluidity below 32° F., and circulating in the refrigerator around and between the ice-moulds, which are filled with fresh water. The water in the moulds is successively frozen, and replaced by fresh moulds filled with water.

Carré's cooling apparatus is based on the fact that water, when cold, absorbs a large quantity of ammoniacal gas, which, when the water is heated, escapes, and is condensed in a cold vessel. On the contrary, when the water just heated becomes cold, a vacuum is formed, and excites a rapid evaporation of the ammonia into the vessel of cooled water, when it is again absorbed. The heat necessary for the evaporation of the ammonia is extracted from the water surrounding the vessel in which the liquid ammonia is contained, and the water consequently is frozen.

FRIGORIFIC MIXTURES.

For the production of intense cold, mixtures of various salts and acids in

various proportions with water are very effective. But more intense degrees of cold are produced with snow or ice.

Table No. 126 contains the ordinary mixtures for the artificial production of cold, known as freezing mixtures. The first part of the table comprises mixtures of salts and acids with each other and with water; the second part, mixtures of salts and acids with snow or ice.

The blanks in the third column of the table indicate that the thermometer sinks to the degrees named in the second column, but never lower, whatever may be the initial temperature of the materials when mixed.

The vessels containing the mixtures should be cooled before the elements are put into them.

If the materials of the mixtures enumerated in the first part of the table be mixed at a higher temperature than that given in the table, namely, 50° F., the fall of temperature is greater. Thus, if the most powerful of these mixtures, No. 11, be made at the temperature 80° F., it will sink the thermometer to + 2°, making a fall of 78 degrees, as against 71 degrees in the table.

The third part of the table contains frigorific mixtures partly selected from the other parts, and combined so as to extend the cold to the extreme degree, − 91° F. The materials should be cooled previously to being mixed to the initial temperature, by mixtures taken from previous parts of the table.

Table No. 126.—Frigorific Mixtures.

First Part.—Proportional mixtures of Salts and Acids with Water.

Mixtures.		Fall of Temperature.	Degrees of cold produced.
		Fahrenheit.	Fahr.
1. Nitrate of ammonia	1	from + 50° to + 4°	46°
Water	1		
2. Muriate of ammonia	5	from + 50° to + 10°	40
Nitrate of potash	5		
Water	16		
3. Muriate of ammonia	5	from + 50° to + 4°	46
Nitrate of potash	5		
Sulphate of soda	8		
Water	16		
4. Sulphate of soda	3	from + 50° to − 3°	53
Diluted nitric acid	2		
5. Nitrate of ammonia	1	from + 50° to − 7°	57
Carbonate of soda	1		
Water	1		
6. Phosphate of soda	9	from + 50° to − 12°	62
Diluted nitric acid	4		
7. Sulphate of soda	8	from + 50° to 0°	50
Hydrochloric acid	5		
8. Sulphate of soda	5	from + 50° to + 3°	47
Dilute sulphuric acid	4		

Table No. 126 (*continued*).

Mixtures.		Fall of Temperature.	Degrees of cold produced.
		Fahrenheit.	Fahr.
9. Sulphate of soda	6	from + 50° to − 10°	60°
Muriate of ammonia	4		
Nitrate of potash	2		
Dilute nitric acid	4		
10. Sulphate of soda	6	from + 50° to − 14°	64
Nitrate of ammonia	5		
Dilute nitric acid	4		
11. Phosphate of soda	9	from + 50° to − 21°	71
Nitrate of ammonia	6		
Dilute nitric acid	4		

SECOND PART.—Proportional mixtures of Salts and Acids with Snow or Ice.

Mixtures.		Fall of Temperature.	Degrees of cold produced.
		Fahrenheit.	Fahr.
12. Muriate of soda (common salt).	1	from any temp. to − 5°	—
Snow, or pounded ice	2		
13. Muriate of soda	2	do. do. to − 12°	—
Muriate of ammonia	1		
Snow, or pounded ice	5		
14. Muriate of soda	10	do. do. to − 18°	—
Muriate of ammonia	5		
Nitrate of potash	5		
Snow, or pounded ice	24		
15. Muriate of soda	5	do. do. to − 25°	—
Nitrate of ammonia	5		
Snow, or pounded ice	12		
16. Dilute sulphuric acid	2	from + 32° to − 23°	55°
Snow	3		
17. Muriatic acid	5	from + 32° to − 27°	59
Snow	8		
18. Dilute nitric acid	4	from + 32° to − 30°	62
Snow	7		
19. Muriate of lime	5	from + 32° to − 40°	72
Snow	4		
20. Crystallized muriate of lime	3	from + 32° to − 50°	82
Snow	2		
21. Potash	4	from + 32° to − 51°	83
Snow	3		

Table No. 126 (*continued*).

THIRD PART.—Mixtures partly selected from the foregoing series, and combined so as to increase or extend the cold to the greatest extremes.

Mixtures.		Fall of Temperature.	Degrees of cold produced.
		Fahrenheit.	Fahr.
22. Sea salt	5		
Muriate of ammonia, Nitrate of potash	5	from $-5°$ to $-18°$	13°
Snow, or pounded ice	1		
23. Sea salt	5		
Nitrate of ammonia	5	from $-18°$ to $-25°$	7
Snow, or pounded ice	12		
24. Phosphate of soda	5		
Nitrate of ammonia	3	from 0° to $-34°$	34
Dilute nitric acid	4		
25. Phosphate of soda	3		
Nitrate of ammonia	2	from $-34°$ to $-50°$	16
Dilute mixed acids	4		
26. Snow	3	from 0° to $-46°$	46
Dilute nitric acid	2		
27. Snow	8		
Dilute sulphuric acid	3	from $-10°$ to $-56°$	46
Dilute nitric acid	3		
28. Snow	1	from $-10°$ to $-60°$	50
Dilute sulphuric acid	1		
29. Snow	3	from $+20°$ to $-48°$	68
Muriate of lime	4		
30. Snow	3	from $+10°$ to $-54°$	64
Muriate of lime	4		
31. Snow	2	from $-15°$ to $-68°$	53
Muriate of lime	3		
32. Snow	1	from 0° to $-66°$	66
Crystallized muriate of lime	2		
33. Snow	1	from $-40°$ to $-73°$	33
Crystallized muriate of lime	3		
34. Snow	8	from $-68°$ to $-91°$	23
Dilute sulphuric acid	10		

COLD BY EVAPORATION.

M. Gay-Lussac directed a current of air, dried or desiccated by being passed through chloride of calcium, upon the bulb of a thermometer wrapped in moist cambric. The temperature was lowered from 10° to 26° F., according to the temperature of the current of air, which varied from 32° to 77° F. It is presumed that the surrounding temperature was the same as that of the current. The following are the falls of temperature for currents of air of given temperatures:—

Temperature of current, Fahrenheit,	32°,	41°,	50°,	59°,	68°,	77°.
Fall of temperature, do.	10°.5,	13°,	16°,	19°.5,	23°,	26°.5.

The most intense cold as yet known was produced by Professor Faraday in the course of his experiments on the liquefaction and solidification of gases, from the evaporation of a mixture of solid carbonic acid and sulphuric ether under the receiver of an air-pump. For the following pressures, measured in inches of mercury, and given also in pounds per square inch, he obtained the corresponding temperatures subjoined:—

Inches of mercury.....	28.4,	19.4,	9.6,	7.4,	5.4,	3.4,	2.4,	1.4,	1.2.
Lbs. per square inch.	14.0,	9.5,	4.6,	3.6,	2.7,	1.7,	1.2,	0.7,	0.6.
Temperatures, Fahr..	−107°,	−112°,	−121°,	−125°,	−132°,	−139°,	−146°,	−161°,	−166°.

Showing that when a perfect vacuum was nearly approached, an intense cold, measured by −166° F., was attained by the evaporation of a mixture of solid carbonic acid and sulphuric ether.

STEAM.

When steam is generated in a boiler, the water is heated till it arrives at the temperature of ebullition, and the elevation of temperature is sensible to the thermometer; next, the water is converted into steam by an additional absorption of heat, which is not measured by the thermometer, and is therefore called latent heat. The heat is not, in fact, latent, but is appropriated in converting water into steam, of the same temperature.

The pressure, as well as the density, of steam which is generated over water in a boiler rises with the temperature; and, reciprocally, the temperature rises with the pressure and density. There is only one pressure and one density for each temperature; and thus it is that steam, produced in a boiler over water, is always generated at the maximum density and maximum pressure corresponding to its temperature. In such condition steam is said to be saturated, being incapable of vaporizing more water into the same space, unless the temperature be raised. Saturation is therefore the normal condition of steam generated in contact with a store of water, and the same density and the same pressure are always to be found in conjunction with the same temperature.

In consequence, saturated steam over water stands both at the condensing point and at the generating point; that is, it is condensed if the temperature falls, and more water is evaporated if the temperature rises.

But, supposing the whole of the water to be evaporated, or that a body of saturated steam is isolated from water, in a space of fixed dimensions, if an additional quantity of heat be supplied to the steam, the state of saturation ceases, the steam becomes superheated, and the temperature and the pressure are increased, whilst the density is not increased. Steam, thus surcharged with heat, approaches to the condition of a perfect gas.

PHYSICAL PROPERTIES OF STEAM.

Relation of the Temperature and Pressure of Saturated Steam.

The results of the experimental observations of M. Regnault on the temperature and pressure of saturated steam, whose observations have superseded in practice those of previous experimentalists, show that the temperature rises more slowly than the pressure. For example, the pressures being advanced at equal intervals of 5 lbs. per square inch, thus:—

1 lb. 6 lbs. 11 lbs. 16 lbs. 21 lbs. 26 lbs. 31 lbs. 36 lbs.,

the temperatures in Fahrenheit degrees are—

102°.1, 170°.2, 197°.8, 216°.3, 230°.6, 242°.3, 252°.2, 260°.9,

which advance by the following diminishing differences,—

$68^\circ.1,\quad 27^\circ.6,\quad 18^\circ.5,\quad 14^\circ.3,\quad 11^\circ.7,\quad 9^\circ.9,\quad 8^\circ.7,$

Without quoting the formula employed by M. Regnault for calculating the pressures due to the temperatures in French measures, it will suffice to give, in a subsequent table (No. 127), the relative pressures in inches of mercury and in pounds per square inch, based on his formula, for low temperatures ranging from 32° F. to 212° F., as given by Claudel.

To define the relation of the temperature and pressure of saturated steam for the higher temperatures comprised in the observations of M. Regnault, the late Mr. W. M. Buchanan arranged a simple formula which applies with accuracy to temperatures ranging from 120° F. to 446° F., the higher limit of the range of Regnault's observations. These limits correspond to pressures of from 1.68 lbs. to 445 lbs. per square inch. The formula is as follows:—

$$t = \frac{2938.16}{6.1993544 - \log p} - 371.85, \quad\quad (1)$$

in which p is the pressure in lbs. per square inch, and t is the temperature of saturated steam in degrees Fahrenheit, as observed by means of an air-thermometer.

Total Heat of Saturated Steam.

The constituent or total heat of steam consists of its latent heat, in addition to its sensible heat. The latent heat of saturated steam at 0° C., the freezing point, was experimentally determined by Regnault to be equal to 606°.5 C.; or such that the total heat of one pound of saturated steam at 0° C. would be capable of raising the temperature of 606.5 lbs. of water one degree. At higher temperatures, the total heat of saturated steam was found to increase uniformly between the temperatures 0° C. and 230° C., at the rate of .305° C. for each increment of temperature of 1°; and, therefore, if the temperature in Centigrade degrees be multiplied by .305, and 606.5 be added to the product, the sum will express the total heat of saturated steam at the given temperature measured from 0° C.; or

$$H = 606.5 + .305\, t\,(\text{Centigrade}), \quad\quad (2)$$

in which H = the total heat of saturated steam of any temperature t° C.

This formula is adapted to the Fahrenheit scale, by taking the total heat at 32° F. equal to 606°.5 C. × 9/5 = 1091°.7 F. For any other temperature t° F., the total heat is equal to 1091°.7 F. + .305 $(t-32)$. The first quantity in this expression, namely 1091°.7, is slightly too much, for whilst Regnault found that the total heat of steam at 100°, his starting point, was 636°.67 C., it was calculated by his formula (No. 2 above) to be 637° C. The above-named quantity should therefore be reduced to 1091.16, and the formula for the total heat of steam in terms of Fahrenheit degrees will stand thus:—

$$H = 1091.16 + .305\,(t-32), \text{ or}$$
$$H = 1081.4 + .305\, t; \quad\quad (3)$$

that is, the total heat of saturated steam of any given temperature in Fahrenheit degrees is equal to 1081°.4 plus the product of the temperature by .305, supposing that the water from which the steam is generated is supplied at the temperature 32° F.

The expression of the total heat represents units of heat when the weight of the steam is one pound.

Supposing that the water to be evaporated is supplied at any higher temperature than 32° F., the total heat to be expended in evaporating it is found by deducting the difference of temperature from the total heat as found by the formula (3). Or, the formula may be modified by deducting the difference of temperature from the first quantity, 1081.4.

For example, if water be supplied at the ordinary temperature, 62° F., which is 30 degrees above 32° F., then $1081.4 - 30 = 1051.4$ will be the proper first quantity. For these and the other cardinal temperatures of water, 100° F. and 212° F., the four equations for the total heat of steam raised from water at the respective temperatures are as follow:—

(Initial temperature 32° F.), $H = 1081.4 + .305t^\circ$ .... (3)
(Do. 62° F.), $H = 1051.4 + .305t^\circ$ .... (4)
(Do. 100° F.), $H = 1013.4 + .305t^\circ$ .... (5)
(Do. 212° F.), $H = 900.5 + .305t^\circ$ .... (6)

In the reduction for the last equation (6), 32° has been deducted from 212°.9, and not from 212°, in order to take into account the item .9°, being the extra specific heat of water at 212° F., compared with that of water at 32° F.

Latent Heat of Saturated Steam.

As the total heat is increased .305°, which is less than a third of a degree, whilst the sensible heat, or temperature, rises 1°, and the sensible heat thus rises faster than the total heat, the latent heat must be reduced as the temperature rises, by as much as .305° is less than 1°, or by $1^\circ - .305^\circ = .695^\circ$, for each degree of temperature, and the latent heat for any temperature t° C. is expressed by the quantity $606^\circ.5 - .695t$.

There is a modifying element, namely, the specific heat of the water, which slightly increases with the temperature, and which requires the fraction $.695\,t$ to be proportionally increased. The equation of Clausius, in which this slight variation is allowed for, is

$$L = 607 - .708\,t \text{ (Centigrade)}, \qquad (7)$$

where L = the latent heat due to the temperature t C.

To adapt this formula to the Fahrenheit scale, take $^9/_5$ths of 607 = 1092°.6 F., and substitute $(t^\circ - 32^\circ)$ F. for t° C.; then the formula becomes

$$L = 1092.6 - .708\,(t - 32); \text{ or}$$
$$L = 1115.2 - .708\,t; \qquad (8)$$

that is, the latent heat of saturated steam at any given temperature in Fahrenheit degrees is equal to 1115.2 less the product of the temperature by .708, supposing that the water which is converted into steam is supplied at 32° F.

Appropriation of the Constituent Heat of Saturated Steam at 212° F.

To trace the appropriation of all the heat that goes to the formation of a pound of steam, in the sensible and the latent state, in terms of thermal units, as well as of dynamic units, or foot-pounds, take one pound of water at 32° F., and convert it into saturated steam at 212° F., the first

instalment of heat is the sensible heat, and it is required for elevating the temperature of the water to 212°, through 180°; in other words, to increase the molecular velocity, and slightly expand the liquid, which appropriates 180.9 units of heat, equivalent to 180.9 × 772, or 139,655 foot-pounds. Secondly, latent heat is applied in overcoming the molecular attraction, and separating the particles; that is to say, in the formation of steam, which appropriates 892.9 units of heat, equal to 689,318 foot-pounds. Thirdly, latent heat is applied in repelling the incumbent pressure, whether of the atmosphere or of the surrounding steam; that is to say, in raising a load of 14.7 lbs. per square inch, or 2116.4 lbs., on a square foot, through a cubic space of 26.36 cubic feet, being the volume of one pound of saturated steam. The work thus done is equal to 2116.4 × 26.36, or 55,788 foot-pounds, or its equivalent, 72.3 units of heat. In strictness, there is the initial volume of the pound of water to be deducted from this total volume, to show the exact volume generated; but it is relatively very small, and is inconsiderable.

The second of the above appropriations of the heat was found by subtracting the sum of the first and third, which are both arrived at by direct observation, from the total heat.

The first appropriation of heat is thus seen to be the sensible heat, and the second and third together constitute the latent heat. The third, it may be added, is simply an expression of the mechanical labour necessary to disengage 26.36 cubic feet of steam, and force it into space against an atmospheric pressure of 2116.4 lbs. per square foot.

The appropriation of the heat expended in the generation of one pound of saturated steam at 212° F., from water supplied at 32° F., may be exhibited thus:—

To Generate One Pound of Steam at 212° F.

	Units of heat.		Mechanical equivalent in foot-pounds.	
The sensible heat:—				
1. To raise the temperature of the water from 32° to 212° F.,		180.9		139,655
The latent heat:—				
2. In the formation of steam...	892.935		689,346	
3. In resisting the incumbent atmospheric pressure of 14.7 lbs. per square inch, or 2116.4 lbs. per square foot,	72.265		55,788	
		965.2		745,134
Total or constituent heat,........		1146.1		884,789

Volume and Density of Saturated Steam.

The density of steam is expressed by the weight of a given constant volume, say, one cubic foot; and the volume is expressed by the number of cubic feet in one pound of steam. The density and volume, which are the reciprocals of each other, have not yet been accurately ascertained by direct experiment. They are, however, determinable in terms of the pressure, temperature, and latent heat of steam, all of which have been experimentally ascertained, by means of the mechanical theory of heat.

Mr. Brownlee has deduced a simple expression for the density of saturated steam in terms of the pressure, as follows:—

$$D = \frac{p^{.941}}{330.36}, \qquad (9)$$

$$\text{or, } \log D = .941 \log p - 2.519, \qquad (10)$$

in which D is the density, and p the pressure in lbs. per square inch. In this expression, $p^{.941}$ is the equivalent of $p^{\frac{16}{17}}$, as employed by Dr. Rankine; and it is simpler to handle. The equation signifies that the logarithm of the pressure is to be multiplied by .941, and that 2.519 is to be subtracted from the product; the remainder is the logarithm of the density, from which the density is found by means of a table of logarithms.

The results presented by the above formula are very accurate; they do not differ from those obtained in terms of the temperature and the latent heat, for pressures of from 1 lb. to 250 lbs. per square inch, by more than one-seventh per cent.

The volume being the reciprocal of the density, then, putting V for the volume,

$$V = \frac{330.36}{p^{.941}}, \qquad (11)$$

$$\text{or } \log V = 2.519 - .941 \log p; \qquad (12)$$

that is, that if the logarithm of the pressure in lbs. per square inch be multiplied by .941, and the product be deducted from 2.519, the remainder is the logarithm of the volume, in cubic feet, of one pound of saturated steam. The nearness of the power, .941, to unity, indicates that the density of saturated steam varies nearly as the pressure, but in a lower ratio; and that the volume of saturated steam varies, for short intervals, nearly in the inverse ratio of the pressure. For example, the pressures per square inch being—

1, 2, 4, 8, 16, 32, 64, 128 lbs.,

the densities, or weights per cubic foot, which are inversely as the volumes, are—

.0030, .0058, .0112, .0214, .0411, .0789, .1516, .2911 lbs.,

being in the ratios of—

1, 1.93, 3.73, 7.13, 13.7, 29.3, 50.5, 97.

Relative Volume of Saturated Steam.

The relative volume of saturated steam is expressed by the number of volumes of steam produced from one volume of water, the volume of water being measured at the temperature 62° F. The relative volume is found by multiplying the volume, in cubic feet, of one pound of steam by the weight of a cubic foot of water at 62° F., which is 62.355 lbs.

Or, it may be found directly in terms of the pressure, by multiplying the second member of the formula (11) by 62.355. Thus, putting n for the relative volume,

$$n = \frac{62.355 \times 330.36}{p^{.941}}, \text{ or}$$

$$n = \frac{20600}{p^{.941}}, \quad (13)$$

$$\text{or, } \log n = 4.31388 - (.941 \times \log p); \quad (14)$$

that is, if the logarithm of the pressure in lbs. per square inch be multiplied by .941, and the product be deducted from 4.31388, the remainder is the logarithm of the relative volume.

Gaseous Steam.

When saturated steam is superheated, or surcharged with heat, it advances from the condition of saturation into that of gaseity. The gaseous state is only arrived at by considerably elevating the temperature, supposing the pressure remains the same. Steam thus sufficiently superheated is known as gaseous steam, or "steam-gas," as Dr. Rankine has named it.

The test of perfect gaseity is the uniformity of the rate of expansion with the rise of temperature; and, whereas, during the first few degrees which follow the temperature of saturation, the rate of expansion is notably greater than that of air, the rate diminishes at still higher temperatures, and ultimately becomes uniform, like that of the expansion of permanent gases.

Dr. C. W. Siemens, experimenting on the expansion of isolated steam, generated at 212°, and superheated and maintained at atmospheric pressure, found that expansion proceeded rapidly until the temperature rose to 220°, and less rapidly up to 230°, or 18° above the saturation point; above which it expanded uniformly, as a permanent gas. Up to 230°, the expansion was five times as much as that of air.

Messrs. Fairbairn and Tate found that for steam generated at low temperatures of saturation,—under 150° F.,—the rate of expansion when the steam was heated was nearly uniform. At 175° F., the expansion for the first five degrees averaged more than three times that of air; above that point, it was nearly the same. For steam generated at the high temperature of 324° F., for a total pressure of 95 lbs. per square inch, the rate of expansion up to 331° was nearly three times that of air; and for the next 25 degrees, one-sixth greater.

M. Regnault concluded from his experiments that saturated steam was nearly gaseous at temperatures below 60° F.

It may be gathered from these observations that saturated steam of ordinary temperatures may be made gaseous by superheating it to the extent of from 10 to 20 degrees. It is thought that the rapidity of expansion by heat, near the boiling point, is to be accounted for by the supposed insensible moisture of steam in the saturated condition, as generated from water, being evaporated and contributing to increase the quantity of steam without raising the temperature. This argument is plausible; but it might be argued, on the contrary, that in the converse process, of abstracting heat from superheated steam, the accelerated reduction of volume when it approaches the point of saturation, is due to incipient condensation, which would be absurd.

It may be inferred, further, that saturated steam of very low temperatures, under 150° or 100° F., is gaseous.

Total Heat of Gaseous Steam.

Regnault found that the total heat of gaseous steam increased, like that of saturated steam, uniformly with the temperature; and at the rate of .475° for each degree of temperature, under a constant pressure. A formula for the total heat of gaseous steam may be constructed on the basis of that for saturated steam, by a modification of the constants; and for the adjustment of these, take the two steams at a low temperature, as 40° F., where they are identical in constitution, both being gaseous. Then, by formula (3) for saturated steam, the total heat at this temperature is

$$1081.4 + (.305 \times 40^\circ) = 1093^\circ.6 \text{ F.}$$

Substituting for the second quantity in this equation, the quantity $(.475 \times 40^\circ)$, and reducing, then

$$1074.6 + (.475 \times 40^\circ) = 1093^\circ.6 \text{ F.}$$

Whence the general formula for the total heat of gaseous steam, produced from water at 32° F.,

$$H' = 1074.6 + .475\,t, \qquad (15)$$

H′ being the total heat, in Fahrenheit degrees, and t the temperature; that is, that to the constant 1074.6, is to be added the product of the temperature by .475, to find the total heat.

By this formula it is found that the total heat of gaseous steam at 212° F., and at atmospheric pressure, is 1175.3° F., which is 29.2 degrees, or 2½ per cent. more than that of saturated steam.

Specific Heat of Steam.

The specific heat of saturated steam is .305, that of water being unity; or, it is 1.281, that of air being unity. It may be noted that .305° is the quantity by which the total heat of saturated steam is increased for each degree of temperature (see formula 3); so that equal intervals of temperature correspond to equal quantities of heat. The expression, .305, for specific heat, is taken in a compound sense, comprising the changes both of volume and of pressure which take place in the production of saturated steam.

The specific heat of gaseous steam is .475, under constant pressure, as found by Regnault. It is upwards of a half more than that of saturated steam. It is identical with the increase of total heat for each degree of temperature (formula 15).

The Specific Density of Steam.

The specific density of gaseous steam has been found by M. Regnault to be .622, that of air being 1. That is to say, that the weight of a cubic foot of gaseous steam is about five-eighths of that of a cubic foot of air, of the same pressure and temperature.

The specific density of saturated steam is usually taken at the same value as that of gaseous steam, as an approximation to the actual value. Thus approximated, it is only correct at very low temperatures, for the specific density increases, though not rapidly, with the temperature, insomuch that though it is practically the same as that of gaseous steam at 100° F., it becomes .643 at 212° F.; and at 303° F., with 70 lbs. absolute

pressure per square inch, it becomes .664, or two-thirds of that of air. At. 358.3° F., with 150 lbs. pressure, it is .681. (See table No. 129.)

Density of Gaseous Steam.

The density or weight of a cubic foot of gaseous steam is expressible by the same formula as for that of air (page 350), except that the multiplier or coefficient is less in proportion to the less specific density, thus:—

$$D' = \frac{2.7074\,p \times .622}{t+461} = \frac{1.684\,p}{t+461}, \qquad (16)$$

in which D′ is the weight of a cubic foot of gaseous steam, p the total pressure per square inch, and t the temperature by Fahrenheit.

Tables of the Properties of Saturated Steam.

The first table, No. 127, of the properties of saturated steam of temperatures ranging from 32° to 212° F. is adapted from a table prepared by Claudel, partly based on Regnault's formulas, and partly on the assumption that the specific density of saturated steam is uniformly .622, or about five-eighths that of air at the same temperature. As already mentioned, the specific density increases, in fact, slightly with the temperature, and this deviation from uniformity explains the small discrepancies between the weights of steam as given in table No. 127, and those as given for temperatures below 212° in the next table.

The table No. 128 gives the properties of saturated steam for pressures of from 1 lb. per square inch to 400 lbs. per square inch, the temperatures ranging from 102° to 445° F. The first column contains the ascending total pressures in lbs. per square inch. The second column, of temperatures, was calculated from the pressures by means of the formula (1):—

$$t = \frac{2938.16}{6.1993544 - \log p} - 371.85.$$

The third column, of the total heat of saturated steam, by formula (3):—

$$H = 1081.4 + .305\,t.$$

The fourth column, of the latent heat of saturated steam, by formula (8):—

$$L = 1115.2 - 708\,t.$$

The fifth column, of the density of saturated steam, by formula (10):—

$$\log D = .941 \log p - 2.519.$$

The sixth column, of the volume of saturated steam, was calculable by finding the reciprocals of the densities, or by formula (12):—

$$\log V = 2.519 - .941 \log p.$$

The seventh column, of the relative volume of saturated steam, by the formula (14):—

$$\log n = 4.31388 - (.941 \times \log p).$$

The table No. 129 contains the comparative densities and volumes of air and saturated steam for pressures up to 300 lbs. total pressure per square inch, and temperatures to 417°.5 F., together with the specific density of saturated steam.

Table No. 127.—Properties of Saturated Steam from 32° to 212° F.

Temperature.	Pressure.		Total heat reckoned from water at 32° F.	Weight of 100 cubic feet.	Volume of one pound of vapour.
	Inches of mercury.	Lbs. per square inch.			
Fahrenheit.	inches.	lbs.	Fahrenheit.	lbs.	cubic feet.
32°	.181	.089	1091.2	.031	3226
35	.204	.100	1092.1	.034	2941
40	.248	.122	1093.6	.041	2439
45	.299	.147	1095.1	.049	2041
50	.362	.178	1096.6	.059	1695
55	.426	.214	1098.2	.070	1429
60	.517	.254	1099.7	.082	1220
65	.619	.304	1101.2	.097	1031
70	.733	.360	1102.8	.114	877.2
75	.869	.427	1104.3	.134	746.3
80	1.024	.503	1105.8	.156	641.0
85	1.205	.592	1107.3	.182	549.5
90	1.410	.693	1108.9	.212	471.7
95	1.647	.809	1110.4	.245	408.2
100	1.917	.942	1111.9	.283	353.4
105	2.229	1.095	1113.4	.325	307.7
110	2.579	1.267	1115.0	.373	268.1
115	2.976	1.462	1116.5	.426	234.7
120	3.430	1.685	1118.0	.488	204.9
125	3.933	1.932	1119.5	.554	180.5
130	4.509	2.215	1121.1	.630	158.7
135	5.174	2.542	1122.6	.714	140.1
140	5.860	2.879	1124.1	.806	124.1
145	6.662	3.273	1125.6	.909	110.0
150	7.548	3.708	1127.2	1.022	97.8
155	8.535	4.193	1128.7	1.145	87.3
160	9.630	4.731	1130.2	1.333	75.0
165	10.843	5.327	1131.7	1.432	69.8
170	12.183	5.985	1133.3	1.602	62.4
175	13.654	6.708	1134.8	1.774	56.4
180	15.291	7.511	1136.3	1.970	50.8
185	17.044	8.375	1137.8	2.181	45.9
190	19.001	9.335	1139.4	2.411	41.5
195	21.139	10.385	1140.9	2.662	37.6
200	23.461	11.526	1142.4	2.933	34.1
205	25.994	12.770	1143.9	3.225	31.0
210	28.753	14.126	1145.5	3.543	28.2
212	29.922	14.700	1146.1	3.683	27.2

Table No. 128.—PROPERTIES OF SATURATED STEAM.

Total pressure per square inch.	Temperature in Fahrenheit degrees.	Total heat, in Fahr. degrees, from water at 32° F.	Latent heat, Fahrenheit degrees.	Density, or weight of one cubic foot.	Volume of one pound of steam.	Relative volume, or cubic feet of steam from one cubic ft. of water.
lbs.	Fahr.	Fahr.	Fahr.	lbs.	cubic feet.	Rel. vol.
1	102.1	1112.5	1042.9	.0030	330.36	20600
2	126.3	1119.7	1025.8	.0058	172.08	10730
3	141.6	1124.6	1015.0	.0085	117.52	7327
4	153.1	1128.1	1006.8	.0112	89.62	5589
5	162.3	1130.9	1000.3	.0138	72.66	4530
6	170.2	1133.3	994.7	.0163	61.21	3816
7	176.9	1135.3	990.0	.0189	52.94	3301
8	182.9	1137.2	985.7	.0214	46.69	2911
9	188.3	1138.8	981.9	.0239	41.79	2606
10	193.3	1140.3	978.4	.0264	37.84	2360
11	197.8	1141.7	975.2	.0289	34.63	2157
12	202.0	1143.0	972.2	.0314	31.88	1988
13	205.9	1144.2	969.4	.0338	29.57	1844
14	209.6	1145.3	966.8	.0362	27.61	1721
14.7	212.0	1146.1	965.2	.0380	26.36	1642
15	213.1	1146.4	964.3	.0387	25.85	1611
16	216.3	1147.4	962.1	.0411	24.32	1516
17	219.6	1148.3	959.8	.0435	22.96	1432
18	222.4	1149.2	957.7	.0459	21.78	1357
19	225.3	1150.1	955.7	.0483	20.70	1290
20	228.0	1150.9	953.8	.0507	19.72	1229
21	230.6	1151.7	951.9	.0531	18.84	1174
22	233.1	1152.5	950.2	.0555	18.03	1123
23	235.5	1153.2	948.5	.0580	17.26	1075
24	237.8	1153.9	946.9	.0601	16.64	1036
25	240.1	1154.6	945.3	.0625	15.99	996
26	242.3	1155.3	943.7	.0650	15.38	958
27	244.4	1155.8	942.2	.0673	14.86	926
28	246.4	1156.4	940.8	.0696	14.37	895
29	248.4	1157.1	939.4	.0719	13.90	866
30	250.4	1157.8	937.9	.0743	13.46	838
31	252.2	1158.4	936.7	.0766	13.05	813
32	254.1	1158.9	935.3	.0789	12.67	789
33	255.9	1159.5	934.0	.0812	12.31	767
34	257.6	1160.0	932.8	.0835	11.97	746
35	259.3	1160.5	931.6	.0858	11.65	726
36	260.9	1161.0	930.5	.0881	11.34	707
37	262.6	1161.5	929.3	.0905	11.04	688
38	264.2	1162.0	928.2	.0929	10.76	671
39	265.8	1162.5	927.1	.0952	10.51	655
40	267.3	1162.9	926.0	.0974	10.27	640
41	268.7	1163.4	924.9	.0996	10.03	625
42	270.2	1163.8	923.9	.1020	9.81	611
43	271.6	1164.2	922.9	.1042	9.59	598
44	273.0	1164.6	921.9	.1065	9.39	585
45	274.4	1165.1	920.9	.1089	9.18	572
46	275.8	1165.5	919.9	.1111	9.00	561
47	277.1	1165.9	919.0	.1133	8.82	550
48	278.4	1166.3	918.1	.1156	8.65	539
49	279.7	1166.7	917.2	.1179	8.48	529
50	281.0	1167.1	916.3	.1202	8.31	518

Table No. 128 (*continued*).

Total pressure per square inch.	Temperature in Fahrenheit degrees.	Total heat, in Fahr. degrees, from water at 32° F.	Latent heat, Fahrenheit degrees.	Density, or weight of one cubic foot.	Volume of one pound of steam.	Relative volume, or cubic feet of steam from one cubic ft. of water.
lbs.	Fahr.	Fahr.	Fahr.	lbs.	cubic feet.	Rel. vol.
51	282.3	1167.5	915.4	.1224	8.17	509
52	283.5	1167.9	914.5	.1246	8.04	500
53	284.7	1168.3	913.6	.1269	7.88	491
54	285.9	1168.6	912.8	.1291	7.74	482
55	287.1	1169.0	912.0	.1314	7.61	474
56	288.2	1169.3	911.2	.1336	7.48	466
57	289.3	1169.7	910.4	.1364	7.36	458
58	290.4	1170.0	909.6	.1380	7.24	451
59	291.6	1170.4	908.8	.1403	7.12	444
60	292.7	1170.7	908.0	.1425	7.01	437
61	293.8	1171.1	907.2	.1447	6.90	430
62	294.8	1171.4	906.4	.1469	6.81	424
63	295.9	1171.7	905.6	.1493	6.70	417
64	296.9	1172.0	904.9	.1516	6.60	411
65	298.0	1172.3	904.2	.1538	6.49	405
66	299.0	1172.6	903.5	.1560	6.41	399
67	300.0	1172.9	902.8	.1583	6.32	393
68	300.9	1173.2	902.1	.1605	6.23	388
69	301.9	1173.5	901.4	.1627	6.15	383
70	302.9	1173.8	900.8	.1648	6.07	378
71	303.9	1174.1	900.3	.1670	5.99	373
72	304.8	1174.3	899.6	.1692	5.91	368
73	305.7	1174.6	898.9	.1714	5.83	363
74	306.6	1174.9	898.2	.1736	5.76	359
75	307.5	1175.2	897.5	.1759	5.68	353
76	308.4	1175.4	896.8	.1782	5.61	349
77	309.3	1175.7	896.1	.1804	5.54	345
78	310.2	1176.0	895.5	.1826	5.48	341
79	311.1	1176.3	894.9	.1848	5.41	337
80	312.0	1176.5	894.3	.1869	5.35	333
81	312.8	1176.8	893.7	.1891	5.29	329
82	313.6	1177.1	893.1	.1913	5.23	325
83	314.5	1177.4	892.5	.1935	5.17	321
84	315.3	1177.6	892.0	.1957	5.11	318
85	316.1	1177.9	891.4	.1980	5.05	314
86	316.9	1178.1	890.8	.2002	5.00	311
87	317.8	1178.4	890.2	.2024	4.94	308
88	318.6	1178.6	889.6	.2044	4.89	305
89	319.4	1178.9	889.0	.2067	4.84	301
90	320.2	1179.1	888.5	.2089	4.79	298
91	321.0	1179.3	887.9	.2111	4.74	295
92	321.7	1179.5	887.3	.2133	4.69	292
93	322.5	1179.8	886.8	.2155	4.64	289
94	323.3	1180.0	886.3	.2176	4.60	286
95	324.1	1180.3	885.8	.2198	4.55	283
96	324.8	1180.5	885.2	.2219	4.51	281
97	325.6	1180.8	884.6	.2241	4.46	278
98	326.3	1181.0	884.1	.2263	4.42	275
99	327.1	1181.2	883.6	.2285	4.37	272
100	327.9	1181.4	883.1	.2307	4.33	270
101	328.5	1181.6	882.6	.2329	4.29	267

Table No. 128 (*continued*).

Total pressure per square inch.	Temperature in Fahrenheit degrees.	Total heat, in Fahr. degrees, from water at 32° F.	Latent heat, Fahrenheit degrees.	Density, or weight of one cubic foot.	Volume of one pound of steam.	Relative volume, or cubic feet of steam from one cubic ft. of water.
lbs.	Fahr.	Fahr.	Fahr.	lbs.	cubic feet.	Rel. vol.
102	329.1	1181.8	882.1	.2351	4.25	265
103	329.9	1182.0	881.6	.2373	4.21	262
104	330.6	1182.2	881.1	.2393	4.18	260
105	331.3	1182.4	880.7	.2414	4.14	257
106	331.9	1182.6	880.2	.2435	4.11	255
107	332.6	1182.8	879.7	.2456	4.07	253
108	333.3	1183.0	879.2	.2477	4.04	251
109	334.0	1183.3	878.7	.2499	4.00	249
110	334.6	1183.5	878.3	.2521	3.97	247
111	335.3	1183.7	877.8	.2543	3.93	245
112	336.0	1183.9	877.3	.2564	3.90	243
113	336.7	1184.1	876.8	.2586	3.86	241
114	337.4	1184.3	876.3	.2607	3.83	239
115	338.0	1184.5	875.9	.2628	3.80	237
116	338.6	1184.7	875.5	.2649	3.77	235
117	339.3	1184.9	875.0	.2652	3.74	233
118	339.9	1185.1	874.5	.2674	3.71	231
119	340.5	1185.3	874.1	.2696	3.68	229
120	341.1	1185.4	873.7	.2738	3.65	227
121	341.8	1185.6	873.2	.2759	3.62	225
122	342.4	1185.8	872.8	.2780	3.59	224
123	343.0	1186.0	872.3	.2801	3.56	222
124	343.6	1186.2	871.9	.2822	3.54	221
125	344.2	1186.4	871.5	.2845	3.51	219
126	344.8	1186.6	871.1	.2867	3.49	217
127	345.4	1186.8	870.7	.2889	3.46	215
128	346.0	1186.9	870.2	.2911	3.44	214
129	346.6	1187.1	869.8	.2933	3.41	212
130	347.2	1187.3	869.4	.2955	3.38	211
131	347.8	1187.5	869.0	.2977	3.35	209
132	348.3	1187.6	868.6	.2999	3.33	208
133	348.9	1187.8	868.2	.3020	3.31	206
134	349.5	1188.0	867.8	.3040	3.29	205
135	350.1	1188.2	867.4	.3060	3.27	203
136	350.6	1188.3	867.0	.3080	3.25	202
137	351.2	1188.5	866.6	.3101	3.22	200
138	351.8	1188.7	866.2	.3121	3.20	199
139	352.4	1188.9	865.8	.3142	3.18	198
140	352.9	1189.0	865.4	.3162	3.16	197
141	353.5	1189.2	865.0	.3184	3.14	195
142	354.0	1189.4	864.6	.3206	3.12	194
143	354.5	1189.6	864.2	.3228	3.10	193
144	355.0	1189.7	863.9	.3250	3.08	192
145	355.6	1189.9	863.5	.3273	3.06	190
146	356.1	1190.0	863.1	.3294	3.04	189
147	356.7	1190.2	862.7	.3315	3.02	188
148	357.2	1190.3	862.3	.3336	3.00	187
149	357.8	1190.5	861.9	.3357	2.98	186
150	358.3	1190.7	861.5	.3377	2.96	184
155	361.0	1191.5	859.7	.3484	2.87	179
160	363.4	1192.2	857.9	.3590	2.79	174

Table No. 128 (*continued*).

Total pressure per square inch.	Temperature in Fahrenheit degrees.	Total heat, in Fahr. degrees, from water at 32° F.	Latent heat, Fahrenheit degrees.	Density, or weight of one cubic foot.	Volume of one pound of steam.	Relative volume, or cubic feet of steam from one cubic ft. of water.
lbs.	Fahr.	Fahr.	Fahr.	lbs.	cubic feet.	Rel. vol.
165	366.0	1192.9	856.2	.3695	2.71	169
170	368.2	1193.7	854.5	.3798	2.63	164
175...	... 370.8 ...	... 1194.4 ...	... 852.9 ...	... .3899...	... 2.56 ...	... 159
180	372.9	1195.1	851.3	.4009	2.49	155
185	375.3	1195.8	849.6	.4117	2.43	151
190...	... 377.5 ...	... 1196.5 ...	... 848.0 ...	... .4222...	... 2.37 ...	... 148
195	379.7	1197.2	846.5	.4327	2.31	144
200	381.7	1197.8	845.0	.4431	2.26	141
210...	... 386.0 ...	... 1199.1 ...	... 841.9 ...	... .4634...	... 2.16 ...	... 135
220	389.9	1200.3	839.2	.4842	2.06	129
230	393.8	1201.5	836.4	.5052	1.98	123
240...	... 397.5 ...	... 1202.6 ...	... 833.8 ...	... .5248...	... 1.90 ...	... 119
250	401.1	1203.7	831.2	.5464	1.83	114
260	404.5	1204.8	828.8	.5669	1.76	110
270...	... 407.9 ...	... 1205.8 ...	... 826.4 ...	... .5868...	... 1.70 ...	... 106
280	411.2	1206.8	824.1	.6081	1.64	102
290	414.4	1207.8	821.8	.6273	1.59	99
300...	... 417.5 ...	... 1208.7 ...	... 819.6 ...	... .6486...	... 1.54 ...	... 96
350	430.1	1212.6	810.7	.7498	1.33	83
400	444.9	1217.1	800.2	.8502	1.18	73

Hypothetical values, calculated by means of the same formulas, for pressures beyond the range of Regnault's observations:—

450...	... 456.7 ...	... 1220.7 ...	... 791.9 ...	... .9499...	... 1.05 ...	... 66
500	467.5	1224.0	784.2	1.0490	.95	59
600	487.0	1229.9	770.4	1.2450	.80	50
700...	... 504.1 ...	... 1235.1 ...	... 758.3 ...	...1.4395...	... .69 ...	... 43
800	519.5	1239.8	747.4	1.6322	.61	38
900	533.6	1244.2	737.4	1.8235	.55	34
1000...	... 546.5 ...	... 1248.1 ...	... 728.3 ...	...2.0140...	... .50 ...	... 31

Note to Table.—This table was originally published in the article "Steam," contributed by the author to the *Encyclopedia Britannica*, 8th edition.

Table No. 129.—Comparative Density and Volume of Air and Saturated Steam.

Total pressure per square inch.	Temperature in Fahrenheit degrees.	Density, or weight of one cubic foot.		Volume of one pound.		Specific density of saturated steam.
		Air.	Steam.	Air.	Steam.	Air = 1.
lbs.	Fahrenheit.	lb.	lb.	cubic feet.	cubic feet.	
1	102.1	.0048	.0030	208.01	330.36	.622
5	162.3	.0217	.0138	46.04	72.66	.635
10	193.3	.0414	.0264	24.17	37.84	.638
14.7	212.0	.0591	.0380	16.91	26.36	.643
20	228.0	.0786	.0507	12.72	19.72	.645
30	250.4	.1142	.0743	8.76	13.46	.651
40	267.3	.1487	.0974	6.73	10.27	.655
50	281.0	.1824	.1202	5.48	8.31	.659
60	292.7	.2155	.1425	4.64	7.01	.661
70	302.9	.2481	.1648	4.03	6.07	.664
80	312.0	.2802	.1869	3.57	5.35	.667
90	320.2	.3119	.2089	3.21	4.79	.670
100	327.9	.3432	.2307	2.91	4.33	.672
110	334.6	.3743	.2521	2.67	3.97	.673
120	341.1	.4051	.2738	2.17	3.65	.676
130	347.2	.4355	.2955	2.30	3.38	.678
140	352.9	.4657	.3162	2.15	3.16	.679
150	358.3	.4957	.3377	2.02	2.96	.681
160	363.4	.5255	.3590	1.90	2.79	.683
170	368.2	.5551	.3798	1.80	2.63	.684
180	372.9	.5844	.4009	1.71	2.49	.686
190	377.5	.6135	.4222	1.63	2.37	.688
200	381.7	.6425	.4431	1.56	2.26	.690
220	389.9	.7000	.4842	1.43	2.06	.692
240	397.5	.7569	.5248	1.32	1.90	.694
260	404.5	.8133	.5669	1.23	1.76	.697
280	411.2	.8691	.6081	1.15	1.64	.700
300	417.5	.9246	.6486	1.08	1.54	.702

MIXTURE OF GASES AND VAPOURS.

If two or more gases or vapours, not having any power of chemical action one upon another, be introduced into the same space, each gas will, after a certain interval, be diffused equally throughout the whole of the space, and will occupy the space exactly as if no other gas were present. The gases thus become intimately mixed.

Moreover, the elastic force or pressure of each gas is the same as if it alone occupied the given space, and the total or resulting pressure of the mixture is equal to the sum of the pressures of the individual gases.

If a vessel be filled with dry air, and a sufficient quantity of water be introduced into the vessel, the water is evaporated, and the vapour occupies the vessel just as if the vessel had been empty, and had previously contained a vacuum. The evaporation proceeds until the vapour becomes saturated; that is to say, until the pressure and density of the vapour arrive at the maximum due to the temperature of the mixture.

And the final pressure of the mixture of air and vapour is equal to the pressure of the contained air plus the pressure of the vapour.

These two propositions, with respect to the mixture of air and vapour, hold with respect to the mixture of vapour with gases generally. They have been practically verified by the results of direct experiment made by M. Regnault; though he found a very slight inferiority of the pressure of vapour to that due to saturation, which he attributed to the hygroscopic affinity of the walls of the vessel.

The process of evaporation is much less rapid in presence of a gas, than when it takes place in a vacuum; owing to the resistance opposed by the pressure of the gas to the disengagement of vapour.

The same law applies for determining the pressure of a mixture of gas and vapour, when the quantity of vapour falls short of the condition of saturation.

Air is said to be saturated with moisture when the moisture or vapour it contains is itself in the condition of saturation, or of maximum density due to the temperature of the air.

HYGROMETRY.

The condition of the air with respect to moisture is called its humidity, or its relative humidity. The degree of humidity is expressed as a percentage of that due to the state of saturation for the temperature. For example, if the proportion of moisture in the atmosphere is just half that which it contains when saturated, the relative humidity is 50 per cent., or 50.

Dew Point.—When atmospheric air containing aqueous vapour is gradually cooled, the temperature of the vapour is lowered whilst its density

is increased, until the vapour arrives at its maximum density for the corresponding temperature. If cooled below this temperature, a part of the vapour is condensed and precipitated as dew. Hence this particular temperature is called the dew-point; and it is different for different degrees of humidity. The dew-point is no other than the temperature of saturated steam, which has arrived at its maximum density in the course of contraction by cooling, whilst, reversely, its temperature has been lowered.

HYGROMETERS.

Daniell's hygrometer, Fig. 123, is an instrument of great precision for ascertaining the dew-point. It consists of a bent tube with a globe at each end, and it is partly filled with ether. The rest of the space is occupied with vapour of ether, the air having been expelled. One of the globes, A, contains a thermometer *t*. This globe is generally made of black glass, which presents a brilliant surface. To use the instrument, the whole of the liquid is first passed into the globe A, and then the other globe B, which is covered with muslin, is moistened externally with ether. The evaporation of this ether from the muslin causes a partial condensation of vapour of ether in the interior of the globe, which produces a fresh evaporation from the surface of the liquid in A, thus lowering the temperature of that part of the instrument. By carefully watching the surface of the globe, the exact moment of the deposition of dew may be ascertained, and then the temperature is

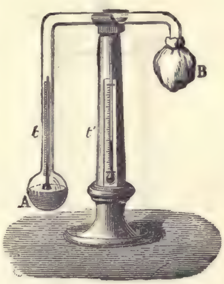


Fig. 123.
Daniell's Hygrometer.

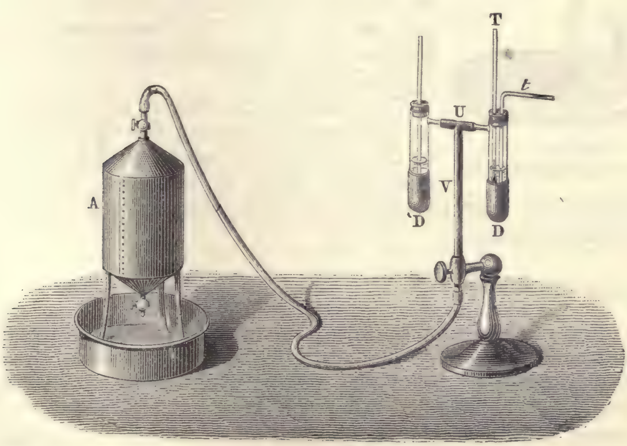


Fig. 124.—Regnault's Hygrometer.

read on the inclosed thermometer. This temperature is a little lower than the dew-point. If the instrument be now left to itself, the exact moment

of the disappearance of the dew may be observed, when the thermometer shows a temperature a little above the dew-point. The mean of the two observed temperatures is taken as the dew-point, and the temperature of the external air is shown by a thermometer t' attached to the stand.

Regnault's hygrometer, Fig. 124, consists of a glass tube closed at the bottom by a very thin silver cup D, and at the top by a cork, through which the stem of a thermometer T is passed, and a glass tube t' open at both ends. The lower end of the tube and the bulb of the thermometer dip into ether contained in the silver cup. The tube D is connected by the tube UV with the inspirator A, which contains water. When the water is allowed to escape from the bottom, a current of air is drawn through the ether, by agitating which the current maintains a uniformity of temperature in it. The cold produced by evaporation speedily causes a deposition of dew; and by the inverse action the dew disappears. The mean of the temperatures observed at the same times is the dew-point. The other tube D′ is not in connection with the aspirator, and it contains a thermometer to measure the temperature of the external air. Alcohol may be used instead of ether.

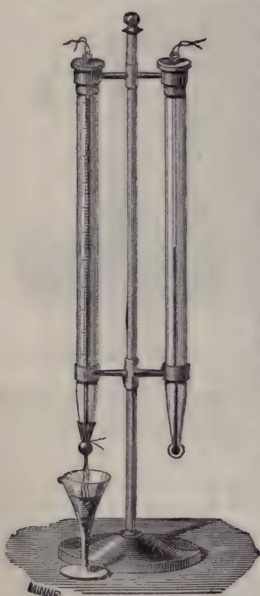


Fig. 125.
Wet and Dry Thermometers.

The wet and dry bulb thermometer, also known as Mason's thermometer, Fig. 125, is in general use. It consists of two thermometers precisely alike. The bulb of one is covered with muslin, which is kept moist by means of a cotton wick leading from a glass of water. The evaporation from the moistened bulb lowers the temperature of it, and the difference of the temperatures read on the two thermometers increases with the dryness of the air. The indications of the two thermometers are interpreted by means of tables specially composed.[1]

The pressure and density of saturated vapour, at temperatures ranging from the freezing point to the boiling point, under atmospheric pressure, have been given in table No. 127, page 386. The temperatures may be called the dew-points of the steams of the corresponding densities.

Properties of Saturated Mixtures of Air and Aqueous Vapour.

The leading properties of saturated mixtures of air and aqueous vapour, under a constant pressure of one atmosphere, or 14.7 lbs. per square inch, at final temperatures or dew-points ranging from 32° to 212° F., are given in table No. 130.

The second and third columns give the total pressures of the vapour and the air, in saturated mixtures, at the temperatures given in the first column. The sum of any pair of these pressures is equal to 14.7 lbs., or one atmos-

[1] These notices of hygrometers are derived from Deschanel's *Natural Philosophy*, English edition. Blackie & Son.

phere: they are complementary to each other. At 32°, for example, the pressure of the vapour, .089 lbs. + 14.611 lbs., the pressure of the air mixed with it, = 14.7 lbs.; and at 210°, the sum of the pressures, 14.126 + .574, is also equal to 14.7 lbs.

Columns 4 and 5 give the respective weights of vapour and air in 100 cubic feet of the saturated mixture, the sum of which, or the total weight of the mixture, is given in column 6.

Columns 7, 8, 9, give the quantities of heat reckoned from 32° F. in 100 cubic feet of the vapour, the air, and the mixture, respectively. The quantity of heat in the vapour is found by multiplying the quantity of heat in one pound of vapour, as given in column 10, by the weight of the vapour in column 4, of the present table.

For example, the total heat of one pound of saturated vapour at 32° F. is 1091.2 units, and the weight of 100 cubic feet of the vapour is .031 lb.: then

$$1091.2 \times .031 = 33.8 \text{ units of heat,}$$

which is the quantity of heat in the vapour given in the fourth column.

The quantities of heat, column 10, are copies of the fourth column of table No. 127, page 386, which are expressions of the quantity of heat in one pound of vapour, reckoned from 32° as the initial temperature of the water converted into vapour.

The quantity of heat in the air, column 8, is found by multiplying the specific heat of air, .2377, by the number of degrees of the temperature in excess of 32°, and by the weight of 100 cubic feet given in the fifth column.

The total heat in the mixture, column 9, is the sum of the heats in columns 7 and 8.

The quantity of dry air, column 11, required for one pound of vapour, in saturated mixture with it, is found by dividing the weight of air, column 5, by the relative weight of vapour, column 4. The volume at 62° F. of the air, column 12, is found by multiplying the weight in column 11, by 13.141 feet, the volume of one pound of air at 62°.

The thirteenth column of the table gives the initial temperature to which the quantities of dry air given in columns 11 and 12, would require to be raised, in order to provide a sufficient quantity of heat, if applied to the water at 32°, to evaporate it, and to form a saturated mixture at each of the temperatures, given in column 1. The product of the weight of air, col. 11, by the specific heat, is equal to the number of units of heat absorbed by the air for one degree elevation of temperature. If, therefore, the quantity of heat in one pound of vapour, column 10, be divided by the weight of air, column 11, and by the specific heat of air, .2377, the quotient, plus the final temperature of the saturated mixture, as given in column 1, is the required initial temperature.

By following the same directions, the various values may be found for any other final temperature of saturated mixture; and, inversely, the final temperature may be found for any given initial temperature.

Table No. 130.—PROPERTIES OF SATURATED MIXTURES OF AIR AND AQUEOUS VAPOUR,

Under a Constant Pressure of one atmosphere, or 14.7 lbs. per square inch, at temperatures of from 32° to 212° F.

Final temperature of the saturated mixture; or Dew-point.	Total pressure per square inch.		Weight of 100 cubic feet.			Quantity of heat in 100 cubic feet, reckoned from 32° F. (water at 32° for vapour).			Quantity of heat in 1 lb. of vapour, reckoned from water at 32° F.	Quantity of dry air required for 1 lb. of vapour, in saturated mixture.		Required initial temperature of the dry air to evaporate 1 lb. of moisture in saturated mixture.
	Vapour.	Air.	Vapour.	Air.	Saturated mixture.	Vapour.	Air.	Saturated mixture.		Weight.	Volume at 62° F.	
Fahr.	lbs.	lbs.	lbs.	lbs.	lbs.	units.	units.	units.	units.	lbs.	cubic feet.	Fahr.
32°	.089	14.611	.031	8.023	8.054	33.8	0.0	33.8	1091.2	258.8	3401	49°.7
35	.100	14.600	.034	7.970	8.004	37.1	5.7	42.8	1092.1	234.4	3080	54.6
40	.122	14.578	.041	7.879	7.920	44.8	15.0	59.8	1093.6	192.2	2526	63.8
45	.147	14.553	.049	7.785	7.834	53.6	24.1	77.7	1095.1	158.9	2088	74.0
50	.178	14.522	.059	7.693	7.752	64.7	32.9	97.6	1096.6	130.4	1714	85.4
55	.214	14.486	.070	7.598	7.668	76.8	41.5	118.3	1098.2	108.5	1426	97.6
60	.254	14.446	.082	7.507	7.589	90.1	50.0	140.1	1099.7	91.6	1203	110.5
65	.304	14.396	.097	7.410	7.507	106.8	58.1	164.9	1101.2	76.4	1004	125.6
70	.360	14.340	.114	7.311	7.425	125.6	64.1	189.7	1102.8	66.0	868	140
75	.427	14.273	.134	7.208	7.342	147.9	73.7	221.6	1104.3	55.0	723	160
80	.503	14.197	.156	7.106	7.262	172.5	81.1	253.6	1105.8	45.6	599	182
85	.592	14.108	.182	6.996	7.178	201.6	88.1	289.7	1107.3	38.4	505	206
90	.693	14.007	.212	6.896	7.108	235.1	95.1	330.2	1108.9	32.5	427	233
95	.809	13.891	.245	6.764	7.009	272.1	101.3	373.4	1110.4	27.6	363	264
100	.942	13.758	.283	6.641	6.924	314.7	107.3	422.0	1111.9	23.5	308	299
105	1.095	13.605	.325	6.505	6.830	361.8	112.9	474.7	1113.4	20.0	263	344
110	1.267	13.433	.373	6.368	6.741	415.8	118.1	533.9	1115.0	17.1	224	385
115	1.462	13.238	.426	6.224	6.650	476.4	122.7	599.1	1116.5	14.6	192	436

Table No. 130 (*continued*).

Final temperature of the saturated mixture; or Dew-point.	Total pressure per square inch.		Weight of 100 cubic feet.			Quantity of heat in 100 cubic feet, reckoned from 32° F. (water at 32° for vapour).			Quantity of heat in 1 lb. of vapour, reckoned from water at 32° F.	Quantity of dry air required for 1 lb. of vapour, in saturated mixture.		Required initial temperature of the dry air to evaporate 1 lb. of moisture in saturated mixture.
	Vapour.	Air.	Vapour.	Air.	Saturated mixture.	Vapour.	Air.	Saturated mixture.		Weight.	Volume at 62° F.	
Fahr.	lbs.	lbs.	lbs.	lbs.	lbs.	units.	units.	units.	units.	lbs.	cubic feet.	Fahr.
120°	1.685	13.015	.488	6.063	6.551	545.6	126.8	672.4	1118.0	12.4	163	499°
125	1.932	12.768	.554	5.900	6.454	620.1	130.4	750.5	1119.5	10.7	140	567
130	2.215	12.485	.630	5.717	6.347	706.2	133.2	839.4	1121.1	9.1	118	667
135	2.542	12.158	.714	5.524	6.238	801.5	135.2	936.7	1122.6	7.74	102	745
140	2.879	11.821	.806	5.325	6.131	906.0	136.7	1042.7	1124.1	6.61	86.8	856
145	3.273	11.427	.909	5.106	6.015	1023.4	137.2	1160.6	1125.6	5.62	73.8	986
150	3.708	10.992	1.022	4.869	5.891	1151.8	136.6	1288.4	1127.2	4.77	62.6	1,145
155	4.193	10.507	1.145	4.619	5.764	1292.3	135.1	1427.4	1128.7	4.03	53.0	1,332
160	4.731	9.969	1.333	4.346	5.679	1506.5	132.2	1638.7	1130.2	3.26	42.8	1,618
165	5.327	9.373	1.432	4.055	5.487	1620.6	128.2	1748.8	1131.7	2.83	37.1	1,847
170	5.985	8.715	1.602	3.739	5.341	1815.0	122.6	1937.6	1133.3	2.33	30.7	2,212
175	6.708	7.992	1.774	3.402	5.176	2013.1	115.6	2128.7	1134.8	1.92	25.2	2,665
180	7.511	7.189	1.970	3.036	5.006	2239.0	106.8	2345.8	1136.3	1.54	20.3	3,280
185	8.375	6.325	2.181	2.651	4.832	2481.7	96.4	2578.1	1137.8	1.22	16.0	4,124
190	9.335	5.365	2.411	2.231	4.642	2747.0	83.8	2830.8	1139.4	0.93	12.2	5,368
195	10.385	4.315	2.662	1.781	4.443	3037.0	69.0	3106.0	1140.9	0.67	8.8	7,370
200	11.526	3.174	2.933	1.300	4.233	3350.6	51.9	3402.5	1142.4	0.44	5.8	12,840
205	12.770	1.930	3.225	0.785	4.010	3689.5	32.3	3721.8	1143.9	0.24	3.2	19,985
210	14.126	.574	3.543	0.232	3.775	4058.2	9.8	4068.0	1145.5	0.065	0.86	73,940
212	14.700	.000	3.683	0.000	3.683	4221.1	0.0	4221.1	1146.1	0.000	0.00	—

COMBUSTION.

The combustible elements of fuel are carbon, hydrogen, and sulphur. There are other elements in fuel—nitrogen, water, and solid incombustible matter—which do not take part in combustion. Fuel is burned with atmospheric air, of which the oxygen combines with the combustible matter, whilst the nitrogen remains neutral. The combining proportions of the elements concerned in or about combustion are given in table No. 131:—

Table No. 131.—Composition and Combining Equivalents of Gases concerned in the Combustion of Fuel. (Old Nomenclature.)

Gases.	Elements of the Gases.		Combining Equivalents. By Weight.	Combining Equivalents. By Measure.
		Equivalents.		One Volume = □
ELEMENTS:—				
Oxygen,	Oxygen,	1	8	▭
Hydrogen,	Hydrogen,	1	1	□
Carbon,	Carbon,	1	6	□
Sulphur,	Sulphur,	1	16	□
Nitrogen,	Nitrogen,	1	14	□
COMPOUNDS:—				
Light Carburetted Hydrogen,	Carbon, Hydrogen,	2 4	12 4 } = 16	□□
Olefiant Gas,	Carbon, Hydrogen,	4 4	24 4 } = 28	□□
Atmospheric Air (mechanical mixture),	Oxygen, Nitrogen,	23 77	8 26.8 } = 34.8	▭ □□ } = □□▭ approximately
Carbonic Oxide,	Oxygen, Carbon,	1 1	8 6 } = 14	▭ □ (ideal) } = □
Carbonic Acid,	Oxygen, Carbon,	2 1	16 6 } = 22	⊟ □ (ideal) } = □
Aqueous Vapour or Water,	Oxygen, Hydrogen,	1 1	8 1 } = 9	▭ □ } = □
Sulphurous Acid,	Oxygen, Sulphur,	2 1	16 16 } = 32	⊟ □ (ideal) } = □

The volume of one pound of the principal gases at 62° F., under one atmosphere is as follows:—

Gas at 62° F.	One Pound. Cubic feet.
Oxygen,	11.887
Hydrogen,	190.000
Nitrogen,	13.501
Air,	13.141
Carbonic Acid,	8.594
Aqueous Vapour, as Gaseous Steam,	21.125
Sulphurous Acid,	5.848

The source of oxygen, as the supporter of the combustion of fuel, is atmospheric air, which consists of oxygen and nitrogen in mechanical combination, in the proportion of 8 to 26.8; or 1 lb. of oxygen to 3.35 lbs. of nitrogen; or, by volume, 1 cubic foot of oxygen to 3.76 cubic feet of nitrogen.

For every pound of oxygen employed in combustion, 4.35 lbs. of air are consumed; or, by measure, for every cubic foot of oxygen employed in combustion, 4.76 cubic feet of air are consumed. For the combustion of one pound of hydrogen, of carbon, and of sulphur, therefore, the quantities of air chemically consumed are as follows:—

One Pound.			
Hydrogen consumes	34.8 lbs., or 457 cubic feet,	of air at 62°.	
Carbon, completely burned, consumes	11.6 lbs., or 152	do.	do.
Carbon, partially burned, consumes	5.8 lbs., or 76	do.	do.
Sulphur consumes	4.35 lbs., or 57	do.	do.

The process of their combustion is indicated in the following tablets:—

Combustion of Hydrogen.

Elements.	Process.	Products.
1 pound hydrogen,	hydrogen, 1 pound,	9 pounds water. (hydrogen and oxygen)
34.8 pounds air,	oxygen, 8 pounds,	
	nitrogen, 26.8 pounds,	26.8 pounds nitrogen.
35.8	35.8	35.8

Complete Combustion of Carbon.

Elements.	Process.	Products.
1 pound carbon,	carbon, 1 pound,	3.66 pounds carbonic acid.
11.6 pounds air,	oxygen, 2.66 pounds,	
	nitrogen, 8.94 pounds,	8.94 pounds nitrogen.
12.6	12.6	12.6

Combustion of Sulphur.

Elements.	Process.	Products.
1 pound sulphur,	sulphur, 1 pound	2 pounds sulphurous acid.
4.35 pounds air,	oxygen, 1 pound	
	nitrogen, 3.35 pounds	3.35 pounds nitrogen.
5.35	5.35	5.35

AIR CONSUMED IN THE COMBUSTION OF FUELS.

Fuels are, for the most part, compounds of carbon, hydrogen, sulphur, and oxygen, in various proportions; and to form a calculation of the quantity of air chemically consumed in the combustion of a fuel, the several quantities required for each combustible element of the fuel are to be calculated. When oxygen is present as a constituent of a fuel, it exists in combination with a portion of hydrogen as water, in solid fuels at least; and in any fuel, liquid or solid, the combined oxygen and hydrogen are driven off, in the process of combustion, as water or steam. Such portion of the constituent hydrogen, therefore, as is thus driven off, already saturated with oxygen, is to be excluded from the calculation for the quantity of air necessary to consume the remainder of the hydrogen, and the carbon and sulphur. Let the constituents of the fuel be expressed proportionally as percentages of the total weight by their initials C, H, O, S, respectively; then the volume of air at 62° F. chemically consumed in the combustion of one pound of a fuel, is expressed for each combustible as follows:—

	Cubic feet.
For the carbon,....................	$152\ C \div 100.$
For the hydrogen,.................	$457\ (H - \frac{O}{8}) \div 100.$
For the sulphur,....................	$57\ S \div 100.$

The quantity $\frac{O}{8}$ signifies the deduction to be made from the constituent hydrogen, for that portion which forms steam with the constituent oxygen, being equal in weight to one-eighth part of the weight of the oxygen. The total volume of air is the sum of these three items:—

$$\frac{152\ C + 457\ (H - \frac{O}{8}) + 57\ S}{100}.$$

Putting A for the total volume of air at 62°, and reducing—

$$A = \frac{152\ (C + 3\ (H - \frac{O}{8}) + .4\ S)}{100}, \text{ or}$$

$$A = 1.52\ (C + 3\ (H - \frac{O}{8}) + .4\ S) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

RULE 1.—*To find the quantity of air at 62° F., under one atmosphere, chemically consumed in the complete combustion of one pound of a given fuel.* Let the constituent carbon, hydrogen, oxygen, and sulphur, be expressed as percentages of the whole weight of the fuel; divide the oxygen by 8, deduct the quotient from the hydrogen, and multiply the remainder by 3; multiply the sulphur by 0.4; add these two products to the carbon, and multiply the sum by 1.52. The final product is the quantity of air in cubic feet.

To find the weight of the air chemically consumed, divide the volume thus found by 13.14; the quotient is the weight of the air in pounds.

Note.—In making ordinary approximate calculations, the sulphur may be omitted.

QUANTITY OF THE GASEOUS PRODUCTS OF THE COMPLETE COMBUSTION OF ONE POUND OF FUEL. 1. *By Weight.*

Pound.		Pounds.		Pounds.	
1 carbon,	and	2.66	oxygen, form	3.66	of carbonic acid.
1 hydrogen,	and	8	oxygen, form	9	of steam.
1 sulphur,	and	1	oxygen, form	2	of sulphurous acid.

Then in the combustion of one pound of fuel the weights of the products are as follows:—

$$3.66\ C \div 100 = .0366\ C = \text{the weight of carbonic acid} \ldots\ldots\ldots (a)$$
$$9\ H \div 100 = .09\ H = \text{the weight of steam} \ldots\ldots\ldots (b)$$
$$2\ S \div 100 = .02\ S = \text{the weight of sulphurous acid} \ldots\ldots\ldots (c)$$

To this is to be added the weight of atmospheric nitrogen separated from the oxygen chemically consumed, and the weight of the constituent nitrogen, N, of the fuel. The quantity of atmospheric nitrogen is 3.35 times, by weight, that of the oxygen consumed; and,

Pound.		Pounds.	
For 1 carbon there are	$2.66 \times 3.35 =$	8.93	nitrogen.
For 1 hydrogen there are	$8 \times 3.35 =$	26.8	do.
For 1 sulphur there are	$1 \times 3.35 =$	3.35	do.

Multiply each of these quantities by their respective percentages of combustible, and divide by 100; the sum of the quotients is the weight of nitrogen separated from the atmospheric oxygen consumed. To this is to be added the constituent nitrogen of the fuel:—

$$8.93\ C \div 100 = .0893\ C.$$
$$26.8\ H \div 100 = .268\ H.$$
$$3.35\ S \div 100 = .0335\ S.$$
$$N \div 100 = .01\ N.$$

Thus, the total weight of nitrogen is equal to

$$(.0893\ C + .268\ H + .0335\ S + .01\ N) \ldots\ldots\ldots (d)$$

Add together the total weights of carbonic acid, steam, sulphurous acid, and nitrogen above noted, and put w for the total weight of the gaseous products of combustion, then—

$$w = .0366\ C + .09\ H + .02\ S + (.0893\ C + .268\ H + .0335\ S + .01\ N);$$
$$\text{or } w = .126\ C + .358\ H + .053\ S + .01\ N \ldots\ldots\ldots (2)$$

RULE 2.—*To find the total weight of the gaseous products of the complete combustion of one pound of a fuel.* Let the elements be expressed as percentages of the fuel; multiply the carbon by 0.126, the hydrogen by 0.358, the sulphur by 0.053, and the nitrogen by .01, and add together those four products. The sum is the total weight of the gases in pounds.

Note.—The weight, in pounds, of the carbonic acid, separately, may be found from the quantity (*a*), above; that of the steam from (*b*), that of the sulphurous acid from (*c*), and that of the nitrogen from (*d*).

2. *By Volume.*

Multiply the weight of each gaseous product, (*a*), (*b*), (*c*), (*d*), by the volume of one pound in cubic feet at 62° F., page 399. Then

Cubic feet.

$$.0366\ C \times 8.59 = .315\ C = \text{Volume of the carbonic acid} \ldots\ldots (e)$$
$$.09\ H \times 21.125 = 1.9\ H = \text{Volume of the steam} \ldots\ldots (f)$$
$$.02\ S \times 5.85 = .117\ S = \text{Volume of the sulphurous acid} \ldots (g)$$
$$(.0893\ C + .268\ H + .0335\ S + .01\ N) \times 13.5 =$$
$$(1.206\ C + 3.618\ H + .45\ S + .135\ N) = \text{Volume of the nitrogen} \ldots (h)$$

Adding together and reducing, and putting V = the total volume of the gases,

$$V = 1.52\ C + 5.52\ H + .567\ S + .135\ N \ldots\ldots\ldots\ldots (3)$$

RULE 3.—*To find the total volume, at 62°, of the gaseous products of the complete combustion of one pound of fuel.* Let the elements be expressed as percentages; multiply the carbon by 1.52, the hydrogen by 5.52, the sulphur by .567, and the nitrogen by .135, and add together the four products. The sum is the total volume, at 62° F., of the gases, in cubic feet.

Note.—The volume of the several gases separately may be found from the respective quantities (*e*), (*f*), (*g*), (*h*).

The volume of the gases at higher temperatures than 62° F. is found by the formula (2), page 347; namely,

$$V' = V\frac{t' + 461}{t + 461} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

As $t = 62°$, this formula becomes, for present purposes,

$$V' = V\frac{t' + 461}{523}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and it appears that the initial or normal volume, as at 62°, under one atmosphere, is doubled when the temperature is raised 523° higher; and that the expanded volume at any other temperature t', in proportion to the normal volume, is found by adding 461 to the temperature, and dividing the sum by 523.

SURPLUS AIR.

If the quantity of surplus air that enters the furnace and passes away unconsumed, be expressed as a percentage of the air chemically consumed, it is found directly from the latter when this is known. If the volume is given, the weight is found by dividing the volume at 62° in cubic feet by 13.14.

HEAT EVOLVED BY THE COMBUSTION OF FUEL.

From the experimental investigations of MM. Favre and Silbermann, the total quantities of heat evolved by the combustion of one pound of some combustibles with oxygen were determined as follows:—

SIMPLE BODIES.	Units of heat.
Hydrogen	62,032
Carbon—Wood charcoal, thoroughly calcined	14,544
Sugar charcoal	14,470
Gas-coke	14,485
Graphite, from blast-furnaces	13,972
Natural graphite	14,033
Diamond (pure carbon)	13,986
Sulphur	4,032

Compound Bodies.	Units of heat.
Carbonic oxide	4,325
Light carburetted hydrogen	23,513
Olefiant gas	21,343
Sulphuric ether	16,249
Alcohol	12,929
Turpentine	19,534
Sulphuret of carbon	6,120
Wax	18,893

The chemical composition of the compound bodies above cited is as follows, and there is added the composition of a few other combustibles:—

Table No. 132.—Chemical Composition of Compound Combustibles.

Combustible.	Combining equivalents.			In 100 parts by weight.		
	Carbon.	Hydrogen.	Oxygen.	Carbon.	Hydrogen.	Oxygen.
				per cent.	per cent.	per cent.
Carbonic oxide	1	—	1	42.9	—	57.1
Light carburetted hydrogen	2	4	—	75.0	25.0	—
Olefiant gas	4	4	—	85.7	14.3	—
Sulphuric ether	4	5	1	64.8	13.5	21.7
Alcohol	4	6	2	52.2	13.0	34.8
Turpentine	20	16	—	88.2	11.8	—
Wax				81.6	13.9	4.5
Olive oil				77.2	13.4	9.4
Tallow				79.0	11.7	9.3

The heating powers of the compound bodies are approximately equal to the sum of the heating powers of their elements. Peclet gives a number of examples in proof of this. Take light carburetted hydrogen, which consists of two equivalents of carbon and four of hydrogen, weighing respectively $2 \times 6 = 12$ and $1 \times 4 = 4$, in the proportion of 3 to 1, or ¾ lb. of carbon and ¼ lb. of hydrogen in one pound of the gas. The elements of the heat of combustion of one pound are, then,

		Units of heat.
For the carbon	$14{,}544 \times \frac{3}{4} =$	10,908
For the hydrogen	$62{,}032 \times \frac{1}{4} =$	15,508
Total heat of combustion, calculated		26,416
Total heat, by direct trial		23,513
Excess by calculation		2,903

Alcohol has 4 of carbon, 6 of hydrogen, and 2 of oxygen. Abstracting the proportion of hydrogen neutralized by the oxygen, there are to be dealt with, 4 of carbon, 4 of hydrogen, and 2 of water, the weights of which are

as 24, 4, and 18; total 46. The quantities of heat evolved in the combustion of one pound of alcohol are, therefore,

		Units of heat.
For the carbon	$14{,}544 \times {}^{24}/_{46} =$	7588
For the hydrogen	$62{,}032 \times {}^{4}/_{46} =$	5394
Total heat evolved, calculated		12,982
Total by direct trial		12,929
Excess as calculated		53

Olive oil consists, in 100 parts, of 77.2 carbon, 13.4 hydrogen, and 9.4 oxygen; or 77.2 carbon, 12.2 hydrogen, and 10.6 water. The heat of combustion is, by calculation,

		Units of heat.
For the carbon	$14{,}544 \times {}^{77.2}/_{100} =$	11,228
For the hydrogen	$62{,}032 \times {}^{12.2}/_{100} =$	7,568
Total heat		18,796

Tallow consists of 79 carbon, 11.7 hydrogen, and 9.3 oxygen, in 100 parts; or 79 carbon, 10.54 hydrogen, and 10.46 water. The heat of combustion is, by calculation,

		Units of heat.
For the carbon	$14{,}544 \times {}^{79}/_{100} =$	11,490
For the hydrogen	$62{,}032 \times {}^{10.54}/_{100} =$	6,538
Total heat		18,028

The successive evolvements of heat in burning carbon,—when carbonic oxide is formed, and when it is converted into carbonic acid,—are deduced from the fact that one pound of carbonic oxide, when burned with oxygen to form carbonic acid, evolves 4325 units of heat. As the oxide consists of 6 of oxygen to 8 of carbon, a pound of it contains ${}^{6}/_{14}$ths of a pound of carbon, the combustion of which has produced 4325 units of heat. In the same ratio, one pound of carbon, as carbonic oxide, would evolve

$$4325 \times 14 \div 6 = 10{,}092 \text{ units of heat}$$

in being converted from oxide into acid. Therefore, the heat of complete combustion, 14,544 units, minus 10,092 = 4452 units, is the heat evolved in the conversion of the carbon into the oxide, and the successive developments of heat by the combustion of one pound of carbon are as follows:—

	Units of heat.	
In the first stage, forming carbonic oxide	4,452, or	30 per cent.
In the second stage, forming carbonic acid	10,092, or	70 „
Heat evolved by complete combustion	14,544, or	100 „

Table of the Heating Powers of Combustibles.

The experimental results of MM. Favre and Silbermann are adopted with some slight revision, recommended by M. Peclet, in table No. 133, column 5. The weight of oxygen, column 2, is calculated from the known equivalents and weights of the elements, as given in table No. 131, page 398.

The weight of air, column 3, is 4.35 times the weight of oxygen, column 2, and the volume of air at 62°, column 4, is 13.14 times the weight, column 3. The equivalent evaporative power, columns 6 and 7, is expressed by the weight of water evaporable at 212° by one pound of combustible—first, if supplied at 62° F., by dividing the total heat of combustion, column 5, by 1116°, which is the total heat of atmospheric steam raised from water supplied at 62°; second, if supplied at 212° F., by dividing by 966°, the total heat of atmospheric steam raised from water supplied at 212°.

Table No. 133.—Total Heat Evolved by Combustibles and their Equivalent Evaporative Power, with the Weight of Oxygen and Volume of Air Chemically Consumed.

Combustibles.	Weight of oxygen consumed per lb. of combustible.	Quantity of air consumed per pound of combustible.		Total heat of combustion of 1 pound of combustible.	Equivalent evaporative power of 1 pound of combustible, under one atmosphere, at 212°.	
1 pound weight.	lbs.	lbs.	cubic feet at 62°.	units.	pounds of water at 62°.	pounds of water at 212°.
Hydrogen	8.0	34.8	457	62,032	55.6	64.20
Carbon, making carbonic oxide	1.33	5.8	76	4,452	4.0	4.61
Carbon, making carbonic acid	2.66	11.6	152	14,500	13.0	15.0
Graphite	2.66	11.6	152	14,040	12.58	14.53
Carbonic oxide	0.57	2.48	33	4,325	3.88	4.48
Light carburetted hydrogen	4.0	17.4	229	23,513	21.07	24.34
Bi-carburetted hydrogen, or olefiant gas	3.43	15.0	196	21,343	19.12	22.09
Sulphuric ether	2.60	11.3	149	16,249	14.56	16.82
Alcohol	2.78	12.1	159	12,929	11.76	13.38
Turpentine	3.29	14.3	188	19,534	17.50	20.22
Sulphur	1.00	4.35	57	4,032	3.61	4.17
Wax	3.24	14.1	185	18,893	16.93	19.56
Olive oil	3.03	13.2	173	18,796	16.84	19.46
Tallow	2.95	12.83	169	18,028	16.15	18.66
(*Supplementary.*)						
Coal, of average composition	2.46	10.7	141	14,133	12.67	14.62
Coke, desiccated	2.50	10.9	143	13,550	12.14	14.02
Wood, desiccated	1.40	6.1	80	7,792	6.98	8.07
Wood-charcoal, desiccated	2.25	9.8	129	12,696	11.38	13.13
Peat, desiccated	1.75	7.6	100	9,951	8.91	10.30
Peat-charcoal, desiccated	2.28	9.9	129	12,325	11.04	12.76

From the table, it appears that when carbon is not completely burned, and becomes carbonic oxide, it produces less than a third of the heat yielded when it is completely burned. For the heating power of carbon an average of 14,500 units will be adopted. The heating power of hydrogen is about four and a quarter times that of carbon.

The calculation for the heating power of a combustible may be reduced to a simple formula. Let C, H, O, and S represent, as before, the percentages of carbon, hydrogen, oxygen, and sulphur, in 100 parts. The elements of the heat evolvable are as follows:—

Of the carbon,............$14{,}500\ C \div 100.$

Of the hydrogen,.........$62{,}032\ (H - \frac{O}{8}) \div 100.$

Of the sulphur,............ $4{,}032\ S \div 100.$

The quantity $\frac{O}{8}$ is a deduction made from the hydrogen to satisfy the constituent oxygen of the fuel: being an eighth of the weight of the oxygen. The total evolvable heat is

$$\frac{14{,}500\ C + 62{,}032\ (H - \frac{O}{8}) + 4032\ S}{100}, \text{ or}$$

$$\frac{14{,}500\ C + (14{,}500 \times 4.28\ (H - \frac{O}{8})) + (14{,}500 \times .28\ S)}{100};$$

or, putting h for the total heat,

$$h = 145\ (C + 4.28\ (H - \frac{O}{8}) + 0.28\ S) \ \ldots\ldots\ldots\ldots\ldots (6)$$

RULE 4.—*To find the total heating power of one pound of a combustible, of which the percentages of the constituent carbon, hydrogen, oxygen, and sulphur are given.* From the hydrogen deduct one-eighth of the oxygen, and multiply the remainder by 4.28; multiply the sulphur by 0.28; add the two products to the carbon; and multiply the sum by 145. The final product is the total heating power of one pound of the combustible, in units of heat.

Note.—The item of sulphur as a combustible may be ignored in calculations for ordinary purposes.

Dividing the second member of the formula (6) by 1116°, the total heat of steam at 212° raised from water at 62°; or by 966° if the water be supplied at 212°; the quotients express the equivalent evaporative power of the combustible. Putting e for the evaporative power, in pounds of water per pound of combustible,—

$$e = 0.13\ (C + 4.28\ (H - \frac{O}{8}) + 0.28\ S), \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

when the water is supplied at 62°; and

$$e = 0.15\ (C + 4.28\ (H - \frac{O}{8}) + 0.28\ S), \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

when the water is supplied at 212°.

RULE 5.—*To find the total evaporative power of one pound of a combustible, of which the percentages of the constituent carbon, hydrogen, sulphur, and oxygen are given.* From the hydrogen deduct one-eighth of the oxygen, and multiply the remainder by 4.28; multiply the sulphur by 0.28; add these two products to the carbon, and multiply the sum by 0.13 when the

water is supplied at 62°, or by 0.15 when the water is supplied at 212°. The final product is the total evaporative power of one pound of the combustible in pounds of water, evaporated at 212°.

Note.—When the total heating power is known, divide it by 1116, when the water is supplied at 62°; or by 966 when the water is supplied at 212°. The quotient is the equivalent evaporative power.

The equivalent evaporative power, from water supplied at 212°, is found roughly by dividing the total heat by 1000.

Temperature of Combustion.

The temperature of combustion is settled by the several quantities and specific heats of the products of combustion. One pound of carbon when completely burned yields 3.66 lbs. of carbonic acid, and 8.94 lbs. of nitrogen. Multiply these by the respective specific heats of the gases—

For Carbon.

For 1 lb. Carbon.		Specific heat.	Units of heat.	
Carbonic acid,.....	3.66 lbs. ×	.2164 =	.792	for 1° F.
Nitrogen,...........	8.94 lbs. ×	.244 =	2.181	,,
	12.60 lbs. ×	.236 =	2.973	,,

showing that the products of combustion absorb 2.973 units of heat in rising 1° F. of temperature. Divide the total heat of combustion, 14,500 units, by 2.973, and the quotient is 4877° F. Add the initial temperature, say 62°, making 4939° F. the temperature of combustion.

For Hydrogen.

For 1 lb. Hydrogen.		Specific heat.	Units of heat.	
Gaseous steam,....	9 lbs. ×	.475 =	4.275	for 1° F.
Nitrogen,...........	26.8 lbs. ×	.244 =	6.539	,,
	35.8 lbs. ×	.302 =	10.814	,,

The total heat of combustion, 62,032 units ÷ 10.8 = 5744°. Add 62° for the temperature of combustion of hydrogen, making 5806° F.

For Sulphur.

For 1 lb. Sulphur.		Specific heat.	Units of heat.	
Sulphurous acid,..	2 lbs. ×	.1553 =	.311	for 1° F.
Nitrogen,...........	3.35 lbs. ×	.244 =	.817	,,
	5.35 lbs. ×	.211 =	1.128	,,

The total heat of combustion, 4032 units ÷ 1.128 = 3575°. Add 62° for the temperature of combustion of sulphur, making 3637° F.

For coal of average composition (the calculation for which will be given in detail) there are 11.94 lbs. of gaseous products, of which the mean specific heat is .246; and

$$11.94 \times .246 = 2.935 \text{ units of heat for } 1^\circ \text{ F.}$$

The total heat of combustion, 14,133 units ÷ 2.935 = 4815°. Add 62° for the temperature of combustion of average coal, making 4877° F.

If surplus air be mixed with the products of combustion, and equal in

quantity to the air chemically consumed, the total weight of gases for one pound of coal is increased to 22.64 lbs., having a mean specific heat, .242; and

$$22.64 \times .242 = 5.478 \text{ units of heat for } 1^\circ \text{ F.}$$

The total heat of combustion, 14,133 units ÷ 5.478 = 2580°. Add 62° for temperature of combustion, making 2642° F.; which is little more than half the temperature of undiluted products of combustion.

From the annexed table, No. 134, it appears that the specific heat of the products of combustion is in general about .250; excepting that for hydrogen, which is .302; and that for sulphur, which is .211.

Table No. 134.—Weight and Specific Heat of the Products of Combustion and the Temperature of Combustion.

(With only the net supply of air chemically necessary.)

One Pound of Combustible.	Gaseous Products for One Pound of Combustible.				
	Weight.	Specific Heat.	Heat to Raise the Temperature 1° F.	Temperature of Combustion, measured from Initial Temperature.	
	pounds.	water = 1.	units.	Fahr.	ratio.
Hydrogen	35.80	.302	10.814	5733°	100
Sulphuric ether	11.97	.256	3.063	5305	92
Olefiant gas	15.90	.257	4.089	5219	91
Petroleum	15.35	.256	3.930	5193	90
Olive oil	14.21	.258	3.666	5128	89.3
Tallow	13.84	.256	3.540	5093	89
Wood, desiccated	8.45	.253	2.136	5138	89
Coal (average)	12.00	.246	2.924	5027	87
Carbon	12.60	.236	2.973	4877	85
Coke	11.77	.236	2.778	4878	85
Wax	15.21	.257	3.914	4826	84
Alcohol	10.09	.270	2.680	4825	84
Light carburetted hydrogen	18.40	.268	4.933	4766	83
Coal, with 10 per cent. more air	12.94	.245	3.189	4432	77
Sulphur	5.35	.211	1.128	3575	62
Coal, with 50 per cent. more air	17.22	.244	4.196	3527	61
Turpentine	12.18	.257	3.127	3470	60
Coal, with 100 p. cent. more air	22.57	.242	5.467	2688	47

FUELS.—COAL.

The fuels, or combustibles, generally used are coal, coke, wood, wood-charcoal, peat, peat-charcoal, and refuse tan-bark. To these may be added petroleum and other oils; recently, straw has been used.

Coal may be arranged in five classes:—

1st. Anthracite, or blind coal, consisting almost entirely of free carbon.

2d. Dry bituminous coal, having from 70 to 80 per cent. of carbon.

3d. Bituminous caking coal, having from 50 to 60 per cent. of carbon.

4th. Long flaming or cannel coal.

5th. Lignite, or brown coal, containing from 56 to 76 per cent. of carbon.

The anthracites have specific gravities varying from 1.35 to 1.92. They retain their form when exposed to a temperature of ignition; though, if too rapidly heated, they fall to pieces. The flame is generally short, of a blue colour. The coal is ignited with difficulty; it yields an intense local or concentrated heat; and combustion generally becomes extinct while yet a considerable quantity of the fuel remains on the grate.

The dry, or free-burning, bituminous coals, are rather lighter than the anthracites; varying in specific gravity from 1.28 to 1.44. They contain a relatively small proportion of volatilizable matter,—about 15 per cent.;—and they soon arrive at the temperature of full ignition. They swell considerably in coking, and thus is facilitated the access of air, and the rapid and complete combustion of their fixed carbon. In some cases, where the combustion is slow, the masses of coke scarcely cohere, and the original forms of the pieces of the coal are in some measure preserved.

The bituminous caking coals have the same range of specific gravity as the dry bituminous coals. They contain the maximum proportion of volatilizable matter, averaging about 30 per cent. of their whole weight. They develop much of the hydrocarbon gases, and burn with a long flame. They swell considerably, and give a coherent coke, which preserves nothing of the original form of the coal.

Small Coal.

In South Wales, where the recognized system of working is by "pillar and stall," upwards of 40 per cent. of the actual contents of the vein of steam-coal is lost.

According as the small coal is raised to the surface or not, the pit is said to be worked on the "altogether coal," or on the "separation" principle. In the former case, from 45 to 50 per cent. of small coal passes through the screen; in the latter case, where only hand-picked coal is sent to the surface, the small amounts to from 5 to 10 per cent. The small coal is screened into three sizes, known as "nuts," "peas" or "beans," and "duff"

or "waste." Small coal generally consists of what passes through screens with spaces between the bars from 1¼ inches, as in South Wales, to ⅝ inch, as in the Newcastle district. The duff consists of what passes through meshes ⅜ inch square; the peas or beans consist of what does not pass through these meshes, but falls between bars 7/16 inch apart. The remainder of the small is nuts.

The relative proportions of large and small coal, on the "altogether" system, and the "separation" system, brought to the surface, in the Newcastle district, may be taken as follows:—

	ALTOGETHER.	SEPARATION.
Round coal	46.1 per cent.	80.29 per cent.
Small coal:—		
Nuts	20.9 ,,	12.50 ,,
Beans	17.6 ,,	3.85 ,,
Duff	15.4 ,,	3.36 ,,
	100.00	100.00

The relative market values of the different sizes of coal as raised, are illustrated by the following list of quotations from a certain colliery, delivered free on board at Sunderland.

Round coals, 10s. to 11s., average	10s. 6d.	per ton,	say	100
Treble-screened nuts	8s.	,,	or	76
Double-screened nuts	7s.	,,	or	67
Pea nuts	6s.	,,	or	57
Single-screened small	6s.	,,	or	57
Pea nuts and duff mixed	4s. 6d.	,,	or	43
Duff	3s. 6d.	,,	or	33

The quantity of small coal separated from the coal brought by railway to London is found, at the end of the journeys, in passing through screens at the staiths, to amount to from 5 to 11 per cent., averaging, probably, 7½ per cent. This represents the breakage of coal between the loading at the pit's mouth and the discharging in London.

The breakage of coal conveyed by sea is also considerable. Between the colliery and the ship, it has been estimated, in one case, at 5 per cent.; and when double-screened at Cardiff, at from 8 to 8½ per cent. Again, the quantity of small coal made by loading into and unloading from the ship is stated to be from 15 to 20 per cent.[1]

In France, at St. Etienne, the quotations were respectively as follows, for round coal, medium coal, and slack:—

Round coal,	2 francs	per 100 kilogrammes,	or 16s.	per ton,	say 100
Medium coal,	1.25 ,,	,, ,,	or 10s.	,,	or 62
Slack,	0.25 to 0.50 ,,	,, ,,	or 2s. to 4s.	,,	12 to 24

Utilization of Small Coal.—It is a matter of national importance to utilize the immense accumulations of small coal, both above and below ground.

[1] See *Coal Economy*, by Mr. F. C. Danvers, 1872; from which the above particulars of small coal are derived.

The best known system is that of Warlich's patent fuel,—a mixture of small coal and tar or pitch moulded into blocks. Each ton of small coal is mixed with 22 gallons, or 242 lbs. of tar, which is over 10 per cent. of the weight of the coal. It is then formed into blocks, and baked at a temperature of 800° F. for nine or ten hours. The volatile matter of the tar is driven off, leaving the pitch as a cement for the coal. In the process of baking, the blocks lose 5 per cent. of their weight.

Wylam's fuel is prepared by mixing with slack about 7 or 8 per cent. of its weight of pitch, in a dry state, ground fine. The mixture is passed by means of an Archimedian screw through a retort maintained at a dull red heat, by which it is softened, when it is moulded under great pressure by a species of brickmaking machine.

Mezaline's fuel, like Wylam's, is a mixture of slack and pitch ground fine, in a pug-mill, where it is at the same time softened by superheated steam introduced into the mass at different points, thus to increase the cohesion of the particles. Fuel thus prepared, when exposed to a high temperature, loses four per cent. of its weight, representing, no doubt, the moisture acquired from the steam.

In Barker's fuel, the binding medium consists of a mucilage formed by the mixture of potato-farina with water, in the proportion of about 1 to 4, with a small quantity of carbolic acid. Thirty gallons of the mucilage was mixed with one ton of coal, and the mixture baked for nine hours at a temperature of 300° F. This mixture was not hard enough to stand rough usage or exposure; and more recently a certain proportion of powdered pitch has been added with good results. The bulk of 1 ton is 33 cubic feet.

In Holland's fuel, lime and cement are mixed with small coal;—making, of course, a large quantity of ash when burned.

Washing of Small Coal.—Coal-washing has long been practised on the Continent for the purpose of separating from the small coal the greater part of the schists, pyrites, and other matters mixed with it when it is extracted from the mine. The clean coal thus obtained is useful principally for the manufacture of coke for metallurgical purposes and for locomotives. The advantages derivable from the washing of coal are beginning to be appreciated in England.

Coal is washed by two different methods. By one system, a wooden box is divided into two compartments, by a partition which descends nearly to the bottom, leaving a communication between the two, of which one is smaller than the other. The larger of the two is fitted with two grates, one above the other, of which the upper one is formed of bars with interspaces of 0.4 inch in width, and the lower is a plate, usually of copper, pierced with numerous small holes. The smaller compartment contains a piston. Coal being filled into the larger compartment upon the grate, the whole box is filled with water, and the piston set in motion. By the action of the piston the water is caused to traverse the coal upwards and downwards, when the heavier particles of schist, &c., fall, and are collected upon the lower grate. When the space is filled up to the level of the upper grate with deposit, the coal is removed.

On the second system, the operation is continuous. Water flows in a long shallow trough, slightly inclined, with cross partitions at intervals from top to bottom, carrying with it the small coal, which is delivered into it at

the upper end. The denser particles are deposited in the first compartments, and the lighter particles in the last ones.

Deterioration of Coal by Exposure.

Coal deteriorates or decays to a greater or less degree by exposure to the atmosphere, by disintegration or crumbling, and also by the gradual combustion of the volatilizable elements. Atmospheric oxygen is absorbed, and converts the hydrocarbons into water and carbonic acid. It has been proved in one case, in Germany, that bituminous coal, after having been exposed for nine months, lost half its value as fuel; coal exposed for three months to a temperature of 284° F., lost all its hydrocarbons. The coke manufactured from coal thus deteriorated is inferior to what is made from coal freshly mined.

The above experimental evidence corroborates the fact that the decay of coal proceeds more rapidly in the hotter climates. Dryness is unfavourable to the change, while moisture accelerates it. When sulphur, or sulphuret of iron (iron-pyrites), is present in considerable quantity in a coal still changing under the action of the air, a second powerful heating cause is introduced, and both acting together may produce "spontaneous combustion." The presence of sulphur, or iron-pyrites alone, if in considerable quantity, is sufficient to excite combustion.

BRITISH COALS.

Composition of Bituminous Coals.—Dr. Richardson's Analyses, 1838.

The first accurate analyses of bituminous coals were made by the late Dr. Richardson, of Newcastle-on-Tyne. The coals submitted to analysis were—1st, Splint coal; 2d, cannel coal; 3d, cherry coal; 4th, caking coal. The following table, No. 135, contains the results of his analyses. The total average composition of the samples analyzed was about 81 per cent. of carbon, 5½ per cent. of hydrogen, 9 per cent. of oxygen and nitrogen, and 5 per cent. of ash.

Table No. 135.—Composition of Bituminous Coals.
By Dr. Richardson, 1838.

Coal, and Locality of Beds.	Carbon.	Hydrogen.	Oxygen and Nitrogen.	Ashes.
	per cent.	per cent.	per cent.	per cent.
Splint, from Wylam	74.82	6.18	5.09	13.91
Do., from Glasgow	82.92	5.49	10.46	1.13
Average	78.87	5.83	7.78	7.52
Cannel, from Wigan, Lancashire	83.75	5.66	8.04	2.55
Do., from Edinburgh	67.60	5.40	12.43	14.57
Average	75.68	5.53	10.23	8.56

Table No. 135 (*continued*).

Coal, and Locality of Beds.	Carbon.	Hydrogen.	Oxygen and Nitrogen.	Ashes.
	per cent.	per cent.	per cent.	per cent.
Cherry, from Jarrow, Newcastle......	84.85	5.05	8.43	1.67
Do., from Glasgow	81.21	5.45	11.92	1.42
Average	83.03	5.25	10.17	1.55
Caking, from Garesfield, Newcastle..	87.95	5.24	5.42	1.39
Do., from South Hetton, Durham	83.27	5.17	9.04	2.52
Average	85.61	5.20	7.23	1.96
Total average	80.80	5.45	8.85	4.90

Weight and Composition of British and Foreign Coals.

By Messrs. Delabèche and Playfair, 1847–50.

An extensive series of analyses and of trials of British coals were conducted by Sir Henry Delabèche and Dr. Lyon Playfair, at the College for Civil Engineers, Putney, in the years 1847–50, to the order of the government. The results of their investigations were published in three Reports on Coals suited to the Royal Navy, in the years 1849, 1850, 1851.

Samples of 98 British coals were analyzed and tried for their evaporative performance, namely:—37 Welsh coals, 18 Newcastle coals (Hartley district), 7 Derbyshire and Yorkshire coals, 28 Lancashire coals, 8 Scotch coals—Total, 98 coals.

In addition to these there were analyzed and tried, one sample of anthracite from Ireland, six patent fuels, and 24 foreign coals.

The chief results of these analyses and trials, compiled from the reports, are averaged and embodied in table No. 136, together with deductions as to the total heat of combustion of the fuels. The specific gravity, and the weight and bulk, of the coals, are given in columns 2, 3, 4, 5; and the chemical composition in columns 6, 7, 8, 9, 10, 11. The quantity of coke produced from each coal is given in column 12. The total heat of combustion is given in units of heat in column 13, and also in equivalent evaporative efficiency in columns 14, 15, when the water is supplied at 62° and at 212° F., and evaporated at atmospheric pressure. These columns, 13, 14, 15, have been calculated by means of formulas (6), (7), (8), page 406. The evaporative efficiency found by the trials is given in column 16.

Table No. 136.—AVERAGE COMPOSITION OF BRITISH AND FOREIGN COALS, WITH THEIR WEIGHT, BULK. HEAT OF COMBUSTION, AND EVAPORATIVE POWER.

Compiled and deduced from the analyses and experiments of Messrs. Delabèche and Playfair, 1847–50.

COALS.	Specific gravity.	Weight and Bulk.			Composition.						Coke produced from the coals.	Total Heat of Combustion of one pound of coal.			Evaporative power of one pound of coal from 212°, by trials.
												Units of heat.	Equivalent evaporative power from 212°.		
		1 cubic foot, solid.	1 cubic foot, heap'd.	Bulk of 1 ton, heap'd.	Carbon.	Hydr'n.	Nitro.	Sulph.	Oxygen.	Ash.			From Water at 62°.	From Water at 212°.	
Averaged groups.		p'nds.	p'nds.	c. ft.	p. cent.	p. c'nt.	p. c'nt.	p. c'nt.	p. cent.	p. cent.	p c'nt.	units.	pounds.	pounds.	pounds.
Welsh, ...37 samples	1.315	82.0	53.1	42.7	83.78	4.79	0.98	1.43	4.15	4.91	73	14,858	13.46	15.52	9.05
Newcastle, ...18 "	1.256	78.3	49.8	45.3	82.12	5.31	1.35	1.24	5.69	3.77	61	14,820	13.29	15.32	8.01
Derbyshire & Yorkshire, 7 "	1.292	80.6	47.2	47.4	79.68	4.94	1.41	1.01	10.28	2.65	59	13,860	12.43	14.34	7.58
Lancashire,..................28 "	1.273	79.4	49.7	45.2	77.90	5.32	1.30	1.44	9.53	4.88	58	13,918	12.62	14.56	7.94
Scotch,...................... 8 "	1.260	78.6	50.0	42.0	78.53	5.61	1.00	1.11	9.69	4.03	54	14,164	12.75	14.77	7.70
Average of British samples...........	1.279	79.8	50.0	44.5	80.40	5.19	1.21	1.25	7.87	4.05	61	14,320	12.83	14.82	8.13
Anthracite, Ireland	1.590	99.6	62.8	35.7	80.03	2.30	0.23	6.76	included in ash.	10.80	90	13,302	12.55	14.50	9.85
Patent fuels,.................6 samples	1.167	73.6	65.2	34.4	83.40	4.97	1.08	1.26	2.79	5.93	average of three, 74.2.	15,000	13.57	15.66	9.20
Foreign—															
Van Diemen's Land, ...9 "	—	—	—	—	65.80	3.50	1.30	1.10	5.58	22.71	—	11,320	10.25	11.83	—
Chili,.........................6 "	—	—	—	—	63.56	5.43	0.82	2.50	14.84	13.31	—	11,030	10.12	11.68	—
Lignite, Trinidad,.....................	—	—	—	—	65.20	4.25	1.33	0.69	21.69	6.84	—	10,438	9.42	10.87	—
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

There are very great variations in the chemical composition and properties of coals. In British coals, the constituents vary in quantity as follows:—

Carbon, from about 70 to 91 per cent. of the gross weight.
Hydrogen, from 3½ to nearly 7 per cent.
Oxygen, from about ½ to 20 per cent.
Nitrogen, from a mere trace to 2 $^1/_5$ per cent.
Sulphur, from nothing to 5 per cent.
Ash, from $^1/_5$ to 15 per cent.
Coke, from 49 to 93 per cent.

The average composition of British coals deduced from the table, is as follows:—

Carbon	about	80	per cent.
Hydrogen	"	5	"
Nitrogen	"	1 $^1/_5$	"
Sulphur	"	1 ¼	"
Oxygen	"	8	"
Ash	"	4	"
	"	100	"
Fixed carbon, or coke	"	61	"

The foreign coals, from Van Diemen's Land, and from Chili, had only from 63 to 66 per cent. of constituent carbon, with 28 per cent. of oxygen and ash.

Welsh Coals.—It may be noted here that Mr. G. J. Snelus, in 1871, made an analysis of Llangennech coal,[1] of which the particulars are subjoined, with those of a few other coals, extracted from the Reports of Delabèche and Playfair, for comparison. "The Ebbw Vale coal," it is said, "may be taken to represent the Monmouthshire steam coals; and Powell's Duffryn represents the Merthyr and Aberdare coals, highly esteemed for locomotives and ocean steamers." There is a close correspondence between the analyses of Llangennech coal made in 1848 and in 1871.

Class of Coal, and Date of Analysis.	Carbon.	Hydrogen.	Nitrogen.	Sulphur.	Oxygen.	Ash.	Coke.
	per cent.	per cent.	per cent.	per cent.	per cent.	p. cent.	p. cent.
Ebbw Vale, 1848.........	89.78	5.15	2.16	1.02	.39	1.50	77.5
Powell's Duffryn, 1848...	88.26	4.66	1.45	1.77	.60	3.26	84.3
Llangennech, 1848........	85.46	4.20	1.07	.29	2.44	6.54	83.7
Llangennech, 1871........	84.97	4.26	1.45	.42	3.50	5.40	86.7
Graigola, 1848.............	84.87	3.84	.41	.45	7.19	1.50	85.5

[1] See Appendix to the Report of the Judges, Mr. F. J. Bramwell and Mr. W. Menelaus, on the Trials of Portable Steam Engines at Cardiff in 1872; for a full Report on the Coal used in the Trials of Steam Machinery by the Royal Agricultural Society.

Patent Fuels.

The patent fuels tried by Delabeche and Playfair consisted of mixtures of bituminous or tarry matter with small bituminous coal. They had an average of 83.4 per cent. of constituent carbon, and 5 per cent. of hydrogen, with less than 3 per cent. of oxygen, and 6 per cent. of ash. Three patent fuels produced an average of 74 per cent. of coke. Warlich's patent fuel was the richest in carbon, of which it contained 90 per cent.; of hydrogen, 5.56 per cent.; of ash, 2.91 per cent. It yielded 85 per cent. of coke; and it evaporated, by trial, 10.36 lbs. of water, per pound of fuel, reckoned from water supplied at 212°.

Weight and Bulk of British Coals.

The average specific gravity of coal, as by the table No. 136, is 1.279; it varies from 1.20 to 1.39.

The average weight of coal is 80 lbs. per cubic foot, solid; the weight varies from 78 to 86 lbs.

The average weight is 50 lbs. per cubic foot, heaped; the weight varying from 45 to 58 lbs.

The average bulk of one ton, heaped, of coal, is 44½ cubic feet; the bulk varying from 38 to 49 cubic feet.

The average specific gravity of patent fuels is 1.167; the average weight is 73½ lbs. per cubic foot, solid, and 65 lbs. per cubic foot, heaped. The bulk of one ton, heaped, is 34½ cubic feet.

These averages show the advantage of the patent fuels in point of compactness, over coals; for though they are the lighter fuel, they occupy less space per ton than coals, on account of the regular forms in which the blocks are manufactured, and the facility for stowing them without much interspace.

Hygroscopic Water in British Coals.

The hygroscopic water in coal,—apart from what is chemically combined with it,—varies considerably. In the analyses of Delabeche and Playfair, in which the specimens were dried at 212° F., it varied from 0.61 to 9.31 per cent. of the weight of the coal. The following are examples:—

	Hygroscopic water.
Powell's Duffryn coal	1.13 per cent.
Mynydd Newydd	0.61 „
Pentrefelin	0.70 „
Park End coals, Lydney	2.78 „
Ebbw Vale	1.34 „
Resolven	1.55 „
Pontypool	1.60 „
Grangemouth coal	6.42 „
Broomhill coal	9.31 „
Wallsend Elgin	2.49 „
Fordel splint	8.40 „
Warlich's patent fuel	0.92 „
Bell's patent fuel	0.90 „
Wylam's patent fuel	1.38 „
Hartley coals	6.19 to 10.17 „
Steamboat Wallsend	1.14 „
Andrew's House, Tanfield	6.58 „

	HYGROSCOPIC WATER.	
Cannel coal, Wigan	1.01	per cent.
Stavely	8.54	"
Vancouver's Island	7.21	"
Chirique	9.11	"
Sydney, New South Wales	3.25	"
Juan Fernandez	6.00	"

It appears from this that the Welsh coals and the patent fuels contained the least proportion of hygroscopic water.

Torbanehill or Boghead Coal.

The Boghead coal is a special mineral found on the estate of Torbanehill, Linlithgowshire. Its colour varies from dark snuff-brown to brownish-black. It is exceedingly hard; the fracture is slaty and conchoidal. When struck with a hammer, it gives a woody sound. Its specific gravity varies from 1.155 to 1.260, the average being 1.189.

In composition, Boghead coal occupies the opposite end of the scale to anthracite,—having a comparatively small percentage of carbon, and a large excess of hydrogen. According to Dr. Penny's analysis of the coal, dried at 212°, the composition is as follows:—

	per cent.
Carbon	63.94
Hydrogen	8.86
Nitrogen	0.96
Sulphur	0.32
Oxygen	4.70
Ash	21.22
	100.00

As the oxygen amounts to only 4.7 per cent., it leaves free a surplus of 8¼ per cent. of hydrogen, to form hydrocarbons with the constituent carbon, when the coal is distilled; and it is found that coal of the above composition yields 67 per cent. of volatile matter, and 31 per cent. of ash. The composition varies in different specimens, as may be observed in the following analyses of four specimens taken from the pit at different dates, table No. 137:—

Table No. 137.—Composition of Boghead Coal.

Coal.	Specific gravity.	Composition.					Coke.
		Fixed carbon.	Volatile matter.	Sulphur.	Ash.	Water.	
		per cent.	per cent.	per cent.	per cent.	per cent.	per cent.
Brown, 1849	1.155	11.3	71.0	0.3	16.8	0.6	28.1
Do., 1851	1.160	7.1	71.00	0.2	21.2	0.5	28.3
Black, 1851	1.218	9.25	62.70	0.35	26.5	1.20	35.75
Do., 1853	1.188	10.52	67.11	0.32	21.0	1.05	31.52
Averages	1.180	9.54	67.95	0.29	21.4	0.84	30.94

From the table it appears that the fixed carbon averages only 9½ per cent.; and that, including ash, the coke averages only 31 per cent., the volatile matter exceeding two-thirds of the whole weight of the coal. When distilled at comparatively low temperatures, Boghead coal affords large quantities of paraffin, paraffin oil, &c.:—a discovery made by Mr. Young.

AMERICAN AND FOREIGN COALS.

By Professor W. R. Johnson, 1843–44.

The results of an investigation of the qualities of American coals, at the Navy Yard, Washington, for the Navy Department of the United States, conducted by Professor W. R. Johnson, were published in "A Report to the Navy Department of the United States, on American Coals," in 1844.

Thirty-nine samples of coal, and three samples of coke, were tried; and the general results are given in table No. 138.

The constituents, so far as the analyses extended, were in the following proportions:—

Volatile matter, other than moisture,......	2⅓ to 34½	per cent.,	average	16.17 per cent.
Hygrometric moisture,..	0 to 3⅛	„	„	1.37 „
Sulphur,..................	0 to 2⅓	„	„	0.49 „
Fixed carbon,............	53 to 91	„	„	73.35 „
Earthy matter,...........	4½ to 15	„	„	9.15 „
			About.........	100.00

Coke (fixed carbon and earthy matter),.................... 82.50 per cent.

The proportions of volatile matter, fixed carbon, ash, and coke, were for the three classes of American coal as follows:—

	Volatile matter.	Fixed carbon.	Ash.	Coke.
Anthracites,........................	3.97	88.54	6.28	94.82
Free burning bituminous coals,....	15.11	73.21	10.27	83.48
Bituminous caking coals,...........	29.43	58.29	10.90	69.19
Averages,...............	16.17	73.35	9.15	82.50

Weight and Bulk of American Coals.

The specific gravity of American coal varies from 1.283 to 1.610, and it averages 1.400.

The weight of solid coal varies from 80 to 100 pounds per cubic foot, and it averages 87½ pounds.

The weight of heaped coal varies from 45 to 56 pounds per cubic foot, and it averages 51¾ pounds.

The bulk of one ton of heaped coal varies from 49⅔ to 40 cubic feet, and it averages 43½ cubic feet.

Table No. 138.—AMERICAN COALS:—AVERAGE WEIGHT, BULK, AND COMPOSITION, 1843.

(Compiled from the Report of Professor W. R. Johnson.)

WEIGHT, BULK, COKE, AND ASH.

Coal.	Specific gravity.	Weight and Bulk.			Coke produced from coal.	Ash and clinkers left by combustion.
		One cubic foot, solid.	One cubic foot, heaped.	Bulk of one ton, heaped.		
		pounds.	pounds.	cubic feet.	per cent.	per cent.
Anthracites	1.500	93.78	53.05	42.35	94.82	8.60
Cokes	—	—	32.13	69.76	—	14.94
Free-burning bituminous	1.358	84.93	52.84	42.42	83.68	11.27
Bituminous caking	1.342	83.90	49.28	45.71	69.01	8.48
Foreign and Western	1.318	82.39	49.31	45.51	65.27	7.98
Average of the three classes of American coals	1.400	87.54	51.72	43.49	82.50	9.42

COMPOSITION.

Coal.	Composition, in Percentages of the Total Weight.				
	Moisture.	Volatile matter, other than moisture.	Sulphur.	Fixed carbon.	Earthy matter.
	per cent.	per cent.	per cent.	per cent.	per cent.
Anthracites, Pennsylvania	1.19	3.97	0.04	88.54	6.28
Coke, two samples from Midlothian and Neff's Cumberland coal, Virginia	—	—	—	—	14.94
Free-burning bituminous, Maryland and Pennsylvania	1.37	15.11	0.42	73.21	10.27
Bituminous caking, Virginia	1.56	29.43	1.01	58.29	10.90
Foreign and Western bituminous.	2.50	32.68	0.24	57.42	7.85
Average of the three classes of American Coals	1.37	16.17	0.49	73.35	9.15

FRENCH COALS.

French coals are divided into five classes, according to their behaviour in the furnace:—

1st. Bituminous caking coals (*houilles grasses maréchales*).

2d. Bituminous hard coals (*houilles grasses et dures*), differing from the first by having less fusibility; the coke is more dense than that of the first, and is best for blast furnaces.

3d. Bituminous coals, burning with a long flame (*houilles grasses à longues flammes*); they are still less fusible or caking than the preceding, and are best for boiler and other furnaces. They are known by the designation *flénu*, and are similar to Lancashire cannel coal.

4th. Dry coals, with a long flame (*houilles sèches à longues flammes*). The coke has not much coherence. These coals are burned on grates; they are less durable than the foregoing.

5th. Dry coals, with a short flame (*houilles sèches à courtes flammes*). These coals burn with some difficulty, and are used chiefly for burning bricks, and in lime-kilns, in breweries for drying malt, and for domestic fires.

Anthracites are classed by themselves.

The coal, as it comes from the mine, large and small together, is known as *tout-venant*—"as it comes." In the market, the coal from a mine is distinguished, according to the size of the pieces, into, 1st, *le gros*, round coal; 2d, *la gaillette*, coal of medium size, in pieces 5 or 6 inches in diameter, which is separated by screening from the third sort; 3d, *le menu*, slack, which is subdivided into three kinds:—*gailletin*, the size of nuts; *tête de moineau*, smaller than gailletin—literally the size of a sparrow's head; and *fine*, which is again distinguished into *fine menue* and *fine poussier*, coal dust.

Utilization of the Small Coal.

The *menu*, or small coal, is made into briquettes, or rectangular blocks; being agglomerated by means of tar, and compressed into moulds, as has already been pointed out in describing English patent fuels, with some slight differences of treatment. 1st. The small coal is mixed with pitch, and compressed in moulds to form blocks. These blocks have great durability, and do not deteriorate by exposure to air. 2d. When the slack is derived from rich bituminous coals it is filled into cast-iron moulds, which are so closed that nothing but gas can escape from them. The moulds are heated in a furnace to upwards of 900° F., where they remain from half an hour to three hours, according to the quality of the coal. By the action of the heat the coal becomes a kind of paste, and tends to swell; but it is on the contrary powerfully compressed by the moulds. 3d. For the slack of dry coals a certain proportion of the slack of bituminous coal is mixed with it, to give cohesive power.

Composition of French Coals.

The table No. 139 contains the specific gravity and composition of a number of French coals. For the first section, comprising Regnault's analyses, compiled from a table by M. Peclet, the samples were dried at a temperature of 120° C., or about 250° F.; and the loss of weight, repre-

senting moisture, varied from 1.36 to 1.60 per cent. The quantity of nitrogen was in general very small in the anthracites; and in the other coals it was from 1.50 to 2.0 per cent. The united weights of oxygen and nitrogen have therefore been taken by M. Peclet to represent the quantity of oxygen, in calculating the heating power, according to the principle already explained, page 403.

Table No. 139.—MEAN DENSITY, COMPOSITION, AND HEATING POWER OF FRENCH COALS.

COALS. Regnault, 22 samples. Marsilly, 79 samples.	Specific gravity.	Quantity of coke.	Composition.				Hydrogen in excess.	Heating Power.
			Carbon.	Hydrogen.	Oxygen and nitrogen.	Ash.		
		per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	units of heat
(Regnault.)								
Anthracites	1.498	88.83	86.17	2.67	2.85	8.56	2.43	14,038
Bituminous hard coals	1.319	74.81	88.56	4.88	4.38	2.19	4.27	15,525
Bituminous caking coals	1.293	67.54	87.73	5.08	5.65	1.54	4.30	15,422
Bitum. coals, long flame	1.303	60.86	82.94	5.35	8.63	3.08	4.15	14,622
Dry coals, long flame	1.362	54.72	76.48	5.23	16.01	2.28	3.09	13,041
(Marsilly.)								
Mons Basin	1.265	71.50	83.85	5.19	8.09	2.85	4.24	14,884
Mons Centre Basin	1.293	81.79	86.38	4.51	5.46	3.66	3.82	14,931
Charleroi Basin	1.197	86.58	86.65	4.18	5.23	3.95	3.52	14,787
Valenciennes Basin	1.289	80.75	86.50	4.52	5.39	3.52	3.85	14,976
Calais Basin	1.280	74.66	84.94	5.15	7.02	2.93	4.22	15,003
Average	1.310	74.20	85.02	4.48	6.87	3.46	3.79	14,723

Note.—The averages are here deduced from averages; being averages of averages, and are to be accepted as approximate, not necessarily exact results.

For the second section, the samples were dried by exhaustion in the receiver of an air-pump during from twelve to twenty-four hours.

It appears from the table that the average composition of French coals is as follows:—

Carbon,	85	per cent.
Hydrogen,	4½	„
Oxygen and nitrogen,	7	„
Ash,	3½	„
Sulphur,	?	„
	100	

The average specific gravity is 1.310, giving a weight of 81.68 lbs. per cubic foot solid.

According to Peclet the weight of heaped coal from different mines is as follows, in table No. 140; to which are added the weight of one cubic foot heaped, and the volume of one ton heaped.

Table No. 140.—Weight and Volume of French Coals, heaped.

Mine.	Weight of one hectolitre, heaped.	Weight of one cubic foot, heaped.	Volume of one ton, heaped.
	kilogrammes.	pounds.	cubic feet.
Labarthe	88	55.0	40.75
Auvergne and Blanzy	87	54.3	41.22
Combelle	86	53.7	41.70
Lataupe	85	53.1	42.19
Saint-Étienne	84	52.5	42.69
Decise	83	51.8	43.21
Mons	80	50.0	44.83
Creusot	79	49.3	45.39
Averages of bituminous coals	84	52.5	42.75
Anthracite	90	56.2	40.00

An abstract of a resumé of analyses of French and other coals and lignites, by MM. Scheurer-Kestner and Charles Meunier-Dollfus, is given in table No. 141, together with the observed heat of combustion. The figures have reference to pure fuel, from which the ash has been separated, in terms of the gaseous constituents only.

Table No. 141.—French and other Coals and Lignites. Analysis of Gaseous Constituents and Observed Heat of Combustion.

(*Scheurer-Kestner and Meunier-Dollfus.*)

The fuel is assumed to be dry and pure—without any ash.

Designation of Combustible.	Gaseous Elements.			Heat of combustion of 1 pound pure (observed).
	Carbon.	Hydrogen.	Oxygen and nitrogen.	
Coal.	per cent.	per cent.	per cent.	units.
Ronchamp, 3 samples	88.59	4.69	6.72	16,416
Sarrebruck, 7 do.	81.10	4.75	14.15	15,320
Creusot, 4 do.	90.60	4.10	5.30	16,994
Blanzy:—Montceau	78.58	5.23	16.19	14,985
Do. Anthracitic	87.02	4.72	8.26	16,400
Angin	84.45	4.21	11.32	16,663
Denain	83.94	4.43	11.63	16,290
English:—Bwlf	91.08	3.83	5.09	15,804
Do. Powell-Duffryn	92.49	4.04	3.47	16,108
Russian:—Grouchefski anthracite	96.66	1.35	1.99	14,866
Do. Miouchi, bituminous	91.45	4.50	4.05	15,651
Do. Goloubofski, flaming	82.67	5.07	12.26	14,438

Table No. 141 (*continued*).

Designation of Combustible	Gaseous Elements. Carbon.	Hydrogen.	Oxygen and Nitrogen.	Heat of Combustion of 1 lb. pure (observed).
LIGNITES.	per cent.	per cent.	per cent.	units.
Rocher bleu	72.98	4.04	22.98	11,670
Manosque, bituminous	70.57	5.44	23.99	13,253
Do. dry	66.31	4.85	28.84	12,584
Bohemia, bituminous	76.58	8.27	15.15	14,263
Russian, Toula	73.72	6.09	20.19	13,837
Lignite, passing to fossil wood	66.51	4.72	28.77	11,444
Fossil wood, passing to lignite	67.60	4.55	27.85	11,360

INDIAN COALS.

In July, 1860, Mr. R. Haines, acting chemical analyst to the Bombay government, reported on samples of coal from Australia, the Nerbudda Valley, and Nagpore. The following are the principal results contained in the report:—

The Australian coal is jet-black and brilliant, very brittle, and breaks with a cubical fracture like Newcastle coal.

The Nerbudda coal is dull black, heavy, very hard, being pulverized with difficulty; it has a laminated structure and slaty cleavage; it has, here and there, interspersed in its substance, small lumps of half-formed coal like charcoal.

The Nagpore coal is very similar in appearance to the Nerbudda coal, and has the same texture, except that the laminæ are alternately dull and glossy.

The Australian coal is bituminous, and it cokes like Newcastle coal. The Nerbudda and Nagpore coals do not even cohere in coking. The ash of the Australian coal is of a dirty white colour, and that of the other coals is similar in appearance.

The results of analysis of these coals are given in table No. 142, together with similar results from English coals. The products are divided into solid, or "coke," and volatile; and in the last two columns are given separately the sulphur and the ash, the first of which is included in the volatile matter and the second in the coke.

The proportions of ash, or incombustible matter, are respectively as follows:—

COAL.	
Australian,	8.38 per cent.
Nerbudda Valley,	18.09 ,,
Nagpore,	18.73 ,,
English,	3.66 ,,

Table No. 142.—Comparative Composition of Australian, Nerbudda, Nagpore, and English Coals. By Mr. R. Haines, 1860.

Locality or Description.	Specific gravity.	Coke.	Volatile matter.	Sulphur.	Ash.
		per cent.	per cent.	per cent.	per cent.
Australia	1.312	68.27	31.73	0.50	8.38
Nerbudda Valley	1.440	66.63	33.37	0.60	18.09
Nagpore	1.417	76.00	24.00	0.34	18.73
Welsh steam coal:—					
From	1.275	62.5	37.5	0.33	1.25
To	1.350	88.1	11.9	5.07	6.94
Average	1.310	80.0	20.0	1.25	3.00
Scotland:—					
From	1.200	49.30	50.70	0.33	1.13
To	1.316	59.15	40.85	1.57	14.57
Average	1.260	54.00	46.00	1.10	4.00
Newcastle:—					
From	1.23	62.70	37.30	0.06	0.20
To	1.31	72.30	27.70	1.85	13.91
Average	1.28	66.00	34.00	1.00	4.00
Average of English coals	1.28	66.66	33.33	1.12	3.66

An official memorandum was addressed to the Indian government, in January, 1867, by Dr. Oldham, superintendent of the geological survey of India, containing the results of analysis of eighty-one samples of Indian coal: showing the volatile matter, the fixed carbon, and the ash. These results are given in table No. 143, and a column is prefixed showing the percentage of coke, which is arrived at by adding that of the ash to that of the fixed carbon. For comparison, the results of a similar analysis of English coals saleable at Calcutta, are added.

The distinguishing characteristic of the Indian coal is the great proportion of ash it contains, varying from 1¾ per cent., though in only one instance, to 59 per cent., and averaging, for 81 samples, 23 per cent. The English coal saleable at Calcutta has only an average of 2.7 per cent. The following are the average compositions of Indian and of English coal at Calcutta, deduced from table No. 143:—

	Indian coals.	English coals.
	per cent.	per cent.
Coke,	70.2	70.8
Fixed carbon,	47.3	68.1
Volatile matter,	29.6	29.2
Ash,	22.9	2.7

showing, notwithstanding the great excess of ash in the composition of the Indian coals, that the quantity of volatile matter is about the same as in the English coals, about 29 per cent. In the absence of a full chemical analysis, it is impossible to say how much of this consists of carbon,

oxygen, hydrogen, and nitrogen individually; and therefore the heating power of the volatile matter cannot be estimated.

Table No. 143.—COMPOSITION OF INDIAN COALS, 1867.

Compiled from a Report by Dr. Oldham.

Locality.	Coke (sum of fixed carbon and ash).	Fixed carbon.	Volatile matter.	Ash.
	per cent.	per cent.	per cent.	per cent.
Kurhurbali Field:—				
From	75.2	50.9	12.6	4.8
To	87.4	73.1	24.8	39.2
Average	79.9	62.8	20.2	17.1
Rajmahal Hills:—				
From	54.4	25.2	28.8	1.5
To	71.2	57.6	44.8	37.6
Average	60.8	44.2	39.3	16.6
Ranigunj Field:—				
From	59.0	39.2	25.6	1.75
To	75.0	63.8	38.7	35.2
Average	65.0	50.0	35.0	15.0
Sherria Field:—				
From	55.4	30.8	18.0	1.7
To	86.0	68.4	44.6	28.8
Average	69.0	56.3	31.0	12.7
Central India (Pench River):—				
From	52.0	30.3	14.0	2.2
To	86.0	61.6	54.0	48.7
Average	65.6	47.4	32.8	18.2
Madras (Godavery River)	81.0	23.2	19.0	57.8
Total averages of Indian coals.	70.2	47.3	29.6	22.9
English coal, saleable at Calcutta: Averages	70.8	68.1	29.2	2.7

It may be added that Dr. Oldham, in 1859, analyzed two specimens of anthracitic coal from Kotlee, in the Punjab, and found their composition as follows:—

	Carbon.	Volatile matter.	Ash.
	per cent.	per cent.	per cent.
No. 1	90.5	4.0	5.5
No. 2	90.0	6.0	4.0

Much of the Indian coal is peculiarly liable to disintegration from exposure to the atmosphere, particularly in the hot seasons. Coal from the new Chanda coalfields is reported to have fallen to so small pieces, after a short period of exposure, as to have become unfit as fuel for locomotives.

COMBUSTION OF COAL.

When coal is exposed to heat in a furnace, the carbon and hydrogen, associated in various chemical unions, as hydrocarbons, are volatilized and pass off. At the lowest temperature, naphthaline, resins, and fluids with high boiling points are disengaged; next, at a higher temperature, volatile fluids are disengaged; and still higher, olefiant gas, followed by common gas, light carburetted hydrogen, which continues to be given off after the coal has reached a low red heat. As the temperature rises, pure hydrogen is also given off, until finally, in the fifth or highest stage of temperature for distillation, hydrogen alone is discharged. What remains after the distillatory process is over, is coke, which is the fixed or solid carbon of coal, with earthy matter, the ash of the coal.

The hydrocarbons, especially those which are given off at the lowest temperatures, being richest in carbon, constitute the flame-making and smoke-making part of the coal. When subjected to degrees of heat much above the temperatures required to vaporize them, they become decomposed, and pass successively into more and more permanent forms by precipitating portions of their carbon. At the temperature of low redness, none of them are to be found, and the olefiant gas is the densest type that remains, mixed with carburetted hydrogen and free hydrogen. It is during these transformations that the great body of smoke is made, consisting of precipitated carbon passing off uncombined. Even olefiant gas, at a bright red heat, deposits half its carbon, changing into carburetted hydrogen; and this gas, in its turn, may deposit the last remaining equivalent of carbon at the highest furnace heats, and be converted into pure hydrogen.

Throughout all this distillation and transformation, the element of hydrogen maintains a prior claim to the oxygen present above the fuel; and until it is satisfied, the liberated carbon remains unburned.

Summary of the Products of Decomposition in the Furnace.

Reverting to the statement of the average composition of coal, page 415, it was found that the fixed carbon or coke remaining in the furnace after the volatile portions of the coal are driven off, averages 61 per cent. of the gross weight of the coal. Taking it, for round numbers, at 60 per cent., the proportion of carbon volatilized in combination with hydrogen, will be 20 per cent.—making up the total of 80 per cent. of constituent carbon in average coal.

Of the 5 per cent. of constituent hydrogen, 1 part is united to the 8 per cent. of oxygen, in the combining proportions to form water, and the remaining 4 parts of hydrogen are found partly united to the volatilized carbon, and partly free.

These particulars are embodied in the following summary of the condition of the elements of 100 pounds of average coal, after having been decomposed, and prior to entering into combustion:—

100 *Pounds of Average Coal in the Furnace.*

Composition.		lbs.		lbs.	Decomposition.
Carbon	Fixed	60	forming	60	fixed carbon.
	Volatilized	20		24	hydrocarbons and free hydrogen.
Hydrogen		5		1¼	sulphur.
Sulphur		1¼		9	water or steam.
Oxygen		8		$1\frac{1}{5}$	nitrogen.
Nitrogen		$1\frac{1}{5}$		4	ash.
Ash		4		100	
About		100			

showing a total useful combustible of 86¼ per cent., of which 26¼ per cent. is volatilized. Whilst the decomposition proceeds, combustion proceeds, and the 25¼ per cent. of volatilized portions, and the 60 per cent. of fixed carbon, successively, are burned.

It may be added that the sulphur and a portion of the nitrogen are disengaged in combination with hydrogen, as sulphuretted hydrogen and ammonia. But these compounds are small in quantity, and, for the sake of simplicity, they have not been indicated in the above synopsis.

Quantity of Air Chemically Consumed in the Complete Combustion of Coal.

Take coal of average composition. Then, applying the rule 1, page 400, the carbon $C = 80$, the available hydrogen $(H - \frac{O}{8}) = 4$, and the sulphur $S = 1.25$, and

$$C + 3\,(H - \tfrac{O}{8}) + .4\,S = 80 + 12 + .5 = 92.5,$$

and $92.5 \times 1.52 = 140.6$ cubic feet of air at 62°, the quantity chemically consumed by one pound of average coal.

To find the proportions in which this quantity of air is appropriated for the volatilized and the fixed portions of the coal, as above divided, for 100 lbs. of the fuel:—

For the Volatilized Portion—

Hydrogen	4 lbs. × 457 =	1828 cubic feet.	
Carbon	20 lbs. × 152 =	3040 ”	
Sulphur	1¼ lbs. × 57 =	71 ”	
			4939 cubic feet.

For the Fixed Portion—

Carbon	60 lbs. × 152 =		9120 ”
Total useful combustible,	85¼ lbs.		14,059 ”

showing that 14,059 cubic feet of air at 62° are required for the complete combustion of 100 lbs. of coal of average composition. It is equivalent

to 140.6 cubic feet of air for one pound of coal, as already found, or in round numbers, 140 cubic feet, of which there are required,

For the volatilized portions	50 cubic feet, or	36 per cent.
For the fixed portion	90 ,, or	64 ,,
	140	100

The weight of this quantity of air, dividing the volume by 13.14, is 10.7 lbs.

The following table, No. 144, gives the composition of, and the quantities of air chemically consumed in the complete combustion of, British coals of the highest, the lowest, and average heating powers, placed together for comparison. It appears from the table that the quantity of air chemically consumed in the combustion of one pound of British coal varies, according to the composition of the coal, from 116 to 163 cubic feet at 62°:—

Table No. 144.—Comparative Statement of Composition, Heat of Combustion, and Air Chemically Consumed by British Coals of the Highest, Lowest, and Average Quality.

Coal (Selected from Delabeche and Playfair's Report).	Carbon.	Hydrogen.	Oxygen.	Sulphur.	Total heat of combustion of one pound of coal.	Air chemically consumed in the complete combustion of one pound of coal.
	per cent.	per cent.	per cent.	per cent.	units.	cubic feet at 62°.
Warlich's patent fuel.	90.02	5.56	—	1.62	16,495	163
Ebbw Vale............	89.78	5.15	0.39	1.02	16,221	161
Haswell Wallsend ...	83.47	6.68	8.17	0.06	15,502	153
Coal of average composition............	80.00	5.00	8.00	1.25	14,133	140
Ince Hall, Pemberton five feet (lowest British)........	68.72	4.76	18.63	1.35	11,525	116
Chirique, Chili (lowest foreign)	38.98	4.01	13.38	6.14	7,349	74

Gaseous Products of the Complete Combustion of Coal.

The quantity of the gaseous products is found by rules 2 and 3, pages 401, 402. Take, for example, the case of coal of average composition.

1. By weight.—The percentages of carbon, hydrogen, sulphur, and nitrogen are respectively 80, 5, 1.25, 1.20. Then, by rule 2, page 402, the weight of the gaseous products, taken collectively, of the combustion of one pound of coal, is

$$(.126 \times 80) + (.358 \times 5) + (.053 \times 1.25) + (.01 \times 1.20) = 11.94 \text{ pounds.}$$

The weights of the gases individually are given by the expressions *a*, *b*, *c*, *d*, page 401, as follows:—

		Pounds.	Per cent.
Carbonic acid	$.0366 \times 80$	$= 2.93$	or 24.5
Steam	$.09 \times 5$	$= .45$	or 3.8
Sulphurous acid	$.02 \times 1.25$	$= .025$	or 0.2
Nitrogen	$= (.0893 \times 80) + (.268 \times 5) + (.0335 \times 1.25) + (.01 \times 1.20)$	$= 8.536$	or 71.5
		11.94	100.0

2. By volume.—The total volume is found by rule 3, page 402; thus:—

$$(1.52 \times 80) + (5.52 \times 5) + (.567 \times 1.25) + (.135 \times 1.20) = 150.07 \text{ cubic feet.}$$

The volumes in detail are, by the expressions *e*, *f*, *g*, *h*, page 401, as follows:—

		Cubic feet at 62°.	Per cent.
Carbonic acid	$.315 \times 80$	$= 25.2$	or 17
Steam	1.9×5	$= 9.5$	or 6
Sulphurous acid	$.117 \times 1.25$	$= 0.15$	trace
Nitrogen	$= (1.206 \times 80) + (3.618 \times 5) + (.45 \times 1.25) + (.135 \times 1.20)$	$= 115.29$	or 77
		150.14	100

showing that the 12 pounds of gaseous products have a volume of 150 cubic feet at 62°, equal to 12½ cubic feet per pound. The element of nitrogen is nearly three-fourths by weight, and fully three-fourths by volume, of the total quantity of gaseous products.

The relatively larger volume of the gaseous products at the higher temperature at which they enter the chimney, is found by the formula (2), page 347, repeated at page 402. If the final temperature be 500° F., the final volume of the gaseous products for one pound of average coal is,

$$150 \; \frac{500 + 461}{62 + 461} = 276 \text{ cubic feet;}$$

or nearly double the volume at 62°. At 585° temperature, the volume would be exactly double, or 300 cubic feet; and at 1108° F. it would be just three times the normal volume at 62°.

Surplus Air.

The quantity of surplus air which passes off with the products of combustion into the chimney, is to be added to that of these products to find the total weight or volume of the gases in the chimney, as already stated, page 402.

Taking the case of coal of average composition, suppose that the quantity of surplus air is equal to that which is chemically consumed by the fuel; then it amounts to 140 cubic feet by volume, or 10.7 pounds by weight, for one pound of coal consumed; adding these to the weight and the volume of the products of combustion above found, there is

	Cubic feet at 62°.	Weight in lbs.
Gaseous products of combustion per lb. of coal,	150	12
Surplus air ,, ,,	140	10.7
Total escaping gases,	290	22.7

When the quantity of surplus air is less than that which is chemically consumed, the volume and weight to be added to those of the products of combustion, are less than 140 cubic feet and 10.7 pounds respectively, in the same proportion.

The total quantity of escaping gases, therefore, produced by the combustion of one pound of average coal, varies according to the proportion of surplus air—

From 150 cubic feet to 290 cubic feet, at 62°;
From 276 „ to 533 „ at 500°;
From 12 pounds to 22.7 pounds in weight.

It is here assumed that the maximum quantity of surplus air does not exceed the quantity of air chemically consumed.

Total Heat of Combustion of British Coals.

The total heat of combustion of coal of average composition, having 80 per cent. of carbon, 5 per cent. of hydrogen, 8 per cent. of oxygen, and 1.25 per cent. of sulphur, is, by rule 4, page 406,

$$145\ (80 + 4.28\ (5 - {}^{8}/_{8}) + (0.28 \times 1.25)) = 14{,}133 \text{ units.}$$

The heating power, expressed in pounds of water evaporable under one atmosphere by one pound of the fuel, is, by rule 5, p. 406, as follows:—

When the water is supplied at 62° the total evaporative power is

$$0.13\ (80 + 4.28\ (5 - {}^{8}/_{8}) + (0.28 \times 1.25)) = 12.67 \text{ pounds of water.}$$

When the water is supplied at 212° the evaporative power is

$$0.15\ (80 + 4.28\ (5 - {}^{8}/_{8}) + (0.28 \times 1.25)) = 14.62 \text{ pounds of water.}$$

The total heat of combustion of British coals is given in table No. 136, page 414; and for contrast in table No. 144, above.

COKE.

The quantity of residuary coke in various coals, was found by laboratory analysis as follows (see previous tables):—

(Excluding anthracites.)	Coke. per cent.	Coke. per cent.
English coals	50 to 72	Average 61.4
American coals	64 to 86	„ 76.4
French coals	53 to 76	„ 64.5
Indian coals	52 to 84	„ 70.2

Anthracite coke scarcely deserves the name; it is without cohesion, and pulverulent. The best coke,—from bituminous coal,—is clean, crystalline, and porous; and it is formed in columnar masses. It has a steel-gray colour, possesses a metallic lustre, with a metallic ring when struck, and is so hard as to be capable of cutting glass.

The quality of coke obviously depends, in a great measure, on the proportions of the constituent hydrogen and oxygen of the coal from which it is made, which regulate the degree of fusibility of the coal when exposed to heat. Taking, for example, the particulars of the coke produced from the French coals, table No. 139, and arranging the averages for each kind of coal in the order of the quantity of hydrogen in excess, the nature of the coke produced, as described by M. Peclet, was as follows:—

Averages.	Hydrogen.	Oxygen and Nitrogen.	Hydrogen in excess.	Nature of the Coke.
	per cent.	per cent.	per cent.	
Anthracites	2.67	2.85	2.43	pulverulent
Dry coals, long flame	5.23	16.01	3.09	in fragments
Bituminous coals, long flame	5.35	8.63	4.15	porous
Bituminous hard coals	4.88	4.38	4.27	porous
Bituminous caking coals	5.08	5.65	4.30	very porous

Showing a series of five coals, with an ascending series of hydrogen in excess, from 2.43 to 4.30 per cent. The nature of the cokes advances correspondingly from pulverulent or powdery, to very porous or excessively fused and raised. The first is, in fact, a failure as a coke, and the second, with 3.09 per cent. of hydrogen in excess, barely coheres, being in fragments; the third and fourth, with about 4.20 per cent. of hydrogen in excess, produce a porous and cohesive coke, and the fifth an excessively porous coke,—bright, but comparatively light for metallurgical operations.

From this it appears that a coal having less than 3 per cent. of hydrogen in excess, is unfit for coke-making; and that, for the manufacture of good coke, coal containing at least 4 per cent of free hydrogen is required.

It is not clear in what manner the presence of free hydrogen operates in fusing the substance of the coal; unless, probably, that the hydrogen being in combination with carbon in various proportions to form tar and oils, softens the fixed carbon, and forms a pasty mass, which is raised like bread by the expansion of the confined gases and vapours seeking to escape. The increasing proportions of volatilized matter which is raised by heat, successively, from anthracites, bituminous, and caking coals, are clearly exemplified by the analyses of American coals in table No. 138, page 419; and they evidently have relation to an increase of the hydrogen in excess above that required to form water with the constituent oxygen. They are as follows:—

American Coals.	Volatilized Matter.	Coke Produced.
	per cent.	per cent.
Anthracite	5.16	94.82
Free burning bituminous coals	16.48	83.68
Bituminous caking coals	30.99	69.01

The increasing volatilized matter explains, as above suggested, the increasing porosity and bulk of the coke yielded by the respective coals.

Anthracitic Coke.

By a process recently introduced at Swansea by Messrs. Penrose & Richards, and described by Mr. W. Hackney,[1] anthracite has been successfully used as the basis of a coke, manufactured from the following mixture:—Anthracite, 60 per cent.; Bituminous coal, 35 per cent.; Pitch, 5 per cent. The materials are crushed and mixed together through a disintegrator. The yield of coke is 80 per cent. of the weight of the charge. The coke is steel-gray in colour, and so hard as to scratch glass easily; and it is about 23 per cent. heavier than the best Welsh coke. It burns in a common fire, or in a blast furnace, without any sign of crumbling.

Quantity of Coke Yielded by Coal.

The quantity of coke produced, on the large scale, from coal varies from 60 to 80 per cent. in weight. The following are examples of the yield:—

Coals.	Coke Produced.
Andrew's House, Tanfield	65 per cent of the coal.
Bristol	60 to 63.5 ,, ,,
Kilsyth	60 ,, ,,
Mons	77 to 80 ,, ,,
Seraing	67 ,, ,,

In general, the yield of good coke is about two-thirds, or 66 per cent. of the coal.

The whole of the coke matter in coal cannot be extracted from it, on the large scale; a portion of it is burned off. Thus, Seraing coal, from which 67 per cent. of coke was made, on the large scale, yielded 80 per cent. of coke, by laboratory analysis.

Weight and Bulk of Coke.

Coal expands in volume in the coking process, insomuch that the volume of the resulting coke is greater by from 10 to 30 per cent. than that of the coal from which it is made. Tanfield coke has 11 per cent. more volume than the coal from which it is made; and as the specific gravity of the coal is 1.26, that of the coke is 0.74, calculated as follows:—

$$1.26 \times \frac{65}{1.11 \times 100} = 0.74.$$

The weight and volume of Tanfield coal, and of coke made from it, are as follows:—

	Specific gravity.	Weight of 1 cubit foot, solid.	Weight of 1 cubic foot, heaped.	Volume of 1 ton, heaped.
Tanfield coal...	1.26	78.57 lbs.	52.19 lbs.	42.92 cubic feet.
Do. coke	0.74	46.14 ,,	30.00 ,,	74.66 ,,

Mickley coke weighs 28 lbs. per cubic foot, heaped, and measures in bulk 80 cub. ft. per ton. Gas coke weighs from 12½ cwt. to 15 cwt. per chaldron.

[1] In a paper read at the meeting of the Iron and Steel Institute, 1875, published in *Engineering*, November 12, 1875.

The American cokes, from Midlothian, Va., and Cumberland, Md., averaged a weight of 32.13 lbs. per cubic foot, heaped, and a volume of 69.8 cubic feet per ton.

The coke used for smelting furnaces in France weighs, ordinarily, 25 lbs. per cubic foot, heaped, and measures, in bulk, about 90 cubic feet per ton. Of the Seraing coking coal, and the coke produced from it, the weight and bulk are as follows,—assuming that the coal is the same as average Newcastle coal, with which it is almost identical in chemical composition:—

	Weight of 1 cubic foot, heaped.	Volume of 1 ton, heaped.
Seraing coal	50 lbs.	45 cubic feet.
Do. coke	31 "	72 "

From the foregoing particulars it may be gathered that coke of good quality weighs from 40 to 50 lbs. per cubic foot solid, and about 30 lbs. per cubic foot, heaped; and that the average volume of one ton is 75 cubic feet, varying from 70 to 80 cubic feet per ton.

Composition of Coke.

For all purposes, the less ash there is in coke the more valuable it is. Pure coke, if such there be, consists entirely of carbon. But in practice, coke consists of carbon, sulphur, and ash. The purest coke known is Ramsay's Garesfield coke. The composition, as ascertained by analysis by Dr. Richardson, is as follows:—

Carbon	97.6	per cent.
Sulphur	0.85	"
Ash	1.55	"
	100.00	

The composition of Durham coke varies within the following limits:—

Carbon	85 to 92 per cent.
Sulphur	¼ to 2 "
Ash	4 to 12 "

Dr. Muspratt gives the results of nineteen analyses of coke of the usual qualities supplied to manufacturers; they are here given in table No. 145, arranged in the order of the percentages of carbon, the first in the list being Ramsay's coke above-mentioned.

For the service of locomotives on railways, coke, besides being dense and hard, should not contain more than 6 per cent. of ash to insure its passing as coke of good quality; with 9 per cent. of ash, it is of mediocre quality; with 12 per cent. of ash, it is decidedly bad coke.

The washing of coal destined for the formation of coke has already been described. Its effect in removing the earthy matter and in improving the quality of the coke has already been referred to. Suppose a coal which, in its ordinary condition, yields a coke containing from 10 to 15 per cent. of ash, the effect of previously washing the coal would be to reduce the quantity of ash in the coke to from 4 to 6 per cent.

Table No. 145.—Composition of Cokes.

Arranged from data given by Dr. Muspratt.

No. of Coke.	Composition.			No. of Coke.	Composition.		
	Carbon.	Sulphur.	Ash.		Carbon.	Sulphur.	Ash.
	per cent.	per cent.	per cent.		per cent.	per cent.	per cent.
1	97.60	0.85	1.55	11	92.70	1.60	5.70
2	96.42	0.83	2.75	12	92.44	1.56	6.00
3	95.51	1.64	2.85	13	91.49	1.46	7.05
4	94.67	1.07	4.26	14	91.16	1.19	7.65
5	94.31	0.72	4.97	15	90.53	1.01	8.46
6	94.21	0.69	5.10	16	89.87	1.78	8.35
7	94.08	0.88	5.04	17	89.69	1.96	8.35
8	93.54	0.76	5.70	18	85.85	2.08	12.07
9	93.41	0.79	5.80	19	84.82	0.78	14.40
10	93.05	1.58	5.37	Average of 19 cokes	93.44	1.22	5.34

Moisture in Coke.

Coke is capable of absorbing from 15 to 20 per cent. of its weight of water. It has been found to absorb as much as 8 per cent. of water on its way from the ovens to its destination in uncovered waggons. Directly exposed to rain, it may absorb as much as 50 per cent. of its weight of water; the most part of which is afterwards quickly evaporated, leaving from 5 to 10 per cent. in the coke.

Loss of Combustible Matter in the Conversion of Coal into Coke.

Peclet quotes the experience with Alais coal, a bituminous hard coal, having the average composition of the coals used for the manufacture of coke at Seraing:—

	Per cent.
Carbon,	89.27
Hydrogen,	4.85
Oxygen and nitrogen,	4.47
Ash,	1.41
	100.00

The yield of coke is 67 per cent., and deducting the ash, 1.41 per cent., there remains 65.59 per cent. as carbon in the coke. The total loss of combustible matter in parts of the coal is then—

	Per cent.
Carbon,	$89.27 - 65.59 = 23.68$
Hydrogen,	4.85

The heat of combustion of the lost carbon and hydrogen is, by rule 4, page 406,

$$145\ (23.68 + 4.28\ (4.85 - \frac{4.47}{8})\ ) = 6096 \text{ units,}$$

showing a loss of 40 per cent. of the total heating power of the coal, which is 15,606 units.

AIR CONSUMED IN THE COMPLETE COMBUSTION OF COKE.

The quantity of air chemically consumed in the complete combustion of coke is found by means of rule 1, page 400. Take, for example, coke of average composition, having 93.44 per cent. of carbon, and 1.22 per cent. of sulphur. By the formula, the volume of air at 62° chemically consumed is

$$1.52\ (93.44 + (1.22 \times 0.4)\ ) = 1.52 \times 93.93 = 142.8 \text{ cubic feet.}$$

To find the weight of this quantity of air, divide the volume by 13.14, and the quotient is the weight, 10.87 lbs.

Similarly, the air chemically consumed by the best and worst cokes, in table No. 145, is found, and the quantities for the three cokes are here placed together for comparison:—

Coke.	Carbon. per cent.	Sulphur. per cent.	Ash. per cent.	Quantity of Air Chemically Consumed. Volume at 62°.	Weight.
No. 1	97.60	0.85	1.55	148.9 cubic feet.	11.33 lbs.
No. 19	84.82	0.78	14.40	128.9 „	9.81 „
Average coke	93.44	1.22	5.34	142.8 „	10.87 „

GASEOUS PRODUCTS OF THE COMBUSTION OF COKE.

The combustible elements of coke—carbon and sulphur—produce carbonic acid and sulphurous acid. These, together with the nitrogen of the air chemically consumed, constitute the products of combustion. By rule 2, page 401, the weight of these products is as follows, for coke of average composition—with 93.44 per cent. of carbon and 1.22 per cent. of sulphur:—

Products.		Pounds.		Per cent.
Carbonic acid,	$93.44 \times .0366$	= 3.42	or	28.4
Sulphurous acid,	$1.22 \times .02$	= 0.24	or	2.0
Nitrogen,	$(93.44 \times .0893) + (1.22 \times .0335)$	= 8.38	or	69.6
Total weight,		12.04	or	100.0

showing a total weight of 12 lbs. of gaseous products for one pound of average coke—the same weight as was found for average coal (page 428).

The volume of the gaseous products, at 62° F., is found from the percentages of the combustibles by the data (*e*) (*g*) (*h*), page 401, respectively as follows:—

Products.		Cubic feet at 62°.		Per cent.
Carbonic acid,	$93.44 \times .315$	= 29.43	or	20.6
Sulphurous acid,	$1.22 \times .117$	= 0.14	or	0.1
Nitrogen,	$(93.44 \times 1.206) + (1.22 \times .45)$	= 113.25	or	79.3
Total volume,		142.82	or	100.0

Showing a total volume, as at 62°, of about 143 cubic feet of gaseous products for one pound of average coke.

Heating Power of Coke.

The heating power of coke is calculated directly from the quantity of constituent carbon, if the sulphur be neglected. Taking, for example, the first and the last samples of coke, of which the analyses are given in table No. 145, with coke of average composition, the percentages of constituent carbon are as follows, to which are added the heating powers of one pound of the fuels, calculated by rules (4) and (5), page 406:—

Coke.	Constituent Carbon. per cent.	Total Heating Power. units of heat.	Total Evaporative Power from water at 212°.
No. 1	97.60	14,150	14.64 pounds.
No. 19....................	84.82	12,300	12.72 „
Average coke	93.44	13,550	14.02 „

Temperature of Combustion of Coke.

The temperature of combustion of carbon, which is the combustible matter of coke, was found, page 407, to be 4877° F. when completely burned. It may therefore be assumed that the temperature of combustion of coke is under 5000° F.

LIGNITE AND ASPHALTE.

Lignite, or as it is occasionally called, brown coal, though it is often found of a black colour, belongs to a more recent formation—the tertiary—than coal. It is in fact an imperfect coal. Brown lignite is sometimes of a woody texture, sometimes earthy. Black lignite is either of a woody texture, or it is homogeneous, with a resinous fracture. Some lignites, more fully developed, are of a schistose character, with pyrites in their composition. The coke produced from various lignites is either pulverulent, like that of anthracite, or it retains the forms of the original fibres. Lignite is less dense than coal.

The table No. 146 contains the composition of lignites of various qualities, including the hygrometric moisture.

Table No. 147 contains the results of analyses and other particulars of lignites and of asphalte, according to Regnault. See also table No. 141, page 423.

Table No. 146.—Density and Composition of various Lignites, including Hygrometric Moisture.

Locality and description.	Specific gravity.	Carbon.	Hydrogen.	Oxygen.	Nitrogen.	Ash.	Moisture.
		p. cent.	p. cent.	p. cent.	p. cent.	p. cent.	p. cent.
Meissner—red brown, woody	1.12	51.24	4.17	52.33	0.17	0.80	10.30
Rheinhardswalde—gray or black, with abundance of resin	1.13	58.78	4.04	20.80	0.15	5.94	10.28
Meissner — brilliant black, fracture fibrous, lustre vitreous	1.32	70.0	3.19	17.59	0.12	5.47	3.63
Hirschberg–brownish black, in tree-like masses.........	1.35	60.30	4.86	20.17	0.12	3.17	11.39

Table No. 147.—Of the Density, Composition, and Heating Power of Lignites and Asphalte.

(By M. Regnault, from a Table by M. Peclet.)

Description.	Locality.	Specific gravity.	Coke produced.	Nature of the Coke.	Composition.				Hydrogen in excess.	Heating Power.
					Carbon.	Hydrogen.	Oxygen & Nitrogen.	Ash.		
			per cent.		per cent.	per cent.	per cent.	per cent.	per cent.	units of heat.
Perfect lignite	Dax	1.272	49.1	Pulverulent.	70.49	5.59	18.93	4.99	3.32	12,312
Do.	Mouths of the Rhone......	1.254	41.1	Do.	63.88	4.58	18.11	13.43	2.41	10,782
Do.	Mont-Mésiner..	1.351	48.5	Do.	71.71	4.85	21.67	1.77	2.25	11,826
Do.	Lower Alps......	1.276	49.5	Do.	70.02	5.20	21.77	3.01	2.59	11,790
Imperfect lignite ...	Greece	1.185	38.9	Like wood charcoal.	61.20	5.00	24.78	9.02	2.03	10,161
Do. ...	Cologne	1.100	36.1		63.29	4.98	26.24	5.49	1.83	10,337
Do. ...	Usnach (fossil wood)......	1.167	—		56.04	5.70	36.07	2.19	1.38	9,005
Bituminous lignite..	Ellebogen	1.157	27.4	Porous.	73.79	7.46	13.79	4.96	5.81	14,337
Do. ..	Cuba	1.197	39.0	Do.	75.85	7.25	12.96	3.94	5.70	14,562
Asphalte	....	1.063	9.0	Do.	79.18	9.30	8.72	2.80	8.26	16,655
				Averages.						
Perfect lignite	....	1.288	47.0	Pulverulent.	69.02	5.05	20.12	5.82	2.64	11,678
Imperfect lignite ...	....	1.151	37.5	Like wood charcoal.	60.18	5.29	29.03	5.57	1.75	9,834
Bituminous lignite..	....	1.177	33.2	Porous.	74.82	7.36	13.38	4.45	5.76	14,449
Asphalte	....	1.063	9.0	Do.	79.18	9.30	8.72	2.80	8.26	16,655

ASPHALTE.

Asphalte, like lignite, has a large proportion of hydrogen. It has less than 9 per cent. of oxygen and nitrogen, and thus leaves 8¼ per cent. of free hydrogen, and it accordingly yields a porous coke.

The average composition of perfect lignite and of asphalte may be taken in whole numbers as follows:—

	Lignite.	Asphalte.
Carbon	69 per cent.	79 per cent.
Hydrogen	5 ,,	9 ,,
Oxygen and nitrogen	20 ,,	9 ,,
Ash	6 ,,	3 ,,
	100	100
Coke, by laboratory analysis	47 ,,	9 ,,

The lignites are distinguished from coal by the large proportion of oxygen in their composition—from 13 to 29 per cent., which goes far to neutralize the hydrogen, so that for the first and second lignites the free hydrogen is less than 3 per cent. For the third—bituminous lignite—the free hydrogen amounts to nearly 6 per cent., and the varied effect of the proportion of free hydrogen is visible on the nature of the coke of lignite, as was found in the case of the coke of coal. Thus,

With 2.64 per cent. of free hydrogen, the coke is pulverulent.
,, 1.75 ,, ,, ,, like wood charcoal.
,, 5.76 ,, ,, ,, raised and porous.

The small yield of coke from asphalte—only 9 per cent.—though the constituent carbon amounts to 79 per cent., is evidently caused by the great amount of free hydrogen volatilizing a large proportion of the carbon.

TOTAL HEATING POWER OF LIGNITE AND ASPHALTE.

The total heating power of lignite and asphalte, in units of heat, and their equivalent evaporative powers in water from 212°, under one atmosphere, are as follows:—

Fuel.	Heating power.	Total evaporative power in water from 212° per pound of fuel.
	units of heat.	pounds.
Perfect lignite	11,678	12.10
Imperfect lignite	9,834	10.18
Bituminous lignite	14,449	14.96
Asphalte	16,655	17.24

It may be observed, with reference to the lignites noted in table No. 141, that the more perfectly converted lignites possess the greatest heating power. There is a fine distinction between lignite passing to fossil wood, and fossil wood passing to lignite; their heating powers are nearly equal to each other, and both are less than the heating powers of the perfect lignites.

WOOD.

Wood, as a combustible, is divisible into two classes:—1st. The hard, compact, and comparatively heavy woods, as oak, beech, elm, ash; 2d. The light-coloured, soft, and comparatively light woods, as pine, birch, poplar. In France, firewood is classed as fresh wood (*bois neuf*), carried by land or water to its destination; raft wood (*bois flotté*), floated to its destination; and peeled wood (*bois pelard*), or oak stripped of its bark.

According to M. Leplay, green wood, when cut down, contains about 45 per cent. of its weight of moisture. In the forests of Central Europe, wood cut down in winter holds, at the end of the following summer, more than 40 per cent. of water. Wood kept for several years in a dry place retains from 15 to 20 per cent. of water.

Wood which has been thoroughly desiccated, will, when exposed to air under ordinary circumstances, absorb 5 per cent. of water in the first three days; and will continue to absorb it, until it reaches from 14 to 16 per cent., as a normal standard. The amount fluctuates above and below this standard, according to the state of the atmosphere.

M. Violette found that, by exposing green wood to a temperature of 212° F., it lost 45 per cent. of its weight, which accords with the observation of M. Leplay. He further found that by exposing small prisms of wood half an inch square and eight inches long, cut out of billets that had been stored for two years, to the action of superheated steam, for two hours, they lost from 15 to 45 per cent. of their weight, according to the temperature of the steam, which varied from 257° F. to 437° F. (125° C. to 225° C.). The following are the particulars for four woods:—

Temperature of Desiccation.	Loss of Weight.			
	Oak.	Ash.	Elm.	Walnut.
	per cent.	per cent.	per cent.	per cent.
125° C. or 257° F.	15.26	14.78	15.32	15.55
150 „ 302	17.93	16.19	17.02	17.43
175 „ 347	32.13	21.22	36.94	21.79
200 „ 392	35.80	27.51	33.38 (?)	41.77 (?)
225 „ 437	44.31	33.38	40.56	36.56

The hardest wood, oak, lost, according to this statement, more weight than the softer woods. The observations queried appear to have been errors of observation. At a temperature of 200° C., or 392° F., wood becomes visibly altered, and the alteration, or decomposition, may likely commence at a lower temperature; and it may be that the losses of weight are not entirely due to a reduction of hygrometric water. A higher temperature than 212° F. appears to be necessary to disengage all the water.

Ordinary firewood contains, by analysis, from 27 to 80 per cent. of hygrometric moisture.

Composition of Wood.

M. Chevandier, in 1844, published the results of analysis of five woods,—beech, oak, birch, poplar, and willow. The woods were reduced to powder, and desiccated at a temperature of 140° C., or 284° F., before being submitted to analysis. The results of analysis are given in table No. 148.

Table No. 148.—Composition of Woods.

(From Analysis by M. Eugène Chevandier, 1844.)

Woods.	Composition.				
	Carbon.	Hydrogen.	Oxygen.	Nitrogen.	Ash.
	per cent.	per cent.	per cent.	per cent.	per cent.
Beech	49.36	6.01	42.69	0.91	1.00
Oak	49.64	5.92	41.16	1.29	1.97
Birch	50.20	6.20	41.62	1.15	0.81
Poplar	49.37	6.21	41.60	0.96	1.86
Willow	49.96	5.96	39.56	0.96	3.37
Twigs and Branches composing the Fagots—					
Beech	50.17	6.12	40.38	1.05	1.77
Oak	49.96	6.02	41.10	1.00	1.90
Birch	51.24	6.22	40.17	1.05	1.32
Poplar	49.50	6.09	40.43	1.00	2.98
Willow	51.54	6.26	36.21	1.41	4.57
Average of woods	49.70	6.06	41.30	1.05	1.80
Average of fagots	50.46	6.14	39.65	1.11	2.50

There is a remarkable nearness to identity in the composition of these woods, and also in the composition of the trunk and the branches.

The results show that the composition of woods is practically as follows:—

Carbon	50	per cent.
Hydrogen	6	"
Oxygen	41	"
Nitrogen	1	"
Ash	2	"
	100	

Showing that there is only 56 per cent. of combustible matter, that there is a large quantity of oxygen, nearly sufficient to neutralize the whole of the hydrogen, and that there is only 2 per cent. of ash.

The above-mentioned analysis is corroborated by the analysis of M. Violette, who ascertained the composition of different parts of the same tree, desiccated at a temperature of 80° C. or 176° F., with the results given in the following table No. 149:—

Table No. 149.—Composition of the Various Members of one Tree.

(From Analysis by M. Violette.)

Members of the Tree.		Composition.			
		Carbon.	Hydrogen.	Oxygen and Nitrogen.	Ash.
		per cent.	per cent.	per cent.	per cent.
Leaves		45.01	6.97	40.91	7.12
Small branches	bark	52.50	7.31	36.74	3.45
	wood	48.36	6.60	44.73	0.30
Medium branches	bark	48.85	6.34	41.12	3.68
	wood	49.90	6.61	43.36	0.13
Large branches	bark	46.87	5.57	44.66	2.90
	wood	48.00	6.47	45.17	0.35
Trunk	bark	46.27	5.93	44.75	2.66
	wood	48.92	6.46	44.32	0.30
Large root	bark	49.08	6.02	48.76	1.13
	wood	49.32	6.29	44.11	0.23
Medium root	bark	50.37	6.07	41.92	1.64
	wood	47.39	6.26	46.13	0.22
Small roots with bark		45.06	5.04	43.50	5.01
Averages:—					
Leaves		45.01	6.97	40.91	7.12
Bark		49.00	6.21	43.00	2.58
Wood		48.66	6.45	44.64	0.25
Small roots with bark		45.06	5.04	43.50	5.01

Here it appears that the composition of the wood is about the same throughout the tree, and that of the bark also; that the wood and the bark have about the same proportion of carbon, 49 per cent., but that the bark has more ash than the wood. The leaves and the small roots have less carbon than the wood,—only 45 per cent.; and more ash,—7 and 5 per cent.

The leaves when dried at 100° C. lost 60 per cent. of water, and the branches 45 per cent.

Composition of Ordinary Firewood.

The respective percentages of the constituent elements of stacked wood in its ordinary state are, of course, reduced in amount when the water is taken into account. Thus, in the following analysis of ordinary firewood, containing 25 per cent. of moisture, the carbon constitutes only 37.5 per cent. of the fuel:—

	per cent.
Hygrometric water	25
Carbon	37.5
Hydrogen	4.5
Oxygen	30.75
Nitrogen	0.75
Ash	1.5
	100.00

Weight and Bulk of Wood.

The density of a large number of woods has already been given in table No. 65, page 208. These values can, in most instances, only be given as approximate, for the density changes with the hygrometric condition of the wood. The specific gravity varies from 1.35, that of pomegranate, to 0.24, that of cork wood.

The density of the ligneous fibre of which wood is formed, has been ascertained by M. Violette, from a great number of observations. The samples of wood were reduced to powder in a mortar, and dried at a temperature of 100° C. He found that the fibre of all woods had the same density, and that its specific gravity was 1.50.

It is said that the quantity of intersticial space in a closely-packed pile of

Table No. 150.—Of the Weight and Bulk of Woods in France.

Woods in ordinary state of dryness.	Weight of one cubic foot, heaped.	Bulk of one ton, heaped.
	pounds.	cubic feet.
Firewood	21.9 to 23.4	102.3 to 95.7
Wood for charring, hard and soft, cut up	18.8	119.1
Do. do. hard wood, cut up	23.4	95.7
Oak, cut up	22.4 to 23.7	100 to 94.4
Do. branches	17.3	132.4
Do. small branches	19.8	113.1
Beech, cut up	23.7	94.4
Do. branches	19.0	118.0
Do. small branches	19.6	114.2
Yoke-elm, cut up	23.1	97.0
Do. branches	18.6	120.3
Do small branches	19.5	114.6
Birch, cut up	21.1	106.1
Do. branches	16.8	133.3
Fir	16.0	140.1
Alder, cut up	18.3	122.4
Willow, cut up	18.0	124.7
Aspen, cut up	17.0	131.4
Pine in the United States	21.0	106.0
Averages	20.0	114.0

wood, consisting of uncloven stems, is 30 per cent. of the gross bulk; for cloven stems, the interstitial space amounts to from 40 to 50 per cent.

A cord of pine wood,—that is, of pine wood cut up and piled,—in the United States, measures 4 feet by 4 feet by 8 feet, and has a volume of 128 cubic feet. Its weight, in ordinary condition, averages 2700 lbs.; or 21 lbs. per cubic foot.

A "corde" of wood, in France, has a volume of 4 cubic metres, or 141 cubic feet.

Firewood is measured, in France, by the *voie*, of which the volume is 2 cubic metres, or 2 *stères*. As the length of the billets is 1.14 metres, or 3.74 feet, the half-*voie*, or *stère*, measures 1.14 metres × 0.88 metre × 1 metre, equal to 1 cubic metre, or 35.3 cubic feet; and the *voie* is equal to 70.6 cubic feet in bulk. The weight of the *voie* of firewood, in Paris, is from 700 to 750 kilogrammes, or from 1544 to 1653 lbs., averaging 1600 lbs.

The *voie* of wood for making charcoal, in the forests of the Ardennes, weighs 1324 lbs.; it consists of one-fourth oak and beech, one-fourth poplar and willow, and one-half elm. The hard wood for charring, of the forests of the Meuse, weighs 1653 lbs. per *voie*.

The above and other particulars given by M. Chevandier are collected and arranged in table No. 150, showing the weight and bulk of ordinarily dry wood.

Quantity of Air Chemically Consumed in the Complete Combustion of Wood.

In terms of the average percentages of carbon, hydrogen, and oxygen, in wood, page 440, the quantity of air consumed is, by the rule 1, page 400,

$$1.52\left(50+3\left(6-\frac{41}{8}\right)\right)=1.52\times 52.625=80 \text{ cubic feet,}$$

or $80 \div 13.14 = 6.09$ lbs.

Gaseous Products of the Combustion of Wood.

For one pound of dry wood the products are, by the expressions (*a*), (*b*), (*c*), page 401,

Products.		Pounds.	Per cent.
Carbonic acid,	$50 \times .0366$	= 1.83 or	21.7
Steam,	$6 \times .09$	= 0.54 or	6.4
Nitrogen,	$(50 \times .0893) + (6 \times .268) + (1 \times .01)$	= 6.08 or	71.9
Total weight of products,		8.45	100.0

says 8½ lbs. weight of products.

The volumes of the products at 62° are, by the expressions (*e*), (*f*), (*h*), page 401,

Products.		Cubic feet.	Per cent.
Carbonic acid,	$50 \times .315$	= 15.75 or	14.4
Steam,	6×1.9	= 11.40 or	10.4
Nitrogen,	$(50 \times 1.206) + (6 \times 3.618)$	= 82.01 or	75.2
Total volume of products for 1 lb. of wood,		109.16	100.0

being about 13 cubic feet per lb. weight of gaseous products.

Total Heat of Combustion of Wood.

The total heat of combustion of dry wood is, by rule 4, page 406.

$$145\ (50 + 4.28\ (6 - \frac{41}{8})\ ) = 145 \times 53.745 = 7792 \text{ units,}$$

which is a little more than half, or 54½ per cent., of that of coal, and is equivalent, by rule 8, page 406, to the evaporation of $0.15 \times 53.745 = 8.07$ lbs. of water at 212°.

When the wood holds 25 per cent. of water, there is only 75 per cent. or three-quarter pound of wood-substance in one pound; and the total heat of combustion is 75 per cent. of 7792 units, or 5844 units, which is only 41 per cent. of that of average coal. Similarly, the equivalent evaporative power is reduced to 6.05 lbs. of water at 212°, of which the equivalent of a quarter of a pound is appropriated to the vaporizing of the contained moisture.

Temperature of the Combustion of Wood.

It is found, in the manner already shown, page 407, that 2.136 units raise the temperature of the products 1° F. The total heat of combustion, 7792 units ÷ 2.136 = 3648° F.; and 3648 + 62 = 3710° F., is the temperature of combustion.

When the wood holds 25 per cent. of water, the weight of the direct products is 75 per cent. of 8.45 lbs., or 6.34 lbs.; and the total heat of combustion is 5844 units, of which 1116° (total heat of steam) ÷ 4 = 279 units, are appropriated to evaporate a quarter of a pound of water from 62°, leaving 5844 − 279 = 5565 units of heat available for raising the temperature of the gases. To raise the direct products one degree of temperature, there are required—

	Units.
$2.136 \times \frac{3}{4}$ = ...	1.602
The evaporated water, as gaseous steam, 0.25 lb. × .475 = ...	.119
Total for 1° F........................	1.721

Then, 5565 ÷ 1.721 = 3234° F., the temperature of combustion. It is only 88.6 per cent. of the temperature for perfectly dry wood.

In order to obtain the maximum heating power from wood as fuel, it is the practice, in some works on the Continent,—as glass works and porcelain works,—where intensity of heat is required, to dry the wood-fuel thoroughly, even using stoves for the purpose, before using it.

WOOD-CHARCOAL.

When wood is exposed to heat it is at first desiccated and afterwards carbonized. Under temperatures up to 300° F., the wood is simply desiccated. Under temperatures over 300° the gaseous elements are driven off, until at 650° the wood yields a charcoal which is black, solid, and brittle. The gases are not completely driven off except under much higher temperatures.

Wood charcoal, completely converted, is black, solid, brittle, and friable; it preserves the form and structure of the wood from which it is made. Though easily pulverized, it makes a very hard powder.

The following are the results of the experiments of M. Violette on the carbonization of wood. He experimented on black elder-wood, formed into prisms 2.4 inches long and 0.4 inch in diameter, made up in sets of twenty prisms. Each set was dried separately at a temperature of 300°, in a current of superheated steam, to which it was subjected during two hours. The carbonization was effected by the same medium, at least up to 662° F.; and in crucibles placed in a furnace, at higher temperatures. The temperatures arrived at when in the furnace were checked by the melting of small pieces of various metals placed in the crucible with the samples.

The table No. 151 gives the weight of the products obtained as the result of carbonization at the given temperatures:—

Table No. 151.—Yield of Charcoal from Black Elder Wood, Carbonized at Different Temperatures.

(By M. Violette.)

Temperature of Carbonization.		Weight of gross product from dry wood.	Observations.
Cent.	Fahr.	per cent.	
150°	302°	100	These products are only wood more and more altered, but they are not charcoal. They are called, in France, *brûlots*.
160	320	98	
170	338	94.55	
180	356	88.59	
190	374	81.99	
200	392	77.10	
210	410	73.14	
220	428	67.50	
230	446	55.37	
240	464	50.79	
250	482	49.67	
260	500	40.23	
270	518	37.14	
280	536	36.16	Brown charcoal (*charbon roux*). Commences to be friable.
290	554	34.09	
300	572	33.61	
310	590	32.87	
320	608	32.23	
330	626	31.77	
340	644	31.53	Very black charcoal.
350	662	29.26	
432	810	18.87	Melting point of antimony. Charcoal very hard.
1023	1873	18.75	Do. silver. Do. do.
1100	2012	18.40	Do. copper. Do. do.
1250	2282	17.94	Do. gold. Do. do.
1300	2372	17.16	Do. steel. Do. do.
1500	2732	17.31	Do. iron. Do. do.
?	?	15.00	Do. platinum. Do. do.

From this table, it appears that charcoal, properly so-called, is not formed until a temperature of 536° F. is reached. From 536° to 644° F., brown charcoal (*charbon roux*), from 36 to 31½ per cent., is formed. Beyond 644° F. the charcoal is black, and the yield diminishes with the increase of temperature, until, at the unknown temperature of melting platinum, it becomes just 15 per cent. of the weight of the dried wood from which it is produced.

Brown charcoal is flexible, unctuous, and soft to the touch; black charcoal is rigid, brittle, and harsh to the touch.

Composition of Charcoal.

The composition of these charcoals varies with the temperatures at which they are produced, as may be seen by the annexed table, No. 152, showing the results of analysis of some of the charcoals obtained:—

Table No. 152.—Composition of Charcoal Produced at Various Temperatures.

(By M. Violette.)

Temperature of Carbonization.		Composition of the Solid Product.				Carbon for a given weight of wood.
		Carbon.	Hydrogen.	Oxygen, Nitrogen, and Loss.	Ash.	
Centigrade.	Fahrenheit.	per cent.	per cent.	per cent.	per cent.	per cent.
150°	302°	47.51	6.12	46.29	0.08	47.51
200	392	51.82	3.99	43.98	0.23	39.88
250	482	65.59	4.81	28.97	0.63	32.98
300	572	73.24	4.25	21.96	0.57	24.61
350	662	76.64	4.14	18.44	0.61	22.42
432	810	81.64	4.96	15.24	1.61	15.40
1023	1873	81.97	2.30	14.15	1.60	15.30
1100	2012	83.29	1.70	13.79	1.22	15.32
1250	2282	88.14	1.42	9.26	1.20	15.80
1300	2372	90.81	1.58	6.49	1.15	15.85
1500	2732	94.57	0.74	3.84	0.66	16.36
Melting point of platinum	—	96.52	0.62	0.94	1.95	14.47

From this table, it is evident how materially a higher temperature operates in driving off the injurious gases, oxygen and nitrogen—injurious, that is, in reducing the heating power; though the useful gas, hydrogen, is likewise driven off—which is a loss for heating power. The carbon and the ash,—the solid constituents,—on the contrary, are proportionally increased. At the same time, there is an absolute loss of carbon, though less in degree than the diminution of the gases, as the temperature rises. The rate of diminution of the absolute quantity of carbon for a given weight of wood, is arrived at

by multiplying the percentages of constituent carbon in the second table (No. 152) by the relative percentages of gross products in the first table (No. 151), for a given temperature, as given in the last column of the second table. Here, the absolute quantity of carbon, which is 47.5 per cent. in the dry wood in its natural state, at 150° C., is reduced to 15.4 per cent., or one-third, at 432° C. or 810° F.; and beyond this temperature, however great the heat may be, there is practically no further diminution of the carbon; that is to say, that no more carbon is driven off by raising the temperature, the gaseous elements alone being driven off.

It is remarkable that the proportion of ash found by M. Violette is only from 1 to 2 per cent., whilst the ash of the original wood averages 1½ per cent.; for, it is naturally supposed that the whole of the ash should be concentrated in the charcoal, and should average 7½ per cent., supposing that the yield of charcoal is one-fifth the weight of wood. The ash-element must have been withdrawn with the gases that escaped during the process of carbonization. It is found that, practically, the ash of the charcoal of commerce amounts to from 7 to 8 per cent.

According to M. Sauvage, the charcoal manufactured in the forests is composed as follows:—

Carbon,	79	per cent.
Hydrogen, free	2	„
Hydrogen, oxygen, and nitrogen,	11	„
Ash,	8	„
	100	

Yield of Charcoal by Laboratory Analysis.

M. Violette ascertained that the greater or less rapidity with which carbonization is effected influence materially the quantity of the yield. He obtained by slow carbonization twice as much charcoal as by rapid carbonization, at the same temperature; but he does not give the details of the experiments by which he arrived at this conclusion.

He further found that when wood was carbonized in close vessels hermetically sealed, the yield was decidedly greater than in open vessels, thus:—

Temperature of Carbonization.	In Open Vessels. yield.	In Closed Vessels. yield.
160° C. or 320° F.	97 per cent.	97.4 per cent.
340° C. or 644° F.	29 „	79.1 „

The charcoal obtained at 180° C., in a close vessel, was brown, friable, and similar to that produced at 280° C. in an open vessel (table No. 151); though differently constituted, as the former held a greater proportion of gaseous matter, and also more ash than the latter.

Finally, M. Violette ascertained by experiments, similarly conducted in open vessels with superheated steam, the quantity of charcoal for various woods and other ligneous substances, dried, in the first place, at 150° C. or 302° F., and then exposed to a temperature of 300° C. or 572° F. These are arranged in table No. 153, in the order of the quantities yielded:—

Table No. 153.—Yield of Charcoal from Various Woods, Dried at 150° C., or 302° F., and Carbonized at 300° C., or 572° F.

(By M. Violette.)

Wood.	Weight of Charcoal.	Wood.	Weight of Charcoal.
	per cent.		per cent.
Cork	62.80	Apple tree	34.69
Decayed Willow	52.17	Elm	34.59
Wheat straw	46.99	Hornbeam	34.44
Oak	46.09	Alder	34.40
Yew	46.06	Birch	34.17
Beech	44.25	Plum tree	34.06
Pine	41.48	Maple	33.75
Poplar (leaves)	40.95	Willow	33.74
Do. (roots)	40.90	Black elder	33.61
Wild pine	40.75	Ash	33.28
Larch	40.31	Pear tree	31.88
Chestnut tree	36.06	Lime tree	31.85
Cherry tree	35.53	Poplar (stem)	31.12
Aspen	34.87	Sweet chestnut tree	30.86

It appears from this table that cork, the lightest of woods, yields the largest percentage of charcoal, about 63 per cent.; and that poplar and sweet chestnut tree yield the lowest, about 31 per cent. But there does not appear to be any definite relation between the density of the wood and the quantity of the yield.

Carbonization of Wood in Stacks—Yield of Charcoal.

Wood has been carbonized, from the remotest times, in heaps on the ground; and this process is still generally followed on the Continent. The stack or pile is covered with a mixture of earth and powdered charcoal, or with turf. A few openings are left in the covering to admit air to the interior, as well as a larger opening at the summit. When the stack is lit it burns rapidly, and so soon as flame appears at the chimney it is partially damped down by a turf. The progress of carbonization is indicated by the colour of the smoke, and when, finally, the mass becomes incandescent, it is covered up with earth and allowed to cool. By this process, the charcoal obtained usually amounts in weight to from 17 to 20 per cent. of the wood, and to more than this in the larger heaps. From 25 to 30 per cent., in volume is obtained in the small heaps, and from 30 to 34 per cent. in the larger heaps. The charring requires from sixty to eighty hours to produce a good quality of charcoal.

In the departments of the Ardennes and the Meuse, in France, according to M. Sauvage, the following are the particulars of the yield of charcoal from wood. In the case of the Ardennes, the wood prepared for carbonization is a mixture of one-fourth oak and beech, one-fourth poplar and

willow, and one-half elm. In the example from the Meuse, hard wood is used. The billets are about 30 inches in length; they are piled on end, in three tiers. The stack contains from 60 to 90 cubic metres, or from 80 to 120 cubic yards:—

	Ardennes. Mixed wood.	Meuse. Hard wood.
Weight of a cubic metre of wood,...	662 lbs.	827 lbs.
Yield of a cubic metre of wood, in weight,........................	132 to 145 lbs.	176 „
Yield of a cubic metre of wood, in volume,........................	10½ to 11½ cub. ft.	12 to 14 cub. ft.
Percentage of yield, in weight,.......	20 to 22 per cent.	21 per cent.
Weight of a cubic metre of charcoal (heaped),....................	440 lbs.	530 lbs.

It is obvious, from the small percentages of yield, averaging 21 per cent. for the mixed woods and the hard wood, that much of the substance of the wood is lost, which would by a better system of carbonization be yielded as charcoal. According to the table No. 153, the maximum yield obtainable from the mixed woods is 38 per cent.; and from the hard woods upwards of 40 per cent.

Manufacture of Brown Charcoal.

The best method of making brown charcoal consists in heating the wood to be charred in a close vessel, by means of superheated steam introduced into the vessel. The required temperature is thus readily obtained, and a homogeneous product is yielded. This process was introduced by Messrs. Thomas & Laurens, and is successfully employed in France and Belgium in the production of brown charcoal for the manufacture of gunpowder, principally for fowling-pieces.

Distillation of Wood.

The distillation of wood in close vessels affords evidence of the increased yield of charcoal obtainable by more careful treatment than in the open-air stack. In France, the wood is distilled in large iron cylinders or retorts capable of holding about 180 cubic feet of wood, as piled; and the operation is completed in from seven to eight hours. By this process, 28 per cent. of charcoal is obtained, with the products of distillation in addition. But 12½ per cent. of wood is consumed as fuel, making a total of 112½ parts of wood for a yield of 28 parts of charcoal; which reduces the available yield to 25 per cent. of the whole quantity of wood consumed, as against 21 per cent. in the open-air stacks of hard wood. There is a gain, in addition, in reduced cost of labour, and in the value of the yield of pyroligneous acid. The gases are directed into the furnace to aid as fuel in heating the retorts.

Charbon de Paris (Artificial Fuel).

Charbon de Paris, or Paris charcoal, is a mixture of two parts of powdered charcoal with one part of gas-tar, formed by powerful compression into

round pieces 4 inches long and 1¼ inch in diameter, and submitted to a high temperature. It takes fire easily, and burns slowly until it is entirely consumed, without making flame or smoke; it makes from 20 to 22 per cent. of ash.

Weight and Bulk of Wood-Charcoal.

It does not appear that the density of wood-charcoal, as manufactured, has been accurately determined. M. Violette determined the density of the matter of the charcoal of black alder, reduced to impalpable powder, so as to extinguish the pores. It varied according to the temperature of carbonization, as shown in table No. 154:—

Table No. 154.—Absolute Density of the Charcoal of Black Alder, Dried at 212° F., as Powder.

(By M. Violette.)

Temperature of Carbonization.		Specific gravity.	Temperature of Carbonization.		Specific gravity.
Centigrade.	Fahrenheit.		Centigrade.	Fahrenheit.	
150°	302°	1.507	330°	626°	1.428
170	338	1.490	350	662	1.500
190	374	1.470	432	810	1.709
210	410	1.457	1023	1873	1.841
230	446	1.416	1250	2282	1.862
250	482	1.413	1500	2732	1.869
270	518	1.402			
290	554	1.406	?	Fusing point of the platinum retort.	2.002
310	590	1.422			

The table shows that the density at 302° F., and of course at inferior temperatures, is that of the natural wood, dried at 100°; that the density of the charcoals produced at from 302° F. to 518° F. was reduced by the increasing temperature from 1.507 to 1.402.

The table shows briefly as follows:—

1. That the density of the charcoal at 302° F., is that of the natural wood dried at 212°, namely 1.507.

2. The density of the charcoals produced at from 302° F. to 518° F. was reduced from 1.507 to 1.402.

3. The density at temperatures above 518° F. increases with the temperature until it reaches 2.000, or double that of water, at the melting point of platinum.

The specific gravity, weight, and bulk of various charcoals are given in table No. 65, page 211, and they are here abstracted for reference—supplemented by the weight and bulk of the charcoal of the Ardennes and the Meuse, derived from the data, page 449:—

WOOD CHARCOAL.	Specific gravity.	Weight of a cubic foot.	Bulk of one ton.
		pounds.	cubic feet.
As powder	1.500	93.5	24.0
In small pieces, heaped	0.405	25.3	88.5
As manufactured, heaped	0.225	14.0	160.0
Ardennes, heaped	0.201	12.5	180.0
Meuse, heaped	0.241	15.0	149.0

MOISTURE IN WOOD-CHARCOAL.

Charcoal absorbs moisture with avidity. The charcoal of commerce is usually exposed to the atmosphere, and open to rain; and it contains generally from 10 to 12 per cent. of moisture.

Charcoal fresh made, from different woods, was exposed by M. Nau, for twenty-four hours, to an atmosphere loaded with moisture, and the weights of water they absorbed during that time are given in the following table, No. 155:—

Table No. 155.—MOISTURE ABSORBED BY CHARCOALS DURING TWENTY-FOUR HOURS.

(By M. Nau.)

Wood from which the Charcoal was made.	Moisture Absorbed.	Wood from which the Charcoal was made.	Moisture Absorbed.
	per cent.		per cent.
White beech	0.8	Horse chestnut	6.06
Ash	4.06	Elm	6.60
Oak	4.28	Alder	7.93
Birch	4.40	Scotch fir	8.20
Larch	4.50	Willow	8.20
Maple	4.80	Italian poplar	8.50
Pine	5.14	Fir	8.90
Red beech	5.30	Black poplar	16.30

Showing a capacity for absorption varying from 0.8 to 16 per cent.

It is certain that the period of exposure was not sufficiently long to saturate the charcoals. For charcoals have been known to absorb increasing quantities of moisture during three months.

M. Violette made some observations on the capacity for moisture of charcoals which had been prepared from black alder at various temperatures. The samples were exposed in a room the air of which was saturated with moisture. Observations were made every eight days, and they lasted three months—until the charcoals ceased to absorb more moisture. The results show that charcoal is less absorbent the higher the temperature at which it is produced. The ordinary black charcoals, produced at tem-

peratures of from 480° to 750° F., are capable of absorbing from 5 to 7 per cent. of water; and taking the extreme observations, the absorption ranges from 21 to 2.2 per cent. between the extreme temperatures. At the lower temperatures, of course, the charcoal was only partially converted.

Air Consumed in the Complete Combustion of Dry Wood-Charcoal.

According to the analysis of M. Sauvage, page 447, there is 79 per cent. of carbon, and 2 per cent. of free hydrogen, in forest-charcoal. By formula (1), page 400, the volume of air at 62° F. chemically consumed in the complete combustion of one pound of charcoal, is

$$1.52\ (79 + (3 \times 2)\ ) = 129 \text{ cubic feet of air at } 62°.$$

The weight of the air is $129 \div 13.14 = 9.8$ pounds.

Gaseous Products of the Complete Combustion of Dry Wood-Charcoal.

The gaseous products of combustion consist of carbonic acid, steam, and nitrogen, and the total weight of them is found by formula (2), page 401, as follows:—

$$(79 \times 0.126) + (2 \times 0.356) = 10.66 \text{ pounds.}$$

The total volume of the gases, as at 62°, by formula (3), page 402, is,

$$(79 \times 1.52) + (2 \times 5.52) = 131 \text{ cubic feet at } 62°.$$

Heat Evolved by the Complete Combustion of Wood-Charcoal.

The total heating power of dry wood-charcoal, having 79 per cent. of carbon and 2 per cent. of free hydrogen, is by formula (6), page 406:—

$$145\ (79 + (4.28 \times 2)\ ) = 12{,}696 \text{ units of heat.}$$

The total evaporative efficiency is, by formula (8), page 406,

$$0.15\ (79 + (4.28 \times 2)\ ) = 13.13 \text{ pounds of water,}$$

evaporated from 212°, under one atmosphere.

For charcoal containing moisture the heating power is less, and may be estimated in the manner already adopted in the case of coke.

PEAT.

Peat is the organic matter, or vegetable soil, of bogs, swamps, and marshes,—decayed mosses or sphagnums, sedges, coarse grasses, &c.,—in beds varying from 1 or 2 feet to 20, 30, or 40 feet deep. The peat near the surface, less advanced in decomposition, is light, spongy, and fibrous, of a yellow or light reddish-brown colour; lower down, it is more compact, of a darker brown colour; and, in the lowest strata, it is of a blackish brown, or almost a black colour, of a pitchy or unctuous feel, having the fibrous texture nearly or altogether obliterated.

Peat, in its natural condition, generally contains from 75 to 80 per cent. of its entire weight, of water. The constituent water occasionally amounts

to 85 or even to 90 per cent., in which case the peat is of the consistency of mire. It shrinks very much in drying; and its specific gravity varies from .22 or .34 to 1.06, the surface peat being the lightest, and the lowest peat the densest. Detailed particulars of the weight and specific gravity of peat are given at page 207.

Table No. 156.—Chemical Composition of Irish Peat, taken as Perfectly Dry.

(Sir Robert Kane.)

Description and Locality of Peat.	Specific Gravity.	Carbon.	Hydrogen.	Oxygen.	Nitrogen.	Ash.
		per cent.	per cent.	per cent.	per cent.	p. cent.
1. Light surface, Philipstown....	.405	57.52	6.83	32.23	1.42	1.99
2. Rather dense, do.	.669	58.56	5.91	31.40	.85	3.30
3. Light surface, Wood of Allen	.335	58.30	6.43	31.36	1.22	2.74
4. Compact and dense, do.	.655	56.34	4.81	30.20	.74	7.90
5. Light fibrous, Ticknevin	.500	58.60	6.55	30.50	1.84	2.63
6. Light fibrous, Upper Shannon	.280	58.53	5.73	32.32	.93	2.47
7. Very dense, compact, do.	.853	59.42	5.49	30.50	1.64	2.97
Averages	.528	58.18	5.96	31.21	1.23	3.43

Table No. 157.—Composition of Sundry Peats, including Moisture.

First, Exclusive of Moisture.

Description of Peat.	Moisture.	Carbon.	Hydrog.	Oxygen.	Nitrog.	Sulphur.	Ash.	Coke.
	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	p. cent.	p. cent.
Good air-dried	—	59.7	6.0	31.9		—	2.4	—
Poor air-dried.......	—	59.6	4.3	29.8		—	6.3	—
Dense, from Galway	—	59.5	7.2	24.8	2.3	.8	5.4	44.3
Averages.........	—	59.6	5.8	29.6		.3	4.7	—
Second, Inclusive of Moisture.								
Good air-dried	24.2	45.3	4.6	24.1		—	1.8	—
Poor air-dried.......	29.4	42.1	3.1	21.0		—	4.4	—
Dense, from Galway	29.3	42.0	5.1	17.5	1.7	.6	3.8	31.3
Averages.........	27.8	43.1	4.3	21.4		.2	3.3	—

When wet, peat is masticated, macerated, or milled, so that the fibre is broken, crushed, or cut. The contraction in drying is much increased by this treatment; and the peat becomes denser, and is better consolidated than when it is dried as cut from the bog. Peat so prepared is known as *condensed peat;* and the degree of condensation varies according to the natural heaviness of the peat. Peat from the lowest beds is but little condensed; but peat from the middle and upper beds is condensed, when dry, to from two to three times its natural density. So effectively is peat consolidated and condensed by the simple process of breaking the fibres

whilst wet, that no merely mechanical force of compression is equal in efficiency to mastication.

The table No. 156 contains the results of chemical analysis of Irish peat of various qualities by Dr. Kane; the samples were desiccated before being submitted to analysis.

Mr. A. M'Donnell gives the composition of average "good air-dried" peat and "poor air-dried" peat, analyzed by Dr. Reynolds, as in table No. 157; to which are added an analysis of dense peat from Galway, made by Dr. Cameron:—

From the above tables, it appears that sulphur is rarely found in Irish peat, and that the average composition of the peat is as follows:—

	Perfectly Dry.	Including 25 per cent. of moisture.
	per cent.	per cent.
Carbon	59	44
Hydrogen	6	4.5
Oxygen	30	22.5
Nitrogen	1¼	1
Sulphur	?	?
Ash	4	3
	100	75
Moisture		25
		100

Ordinary air-dried peat contains from 20 to 30 per cent. of its gross weight of moisture. If dried in air in the most effective manner, it contains at least 15 per cent. of moisture; and even when dried in a stove, it seldom holds less than 7 or 8 per cent.

The peats named in table No. 156 were subjected to distillation, when they yielded water, tar, charcoal, and gas, in the proportions shown in table No. 158:—

Table No. 158.—Products of Distillation of Irish Peat.

Description and Locality of Peat.	Water.	Crude Tar.	Charcoal.	Gas.
	per cent.	per cent.	per cent.	per cent.
Nos. 1 and 2, Philipstown	23.6	2.0	37.5	36.9
" 3, Wood of Allen	32.3	3.6	39.1	25.0
" 4, do.	38.1	2.8	32.6	26.5
" 5, Ticknevin	33.6	2.9	31.1	32.3
" 6, Upper Shannon	38.1	4.4	21.8	35.7
" 7, "	21.8	1.5	19.0	57.7
Averages	31.4	2.8	29.2	36.6

The tar, when re-distilled, yielded water, paraffine, oils, charcoal, and gas. The water yielded chloride of ammonium, acetic acid, and wood-spirit.

Heating Power of Irish Peat.—In peat of average composition, as given above, the heating power is by rule 4, page 406,

perfectly dry,.................... $145\ (59 + 4.28\ (6 - \frac{30}{8})\ ) = 9951$ units of heat;

containing 25 per cent. of moisture,........... $145\ (44 + 4.28\ (4.5 - \frac{22.5}{8})\ ) = 7435$ units of heat.

Deduct for evaporating the moisture, ¼ lb., supplied at 62°; 1116° ÷ 4 .... = 279 do. do.

Effective heating power.............. 7156 do. do.

The total evaporative power of 1 lb. of fuel, evaporating at 212°, is as follows:—

	Perfectly dry.	Containing 25 per cent. of moisture.
When water is supplied at 62°, divisor 1116°...	8.91 lbs.	6.41 lbs.
Do. do. 212°, do. 966°...	10.30 lbs.	7.41 lbs.

British and foreign peats are very much like Irish peat in composition; the principal variation takes place in the proportion of ash.

PEAT-CHARCOAL.

The charcoal of ordinary dried peat is very porous, and, in general, light and fragile; but the charcoal of condensed peat is dense and solid. It burns easily but slowly: small incandescent pieces separated from the fire continue to burn until the whole of the carbon disappears. Good peat yields from 30 to 40 per cent. of charcoal; and the charcoal when perfectly dry consists generally of from 85 to 90 per cent. of carbon, and 10 to 15 per cent. of ash.

The heating power of one pound of peat-charcoal, containing 85 per cent. of carbon, by rule 4, page 406, is, $145 \times 85 = 12{,}325$ units of heat; equivalent to the evaporation, at 212°, of 11.04 lbs. of water supplied at 62°, or of 12.76 lbs. supplied at 212°. The temperature of combustion is that of carbon, 4877° F.

In France, the peat-charcoal of Essonne contains 18.2 per cent. of ash. In the Ardennes, Bar peat carbonized in ovens yields 44 per cent. of charcoal; but this contains one-third volatile matter, one-fourth ash, and only 43 per cent. of carbon.

TAN.

Tan, or oak-bark, after having been used in the processes of tanning, is burned as fuel. The spent tan consists of the fibrous portion of the bark. According to M. Peclet, five parts of oak-bark produce four parts of dry tan; and the heating power of perfectly dry tan, containing 15 per cent. of ash, is 6100 English units; whilst that of tan in an ordinary state of dryness, containing 30 per cent. of water, is only 4284 English units. The weight of water evaporated at 212° by one pound of tan, equivalent to these heating powers, is as follows:—

	Perfectly dry.	With 30 per cent. of moisture.
Water supplied at 62°	5.46 lbs.	3.84 lbs.
” ” 212°	6.31 ”	4.44 ”

STRAW.

The average composition of wheat-straw is as follows:—

Water	14.23	per cent.
Organic or combustible matter; consisting of carbon, hydrogen, oxygen, and nitrogen..	78.30	”
Ash	7.47	”
	100.00	

Chemists have not, so far as the author has learned, thought it worth while to record the proportions of the organic elements. But it may be supposed that the composition of straw is similarly proportioned to that of peat. The weight of pressed straw is from 6 to 8 lbs. per cubic foot.

LIQUID FUELS.

Petroleum is a hydro-carbon liquid which is found in abundance in America and Europe. According to the analysis of M. Sainte-Claire Deville, the composition of fifteen petroleums from different sources was found to be practically constant. The average specific gravity was .870. The extreme and the average elementary composition were as follows:—

Carbon	82.0 to 87.1 per cent.	Average	84.7 per cent.
Hydrogen	11.2 to 14.8 ”	”	13.1 ”
Oxygen	0.5 to 5.7 ”	”	2.2 ”
			100.0

The total heating and evaporative powers of one pound of petroleum having this average composition are, by rules 4 and 5, page 406, as follows:—

Total heating power $= 145\left(84.7 + 4.28\left(13.1 - \frac{2.2}{8}\right)\right) = 20{,}240$ units.

Evaporative power: evaporating at 212°, water supplied at 62° = 18.13 lbs.
Do. do. do. 212° = 20.33 lbs.

Petroleum-Oils are obtained in great variety by distillation from petroleum. They are compounds of carbon and hydrogen, ranging from $C_{10}H_{24}$ to $C_{32}H_{64}$; or, in weight,

$$\text{from}\left\{\begin{array}{l}71.42 \text{ carbon}\\28.58 \text{ hydrogen}\\\hline 100.00\end{array}\right\} \text{to} \left\{\begin{array}{l}73.77 \text{ carbon}\\26.23 \text{ hydrogen}\\\hline 100.00\end{array}\right.$$

The specific gravity ranges from .628 to .792. The boiling point ranges from 86° to 495° F. The total heating power ranges from 28,087 to 26,975 units of heat: equivalent to the evaporation, at 212°, of from 25.17 lbs. to 24.17 lbs. of water supplied at 62°, or from 29.08 lbs. to 27.92 lbs. of water supplied at 212°.

Schist-Oil, like petroleum, consists of carbon, hydrogen, and oxygen; but there is less hydrogen and more oxygen, as may be seen from the following analysis by St.-Claire Deville:—

	From Vagnas Schist.	From Autun Schist.
Carbon	80.3	79.7
Hydrogen	11.5	11.8
Oxygen	8.2	8.5
	100.0	100.0

Pine-wood Oil, analyzed by the same chemist, contains 87.1 per cent. of carbon, 10.4 per cent. of hydrogen, and 2.5 per cent. of oxygen.

COAL-GAS.

Mr. Vernon Harcourt made an analysis of coal-gas,[1] one pound of which

COAL-GAS. (Mr. V. Harcourt.)	Carbon.	Hydrogen.	Oxygen.	Total.
	per cent.	per cent.	per cent.	per cent.
Olefiant gas	10.5	1.7	—	12.2
Marsh gas	39.7	13.2	—	52.9
Carbonic oxide	5.9	—	7.9	13.8
Carbonic acid	1.9	—	5.0	6.9
Hydrogen	—	8.1	—	8.1
Nitrogen	—	—	—	5.8
Oxygen	—	—	—	0.3
	58.0	23.0	—	100.0

had a volume of 30 cubic feet at 62° F. The heating power, calculated for the three elements, is as follows:—

			Units.
Carbonic oxide	13.8 per cent	× 4,325 ÷ 100 =	597
Carbon	50.2 „	× 14,500 ÷ 100 =	7,279
Hydrogen	23.0 „	× 62,000 ÷ 100 =	14,260
Total heat of combustion			22,136

This is equivalent to the evaporation of 19.84 lbs. of water from 62° at

[1] See a paper on "Petroleum and other Mineral Oils, applied to the Manufacture of Gas," by Mr. Owen C. D. Ross, in the *Proceedings of the Institution of Civil Engineers*, vol. xl. page 150.

212° F., or to 22.92 lbs. from and at 212°. The heating power of 1 cubic foot is 738 units, equivalent to the evaporation of .66 lb. or .76 lb. of water.

Mr. F. W. Hartley[1] tested the heating power of gas manufactured by the South Metropolitan Gas Company, London. By means of his gas calorimeter he determined the heating power of one cubic foot of gas at 60° F., under 30 inches of mercury, to be 622.15 units, equivalent to the evaporation of .56 pound of water from 60° at 212°, or .64 pound from and at 212°.

Taking the means for these two gases, the heating power of 1 cubic foot at 62° F. is equivalent to the evaporation of .70 pound of water from and at 212° F.

[1] *Report on the Gas Section of the International Electric and Gas Exhibition at the Crystal Palace*, 1882–83, page 23. It may here be stated that according to the results of tests of several "Instantaneous Gas Waterheaters," made by Mr. D. K. Clark, in the same connection, much more heat was generated than was deduced from Mr. Hartley's determinations.

It is due to Mr. Hartley to state that he was cognizant of the usual presumption that the potentiality of gas as a heating power is greater than what is deduced from such experiments. "The problem," he says, "of determining with ease and certainty when a quantity of two to three cubic feet can be had, the absolute calorific value of a combustible gas, within about ½ per cent. of the truth, is, I think, completely solved; and the fact that the calorific power indicated for the gas in question is much below that which ordinary coal-gas is presumed to possess, in no degree disturbs my belief in the accuracy of the results which I have now the honour to submit."—*Report*, page 23.

APPLICATIONS OF HEAT.

This section on the applications of heat comprises the principles of the transmission of heat through solid bodies. The consideration of the application of the heat of furnaces for the generation of steam in boilers will be taken up in the section on steam-boilers. In the present section, the subjects dealt with relate to the heating and evaporation of water by steam, the condensing of steam by water, the heating of air by hot water and by steam, the warming and ventilation of buildings, distillation, cooling, drying, blast-furnaces, and cognate subjects.

TRANSMISSION OF HEAT THROUGH SOLID BODIES—FROM WATER TO WATER THROUGH SOLID PLATES OR BEDS.

With a view to educe the general principles of the transmission of heat through solid bodies, M. Peclet made a series of experiments on the transmission of heat through plates of metal, heated on one side by heated water, and cooled on the other side by water at a low temperature. He found from experiments made with wrought iron, cast iron, copper, lead, zinc, and tin, that when the fluid in contact with the surface of the plate was not changed by artificial means, the rate of conduction of metals was not only the same for different metals, but also for different thicknesses of the same metal. Correctly ascribing this uniformity of performance to the presence of a stagnant film of water adhering to the surfaces of the plates, which, by its inferior conductivity, negatived in a greater or less degree the conductivity of the plates themselves, he made a new series of experiments with lead plates, in which the water was thoroughly circulated over the surface; and he found that the quantity of heat transmitted through the plates was inversely proportional to the thickness. Having by this means settled the constant for lead, he adopted the results of Depretz's experiments on the conducting power of metals (see page 331), and calculated the constants for other metals from these data. He, further, made a series of experiments on the conducting or transmissive power of "bad conductors" of heat, between two surfaces of water:—stone, and wood and other vegetable substances, which were incased in two thin coats of copper, to prevent the absorption of water by them.

M. Peclet lays down the elementary law of the transmission of heat as follows:—The flow of heat which traverses an element of a body in a unit of time is proportional to its surface, and to the difference of temperature of the two faces perpendicular to the direction of the flow; and is in the inverse

ratio of the thickness of the element. This law, he maintains, is rigorously deduced from the nature of the motion of heat, and he embodies it in the following formula:—

$$M = (t - t')\frac{C}{E}; \qquad (1)$$

in which t and t' are the temperatures of the surfaces, C the quantity of heat transmitted per hour for one degree of difference of temperature through one unit of thickness, and E the thickness. That is to say, using English measures:—if the difference of temperatures in degrees Fahrenheit be multiplied by the constant C for the given material, one inch thick, and divided by the thickness in inches, the quotient is the quantity of heat in English units passed through the plate per square foot per hour.

The quantities of heat transmitted through plates or beds of metals and other solid bodies, one inch in thickness, for 1° F. difference of temperature per hour, as determined by M. Peclet, are given in table No. 159, being the values of the constant C in formula (1). The conditions are, that the surfaces of the conducting material must be perfectly clean, that they be in contact with water at both faces of different temperatures, and that the water in contact with the surfaces be thoroughly and constantly changed. M. Peclet found that when metallic surfaces became dull, the rate of transmission of heat through all metals became very nearly the same.

Table No. 159.—Quantities of Heat Transmitted from Water to Water through Plates or Beds of Metals and other Solid Bodies 1 Inch Thick, per Square Foot, for 1° F. difference of Temperature between the Two Faces per Hour.

Selected from M. Peclet's tables, and converted for English measures.

Substance.	Quantity of heat.	Substance.	Quantity of heat.	Substance.	Quantity of heat.
	units.		units.		units.
Gold	620	Fir, across fibre	.74	Coke, powd.	.80
Platinum	604	Fir, along the fibre	1.36	Iron filings	1.26
Silver	596	Caoutchouc	1.36	Cotton wool	.32
Copper	555	Gutta-percha	1.37	Calico	.40
Iron	225	Glass	6.56	Carded wool	.35
Zinc	225	Sand	2.16	Eider-down	.31
Tin	177	Brick, powder'd	1.12	Canvas	.42
Lead	112	Chalk, do.	.69	White writing-paper	.34
Marble	24	Ashes of wood.	.53	Gray paper unsized	.19
Plaster	2.6	Wood-charcoal, powdered	.63		
Terra cotta	4.8				
Oak, across fibre	1.69				

Mr. James R. Napier made experiments with experimental boilers of iron and copper of various thicknesses, over a gas flame, and he found only a

small difference in evaporating power of about a twentieth or a thirtieth in favour of the copper: results which are corroborative of M. Peclet's deductions.

Professor Rankine states that in all experiments of this kind the condition of the heating surface is important, whether smooth or rough, and whether perfectly clean or incrusted to any extent.

But, the rate of transmission of heat through metallic plates also differs very much according to the substances in contact with the plate, between which the heat is transmitted:—as between water or steam and water, or between water and air, or gaseous matter and water, and so on. Mr. Thomas Craddock, at an early period, proved that the rate of cooling by transmission of heat through metallic surfaces, was almost wholly dependent upon the rate of circulation of the cooling medium over the surface to be cooled, and that water was enormously more efficient than air for the abstraction of heat. He suspended a tube filled with hot water having a thermometer suspended in the water. The water was cooled from a temperature of 180° to 100° F., in still air, in 25 minutes, and in still water in one minute. Again, when he moved the tube filled with hot water, by rapid rotation, at the rate of 40 miles per hour through air, it lost as much heat in 1 minute as it did in still air in 12 minutes. In water, at a velocity of two miles per hour, as much heat was abstracted in half a minute as was absorbed in one minute when at rest in the water. Mr. Craddock concluded that the circulation of the cooling fluid becomes of greater importance as the difference of temperature on the two sides of the plate becomes less.

HEATING AND EVAPORATION OF LIQUIDS BY STEAM THROUGH METALLIC SURFACES.

Mr. John Graham heated water in a square wooden cistern, having a double iron bottom, into which steam of 16½ lbs. per square inch, absolute pressure, having a temperature of 218° F., was admitted. When the water in the cistern stood at 60° F., the steam was admitted, and the following were the successive temperatures at equal intervals of time, as reported by Mr. Graham:—

Time from the commencement. seconds.	Temperature of the water. Fahrenheit.	Increments of temperature. Fahrenheit.
0	60°	0°
10	100	40
20	134	34
30	158	24
40	174	16
50	183	9
60	192	9
70	198	6
80	201	3
90	206	5
100	210	4

It was found to be difficult to raise the temperature above 210° F. The increased activity in the rise of temperature towards the end, was no doubt

due to the increased movement in the water as it approached the boiling point. To show the rate of the passage of heat with respect to the mean difference of the temperatures of the steam and the water during each interval of ten seconds, the mean temperature of the water during each interval is given in the second column below, the difference of these mean temperatures and that of the steam in the third column, the increments of temperature in the fourth column; and, in the last column, the rise of temperature per degree of difference of temperature is given:—

Times.	Mean temperatures of the water.	Difference of temperatures of the water and the steam.	Increments of temperature.	Increments of temperature per degree of difference.
seconds.	Fahrenheit.	Fahrenheit.	Fahrenheit.	Fahrenheit.
10	80°	138°	40°	.290
20	117	101	34	.336
30	146	72	24	.333
40	166	52	16	.308
50	178	40	9	.225
60	187	31	9	.290
70	195	23	6	.261
80	199.5	18.5	3	.162
90	203.5	14.5	5	.345
100	208	10	4	.400

The quantity of heat transmitted per degree of difference of temperature is in proportion to the increments of temperature in the last column. Though irregular, they are, taken together, practically uniform per degree of difference of temperature. At the same time, the quantity per degree in the middle stages appears to be slightly reduced as the total difference of temperature is reduced.

M. Clement found that a sheet of copper, 1 metre square and about ⅛ inch thick, when heated on one face by steam of 212° F., and cooled on the other face by water at 82°.4 F., making an excess of temperature of 129°.6, condensed 20.5 lbs. of steam per square foot per hour, equivalent to

$$20.5 \div 129^\circ.6 = 0.160 \text{ lbs. of steam}$$

condensed per square foot per degree of difference of temperature per hour. The total heat of steam at 212° F. is 1095°.6 above 82°.4, and for 20.5 lbs. of steam there are $1095.6 \times 20.5 = 22{,}460$ units of heat, and

$$22{,}460 \div 129^\circ.6 = 173 \text{ units of heat,}$$

which is the quantity of heat passed through the plate per square foot per degree of difference of temperature per hour.

M. Peclet gives the performance of a copper boiler with double bottom for boiling beet-root juice by steam of three atmospheres, or 275° F., admitted into the bottom. The area exposed to steam amounted to 25.82 square feet. A quantity of juice weighing 1984.5 lbs. was delivered into the copper at a temperature of 39° F., and heated to 212° F., through 173°

in sixteen minutes, equivalent to a rise of $(173 \times 60 \div 16) = 649°$ of temperature per hour. The total heat transmitted per square foot per hour was

$$649° \times 1984.5 \div 25.82 = 49{,}880 \text{ units of heat.}$$

The mean temperature of the juice was $(212 + 39) \div 2 = 126°$, and the mean difference of temperature was $275° - 126° = 149°$; then the total heat transmitted per square foot per degree of difference of temperature per hour was

$$49{,}880 \div 149° = 335 \text{ units of heat.}$$

The total heat of the steam was 1071° above 126°, the mean temperature of the juice, and the quantity of steam condensed per square foot per degree per hour was

$$\frac{335}{1071} = .313 \text{ lb. of steam.}$$

M. Peclet quotes the results of experiments made by Laurens and Thomas on the heating power of steam operating through coils of pipe. In the first experiment, the pipe was 137.8 feet long and 1.36 inches in diameter externally, presenting 48.20 square feet of surface. Steam of three atmospheres, or 275° F., was freely admitted into the pipe, and it raised the temperature of 882 lbs. of water from 46° F. to 212° through 166° in four minutes, equivalent to a rise of $(166° \times 60 \div 4) = 2490°$ in an hour. The mean temperature of the water was $(212 + 46) \div 2 = 129°$, and the difference of temperature was $275 - 129 = 146°$. Hence the total heat transmitted per square foot per hour was

$$2490° \times 882 \div 48.20 = 45564 \text{ units of heat,}$$

and the total heat per square foot per degree per hour was

$$45564 \div 146 = 312.1 \text{ units of heat.}$$

The total heat of the steam was 1068° above 129°, and the quantity of steam condensed per square foot per degree per hour was, therefore,

$$\frac{312.1}{1068} = .292 \text{ lb. of steam.}$$

Next, 551.25 lbs. of water was evaporated from 212° by the same steam, in 11 minutes, being at the rate of 3007 lbs. per hour, or 62.38 lbs. per square foot per hour. The total heat of the atmospheric vapour was 966° above 212°, and the heat transmitted per square foot per hour was

$$966 \times 62.38 = 59{,}710 \text{ units of heat.}$$

The difference of temperature was $275 - 212 = 63°$, and the quantity of heat passed per square foot per degree per hour was

$$59{,}710 \div 63 = 948 \text{ units of heat.}$$

The quantity of steam condensed per square foot per degree per hour was

$$\frac{948}{966} = .981 \text{ lb. of steam.}$$

In another experiment, two coils of steam-pipe 49.2 feet long, and 1.34 inches in diameter, presenting a surface of 34.52 square feet, with steam of two atmospheres, or 250°.4 F., evaporated 1587.6 lbs. of water at 212° per hour; or 46 lbs. per square foot per hour, with a difference of temperature $(250.4 - 212) = 38°.4$ F. The quantity of heat passed per square foot per hour was,

$$966 \times 46 = 44{,}430 \text{ units,}$$

and, per degree per square foot per hour, was,

$$44{,}430 \div 38.4 = 1157 \text{ units.}$$

The quantity of steam condensed per degree per square foot per hour was

$$\frac{1157}{966} = 1.20 \text{ lbs.}$$

The following are the results of experiments by M. P. Havrez in heating water by steam with a coil of copper pipe, given in *Engineering*, vol. vi. The coil was 14.21 feet long, and 1.57 inches in diameter; superficial area, 5.85 square feet. The pipe was incrusted to some extent:—

Total pressure and temperature of steam.		Water heated.	Time to heat water.
1. 67 lbs.	300° F.	232.65 lbs. from 68° to 212°	— — — —
2. 89 „	319.4	232.65 „ „ „	to 122°, 4 min.; 212°, 10 min.
3. 89 „	319.4	217.80 „ 104 to 212	„ 3 „ „ 7½ „

	Actual weight of steam condensed.	Weight per square foot per hour.	Weight per 1° F. difference of temperature.
1.	41.25 lbs.	42.25 lbs.	.264 lb.
2.	44.00 „	45.00 „	.271 „
3.	28.60 „	39.00 „	.270 „
	Averages,	42.08 „	.268 „

It may be noted that the water was heated to 122°, and then to 212°, in the second and third experiment, at the following rates:—

2d Experiment,	to 122° at	13°.5 per minute.	To 212° at	15° per minute.
3d do.	„ „	6.0 „	„ „	20 „

In continuation of the experiments, with the same pressure of steam, portions of the water were evaporated:—

	Water evaporated in 10 minutes.	Ebullition.	Evaporated per square foot per hour.	And per degree.
1.	9.9 lbs.	soft,	10.15 lbs.	.115 lb.
2.	15.95 „	violent,	16.35 „	.174 „
3.	7.7 „	very soft,	7.89 „	.084 „
			11.46 „	.126 „

There are, as *Engineering* remarks, inconsistencies in these results. The scale, no doubt, impeded the activity of the heat.

The results of experiments by M. Havrez, with a cast-iron boiler having a double bottom, are also given by *Engineering*. The boiler was 18.5 inches in diameter, and 13.5 inches deep, and had a jacketted surface of 6.576 square feet:—

	Total pressure and temperature of steam.		Water heated.	
1.	67 lbs.	300° F.	229.7 lbs.	from 80° to 212°, in 25 minutes.
2.	67 „	300	237.6 „	„ 68 to 212, in 22 „

	Actual weight of steam condensed.	Per square foot per hour.	And per degree difference of temperature.
1.	31.24 lbs.	10.14 lbs.	.066 lb.
2.	34.10 „	14.10 „	.088 „
	Averages,	12.12 „	.077 „

In evaporating the water at 212°, from 8.8 to 11.44 lbs. of steam was condensed in 10 minutes; being at the rate of 8.02 to 10.4 lbs. per square foot per hour; or .091 to .118 lb. per square foot per degree of difference of temperature per hour.

The quantities of heat, in M. Havrez's experiments, transmitted per square foot per degree of difference of temperature per hour are found from the quantities of steam condensed, as follows:—

	Heat utilized from 1 lb. of steam.		Steam condensed per square foot per degree per hour.		Heat transmitted per square foot per degree per hour.
Coiled pipe; heating water:—					
1st Experiment,	1005 units	×	.264 lb.	=	265.3 units.
2d Do.	1071 „	×	.271 „	=	290.2 „
3d Do.	1053 „	×	.270 „	=	284.3 „
Averages,			.268 „		280.0 „
Coiled pipe; evaporating water:—					
1st Experiment,	961 units	×	.115 lb.	=	110.5 units.
2d Do.	967 „	×	.174 „	=	168.2 „
3d Do.	967 „	×	.084 „	=	81.2 „
Averages,			126		120.0 „
Cast-iron boiler; heating water:—					
1st Experiment,	1059 units	×	.066 lb.	=	69.90 units.
2d Do.	1063 „	×	.088 „	=	93.54 „
Averages,			.077 „		81.72 „
Cast-iron boiler; evaporating water:—					
1st Experiment,	961 units	×	.091 lb.	=	87.45 units.
2d Do.	961 „	×	.118 „	=	113.40 „
Averages,			.105 „		100.43 „

Mr. William Anderson[1] gives the results of experiments on the power of sugar-clarifiers in heating water. They were 6 feet 6½ inches in diameter, and 2 feet 6 inches deep; containing a copper pan 18 inches deep, bolted into cast-iron steam-jackets, with a working capacity of 450 gallons, and having a heating surface of 52.58 square feet. The

[1] "On the Aba-el-Wakf Sugar Factory, Upper Egypt." *Proceedings of the Institution of Civil Engineers*, vol. xxxv., 1872-73.

average results of three experiments in heating water to 212° are as follows:—

Mean duration of the experiments............24 minutes.
Mean initial temperature of water............67° F.
Mean steam pressure above atmosphere ...42.1 lbs., 289° F.
Mean weight of condensed steam ....742 "
Mean weight of water heated................4558 "

				units.
Units of heat in condensed steam 742	" × 990°	=		734,580
Heat spent in heating copper.................840	" × 145° × .095 =	11,571		
" " cast iron............2828	" × 145° × .129 =	52,900		
" " wrought iron.........567	" × 145° × .113 =	9,200		
" " water 4558	" × 145°	= 660,910		
				734,671

	units.
Units of heat per square foot per difference of 1° per hour in heating water..	210.2
Loss in heating clarifier, radiation, &c..................................	11.1 per cent.

The mean temperature of the water was $\frac{67+212}{2} = 140°$, and the heat utilized per pound of steam was 1062° ($= 1170 + 32 - 140$). Then,

$$\frac{210.2}{1062} = .198 \text{ lb. of steam,}$$

condensed per square foot per degree per hour.

In other experiments, with a smaller clarifier, similar in construction, of 12 gallons of capacity, the trials were carried further, and the rate of boiling was ascertained, both for water and for sirup, the latter consisting of a solution of 9 lbs. of molasses and 4 lbs. of sugar in 90 lbs. of water, equal to juice at about 8° Beaumé. The quantities of heat passed through the metal were as follows:—

	Water.	Juice.
	units.	units.
In heating, per square foot per difference of 1° F. per hour........	260	219
In evaporating, " " "	606	521

showing a greatly accelerated passage of heat when evaporating, 2⅓ times as much as in only heating the water; also, that the addition of 14½ per cent. of sugar reduced the efficiency of the surface by about 15 per cent.

Mr. Anderson made similar trials to test the efficiency of the concentrators, for their evaporating powers. It is only necessary to state here that each of the concentrators consists of a copper tray 23 feet long by 6 feet wide, ¼ of an inch thick, heated by a steam-boiler beneath it, and forming part of it. The boiler is 12½ inches deep, flat-bottomed, and stayed to the tray at 6 inches pitch. The heating surface of the tray is increased by 495 upright hollow nozzles of brass, screwed into it, very thin, and slightly taper; average external diameter 2⅛ inches, vertical projection 4½ inches. The tray is inclosed by a sheet-iron cover. The heating surface of the tray consisted of 138 square feet horizontal surface, and 187 feet of vertical surface, together, 325 square feet. By experiment, it was found that surfaces similar to those of the tray performed as follows:—

In heating water to the boiling point, 5.8 lbs. effective pressure per square inch, 228° F., per square foot per 1° F. difference per hour.. 368 units.
In evaporating .... 660 „

Here the passage of heat for evaporation was 1.8 times as much as in heating without evaporation. Applying this ratio to the performance of the tray itself, Mr. Anderson calculates that the efficiency of the tray by experiment was—

For heating.. 271 units.
For evaporation ....491 „

The obviously superior efficiency of the model is accounted for by its having been fully charged with steam from the factory boilers; "whilst in the actual tray the generator was evidently unequal to the work."

The mean pressure in the generator was 47 lbs. effective, with the temperature 294° F.; and that in the tray was 5.8 lbs., temperature 228° F. The total heat of the first steam was 1171° from 32° F., or 975° above 228°; and the quantity of steam condensed per square foot per degree per hour was—

	Condensed.
For heating	$271 \div 975 = 0.278$ pound of steam.
For evaporating..................	$491 \div 975 = 0.504$ „ „

Mr. F. J. Bramwell, in discussing Mr. Anderson's paper, gave particulars of similar experiments made by him with a jacketed copper pan, having a working capacity of 100 gallons, and a heating surface of 25 square feet. The pan had been at work for eight or nine years, and probably was incrusted on the steam side. He tried the performance of the pan with steam successively of 5 lbs., 10 lbs., 15 lbs., and 20 lbs. effective pressure, raising the temperature of the water from 58° to 212°, and evaporating it. In the first experiment, with 5 lb. steam, he found that the rates of transmission of heat per square foot per degree of difference per hour, taking observations every five minutes, in raising the temperature to 200°, were successively 161, 151, 176, 160 units of heat, whilst in heating from 200° to 212° the rate advanced to 327 units; and, when ebullition commenced, to 427 units. The observed rates at the different pressures are subjoined for comparison:—

Effective pressure of steam.	Initial temperature of water.	Average rate of transmission up to 212°.	Average rate of transmission, evaporating at 212° F.
5 lbs.	58° F.	—	427 units.
10 lbs.	58	186 units	435 „
15 lbs.	58	—	458 „
20 lbs.	58	205 „	488 „
Averages,		196 „	452 „

From these results it appears that the rate of transmission for evaporation is more than double the rate for heating; and the detailed observations, at 5 lbs. pressure, show a marked acceleration of transmission when the water was within 12° of the boiling point. These experiments confirm those of Mr. Anderson, and it is very probable that the greater agitation

and quicker circulation of the water as it neared the boiling point, and whilst boiling, was the cause of the increased rate of transmission of the heat. The average rates above given show that, per square foot per degree per hour, the quantities of steam condensed were:—

	Condensed.
In heating up to 212°,	.201 lb.
In evaporating at 212°,	.463 ,,

The various results of performance above detailed are numbered and collected in table No. 160, and the averages for copper-plate surfaces, copper-coil surfaces, and cast-iron surfaces are given in the lower part of the table.

Table No. 160.—Results of Performance of Coiled Pipes and Boilers in Heating and Evaporating Water by Steam.

Authority.	Apparatus.	Steam condensed per square foot, for 1° F. difference of temperature per hour.		Heat transmitted per square foot, for 1° F. difference of temperature per hour.	
		Heating.	Evaporating.	Heating.	Evaporating.
		lbs.	lbs.	units.	units.
1. Clement....	Copper plate...........	.160	—	173	—
2. Peclet.......	Copper boiler...........	.313	—	335	—
3. Laurens.....	Copper coil.............	.292	.981	312	948
4. Do.	2 Do. do.............	—	1.20	—	1120
5. Havrez......	Copper coil.............	.268	.126	280	120
6. Do.	Cast-iron boiler.........	.077	.105	82	100
7. Anderson...	Copper clarifier.........	.198	—	210	—
8. Do. ...	Copper concentrator..	.278	.504	271	491
9. Do. ...	Copper concentrator (model)......	—	—	368	660
10. Bramwell...	Copper pan.............	.201	.463	196	452
Averages for copper-plate surface, Nos. 2, 7, 8, 9, 10..................		.248	.483	276	534
Averages for copper-pipe surface, Nos. 3, 4..............................		.292	1.090	312	1034
Cast-iron plate surface, No. 6............		.077	.105	82	100

Note.—Nos. 1 and 5 are omitted from the averages as the information is incomplete, and for No. 5, the results are not consistent.

It appears that the efficiency of copper-plate surface for evaporation is double its efficiency for heating water; for copper-pipe surface the efficiency is more than three times as much; and for cast-iron plate surface, a fourth more.

That the efficiency of pipe-surface is a fifth more than that of plate-surface for heating, and more than twice as much for evaporation.

That, in round numbers, copper-plate surface condenses half a pound of steam, copper pipe condenses a pound of steam, and cast-iron plate-surface a tenth of a pound, per square foot per degree of difference of temperature per hour, for evaporation.

That the quantity of heat transmitted is at the rate of about 1000 units per pound of steam condensed.

These are the results to be expected when the surfaces are in good condition.

COOLING OF HOT WATER IN PIPES.

M. Darcy states that the water from the artesian wells at Grenelles passed underground through cast-iron pipes of from 6½ to 10 inches diameter, for a length of 2530 yards, in 8½ hours, equivalent to an average velocity of 3 inches per second, discharging about 50 gallons per minute. The water was cooled from 80° to 69°.5 F., or 10°.5; being at the rate of 1°.24 per hour. The loss of heat amounted to 307,600 units per hour, which passed through 16,424 square feet of surface: at the rate of 18.7 units per square foot per hour, for a mean temperature of 75°. When at rest in the pipe, the water was cooled at the rate of 10° F. in 7 hours, or 21.6 units per square foot per hour. Taking the temperature of the ground at 62°, the mean difference of temperature was 13° F., and the heat transmitted per square foot per degree per hour was $18.7 \div 13 = 1.44$ units when the water was in motion, and $21.6 \div 13 = 1.66$ units when the water was at rest.

Taking the results of experiments by Mr. Tredgold on the rate of cooling of water in pipes, in air, as corrected by Mr. Hood, a cast-iron pipe 30 inches long, 2½ inches in diameter internally, and ¼ inch thick, was filled with water at 152° F. It exposed a surface of 2 square feet, with a surrounding temperature of 67° F.; and the quantity of water, including an equivalent for the heated iron, was 172 cubic inches, or 6 lbs. weight. The water was cooled at a nearly uniform rate, from 152° to 140° F., in the following times, to which are added the cooling and the units of heat passed per minute:—

State of cast-iron surface.	Cooled 12° F. in	Cooled per minute.	Heat passed per square foot per minute.
1. Ordinary brown (rusty),........	15 minutes ...	0°.8 F....	$0.8 \times 3 = 2.4$ units.
2. Black varnished,................	14.53 „	... 0.83	... $0.83 \times 3 = 2.5$ „
3. White, two coats of lead paint,	15.33 „	... 0.78	... $0.78 \times 3 = 2.34$ „

To reduce these results to the general standard for comparison:—

	Mean temperature of water.	Mean difference of temperature of water and air.	Heat passed off per square foot per hour.	Heat passed off per square foot per degree of difference per hour.
	Fahr.	Fahr.	units.	units.
1.	146°	79°	$2.4 \times 60 = 144$	1.823
2.	146	79	$2.5 \times 60 = 150$	1.900
3.	146	79	$2.34 \times 60 = 140.4$	1.778

From other experiments by Tredgold, hot water was cooled in vessels made of tinned plate, sheet iron, and glass from 180° to 159° F., in a room at 56°, showing an average excess of temperature of 114° F. They con-

tained 2.2 lbs. of water, including an equivalent for the metal. The results were as follows:—

Surface.	Area of surface. square feet.	Time to cool 30° F. minutes.	Cooled per minute. Fahr.	Heat passed per square foot per minute. units.
4. Tinplate,.....	.55	46	0°.65	$.65 \times 2.2 \div .55 = 2.60$
5. Sheet iron,..	.533	29	1°.03	$1.03 \times 2.2 \div .533 = 4.26$
6. Glass,........	.500	31½	0°.94	$.94 \times 2.2 \div .50 = 4.14$

The heat passed off per hour was,

4. 156.0 units per square foot, and 1.37 units per degree of difference.
5. 255.6 „ „ „ 2.24 „ „ „
6. 248.4 „ „ „ 2.18 „ „ „

To group the experimental results adduced for the transmission of heat from hot water in iron pipes and vessels to the external air:—

	Per square foot per degree difference of temperature per hour.
2½ inch cast-iron pipe, ¼ inch thick, naked,.....	1.82 units.
Sheet-iron vessel,.....................................	2.24 „
Mean,..............................	2.03 „

Cooling of Hot Wort on Metal Plates in Air.

The results of experiments on the cooling of wort at Trueman's brewery are recorded in *Engineering*, vol. vi. Two coolers, 110 feet by 25 feet, made of thin copper, No. 15 wire-gauge, or $^1/_{13}$ inch thick, were supported on open joists, and air was free to circulate above and below the coolers. The total cooling surface amounted to 5500 square feet. The wort was run over the coolers in a thin stream, of which 50 barrels of 360 lbs. each were cooled from 212° to 110° F. per hour. The total heat passed off by evaporation and by conduction through the metal was $50 \times 360 \times (212° - 110°) = 1,836,000$ units per hour; being at the rate of 334 units per square foot per hour.

When the wort was left to stand on the coolers, from 2 to 2½ inches deep, it was cooled 140° in from six to eight hours. Taking 10 lbs. per square foot as the weight of the water, the quantity of heat passed off was $\frac{140 \times 10}{7} = 200$ units per square foot per hour.

The mean temperature of the wort was, in the first case, $\frac{212 + 100}{2} = 161°$; and in the second case $212° - \frac{140}{2} = 142°$. The mean differences of temperature, taking that of the air at 62°, were 99° and 80°, and the heat passed off per square foot per degree of difference of temperature per hour was—

For the flowing wort,..........$334 \div 99 = 3.37$ units.
For the still wort,...............$200 \div 80 = 2.50$ „

Cooling of Hot Wort by Cold Water in Metallic Refrigerators.

From the instructive discussion of the principles of brewery engineering in *Engineering*, vol. vi., the following particulars are derived of the performance of tubular refrigerators, in which cold water is passed through thin metallic tubès, which are surrounded by the wort to be cooled. The water and the wort are moved in opposite directions in such a manner that the cold water, on its entrance into the refrigerator, meets the cooled wort just before it leaves the refrigerators, and the warmed water passes away from the refrigerator where the hot wort enters. The following are particulars of the performance in five experiments:—

Table No. 161.—Results of Performance of Metallic Refrigerators in Cooling Hot Wort with Cold Water.

Area of cooling surface of refrigerator.	Wort.					Water.			
	Specific gravity.	Quantity passed through per hour.	Initial temperature.	Final temperature.	Cooled down.	Quantity passed through per hour.	Initial temperature.	Final temperature.	Warmed up.
Square feet.		Barrels.	Fahr.	Fahr.	Fahr.	Barrels.	Fahr.	Fahr.	Fahr.
1. 881	—	33.9	212°	72°	140°	61.1	65°	169°	104°
2. 514	1.104	36.1	155	59	96	75.5	54	100	46
3. 514	1.088	36.6	191	59	132	99.5	54	100	46
4. 514	1.035	47.3	193	59	134	90.7	54	100	46
5. 514	1.018	48.0	178	59	119	102.0	54	100	46

Note 1.—A barrel contains 36 gallons, or 360 lbs. of water.

2.—The temperature of the air in Nos. 2 and 4 was 44° F., and in Nos. 3 and 5, 40°.

Dealing with the data of this table, the following are the mean temperatures and differences of temperature of the wort and the water, with the quantities of heat transmitted per unit of surface, temperature, and time:—

No. of experiment.	Mean temperatures		Mean difference of temperature.	Heat transmitted per square foot per degree per hour.	
	Of wort.	Of water.		Measured by reduction of temperature of wort.	Measured by increase of temperature of water.
	Fahr.	Fahr.	Fahr.	units.	units.
1	142°	117°	25°	78	104
2	107	77	30	81	81
3	125	77	48	71	67
4	126	77	49	91	59
5	118.5	77	41.5	96	79
Averages,				83.4	78

To show how the quantities of heat in the last two columns are calculated, take the first example. The quantity of wort passed through per hour was 33.9 barrels of, say, 360 lbs. each, neglecting the extra specific gravity; cooled down through 140° F., the cooling surface was 881 square feet, and the mean difference of temperature of the wort and the water was 25° F. Then,

$$\frac{33.9 \times 360 \times 140}{881 \times 25} = 78 \text{ units of heat,}$$

passed from the wort, per square foot per 1° F. difference of temperature per hour. Again, 61.1 barrels of water were warmed up through 104° F. Then,

$$\frac{61.1 \times 360 \times 104}{881 \times 25} = 104 \text{ units of heat,}$$

absorbed by the water, per square foot per 1° F. per hour, and similarly for the other examples. There is an inconsistency in the excess of heat taken up by the water, as calculated, above that which was passed from the wort in the first example, indicating that there was an error of observation. For the second example, the quantities are equal. The remaining observations show, reversely, that more heat passed from the wort than was taken up by the water. The averages of all the examples show that 83.4 units of heat were passed from the wort, and 78 units were absorbed by the water, per square foot per 1° F. difference of temperature per hour.

It is well to note, as observed in *Engineering*, that the rate at which the wort parts with its heat increases generally as the specific gravity is less. This acceleration points to the conclusion, that if water be substituted for wort, the rate of transmission would be 100 units per square foot per 1° F. difference per hour; although, conversely, the cooling water would absorb only 80 units. The difference, 20 units, would be passed off by radiation and conduction.

CONDENSATION OF STEAM IN PIPES EXPOSED TO AIR.

Tredgold found, by experiment, that steam of an absolute pressure of 17.5 lbs. per square inch, temperature 221° F., produced one cubic foot of water per hour by condensation in iron pipes exposing 182 square feet of surface in a room at 60° F. The difference of temperature was 161°, and the condensation per square foot per hour was .352 lb. of water; or, per degree of difference of temperature, .0022 lb.

Experiments made in 1859 by M. Burnat, on the efficiency of coating for cast-iron steam pipes, afford valuable data in this connection.[1] The pipes were 4.72 inches in diameter externally, and ¼ inch thick; they were arranged in five groups of four pipes each, each group presenting an aggregate surface of 58½ square feet. The groups were placed at 40 inches apart, and inclined at an angle of 1 in 20, in a large unheated hall free from air-currents. The pipes of the first group were covered with straw laid lengthwise to the thickness of 0.6 inch, bound with straw rope laid closely round it. The second group were left bare as they came from the

[1] Reported in *Proceedings of the Institution of Civil Engineers*, vol. xli., 1874–75.

foundry. In the third group, each pipe was laid in a pottery pipe, with an air-space between the two, and coated with a mixture of loamy earth and chopped straw, covered with tresses of straw. In the fourth group, the pipes were covered with cotton waste to a thickness of an inch, wrapped in cloth bound with string. In the fifth group, the pipes were coated with a composition of clay and cow's hair to a thickness of 2.36 inches. Finally, trials were made with the second group of pipes by coating them with some old felt which had been treated with caoutchouc; and a second trial of the fifth group, after the composition had received a coat of white paint. The pipes were supplied with steam of from 16½ lbs. to 30 lbs. absolute pressure per square inch; and each experiment lasted from 40 to 56 minutes. The results of the experiments are given in the annexed table No. 162.

Table No. 162.—Results of Experiments on the Condensation of Steam in Cast-Iron Pipes.

(M. Burnat.)

Absolute pressure of steam per square inch.	Temperatures.			Steam condensed per square foot of external surface of pipes per hour.				
	Steam.	Air.	Difference.	Straw coat, 1st.	Bare, 2d.	Pottery coat, 3d.	Waste coat, 4th.	Plaster coat, 5th.
lbs.	Fahr.	Fahr.	Fahr.	lb.	lb.	lb.	lb.	lb.
16.5	218°.0	46°.4	171°.6	.139	.496	.170	.217	.254
16.5	218.0	33.8	184.2	.152	.485	.166	.205	.262
18.4	223.4	33.7	189.7	.164	.555	.186	.229	.287
18.4	223.4	27.1	196.4	.182	.571	.264	.287	.344
22.0	233.2	41.5	191.7	.246	.576	.258	.244	.320
22.0	233.2	36.5	196.7	.164	—	.158	.250	—
22.0	233.2	36.1	197.1	.162	.557	.178	.260	—
22.0	233.2	28.9	204.3	.201	.586	.264	.328	.346
25.7	241.6	43.3	198.4	.244	.645	.301	.375	.389
25.7	241.6	36.5	205.1	.274	—	.285	.369	—
29.4	249.1	43.3	205.8	.252	.721	.270	.342	.379
29.4	249.1	30.6	218.4	.225	.621	.250	.328	.336
Averages, 22.0	233.1	36.5	196.6	.200	.581	.229	.286	.324

When the plaster coat of the fifth group was painted white, an average of 0.307 lb. of steam was condensed per square foot of pipe per hour; and the second group, with the felt coating, condensed 0.313 lb. of steam per square foot per hour.

From these data the following constants have been derived, for an absolute pressure of steam of 22 lbs. per square inch; for the quantity of steam condensed, and the quantity of heat passed off, per square foot of external surface of pipe per hour for 1° F. difference of temperature. The quantity of heat transmitted per pound of steam is the difference of the total and sensible heats of the steam, or $(1152.5 + 32) - 233.1 = 951.4$ units:—

Condition of Surface.	Steam Condensed per Squ. Foot per degree per hour. lb.	Heat passed off. units.
Bare, or uncovered pipe	.00300	2.812
Coated with straw	.00102	0.968
Cased in pottery pipes, with air space	.00115	1.108
Coated with cotton-waste, 1 inch thick	.00146	1.384
Coated with old felt	.00159	1.515
Coated with plaster of loamy earth and hair	.00165	1.568
The same, painted white	.00156	1.486

The most effective coat for the prevention of condensation was the straw coat, and that it had the effect of reducing the loss by condensation to one-third of that which took place with the naked pipe. With the naked pipe, 2.812 units of heat were transmitted per square foot per degree per hour.

In experiments by Mr. B. G. Nichol, a wrought-iron pipe 3¾ inches in diameter outside, ¼ inch thick, and lagged to half an inch thick with felt and spun yarn, condensed steam at 245° F. at the rate of .262 lb. per square foot per hour, in an external temperature of 60°, equivalent to 1.26 units of heat per square foot per 1° difference of temperature.

According to M. Clément's experiments, the quantities of steam given in the second column below, were condensed per square foot of pipe-surface per hour, in a temperature of 77° F. Assuming that the steam condensed was of 20 lbs. absolute pressure, the difference of temperature was 151° F., and the weight of steam condensed per 1° F. is given in the third column.

Surface.	Steam condensed per square foot per hour. total.	per 1° F.
Bare cast-iron pipe, horizontal	.328 lb.	.00217 lb.
Blackened do. do.	.308 "	.00204 "
Bare copper pipe, do.	.267 "	.00177 "
Blackened do. do.	.308 "	.00204 "
Do. do. upright	.359 "	.00238 "

Here it appears that the blackened surfaces of iron and of copper were equally active; and that the upright pipe condensed more steam than the same pipe laid horizontally.

Mr. Grouvelle found that, in a temperature of 60° F., a square foot of pipe heated by steam condensed 0.328 lb. of steam per square foot per hour. Assuming that the steam was of 20 lbs. absolute pressure, the difference of temperatures was 228° − 60° = 168° F.; and 0.0020 lb. of steam was condensed per square foot per 1° F.

Summarizing the several results above for bare cast-iron pipes:—

	Difference of temperature.	Steam condensed per square foot per hour. total.	per 1° F.
Tredgold	161° F.	0.352 lb.	.0022 lb.
Burnat	196.6	0.581 "	.0030 "
Clément	151	0.328 "	.00217 "
Grouvelle	168	0.328 "	.0020 "
Average, say, for steam of 20 lbs. absolute pressure	169	0.400 "	.00235 " say 1/420 lb.

To find the quantity of heat dissipated by the condensation of $^1/_{420}$ lb. of steam:—the difference of the sensible and total heats of one pound of steam of 20 lbs. absolute pressure, is the latent heat, 954 units; and $954 \div 420 = 2.27$ units, the heat dissipated per square foot of surface per 1° F. difference of temperature per hour.

To compare the condensing power of still air with that of still water, and referring to the contents of table No. 160, page 468, in the absence of records of experiments made under exactly the same conditions, it may be inferred that the rate of condensation in thin pipes in air is to that in water below the boiling point, per unit of surface, temperature, and time, as 2.26 units to 312 units, or as 1 to 138. M. Peclet takes the ratio as 1 to 200, though so high a ratio is scarcely warranted by the evidence.

Condensation of Steam in a Boiler Exposed in Open Air.—Messrs. Fox, Head, & Co., Middlesborough, made comparative experiments with a steam-boiler on their premises, in two conditions—naked, and covered with non-conducting cement. From an account of the experiments in *Engineering*, vol. vi., it appears that steam of 50 lbs. absolute pressure per square inch was maintained, and that the effect of removing the covering was that one cubic foot of water converted into steam was condensed by 50 square feet of exposed boiler-surface per hour. This is equivalent to 1.25 lbs. of steam per square foot per hour. The weather was fine, and taking the temperature of the open air at 62° F., that of the steam was $298 - 62° = 236°$ above the atmospheric temperature; and the rate of condensation per square foot per degree of difference of temperature per hour was,

$$1.25 \div 236 = .0053 \text{ lb.}$$

The latent heat of one pound of the steam was 904° and $904 \times 1.25 = 1130$ units of heat transmitted per square foot per hour. The quantity of heat transmitted per square foot per degree of difference of temperature per hour was,

$$1130 \div 236 = 4.79 \text{ units.}$$

This is more than three times as much as was found to be transmitted in the still air of a room.

Condensation of Vapours in Pipes or Tubes by Water.

The condensation of vapours by the application of cold water or air, is in principle the same as the heating of water or air by steam; and the same proportions for condensing surface, when steam is to be condensed, are applicable in the two cases.

The surface-condenser of a steam-engine is a case in point. To educe the constant quantity of heat transmitted per unit of surface, temperature, and time, close analysis of the indicator-diagram would be required, to follow exactly the variations of pressure and temperature of the condensing steam. From the investigations of M. Audenet,[1] of the action of the surface-condensers on board the transport ship *Dives*, it appears that 500 English units of heat were transmitted per square foot per 1° F. difference of temperature per hour. These condensers were arranged in three groups

[1] *Proceedings of the Institution of Civil Engineers*, vol. xxxix., 1874–75, p. 399.

of tubes, successively traversed by the water. For the condensers, arranged in two groups, on board the *Rochambeau*, the constant was only from 220 to 240 English units.

A valuable series of experiments on the surface-condensation of steam was made, in 1875, by Mr. B. G. Nichol, at the Ouseburn Engine Works, Newcastle.[1] A brass tube, ¾ inch in diameter outside, and No. 18 wire-gauge in thickness, was inclosed in an iron pipe 3¾ inches in diameter outside, ¼ inch thick, and 5 feet 5⅛ inches long between the ends. The brass tube exposed an external condensing surface of 1.0656 square feet. Steam was admitted into the pipe, and was condensed by cold water passed through the tube. The pipe was lagged with felt and wrapped with white spun yarn to a diameter of 4¾ inches. It was tested for the radiation of heat from its external surface, which had an area, including the ends, of 5.48 square feet; the inner tube having been sealed up during the test. It was found that steam of an average temperature of 245° F. was condensed in the pipe at the rate of 1.4375 lbs. per hour, equivalent to .262 lb. per square foot of surface. The heat transmitted was (total heat $1154+32$) $-245=941$ units per pound of steam condensed; and it was ($941 \times .262$) $=246.5$ units per square foot per hour. The external temperature in the workshop was 60° F.; the difference of internal and external temperatures was $245°-60°=195°$; thence the radiation per degree of difference of temperature was $246.5 \div 195 = 1.26$ units per square foot.

The temperature of the steam introduced for experiment into the pipe, was about 255° F., for a total pressure of 32.5 lbs. per square inch, and the initial temperature of the condensing water was 58°. Two series, of three experiments each, were made with the pipe in a vertical and in a horizontal position. The following are the principal results of the six experiments:—

	Vertical Position.			Horizontal Position.			
	1	2	3	4	5	6	
Steam condensed per square foot of tube per hour,—	52.32,	78.18,	84.34,	67.8,	104.6,	121.3	pounds.
Condensing water passed through tube per square foot per hour,—	659,	2272,	3184,	633,	2505,	3390	pounds.
Condensing water per pound of steam condensed,—	12.6,	29,	37.7,	9.3,	24,	27.9	pounds.
Velocity of water through the tube in feet per minute,—	81,	278,	390,	78,	307,	415	feet.
Final temperature of condensing water,—	140°,	93°.5,	85°,	165°,	101°,	94°.5	F.
Rise of temperature of condensing water,—	82°,	35°.5,	27°,	107°,	43°,	36°.5	F.

[1] An excellent account of these experiments was published in *Engineering*, of December 10, 1875, from which the principal data are derived for this notice.

	Vertical Position.			Horizontal Position.		
	1	2	3	4	5	6
Mean temperature of condensing water,—	99°,	75°.7,	71°.5,	111°.5,	79°.5,	76°.2 F.
Mean difference of temperature of steam and condensing water,—	156°,	179°.3,	184°.5,	141°.5,	173°.5,	177°.8 F.
Heat transmitted from steam, reckoned from its temperature, per square foot per hour,—	45,960,	68,670,	74,040,	59,650,	91,950,	106,700 units.
Heat transmitted from steam, reckoned from its temperature, per square foot per hour, per 1° F. difference of temperature,—	295,	383,	401,	422,	530,	600 units.
Heat absorbed by the water per square foot per hour,—	54,038,	80,656,	85,968,	67,731,	107,715,	123,735 units.
Heat absorbed by the water, per square foot per hour, per 1° F. difference of temperature,—	346,	449,	466,	479,	621,	696 units.

The condensing tube acted more efficiently in the horizontal position than in the vertical position: a result the reverse of what was found by M. Clément, condensing in air (p. 474). There is a large excess of heat as carried off by the water, above the heat as calculated from the quantity of steam condensed. In this calculation, it is assumed that the condensed steam left the pipe at the temperature of the steam; but very probably the water was reduced within the pipe more nearly to the temperature at which it was discharged—about 200° F.

It appears, further, that the efficiency of the condensing surface was very much increased by an increase of velocity of the water through the tube.

When other vapours, as those of alcohol, are to be condensed, it may be assumed for purposes of general comparison, that the weight of vapour that may be condensed per unit of surface, temperature, and time, will be inversely as the total heat of the vapour. The total heat of vaporized alcohol, by table No. 125, page 372, is 461.7 units, which is about $^4/_{10}$ths of that of steam at one atmosphere; and the relative weights of steam and alcoholic vapour, at this pressure, that may be condensed per unit of surface, temperature, and time, are as

461.7 to 1146.1, or as 1 to 2.5 nearly.

WARMING AND VENTILATION.

Ventilation.

Mr. Hood finds that in winter from 3½ to 5 cubic feet of air per head per minute are sufficient, under ordinary conditions, for the proper ventilation of apartments; and in summer, from 5 to 10 cubic feet per minute. With these proportions the wholesomeness and purity of the atmosphere are maintained.

These proportions agree with those deduced by M. Peclet; according to

his deductions, 3½ cubic feet of air per head per minute is the minimum that should be provided, in ordinary circumstances. When the ventilation takes place by numerous apertures from below upwards, from 4 to 6½ cubic feet maintains the air of the room sufficiently pure. In peculiar cases, as in hospitals, from 30 to 60 cubic feet of air per bed per minute are admitted.

Ventilation is produced by natural draft, or by artificial draft produced by mechanical means. The second method will be considered in a subsequent section. With respect to the first, the ascensional force is measured, as it is with hot water, page 484, by the difference in weight of two columns of air of the same height, the height being measured by the total difference of level between the inlets for warm air and the outlets into the atmosphere. The difference of weight is ascertained from the difference of the temperatures of the ascending warmer air and the external atmosphere, by the aid of table No. 115, page 351; or for intermediate temperatures, by the formulas (9), page 350, and (2), page 347. The reasoning that is applied to the question of the circulation of water-columns, page 485, is applicable to that of air-columns. Suffice it for the present to reproduce the following table, No. 163, by Mr. Hood, showing the rate of discharge through a ventilating opening one foot square, for various heights and differences of temperature, calculated by a rule like that for water at page 485; and subjected to a reduction of one-fourth the calculated quantities, to comprise the necessary corrections for the contaminations, chiefly carbonic acid, which go to increase the specific gravity of the current, for frictional resistance, and for the resistance of angular deviations:—

Table No. 163.—Air Discharged through a Ventilator per Square Foot of Opening, for Various Heights and Differences of Temperature.

Height of Ventilator from the Floor.	Excess of Temperature of the Room above that of the External Air, in Fahrenheit degrees.					
	5°	10°	15°	20°	25°	30°
feet.	cubic feet.	cubic feet.	cubic feet.	cubic feet.	cubic feet.	cubic feet.
10	116	164	200	235	260	284
15	142	202	245	284	318	348
20	164	232	285	330	368	404
25	184	260	318	368	410	450
30	201	284	347	403	450	493
35	218	306	376	436	486	531
40	235	329	403	465	518	570
45	248	348	427	493	551	605
50	260	367	450	518	579	635

The velocity of the draft having been found for any particular case, together with the quantity of air to be supplied per minute, the sectional area of the air passages, inlet and outlet, may be simply calculated from those data.

"In all methods of ventilation," says Mr. Hood, "it is advisable to make the aggregate area of the openings that admit the fresh air larger than the aggregate openings for the efflux of the vitiated air. This becomes necessary notwithstanding the increase of volume which takes place in the heated and vitiated air. If the opposite course be adopted, and the eduction-tubes be larger than the induction-tubes, then a counter-current takes place in the hot-air or ventilating tubes, and the cold air descends through them; but by making the induction-tubes numerous, and of a large total area, the velocity of the entering current is reduced, and unpleasant drafts are avoided. It is also expedient to divide the entering current as much as possible; for by so doing, it prevents the dangerous effects of cold draughts, when the entering current is colder than the air of the room; and when it is hotter than the air of the room it prevents the air from rising too rapidly towards the ceiling, and therefore distributes it more equally throughout the apartment. Provided the aggregate openings for the admission of cold air be not less in size than those for the emission of the heated air, the quantity of air which enters a room depends less upon the size or number of the openings which admit the fresh air than upon the size of those by which the vitiated air is carried off."

In very hot weather and with crowded assemblies, the draft is assisted in theatres and some other large buildings, by heating the air in the upper part of the ventilating tube, which materially accelerates the upward current, and increases the influx of fresh air. The heat of the large gasalier in the centre of the house near the ceilings of theatres is thus utilized for ventilation.

Another mode of accelerating the draft is to conduct the spent air into the lower part of a vertical shaft, where a furnace is maintained in active combustion, and a very hot column of air is maintained.

Ventilation of Mines by Heated Columns of Air.

Reserving for a subsequent section the consideration of mechanical ventilation, the ventilation of a mine by the assistance of a furnace placed at the bottom of the upcast shaft is effected by the heating of the ascending column of air and other gases discharged from the mine, just before entering the shaft, by burning fuel. The furnace should be as low down as possible, so as to afford the longest column of heated air that may be got, since the velocity of draft increases as the square root of the height of the column. The furnace should be so constructed that all the air from the mine should pass freely under and over the grate. The grate may be six feet in length from front to back, but only the first four feet of bar-surface are covered with fuel; and with air-space round the arch, the radiant heat of the furnace is economized. There is a great loss of heat by lateral conduction through the rock and the walls of the shaft. When shafts are dry and bricked throughout, a temperature of 200° F. is the greatest that can be had economically. Even in such shafts the loss of heat laterally often amounts to a fifth; and in shafts which are wet and unwalled, the loss amounts occasionally to four-fifths of the whole of the heat communicated. According to Mr. Mackworth, 100° F. should be a sufficiently high temperature for good ventilation; it is relatively economical, and does not do much injury to machinery. With a powerful furnace, and in the absence of obstructions,

the greatest velocity of the current is 30 feet per second; but when there is machinery in the shaft, the velocity seldom exceeds 10 feet per second. The first object in ventilation is to produce a slow perceptible motion in the whole of the air of the mine. At a velocity of 30 feet per minute, the flame of a candle is just perceptibly deflected. The air should not, if possible, be made to travel faster; for the resistance of the sides, and leakages, increase rapidly as the velocity is increased. The air should be heated uniformly, but slightly; and that it may not be impeded, the furnace-drift should be 5 or 10 fathoms in length, and should rise at an inclination of 1 in 4.

One pound of coal of average composition, when completely burned, is capable of raising, in round numbers, 600,000 cubic feet of air 1° F. in temperature. At Hetton Colliery, where there are three furnaces, of which one is 9 feet wide, and two are 8 feet wide, one pound of coal raises the temperature of 11,066 cubic feet of air 62° F., equivalent to the raising of $11{,}066 \times 62 = 686{,}092$ cubic feet 1° in temperature.

One of the best examples of furnace-ventilation is, or was, to be found at Morfa Colliery, South Wales. The furnace is 6 feet 2 inches wide, at the base of a shaft 10 feet in diameter, and 60 fathoms deep; it delivers 62,000 cubic feet of air per minute, raised to a temperature of 198° F. by the combustion of 5¼ lbs. of coal. The average temperature of the ascending column at a depth of 25 yards down the shaft was observed to be 188°, just before coals were charged on the grate; two minutes after charging the temperature was 196°; three minutes after charging, 196°; and eight minutes after, 191°. The "drag" or draft was 3½ lbs. per square foot, not including the shafts. The useful effect was, therefore,

$$\frac{62{,}000 \times 3.5}{33{,}000} = 6.58 \text{ horse-power,}$$

or 6½ horse-power, as estimated by Mr. Mackworth; from which he infers that 1¼ horse-power was obtained by one pound of coal per minute. This, reduced to the ordinary form for comparison, is equivalent, for (5.25 lbs. $\times 60 =$) 315 lbs. of coal consumed per hour, to 48 lbs. of coal per horse-power per hour. At Hetton Colliery it is found, by a similar calculation, that 40 lbs. of coal was consumed per horse-power per hour, in a shaft 150 fathoms deep.

It is stated that a consumption of one pound of coal per minute for furnace-ventilation is sufficient for a mine employing 300 men, in the hottest summer day.

In collieries at Wrexham, the waste-steam of the engine is employed to heat the air in the upcast shaft. A cage, consisting of 150 gas-pipes united at top and bottom by hollow cast-iron rings, is placed in the lower part of the shaft, or in the return drift, the exhaust steam is condensed in the cage, and a temperature of 80° F. is thereby maintained.[1]

Cooling Action of Window Glass.

Mr. Hood states that one square foot of window glass will cool 1.28 cubic feet of air (say at 62° F.) 1° F. per minute, or 76.8 cubic feet per

[1] The data contained in the above notice of the ventilation of mines are derived from a lecture by Mr. H. Mackworth, reported in the *Colliery Guardian*, in 1858.

hour, per degree of difference of temperatures of the internal and external air. One unit of heat will raise the temperature of 55½ cubic feet of air at 62° F. by 1° F., from which it follows that heat is transmitted through window-glass from the air of a room to the external air, at the rate of

$$\frac{76.8}{55.5} = 1.40 \text{ units,}$$

per square foot per degree of difference of temperature per hour.

The relative cooling influence of wind, or air in motion, on glass, was tested by exposing the bulb of a thermometer, which was raised to a maximum temperature of 120° F., to a current of air at 68°, moving at various velocities. The time required to cool the thermometer 20°, varied inversely as the square root of the velocity.

Heating Rooms by Hot Water.

The effect of hot water in heating air is a function of the respective specific heats.

The average specific heat of water between 32° and 212° F. is		1.005
The specific heat of air is		.2377
Ratio of densities of water and air at 62° F.	1 to	819.4
Ratio of the volumes of water and air raised 1° F. by equal quantities of heat (1 to 819.4 ÷ .2377).	1 to	3465.

From this it appears that one cubic foot of water will, by parting with 1° F. of heat, raise the temperature of 3465 cubic feet of air at 62° by 1° F.; or one unit of heat will raise 55½ cubic feet of air at 62° by 1° F.

Mr. Hood estimates, from experiments made by Tredgold, that the water contained in an iron pipe of 4 inches diameter internally and 4½ inches externally, loses 0.851° F. of heat per minute when the excess of its temperature is 125° F. above that of the surrounding air, and that one foot in length of the pipe will heat 222 cubic feet of air one degree per minute when the difference of temperature is 125° F. This estimate is too low, as it is based upon too high a value for the specific heat of air, namely, .2767. If the quantity be increased in the inverse ratio of the assumed and the actual specific heat of air, the volume of air raised 1° by one foot length of four-inch pipe, when the excess of temperature is 125° F., will be

$$222 \times \frac{.2767}{.2377} = 258 \text{ cubic feet.}$$

Assuming that the rate of cooling of a hot-water pipe is proportional to the excess of temperature, it would follow from the observation above recorded that when the temperature of the pipe is 147° F. above that of the air in the room, it falls 1° in a minute.

Let t = the temperature of the pipes, t' = the required temperature of the room, t'' = the temperature of the external air, V = the volume of air in cubic feet to be warmed per minute, and l = the length of the pipe in feet. Then, according to the preceding data,

$$l = \frac{125}{222}\frac{(t'-t'')\,V}{(t-t')};\ \text{or,}$$

$$l = .56\ V\,\frac{t'-t''}{t-t'} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

using Mr. Hood's divisor 222. But

$$l = .50\ V\,\frac{t'-t''}{t-t'} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

using the divisor 258. Whence the rule:—

RULE.—*To find the length of four-inch pipe required for heating the air in a building.* Multiply the volume of air in cubic feet to be warmed per minute, by the difference of temperature in the room and the external temperature, and by 0.56 (Mr. Hood), or by 0.50 (the author), and divide

Table No. 164.—LENGTH OF FOUR-INCH PIPE TO HEAT 1000 CUBIC FEET OF AIR PER MINUTE.

Temperature of the Pipe, 200° F.

EXTERNAL TEMPERATURE.	TEMPERATURE OF THE ROOM.									
	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
Fahrenheit.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
10°	126	150	174	200	229	259	292	328	367	409
12	119	142	166	192	220	251	283	318	357	399
14	112	135	159	184	212	242	274	309	347	388
16	105	127	151	176	204	233	265	300	337	378
18	98	120	143	168	195	225	256	290	328	368
20	91	112	135	160	187	216	247	281	318	358
22	83	105	128	152	179	207	238	271	308	347
24	76	97	120	144	170	199	229	262	298	337
26	69	90	112	136	162	190	220	253	288	327
28	61	82	104	128	154	181	211	243	279	317
30	54	75	97	120	145	173	202	234	269	307
32	47	67	89	112	137	164	193	225	259	296
34	40	60	81	104	129	155	184	215	249	286
36	32	52	73	96	120	147	175	206	239	276
38	25	45	66	88	112	138	166	196	230	266
40	18	37	58	80	104	129	157	187	220	255
42	10	30	50	72	95	121	148	178	210	245
44	3	22	42	64	87	112	139	168	200	235
46	—	15	34	56	79	103	130	159	190	225
48	—	7	27	48	70	95	121	150	181	214
50	—	—	19	40	62	86	112	140	171	204
52	—	—	11	32	54	77	103	131	161	194

the product by the difference of the internal temperature and that of the pipes. The quotient is the length of pipe in feet.

		Mr. Hood.	Author.
Note.—For three-inch pipes, use the multiplier		0.75, or	0.67.
For two-inch pipes,	do. do.	1.12, or	1.00.

The table No. 164, composed by Mr. Hood, shows the length of four-inch pipe required to heat 1000 cubic feet of air per minute, when the temperature of the pipe is 200° F.

Total Quantity of Air to be Warmed per Minute.—In habitable rooms the

Table No. 165.—Length of Four-inch Pipe required to Warm any Building.

Building.	Length of Pipe per 1000 cubic feet.	Temperature maintained.	Remarks.
	feet.	Fahrenheit.	
Churches and large public rooms	5	55°	In very cold weather. If the air is regularly changed, from 50 to 70 per cent. more pipe is required.
Dwelling-rooms	12	65	
Do.	14	70	
Halls, shops, waiting-rooms, &c.	10	55	
Do. do.	12	60	
Work-rooms, manufactories, &c.	6	50 to 55	
Do. do.	8	60	
Schools and lecture-rooms..................	6 to 7	55 to 58	
Drying-rooms for wet linen, &c.—When empty.................	150 to 180	120	
Do., when filled	,, ,,	80	
Drying-rooms for curing bacon, drying paper, leather, hides	20	70	
Greenhouses and conservatories	35	55	In coldest weather.
Graperies and stove-houses	45	65 to 70	Do., do.
Do. do.	50	70 to 75	Do., do.
Pineries, hot-houses, and cucumber pits.	55	80	

Note to Table.—The lengths of pipe are only suitable for buildings on the usual plan and of ordinary proportions.

total quantity is equal to from 3½ to 5 cubic feet per minute for each person, plus the equivalent of 1¼ cubic feet for each square foot of glass.

For conservatories, forcing-houses, and like buildings, the quantity of air to be warmed is 1¼ cubic feet per square foot of glass per minute. The radiation of heat from frames and sashes made of metal is as great as from glass. The surfaces of these are to be included in the calculation. For wood frames, deduct one-eighth from the gross area of surface.

Approximate Rules for the Length of Four-inch Pipe required to Warm any Building.—Rules are deduced by Mr. Hood from the results of experience, and they are generally useful in practice. The multipliers are collected in the table No. 165.

Proper Diameter of Pipe.—The four-inch pipe is of the best size for all horticultural purposes. For most other purposes, smaller pipes may generally be more advantageously employed.

Loss by Sinking Heating Pipes in Trenches.—When pipes are placed in trenches covered with grating, the loss of heat, as estimated by Mr. Hood, amounts to from 5 to 7 per cent., which passes into the ground.

Motive Power of Water in Circulation through Heating Pipes.—The ascensional force is measured by the difference in weight of the two columns of water of the same height, ascending and descending from and to the boiler. The difference of weight is ascertained from the difference of the average temperatures of the columns from which the respective densities are deduced by the aid of table No. 109, page 339.

The following table showing the difference of weight of two columns of water one foot high at various temperatures, which is calculated by Mr. Hood by Dr. Young's formula, and gives practically the same results as Rankine's formula, table No. 109, page 339.

Table No. 166.—Difference of Weight of Two Columns of Water, each One Foot High, at various Temperatures.

Assumed actual Temperatures from 170° to 190° F.

Difference of Temperature of the two Columns.	Diameter of Pipe.				Difference of weight per square inch.
	1 Inch.	2 Inches.	3 Inches.	4 Inches.	
Fahrenheit.	grains.	grains.	grains.	grains.	grains.
2°	1.5	6.3	14.3	25.4	2.028
4	3.1	12.7	28.8	51.1	4.068
6	4.7	19.1	43.3	76.7	6.108
8	6.4	25.6	57.9	102.5	8.160
10	8.0	32.0	72.3	128.1	10.200
12	9.6	38.5	87.0	154.1	12.264
14	11.2	45.0	101.7	180.0	14.328
16	12.8	51.4	116.3	205.9	16.392
18	14.4	57.9	131.0	231.9	18.456
20	16.1	64.5	145.7	258.0	20.532

The velocity of circulation is that of a falling body due to the difference of height of two columns of water of equal weights or pressures on the base, and it varies as the square root of the difference of height. The velocity may be found by the aid of table No. 85, page 280. The difference of height is proportional to the difference of volumes, table No. 109; and if the mean height be increased in the same proportion, the increase will be the height from which the velocity is to be calculated. For example, let the mean height be 10 feet, and the difference of average temperatures of the two columns 10° F., say between 170° and 180°. The respective volumes are as 1.0269 and 1.031, and

$$10 \text{ feet} \times \frac{1.031}{1.0269} = 10.04 \text{ feet.}$$

Then $10.04 - 10 = .04$ foot, the difference of height; and the velocity due to this height is 1.61 feet per second, or 96.6 feet per minute.

If the height be 20 feet, the difference is .08 foot, for which the velocity due is 136.20 feet per minute.

In practice, of course, the velocities due are not attained, nor, at least in the more complex forms, nearly attained. The actual velocities are, in some cases, not more than a half or even a ninth of the velocities due to gravity.

Quantity of Coal Required to Heat the Pipes.—Mr. Hood gives the following table, No. 167, showing the quantities of coal consumed in heating 100 feet of pipe for various differences of temperatures. These quantities are based on the results of experiments by Rumford and others in heating water with coal as fuel, and are no doubt approximately correct.

Table No. 167.—Coal Consumed per Hour to Heat 100 Feet of Pipe.

For given differences of temperature of the pipe and the air.

Diameter of Pipe.	Difference of Temperature of the Pipe and the Air in the Room in Fahrenheit Degrees.														
	150	145	140	135	130	125	120	115	110	105	100	95	90	85	80
inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
4	4.7	4.5	4.4	4.2	4.1	3.9	3.7	3.6	3.4	3.2	3.1	2.9	2.8	2.6	2.5
3	3.5	3.4	3.3	3.1	3.0	2.9	2.8	2.7	2.5	2.4	2.3	2.2	2.1	2.0	1.8
2	2.3	2.2	2.2	2.1	2.0	1.9	1.8	1.8	1.7	1.6	1.5	1.4	1.4	1.3	1.2
1	1.1	1.1	1.1	1.0	1.0	0.9	0.9	0.9	0.8	0.8	0.7	0.7	0.7	0.6	0.6

Boiler-Power.—One square foot of boiler-surface exposed to the direct action of the fire, or three square feet of flue-surface, will suffice, with good coal, for heating, in round numbers, 50 feet of pipe. Mr. Hood fixes the proportion at 40 feet of four-inch pipe for all purposes. The usual rate of combustion of coal is about 10 lbs. or 11 lbs. of coal per square foot of fire-grate, and at this rate, 20 square inches of grate suffice for heating 40 feet of four-inch pipe.

Four square feet of boiler-surface exposed to the direct action of a good fire are capable of evaporating one cubic foot of water per hour. The best form of boiler for heating purposes is shown in Fig. 126 annexed. It is

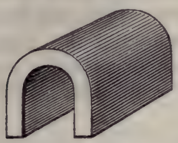


Fig. 126.—Boiler for heating purposes.

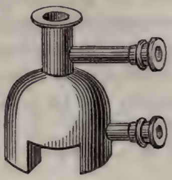


Fig. 127.—Boiler for heating purposes.

generally made of wrought-iron plates rivetted together. Another good form is shown in Fig. 127.

French Practice.—M. Claudel states that to warm a factory 13 metres wide by 3.25 metres high (43 feet by 10.5 feet), a single line of hot-water pipe 6¼ inches in diameter along the room appears to be sufficient, the temperature in the pipe being from 170° to 180° F. He adds that, in practice, the water being at 180° F., and the air at 60° F., making a difference of 120° F., it is convenient to reckon from 1.5 to 1.75 square feet of water-heated surface as equivalent to one square foot of steam-heated surface, and to allow from 8 to 9 square feet of hot-water pipe-surface per 1000 cubic feet of room.

M. Grouvelle affirms that four square feet of cast-iron pipe-surface, whether heated by steam or by water at 80° or 90° C., or 176° to 194° F., will warm 1000 cubic feet of workshop, maintaining a temperature of 60° F. Steam is condensed at the rate of 0.328 lb. per square foot per hour.

Perkins' System.—This system consists of the continuous circulation of water through endless wrought-iron tubes of ½-inch bore and 1 inch outside diameter, proved under a pressure of 200 atmospheres. The temperature of the water at the upper part of the circuit, varies from 300° to 400° F., corresponding to pressures of from 4½ to 15 atmospheres. The tubes become red-hot in the furnace. The length of tube in the furnace is a sixth of the total length of the circuit. Twenty feet of length are allowed for heating 1000 cubic feet of capacity. Taking the mean diameter ¾ inch, this gives four square feet of surface per 1000 cubic feet. Though the heater is apparently water-tight, the larger sizes are subject to a loss of about a pint of water in eight or ten days, which is restored by means of a force-pump.

M. Gaudillot, in France, manufactures heaters on this system with tubes of from 1.20 to 1.60 inches in external diameter. They support a pressure of 40 atmospheres very well.

Heating Rooms by Steam.

To find the length of pipe required for heating a room by steam, the temperature of the steam, which varies with the pressure, and may be found in table No. 128, page 387, is to be employed for the value of t in the formulas (1) and (1 a), page 482. The length of pipe required for heating by steam, is of course less than that required with water, as the temperature

is much higher. Taking a standard absolute pressure of steam of 20 lbs. per square inch, the temperature is 228°; and if the room is to be heated to 60°, the difference is 168°, and the formula (1 a), page 482, becomes

$$l = .50\ \mathrm{V}\frac{t'-t''}{168}, \text{ or}$$

$$l = \mathrm{V}\frac{t'-t''}{336} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

RULE.—*To find the length of four-inch pipe required for heating the air in a building by steam of 20 lbs. absolute pressure per square inch.* Multiply the volume of air in cubic feet to be warmed per minute, by the difference of the external and internal temperatures, and divide the product by 336. The quotient is the length of pipe in feet.

Note.—For three-inch pipes use the divisor............252
For two-inch pipes „ „ 168
For one-inch pipes „ „ 84

The boiler for a steam-heating apparatus should be capable of evaporating as much water per hour as the pipes would condense in the same time. Mr. Hood recommends that six square feet of direct surface of boiler should be provided to evaporate a cubic foot per hour. Now, adopting the mean weight of steam of 20 lbs. absolute pressure condensed per square foot of pipe per degree of difference of temperature per hour, namely .00235 lb., the quantity of pipe-surface that would form a cubic foot of condensed water per hour, taking the weight of this volume of water at 62.4 lbs., would be, per 1° difference of temperature,

$$62.4 \div .00235 = 26,550 \text{ square feet.}$$

For a difference of 168° the required surface would be

$$26,550 \div 168° = 158 \text{ square feet, say } 160 \text{ square feet.}$$

Four square feet of direct boiler-surface, or its equivalent of flue-surface, should, therefore, be provided for every 160 square feet of steam-pipe containing steam of 20 lbs. absolute pressure per square inch, and maintaining a temperature of 60° F. in a room.

The following lengths of pipe are required to present 160 square feet of surface:—

	Length for 1 square foot.	Length for 160 square feet.
4-inch pipe, ¼ inch thick,	............10.2 inches,	136 feet.
3 „ „ „	............13.0 „	173 „
2 „ „ „	............18.3 „	244 „
1 „ ⅛ „	............36.6 „	488 „

French practice.—According to M. Grouvelle, one square metre of pipe-surface, heated by steam, sufficed to heat and maintain at 15° C., or say 60° F., a room with ordinary proportions of walls and windows, such as a library or an office, of from 66 to 70 cubic metres of capacity, or a workshop of from 90 to 100 cubic metres. If the workshop is to be maintained at a high temperature, a square metre of surface is allowed for 70 cubic metres. The Exchange at Paris is sufficiently heated by one square metre

for 67 cubic metres. The allowance of one square metre for 70 cubic metres is equivalent to 4.35 square feet per 1000 cubic feet of capacity; or to 5.11 lineal feet of four-inch pipe per 1000 feet.

For heating workshops, 8 metres wide by 3 metres high, having 260 square feet of section, with a window-surface one-sixth of the total surface, engineers in France allow an iron pipe of 16 inches in circumference, or 5 inches in diameter, passing once through the shop, presenting 1.33 square feet of surface per foot run, or 5.2 square feet per 1000 cubic feet, the same as has just been calculated.

According to the observations of M. Peclet on steam-heating apparatus, particularly in a large factory, for a maximum difference of 36° F. between the interior and exterior temperatures, it was necessary to reckon on a delivery of 26 units of heat per hour per square foot of wall of 13 or 14 inches in thickness, and 30 units of heat per square foot of glass.

Heating by Ordinary Open Fires and Chimneys.

M. Claudel says that the quantity of heat radiated into an apartment from a fireplace is about one-fourth of the total heat radiated by the combustible. The heat radiated into an apartment from wood when burned amounts to only 6 or 7 per cent. of the total heat of combustion. For coal and for coke, the heat thus utilized amounts to about 13 per cent.

In burning wood, ordinary chimneys draw about 1600 cubic feet of air per pound of fuel; and better constructed chimneys about 1000 cubic feet. A sectional area of from 50 to 60 square inches is sufficient for the chimneys of ordinary apartments. For apartments designed to hold a great number of persons, a section of 400 square inches, say 32 by 13 inches, is usually employed.

From experiment it appears that the proportions of fuel required to heat an apartment are as 100 for ordinary fire-places, 63 for metal stoves, and from 13 to 16 for apparatus similar to stoves, with open fires.

Heating by Hot Air and Stoves.

Sylvester's cockle-stove is constructed of wrought-iron, ¼ inch thick, formed with an arch and two sides, closed at the ends, through one of which the furnace-mouth is made. The furnace is formed of fire-brick within the case, and the products of combustion are drawn off by flues below the furnace. The case is inclosed in fire-brick, with about 5 inches clear space for the circulation of the air to be heated. The air is introduced through the brickwork at the lower part of the sides, through numerous iron tubes, which are laid to within an inch clear of the sides of the case, and cause the fresh air to impinge upon the heated surface. The air thus brought in passes over the entire surface of the cockle into the upper part of the envelope, whence it is led away through any required number of pipes to the different rooms to be warmed. The ends of these exit pipes are placed within an inch

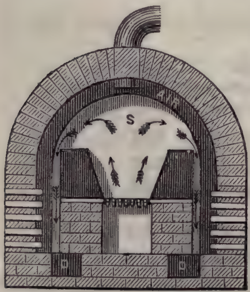


Fig. 128.—Sylvester's Cockle-stove.

above the top of the case. One of these cockle-stoves is illustrated by Fig. 128; the wrought-iron case is 5 feet square by 5 feet high. There are from 150 to 200 air pipes 2 inches in diameter, or 2 inches square, at the sides. The grate contains about 5 square feet of area, and the flues at the bottom are 9 by 6 inches.

From the results of Mr. Sylvester's experiments with a smaller cockle-stove, it was found that with a consumption of 5 lbs. of coal per hour, and a heating surface of 17 square feet, the temperature of 344,600 cubic feet of air was raised 56° F. in twelve hours, with 60 lbs. of coal; being equivalent to the heating of 95,000 cubic feet of air 1° F. per square foot of surface per hour; or to the heating of 321,626 cubic feet of air 1° F. by one pound of coal. It thus appears that each square foot of cockle-surface is equal to 7 square feet of hot-water pipe.

French Practice.—From results obtained by M. Peclet, it is ascertained that when the flue-pipes of stoves, conveying hot products of combustion, heat directly the air of a room, the quantities of heat passed off per square foot per hour for 1° F. difference of temperature, vary according to the material of the pipe as follows:—

Cast-iron,..............................	3.65 units of heat.
Sheet-iron,..............................	1.45 ,, ,,
Terra-cotta, 0.4 inch thick,........	1.42 ,, ,,

It may be noted that the great difference here observable between cast and wrought iron in passing heat from a flue to the outer air, does not exist when the pipe is occupied by steam or hot water. If an excess of temperature equal to 800° F. be assumed, as between the inside and outside of the pipe, the quantities of heat given off per square foot per hour would be,

For cast-iron,.............	3.65 × 450 = 1642 units of heat.
For sheet-iron,...........	1.45 × 450 = 652 ,, ,,
For terra-cotta,...	1.42 × 450 = 639 ,, ,,

Yet, in practice, the same surface is allowed for cast and for sheet iron; at the rate of one square foot for 328 cubic feet of space to be heated. The diameters of stove-pipes vary from 4 to 8 inches.

The air thus heated receives a degree of humidity from a vase full of water placed on the stove. The water so dissipated amounts to a little more than 2½ pints per day for a room of from 2500 to 3000 cubic feet of capacity.

House-stoves placed in the Room to be Warmed.—M. Claudel says that inside stoves are employed in schools and hospital-wards; they consist of an upright column, square or cylindrical, from 5 to 7 feet high, inclosing the furnace; surmounted by a pipe which rises vertically, and is then carried nearly horizontally through the apartment to a chimney. The column is inclosed in an outer casing of sheet iron or brickwork, with an interspace into which the external air is admitted, and from the upper part of which the air passes into the room. The temperature of the furnace does not exceed from 1100° to 1300° F. In practice, it is found convenient to assume that the products of combustion leave the stove at a temperature of about 950° F., or 500° C., that they are completely cooled in their course, that the temperature of the room is 60° F., and that the quantity of

heat emitted is the same as if the pipe had an average temperature of 480° F., or 250° C. A heating surface of from 20 to 30 square feet is allowed per pound of coal burned per hour, not reckoning the surface of the stove. Large grates are preferred, with slow combustion.

House-stoves placed Outside the Room to be Heated.—The useful effect of these stoves may be taken at from 60 to 70 per cent. of the heating power of the fuel. The surface of grate should be 15 square inches per pound of coal consumed per hour. The heating surface is two square metres per kilogramme of coal, or per two kilogrammes of wood: equivalent to 10 square feet per pound of coal per hour. From 2½ to 3½ pints of water are consumed per 1000 cubic metres; or 1 pint for from 1000 to 1400 cubic feet of space.

The spent air of the room is passed off into a chimney.

HEATING OF WATER BY STEAM IN DIRECT CONTACT.

The heating of water by steam, when the elements are brought into direct contact, is practically instantaneous. The author made experiments on this subject by admitting steam at 90 lbs. effective pressure from a locomotive-boiler into a body of cold water contained in a cylindrical reservoir, 3 feet 6 inches in diameter, and 15 feet long, made of ½-inch iron plate, having a total capacity of 144 cubic feet. The steam was conveyed from the boiler to the reservoir by a 1-inch pipe, from which it was freely discharged into a 2-inch iron pipe, open at the extremity, laid in the water along the bottom of the reservoir. The reservoir lay horizontally, without any covering, in a factory. Fifty-five and a half cubic feet, or 3464 lbs. of cold water at 60°, were delivered into the reservoir, and the water was heated by the steam blown into it to a pressure of 85 lbs. effective per square inch, in two hours, with a temperature of 328° F., or through $328 - 60 = 268°$ F. The quantity of heat communicated in two hours was, therefore, $3464 \times 268 = 928,352$ units, at the rate of 464,176 units per hour. Taking the initial temperature of the steam of the boiler, 331° F., and the mean temperature of the heated water $\frac{328 + 60}{2} = 194°$; the mean difference of temperature was $331 - 194 = 137°$, and the quantity of heat communicated per 1° F. of difference per hour was,

$$\frac{464176}{137} = 3388 \text{ units of heat.}$$

To communicate the whole of this quantity of heat through the surface of a pipe at the rate of 300 units per foot per 1° F. per hour, there would have been required $3388 \div 300 = 11.3$ square feet of surface. It is probable, as a matter of fact, that, though the 2-inch pipe was open to the water at the end, the most of the steam was condensed within the pipe before it could reach the end. The surface of the pipe had about 8 square feet of area.

There was, of course, a loss of heat by radiation from the surface of the reservoir; but it is not material to the purpose of this notice.

EVAPORATION (SPONTANEOUS) IN OPEN AIR.

So-called "spontaneous" evaporation from water exposed to air proceeds at all temperatures, when the conditions are suitable. The total rate of evaporation is in proportion to the extent of the surface exposed to the air. An increase of the temperature of the liquid is attended by an increase of the rate of evaporation, though not in direct proportion. The rate of evaporation is greater when the air is in motion over the surface of the water than when it is at rest. The rate of evaporation is also greater in proportion as the air is dryer, or the less the moisture previously existing in the air; and on the contrary, when the air is saturated with moisture, the evaporation is reduced to nothing.

When the atmosphere is perfectly dry, the rapidity of evaporation is proportional to the pressure of the vapour due to the temperature of the water, for which reference may be made to tables No. 127, page 386, and No. 130, page 396. This law was discovered by Dr. Dalton, who gives the following illustration:—

At the temperatures	212°,	180°,	164°,	152°,	144°,	138°,	
The pressures are	30,	15,	10,	7½,	6,	5	inches of mercury;

And the weights of water evaporated at these temperatures are proportional to 30, 15, 10, 7½, 6, 5.

But the atmosphere impedes the diffusion, and, consequently, the generation of vapour; although, ultimately, the full charge of saturated vapour due to the temperature is absorbed by it. When vapour is present in the air, which it usually is to a greater or less degree, the pressure of this vapour is to be deducted from that of the vapour due to the temperature of the water; and the residual force is the active "evaporating force." Dr. Dalton found that with the same evaporating force, thus determined, the same rapidity of evaporation is maintained, whatever be the temperature of the air.

But when a current of air blows over the surface of the water, the rapidity of evaporation is greater than when the air is still, because the air in motion sweeps away the vapour as it rises, and a continuous supply of comparatively dry air is secured. With the same evaporating force, a strong wind will double the production of vapour, compared with the quantity produced in a still atmosphere.

Dr. Dalton's experiments were made with an evaporating surface of 6 inches in diameter, in still air and in wind, and he gives a table of the rates of evaporation in grains per minute, for temperatures up to 85° F., on the assumption that the air is perfectly dry.[1] The following table, No. 168, is calculated to show the rate of evaporation, in pounds per square foot per hour, extended up to 212° F., and for three states of the air:—when still, when there is a gentle wind, and when there is a brisk wind. The pressures are given in inches of mercury, and are those adopted by Dr. Dalton, which, for the purpose of the table, do not materially vary from those given in table No. 127.

[1] *Memoirs of the Literary and Philosophical Society of Manchester*, vol. v. p. 579.

Table No. 168.—"Spontaneous" Evaporation of Water in Still Air and in Wind, Assuming the Air to be Perfectly Dry, for Temperatures from 32° to 212° F.

(Founded on Dr. Dalton's tables.)

Temperature of the water.	Pressure of the vapour.	Water evaporated per square foot of surface per hour.			Temperature of the water.	Pressure of the vapour.	Water evaporated per square foot of surface per hour.		
		Air still.	Gentle wind.	Brisk wind.			Air still.	Gentle wind.	Brisk wind.
Fahr.	inches of mercury.	lbs.	lbs.	lbs.	Fahr.	inches of mercury.	lbs.	lbs.	lbs.
32°	.200	.0349	.0448	.0550	125°	3.79	.6619	.8494	1.043
35	.221	.0386	.0495	.0608	130	4.34	.7580	.9727	1.194
40	.263	.0459	.0589	.0723	135	5.00	.8730	1.121	1.376
45	.316	.0552	.0708	.0869	140	5.74	1.003	1.286	1.579
50	.375	.0655	.0841	.1032	145	6.53	1.140	1.463	1.796
55	.443	.0774	.0993	.1218	150	7.42	1.296	1.663	2.043
60	.524	.0917	.1175	.1441	155	8.40	1.467	1.882	2.310
62	.560	.0979	.1255	.1540	160	9.46	1.652	2.120	2.602
65	.616	.1076	.1381	.1694	165	10.68	1.865	2.394	2.938
70	.721	.1257	.1616	.1983	170	12.13	2.118	2.719	3.336
75	.851	.1486	.1907	.2341	175	13.62	2.378	3.053	3.746
80	1.000	.1746	.2241	.2751	180	15.15	2.646	3.395	4.167
85	1.17	.2043	.2622	.3218	185	17.00	2.969	3.810	4.676
90	1.36	.2375	.3048	.3745	190	19.00	3.318	4.258	5.226
95	1.58	.2760	.3541	.4346	195	21.22	3.706	4.758	5.837
100	1.86	.3248	.4169	.5116	200	23.64	4.128	5.298	6.502
105	2.18	.3807	.4886	.5996	205	26.13	4.563	5.856	7.187
110	2.53	.4418	.5670	.6959	210	28.84	5.034	6.464	7.933
115	2.92	.5100	.6544	.8030	212	30.00	5.239	6.724	8.252
120	3.33	.5815	.7463	.9160					

It appears from the table that the rates of evaporation, for each of the three conditions of the air, when perfectly dry, are in simple proportion to the pressure of the steam; and, as affected by the stillness or the motion of the air, they are—

for still air,	a gentle wind,	a brisk wind,
as 1,	1.28,	1.57.

It is to be understood that the temperature 212° is a limiting temperature, which cannot be actually reached without displacing the air entirely.

It is also to be remarked that, though Dr. Dalton lays down the proposition that the rapidity of the evaporation is the same, whatever may be the temperature of the air, yet it is clear that water is evaporated more rapidly when a warm current blows over it than when it is traversed by a cold current. Such increase of evaporation is probably the result of the reflex action of heat imparted by the air to the superficial water, the "evaporative

force" of which is increased by the rise of temperature due to the heat abstracted from the air. The cooling of air by passing it over or through water is a well-known expedient. In India, the air of apartments is cooled by passing it, as it enters, through and over the "tatta," a bamboo frame or trellis, over which water is suffered to trickle.

From what has been stated, with respect to "mixtures of gases and vapours," page 392, it appears that the condition of "saturation," attributed to the mixture of vapour and air, properly belongs to the vapour itself, as vapour, when it has arrived at its maximum density and pressure for the temperature of the air.

Use of the Table No. 168.—Dr. Dalton gives the solution of the problems based upon the original tables, of which the first is here rendered into a rule in relation with the table No. 168.

RULE. *To find the quantity of water exposed to air that would be evaporated per square foot of surface per hour, at a given temperature of air, with a given dew-point.* Subtract the tabulated weight of water corresponding to the dew-point from the weight corresponding to the temperature of the air; the remainder is the weight of water that would be evaporated per square foot of surface per hour.

The weights of water are to be selected from the 3d, 4th, or 5th columns, according to the state of the wind.

To find the dew-point, Dr. Dalton used a very thin glass vessel, into which he poured cold water, of which he noted the temperature. If the vapour in the atmosphere was instantly condensed on the glass, he changed the water for warmer water, and so proceeded until he ascertained the proper temperature—the dew-point—when he could just perceive a slight dew deposited on the glass. The dew-point may be found with much greater precision by means of hygrometers, described at page 393.

Dr. Pole has constructed an empirical formula which roughly represents the results of Dr. Dalton's experiments. Let T = the temperature of the atmosphere in degrees Fahr., t = the dew-point, V = the velocity of the wind in miles per hour, E = the rate of evaporation in inches per day from a water surface, and A = a numerical coefficient. Then,

$$E = \frac{T^2 - t^2}{A(100 - V)} \qquad (3)$$

The value of A = 80 for high or summer temperatures, and A = 100 for low or winter temperatures. Dr. Pole remarks that Dalton's tables do not provide for cases where the temperature of the water differs materially from that of the air; and that, probably, in such cases, T should be made to represent the temperature of the water-surface, and not that of the air.[1]

DESICCATION.

The drying of wet or moist materials, by means of currents of air, is based on the principles already announced which regulate the evaporation of water from the surface. If a current of air be saturated with moisture

[1] *Minutes of Proceedings of the Institution of Civil Engineers*, vol. xxxix. page 36.

or vapour, its efficiency for drying out moisture from bodies with which it comes in contact, is exactly nothing. To act as a dryer, in other words, to assist in evaporating and carrying off moisture, it must be either perfectly dry, or, at the least, sub-saturated; and inasmuch as its capacity, in the conventional language already explained, for absorbing moisture—in the state of vapour, of course—increases with its temperature, it is obvious that the higher the temperature of the air, the greater is its efficiency. If, then, the air-current be surcharged with heat, it stimulates evaporation in two forms—by imparting a portion of its heat to the wet or moist surface, which is utilized in the evaporation of the moisture, and by tolerating the presence of a greater quantity of moisture in mixture with it, which is carried away as it rises from the surface by the current.

The drying, or vaporization of moisture, by such means, involves, of course, a lowering of the temperature of the air, or the moist body, or both; and the problem arises: What is the initial temperature of dry air required?

The first problem for solution is twofold:—Given the final temperature at which the saturated mixture is to be discharged, what is the quantity of dry air required for a given weight of vapour in saturated mixture? and to what initial temperature is the air required to be raised in order to supply heat for the evaporation of the given weight of steam? The answer is to be found in the sub-section on the "Properties of Saturated Mixtures of Air and Aqueous Vapour," with table No. 130, page 394.

But, in ordinary practice, the artificially heated air-current does not arrive at the condition of saturation before it is discharged; and a large surplus of air is therefore to be provided, the proportional amount of which varies with the circumstances under which the current is applied. The standard of perfect efficiency is presented in the table No. 130. M. Peclet notices a process employed by M. Montgolfier for drying the skins of grapes after having been pressed, by means of a forced current of air. It was found that, in autumn, 5340 cubic feet of air, moving at a velocity of about 16 feet per second, were required for the evaporation of one pound of water from the pressed grapes. Let the initial temperature of the air be assumed at 64° F., then, by the table No. 130, a volume of dry air equal to 2526 cubic feet would have sufficed to evaporate one pound of water; but if, as is probable, the air had already been loaded with half the quantity of moisture it could carry, then at least double the tabular quantity would have been necessary, or $2526 \times 2 = 5052$ cubic feet, which is nearly equal to the quantity actually employed.

In the design of a drying-chamber, it is of the first importance that the air-current should be admitted at the highest point of the chamber and discharged at the level of the floor. The reverse process, of admitting it at or near the floor, and discharging it at the upper part, is vicious practice. In the latter case the circulation is imperfect, for the hot air seeks the most direct route to the points of egress; in the former case the hot air is uniformly distributed, and if the points of discharge are properly placed, the descending current is applied equally over the area of the chamber. In a drying-chamber, noticed by M. Peclet, for drying vermicelli, at Saint Ouen, having a capacity of upwards of 6000 cubic feet, and heated by air of from 86° to 104° F., the following were the results of heating by ascending and by descending currents:—

Discharge of Current.	Vermicelli Produced. First Quality.	Second Quality.	Fermented.
Above (mean of 5 trials)......	540 lbs.	400 lbs.	4.5 lbs.
Below " "	3,200 "	143 "	86.0 "

These results prove decisively the superiority of the descending current. The consumption of fuel with the descending current was also much the less.

Drying-house for Calico (Peclet).—In a drying-house used by M. René Duvoir, the pieces of calico were suspended from bars ranged horizontally across the upper part of the house. Air heated to 250° F. was admitted through a number of openings from a brick flue at the floor, regulated by dampers, from which it rose to the upper part, and thence descended to the floor, where it was discharged. The external temperature was 77° F., and the temperature of the discharged current in the chimney was 100° F. In six hours, 150 pieces of calico, holding 2490 lbs. of water, were dried, with a consumption of 706 lbs. of coal, corresponding to an evaporation of 3.52 lbs. of water per pound of coal. The quantity of air heated to 250° F., for this duty, was 1,943,000 cubic feet, the weight of which, at the rate of 13.52 cubic feet to the pound (by Rule 9, page 350), was 143,713 lbs. Then $143713 \times .2377 = 34160$ units of heat for 1° F. elevation of temperature, and for $250 - 77 = 173°$, the total elevation of temperature, the total heat consumed was $34160 \times 173° = 5,909,250$ units, being at the rate of

$$\frac{5,909,250}{706} = 8370 \text{ units per pound of coal.}$$

The water evaporated per pound of coal was 3.52 lbs., for which 1067 units of heat, reckoned from 77° F., were absorbed per pound of water; and for 3.52 lbs.,

$$1067 \times 3.52 = 3756 \text{ units of heat,}$$

was the quantity of heat utilized for evaporation per pound of coal, being 45 per cent. of the total heat communicated to the air.

The temperature at which the air was discharged being 100°, it was 23° above the external temperature, or the loss by the excess was $\frac{23}{173} \times 100 = 13$ per cent. of the whole heat communicated to the air.

The distribution of the heat communicated to the air was, therefore, approximately as follows:—

In evaporating moisture,........................	45	per cent.
Carried off by the air, ...	13	"
Loss by radiation and conduction,...........	42	"
	100	

It is easy to show that the air when discharged was not nearly saturated. At 100° F., the temperature of discharge, the proportions of moisture and air in one pound of a saturated mixture, by table No. 130, are as .283 to 6.641, or as 1 to 23.5. In 143,713 lbs. of air, therefore, when in a state of saturation, there would have been

$$143,713 \div 23.5 = 6,115 \text{ pounds of moisture.}$$

But there was only 2490 lbs. of moisture in the air, or about two-fifths of the proportion for saturation.

The single good feature in this drying-house, is the extraction of the spent current at the level of the floor. No provision was made to effect the distribution of the heat uniformly through the room; and there can be no doubt that the condition of sub-saturation of the air was, for the most part, the result of the absence of such provision.

Drying Linen.—The maximum evaporative performance of coal in drying linen, does not exceed an evaporation of 3 lbs. of water per pound of fuel; and it is sometimes as low as 1.36 lbs.

Drying Various Stuffs.—According to M. Rouget de Lisle, in one pound of wet cloth, after having been wrung or pressed, or passed through the hydro-extractor, there remained the respective quantities of water as follows:—

	Water Left in One Pound of			
	Flannel.	Calico.	Silk.	Linen.
When twisted,..................	2.00 lbs.	1.0 lb.	.95 lb.	.75 lb.
When pressed,..................	1.00	.6	.5	.40
When passed through the hydro-extractor,..........	.60	.35	.3	.25

In these instances, the centrifugal machine was 26 inches in diameter, and made from 500 to 600 turns per minute.

M. Penot made experiments on drying-houses at Mulhouse, and found that one pound of coal evaporated from 1.02 to 2.86 lbs. of water: the latter under favourable circumstances.

According to M. Royer, in a drying-house 31½ feet long, 26 feet wide, and 62½ feet high, the heating surface of the stove amounted to 758 square feet, with a consumption of 55 lbs. of coal per hour. There were during these trials, lasting fifteen days each, evaporated successively 2.37, 2.53, and 2.18 lbs. of water per pound of coal.

Mr. J. R. Napier, in drying stuffs by air heated to 240° F., with a descending draft, evaporated 3 lbs. of water per pound of coal.

Drying Stuffs by Contact with Heated Metallic Surfaces.—M. Clément applied a piece of calico, weighing 2½ lbs., holding an equal weight of water, to a plate of copper of the same extent, heated by steam at 212° F.; and it was dried in one minute. The evaporation was effected at the rate of 1.42 lbs. of water per square foot per hour.

When stuffs are dried by passing them over cast-iron cylinders, heated by steam internally, it appears from experiments made by M. Royer, that in drying calico which held its weight of water, 74 lbs. of water were evaporated by the condensation of 102 lbs. of steam. In other experiments made with a machine of six cylinders, the efficiency in drying was only two-thirds of that attained in the first-described experiment. The experiments were made in winter in a place which was imperfectly closed.

Drying Grain.—It is reported in the *Engineer*, that Messrs. Crighton & Co., Abo, dried 450 lbs. of grain, extracting 15 per cent. of its weight, or 67½ lbs. of water, by the consumption of 18 lbs. of birchwood, being at the rate of 3.75 lbs. of water per pound of wood.

Drying Wood.—In the forges of Lippitzbach, Carinthia, according to M. Leplay, wood is piled and dried in close chambers by burning a part of

the wood, averaging a fourth of the total quantity. The furnaces are below the floor, between which and the furnaces a space is provided for the circulation of the products of combustion under the floor. Air in considerable quantity is admitted to and mixed with the products of combustion to moderate their temperature to 350° F.; when the current passes into the upper chamber amongst the wood to be dried. On this system, there is considerable loss of heat by radiation and by the excessive dilution of the products of combustion with air.

At the Neuberg factory, the products of combustion circulate in a species of stove constructed of thin masonry, and pass thence through cast-iron pipes by which the air is heated for drying the wood. On this system the wood consumed does not exceed an eighth of the total quantity.

The limit of temperature at which wood should be dried ought not to exceed 340° or 350° F. M. Leplay states that the wood to be dried contains 40 per cent. of water; whence it appears that one pound of the fresh wood evaporates 1.20 lbs. of water in the first of the above-described processes; and in the second process, 2.80 lbs. of water.

In a system of drying-furnace recently adopted in France for wood, peat, &c., a chamber 62 feet long and 14 feet wide is employed. The wood, in billets, is loaded into waggons, having a capacity of about 100 cubic feet each, on rails. Each waggon-load successively is introduced at one end and withdrawn at the other, whilst the mixture of hot gases and air is introduced at the other end, and passes to the end at which the waggons are introduced. A temperature of 270° F. is maintained at the middle of the chamber. The maximum temperature is 320° F. The wood remains sixty-four hours in the chamber, and the usual quantity of moisture it contains, from 20 to 25 per cent., is evaporated by the combustion of 1½ cords of wood for every 16 cords to be dried; that is, 1½ lbs. are burned to dry 16 lbs., evaporating 4 lbs. of water; being at the rate of 2.66 lbs. of water per pound of fresh wood.

HEATING OF SOLIDS.

Cupola Furnace.—M. Peclet estimates that, in melting pig iron in an ordinary cupola, by the combustion of 30 per cent. of its weight of coke, 14 per cent. only of the heat of combustion is actually utilized. This estimate is based on the result of an experiment by Clement, showing that to heat and melt 1 pound of pig iron, 504 English units of heat are necessary.

Plaster Ovens.—He also states that to dry plaster, the heat of combustion of 7 per cent. of its weight in wood is absorbed, whereas the actual consumption of wood amounts to from 9 to 14 per cent.,—showing that from 50 to 80 per cent. of the total heat generated is utilized.

Metallurgical Furnaces.—Dr. Siemens states that, in an ordinary reheating furnace, employed in metallurgical operations, one ton of coal is consumed in heating 1⅔ tons of wrought iron to the welding point, 2700° F.; whilst he estimates, in terms of the specific heat of iron, .114, and the heating power of coal, 14,000 units of heat, that a ton of coal is capable of heating up 39 tons of iron. From this it appears that only 4¼ per cent. of the whole heat generated is appropriated by the iron. Similarly, he estimates that

barely 1½ per cent. of the whole heat generated is utilized in melting pot-steel, in ordinary furnaces; whilst, in his regenerative furnaces, a ton of steel is melted by the combustion of 12 cwts. of small coal, showing that 6 per cent. of the heat produced is utilized.

Blast-Furnace.—Mr. J. Lothian Bell[1] has formed detailed estimates of the appropriation of the heat of Durham coke in the Cleveland blast-furnaces; from which the following abstract has been prepared:—

Durham coke, it is assumed, consists of 92.5 per cent. of carbon, 2.5 per cent. of water, and 5 per cent. of ash and sulphur. To produce 1 ton of pig-iron, there are required 11 cwts. of limestone, and 49 cwts. of calcined iron-stone; the iron-stone consists of 18.6 cwts. of iron, 9 cwts. of oxygen, and 21.4 cwts. of earths. There is formed 7.26 cwts. of slag, of which 1.1 cwt. is formed with the ash of the coke, and 6.16 cwts. with the limestone. There are 21.4 cwts. of earths from the iron-stone, less .74 cwt. of bases taken up by the pig-iron and dissipated in fume; say, 20.66 cwts. Total of slag and earths, 27.92 cwts.

Mr. Bell assumes that 30.4 per cent. of the carbon of the fuel, which escapes in a gaseous form, is carbonic acid; and that, therefore, only 51.27 per cent. of the heating power of the fuel is developed, and the remaining 48.73 per cent. leaves the tunnel-head undeveloped. He adopts, as a unit of heat, the heat required to raise the temperature of 112 lbs. of water 1° Centigrade.

Distribution of the heat generated in the blast-furnace for the production of 1 ton of pig-iron:—

		UNITS.	PER CENT.
Evaporation of water in coke, and chemical action, in smelting,		48,354	54.1
Fusion of pig-iron,		6,600	7.4
Fusion of slag,		15,356	17.2
Expansion of blast,		3,700	4.1
Appropriated for the direct work of the furnace,		74,010	82.8
Loss by radiation through the walls,	3,600		4.0
Carried away by tuyere-water,	1,800		2.0
Sensible heat of gaseous products,	10,000		11.2
Waste,		15,400	17.2
Total heat generated in the furnace,		89,410	100.0

The undeveloped heat of the fuel amounts proportionally to $89{,}410 \times \frac{48.73}{51.27} = 84{,}980$ units. Add to this, the sensible heat of the gaseous products, 10,000 units, and the sum, 94,980 units, is disposed of as follows:—

[1] *The Journal of the Iron and Steel Institute*, 1872, 1875. The abstract given in the text affords but a meagre notion of the variety and extent of Mr. J. Lothian Bell's investigations, the value and importance of which are highly and justly appreciated by manufacturers of iron.

Distribution of the waste and undeveloped heat of the fuel required for the production of 1 *ton of pig-iron.*

		UNITS.	PER CENT.
Generation of steam for blast-engine and various pumps connected with the work,........................		28,080	29.6
Heating the blast to 905° F.,..............................		11,920	12.5
Appropriated for direct work,.........................		40,000	42.1
Loss by radiation from the gas tubes, ...	3320		3.5
Loss of heat escaping by the chimneys, ... (temperature, 770° F., from boilers) (Do. 640° F., from stoves)	21,660		22.8
Radiation at boilers and stoves, 25 per cent.,	16,240		17.1
Waste, ...		41,220	43.4
Loss of gases from blast-furnaces, in charging, 5 per cent.,		4,740	5.0
Sundry,..		9,020	9.5
Total waste and undeveloped heat,.....		94,980	100.0

For the performance of the duty according to these analyses, Mr. Bell states that 19.08 cwts. of carbon, or 20.62 cwts. of coke, are required, per ton of iron produced from ore yielding 41 per cent. of iron. In a furnace having 18,000 cubic feet of capacity, 80 feet high, 1 ton of No. 3 pig-iron was produced with 21½ cwts. of ordinary Durham coke, from Cleveland iron-stone.

In recent years, by raising the temperature of the blast to 485° C., or 905° F., the consumption of coke, with a furnace 48 feet high, was reduced to 28 cwts. per ton of iron. With a cold blast, more than 60 cwts. would probably have been required.

It is stated, that at Barrow works, where the Siemens-Cowper regenerative stove is employed for heating the blast to 1100° F., the quantity of coke consumed is 20.08 cwts. per ton of iron.

THE STRENGTH OF MATERIALS.

The strength of materials is measured by the resistance which they oppose to alteration of form, and ultimately to rupture, when subjected to force, pressure, load, stress, or strain. The exigencies of scientific precision have caused the general substitution of the word "stress" for the good old engineer's word "strain," as expressive of force, though "strain" may still be employed to express alteration of form.[1]

Stress is applied in five recognized modes:—

1st. *Tensile stress*, tending to draw or pull the body asunder. The immediate effect is *elongation.*

2d. *Compressive stress*, tending to crush it. The immediate effect is *compression.*

3d. *Shearing stress*, tending to cut it through. The immediate effect is *lateral compression, elongation, and deflection.*

4th. *Transverse or lateral stress*, tending to bend it and break it across, the force being applied laterally, and acting with leverage. The immediate effect is *lateral deflection.*

5th. *Torsional stress*, tending to twist it asunder, the force acting with leverage. The immediate effect is *angular deflection.*

Mr. Callcott Reilly aptly reduces the varieties of stress to three kinds of simple stress:—Tensile stress, compressive stress, and shearing stress. These are the ultimate forms of stress; they are combined in transverse stress, and the third is substantially the form of torsional stress, where the strain is applied over a very short length. Or, where torsional stress is applied over a considerable length, the tensile form of stress is combined with shearing stress.

When stress is applied gradually to a solid body, the *strain*, or alteration of form, is proportional to the intensity of the stress, so long as the inherent elastic force of the body is not overbalanced by the stress—so long, that is to say, as the alteration of form remains within the *elastic limit*, the stress, at the same time, remaining within the limit of *elastic strength.* When the elastic limit is turned and exceeded, the body begins to *yield* under gradually accumulating stress, and the strain or alteration of form becomes proportionally greater and greater with the intensity of the stress, until, finally, *rupture* or *breakage* takes place.

[1] As Dr. Pole says, in his lectures on *Iron as a Material of Construction*, this word *strain* "appears to convey its idea so clearly that there must be little chance of expunging it from the practical mechanic's vocabulary." Mr. Stoney employs it exclusively in his work on *The Theory of Strains.* Mr. Kirkaldy uses the word "stress" in his reports of his experiments on the strength of materials.

When a body is loaded in excess of the elastic limit, without breakage, it returns, when unloaded, towards its normal form, but it fails to regain it. It is, in so far, deformed, and it has acquired a *permanent set* or a *set.*

There are five data of importance to be observed in the measurement of the strength of materials.

1st. The limit of elasticity, or the *elastic limit.*

2d. The greatest stress which the material is capable of sustaining within the elastic limit, or the *elastic strength.*

3d. The *strain,* or alteration of form—elongation, compression, deflection, or torsion—within the elastic limit.

4th. The total extent of the *strain,* or alteration of form, with the *set,* before rupture takes place.

5th. The greatest stress which the material is capable of supporting before rupture takes place; or, the *absolute strength.*

The first and second data are of prime importance; the others are subsidiary. For, in practice, it is necessary, in order to insure the permanency of a structure, that its proportions should be such that, under the maximum stress to which any piece is to be subjected, it should not be strained beyond the elastic limit of its strength.

WORK OF RESISTANCE OF MATERIAL.

Under a Quiescent Load.—Since the intensity of the elastic resistance increases uniformly with the total space through which the action of the stress takes effect, it may be represented by the triangular space ABC, Fig. 129, in which AB is the total space described, and BC is the measure of the stress applied. Suppose the stress BC = 10,000 lbs., and the space AB = 1 inch, then the stress, 10,000 lbs., which has been applied, operates through a space of 1 inch, and has been opposed by an elastic resistance which commenced at A, and increased uniformly, from 0 at A, to 10,000 lbs. at B. The intensity of the resistance at different points along the space AB, is measured by the ordinates of the triangle parallel to the base through the given points; and if the space be divided into four parts, for example, at the points a, b, c, the values of the ordinates $a\,a'$, $b\,b'$, $c\,c'$, or the intensities of the resistance at the points of elongation a, b, c, are respectively 2500, 5000, and 7500 lbs. If an indefinitely great number of ordinates be drawn, they will occupy the whole area of the triangle, and the average length of the ordinates will be half the base BC, equivalent to 5000 lbs.

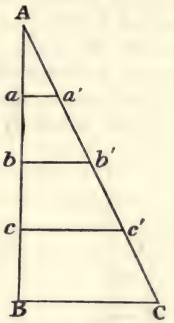


Fig. 129.—Deflection under a Quiescent Load.

Hence a ready means of calculating the quantity of work necessary to strain a piece to its elastic limit:—Multiply half the elastic strength in pounds by the space in feet described by the resistance or by the stress. The product is the work expended. If R = the elastic strength in pounds,

s = the space described by the load or stress in feet, and w = the work done, the rule is formulated thus:—

$$w = \tfrac{1}{2}\, R\, s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

For example, using the above data, if 10,000 be the elastic strength, and 1 inch or .0833 foot be the space described, then the work done in straining the piece to the limit of its elastic strength is

$$\tfrac{1}{2}\,(10{,}000 \times .0833) = 416.7 \text{ foot-pounds.}$$

Under a Load suddenly applied.—In these calculations of stress and work, it is assumed that the stress is applied gradually, so that no appreciable velocity and momentum be generated as the stress is applied. If, on the contrary, a weight equal to the total load be applied suddenly and all together, the momentary deflection under the load amounts to twice the permanent deflection, or twice that which is effected by the load when gradually applied, supposing that the total deflection does not exceed the elastic limit. Let A B, Fig. 130, be the deflection caused by the gradual application of a weight W, and A B′ the momentary deflection caused by the sudden application of it. Draw the ordinates B C and B′C′ to measure the resistance at the points B and B′, and complete the triangle A B′C′. Through C draw the vertical D D′. Then the rectangle A B′ D′ D measures the work done by the load in falling through the height A B′, and the triangle A B′C′ measures the work of resistance to deflection. These are equal to each other; and as B′C′ must be twice B′D′, so A B′ is twice A B; that is to say, the momentary deflection under a load suddenly applied is twice the steady deflection under the same load very gradually applied.

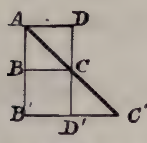


Fig. 130.—Deflection under a Sudden Load.

It follows that in proportion to the rapidity with which loads are applied, as when railway trains run upon a bridge, of course in the absence of percussive action, the deflection is greater than that due to the same load at rest on the bridge, and increases with the speed of transit. But it does not amount to twice the deflection due to a quiescent load, though it approaches to this limit as the speed increases.

Under Stress by Percussion.—When a solid material is exposed to percussive stress, as, for instance, when a heavy weight falls upon a beam transversely, the work of resistance is measured by the product of the weight by the total fall—the total fall being equal to the height of fall above the beam plus the deflection. To exemplify percussive action within the elastic limit, reproduce Fig. 130 on the same scale, in thick lines, in Fig. 131, with the same letters of reference, and let A′A be the height of fall above the beam, and B′ B″ the additional deflection under the weight. A′B″ is the total fall, and A B″ the total deflection. Draw the horizontal line B″C″, and produce A C′ and D D′ to meet it at C″ and D″. Then the rectangle A′ B″ D″ D‴ measures the work for the total fall, and the triangle A B″C″ measures the work of resistance to deflection; and these quantities of work are equal to each other. The scale of the diagram above the normal level of the beam, A D,

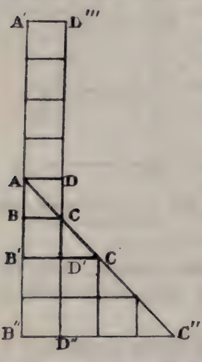


Fig. 131.—Deflection under Stress by Percussion.

is the same as that of the portion below A D, for the sake of simplicity of illustration, and for ready comparison of the squares representing quantities of work.

COEFFICIENT OF ELASTICITY.

The elasticity of a bar of any solid material subjected to a direct tensile or a direct compressive force, within the elastic limits, is measured by a constant fraction of the length per unit of force per unit of sectional area. The unit of force and area is usually taken as one pound per square inch, but it is sometimes taken as one ton per square inch. E is used to symbolize the denominator of the fraction. For example, if a bar of iron be extended $^1/_{12,000}$th part of its length per ton of stress per square inch of section,

$$\frac{1}{12,000}=\frac{1}{E}.$$

The bar would therefore be stretched to double its normal length by a force of 12,000 tons per square inch, if the material were perfectly elastic. The supposition, though imaginary, is convenient; and the coefficient of elasticity is usually defined as the weight which would stretch a perfectly elastic bar of uniform section to double its length. It is represented by E, which may be employed to express pounds, tons, or any other measure of weight.

The coefficient of elasticity may also be expressed in terms of the length in feet of a bar of the given material, the weight of which would be equal to the force required to stretch it to twice its normal length. For example, a 1-inch square bar of iron weighing $3\frac{1}{3}$ pounds per lineal foot would require to be $(12,000 \times 2240 \div 3\frac{1}{3} =)$ 8,064,000 feet in length to stretch it at the upper end to twice the normal length, and this is another expression for the coefficient of elasticity.

The same methods of expressing the coefficient of elasticity are applied to the elastic resistance to compression. That is, the coefficient, in weight, is expressed by the denomination of the fraction of its length by which a bar is compressed per unit of weight per square inch of section.

TRANSVERSE STRENGTH OF HOMOGENEOUS BEAMS.

Tensile resistance is selected as the basis of the following formulas, which are constructed on the assumption that, within elastic limits, extension is equal to compression, under equal stresses; and their strict application is confined to the calculation of stress, strain, and strength, within elastic limits. At the same time, they are practically applicable for calculating ultimate strength.

1. SYMMETRICAL SOLID BEAMS.

Let a homogeneous beam, A B, Fig. 132, of rectangular section, be freely supported horizontally at both ends, at *a* and *b*. Bisect the depth *c d* at *o*, and draw the horizontal line *n o p*. Let the beam be loaded by the weight W, applied at the middle, *c d*, of the beam. Then the beam is deflected under the load, and the upper half, above the line *n o p*, is compressed, and the lower half is extended, in such a manner that, having regard to the vertical section *c d*, the compression and elongation respectively increase uniformly from zero at the central point, *o*, to a maximum at the upper and lower surfaces, *c* and *d*. The proportional increase may be

represented by the triangles *o e f* and *o g h*, formed by the lines *e h* and *f g*, intersecting at the central point *o*; in which the graduated shortening and lengthening of the fibres at any given height are represented by horizontal lines drawn across the triangles.

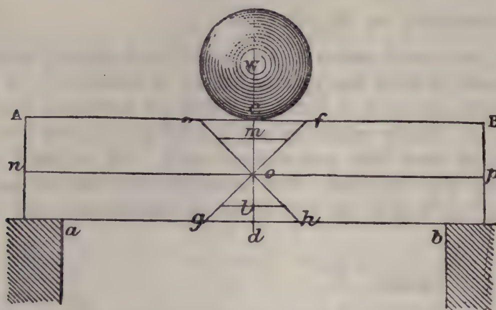


Fig. 132.—Transverse Stress on a Rectangular Beam.

The horizontal compressive stress in the upper half, and the tensile stress in the lower half of the beam, with respect to the section *c d*, likewise increase uniformly from the central point *o*, where they are zero, to the upper and lower surfaces at *c* and *d*. Such is the ordinary theory of transverse stress in a rectangular beam, and it is assumed that, throughout the whole length of the beam, as at the section at the middle, there is no horizontal stress in the central line *n o p*, with respect to any vertical section. This line is therefore called the *neutral line* or *neutral axis;* and it is a line of demarcation between the directly horizontal compressive stress above and the tensile stress below.

Further, the sum of the compressive stress above the neutral axis is equal to the sum of the tensile stress below, and each may be replaced by its resultant stress at the resultant centre without affecting the equilibrium. If the cross section of the beam, Fig. 133, No. 1, be divided into a number of strips of equal thickness, the moment of stress for each strip in the upper and in the lower group, with respect to the neutral axis *n o p*, may be calculated, and the sum of the moments for each group divided by the sum of the

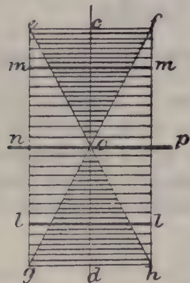


Fig. 133, No. 1.—Longitudinal Resistance in Loaded Beams.

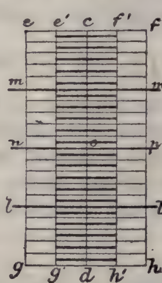


Fig. 133, No. 2.—Diagonal Resistance in Loaded Beams.

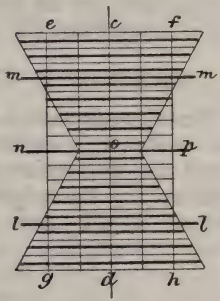


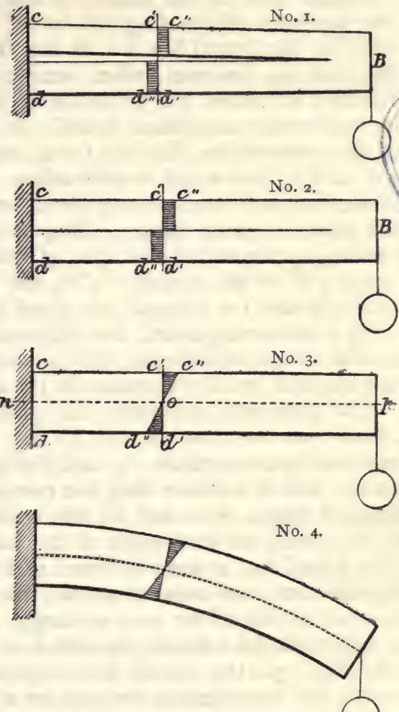
Fig. 133, No. 3.—Combined Resistance in Loaded Beams.

resistances, when the quotient will be the resultant radius. But this calculation may be saved by drawing the diagonals *e h* and *f g*, when it is apparent that the shaded triangles formed by them exhibit the relative quantities of stress for each strip, and that the resultant lines of stress, *m m* and *l l*, pass through the centres of gravity of the triangles, each at a distance from the neutral axis equal to two-thirds of the half-depth of the beam. (Fig. 133, 1.)

These, the moments of the normal stresses or resistances due to the abso-

lute horizontal compression and extension of the beam, are supplemented by diagonal resistances, by which each of them is augmented 75 per cent.

To elucidate the origin of diagonal resistance, it may be observed that the upper and lower portions of a beam—above and below the neutral axis—may be considered as two individual members of a frame, united at their surface of contact—the neutral axis. Let *c* B *d*, Figs. 134, No. 1, be a triangular frame, fixed at *cd* and loaded at B. The pieces *c c′* and *d d′* of the upper and lower members are respectively extended and compressed, when the load is applied, to the lengths *c c″* and *d d″*. If the members of the frame are placed parallel to each other, in close contact, as in Figs. 134, No. 2, extension and compression take place as before. Let, now, the two members be united in the line *n o p*, No. 3, and so consolidated as to form a semi-beam; the extension and the compression partially neutralize each other:—at the neutral line *n o p* they are absolutely neutralized, and the amounts of extension and of compression are represented by the triangles *c′ c″ o* and *d′ d″ o*. The structure is thus, in a certain sense, crippled; and the extension and compression, instead of being rectilineal, are curvilineal, and the semi-beam is deflected, as in No. 4.



Figs. 134.—Stress in a Loaded Beam.

The counteraction here pointed out is necessarily exerted diagonally, at an angle of 45° with the neutral line; as in the line *o′ c′*, Fig. 135, at 45° with the transverse section *c′ d′*. The diagonal forces, as applied to the

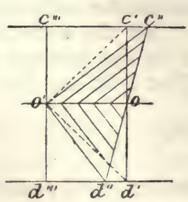


Fig. 135.—Diagonal Stress in a Beam.

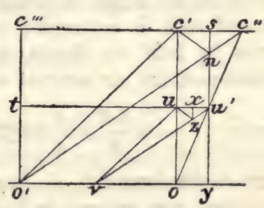


Fig. 136.—Diagonal Stress in a Beam.

transverse section *c′ d′* strained into the position *c″ d″*, are represented by diagonals at 45° drawn from points in the upper half-depth *c″ o* to the neutral line *o o′*, for tensile resistance; and from points in the lower half-

depth $o d''$, for compressive resistance. Reproduce the upper half-depth to a larger scale, in Fig. 136. Draw the perpendicular $c' n$, and thence the perpendicular $n s$; the diagonal extension is measured by $n c''$, and the horizontal component $s c''$ is the horizontal extension at the upper side of the beam, due to the length of the portion $c' c'''$. These extensions are also measures of the diagonal force, and its horizontal component. Take next the horizontal line $t u$, at any other position in the half-depth. The diagonal $v u$ becomes, when strained, $v u'$, at 45°, and, by the same construction as before, the extension of the diagonal is measured by $z u'$, whilst its horizontal component is $x u'$. It is easily deduced from the similarity of the construction, that the forces measured by the horizontal components $x u'$ and $s c''$ are equal to each other. Similarly, the horizontal components of the diagonal forces acting throughout the whole depth of the section $c' o$, are equal to each other. They may, therefore, be represented in their entirety by the rectangle $c' s y o$, of which the length $c' s$ is equal to half the length $c' c''$ of the triangle $c' c'' o$; and the areas of force represented by the rectangle and the triangle, are equal to each other.

By a similar argument, the diagonal compressive resistance in the lower section of the semi-beam, may be analyzed. It is the simple converse of the diagonal tensile resistance in the upper section.

The horizontal resistance due to diagonal stress, is represented diagrammatically, on the cross section, by Fig. 133, No. 2, in which the shaded area inclosed by the verticals $e' g'$ and $f' h'$ represents, in its upper half, the tensile stress; and in its lower half, the compressive stress; for which the resultant lines of stress, $m m$ and $l l$, are each at a distance from the neutral axis equal to half the half-depth of the beam. The two elements of resistance (No. 1 and No. 2) are combined in Fig. 133, No. 3, showing in deep shading the combined areas of stress of uniform intensity; the amount of which is equal to that of the semi-rectangle. This investigation for a semi-beam is applicable for a beam supported at the ends, and loaded at the middle, like Fig. 132, the tensile and compressive stresses being inverted. The tensile and compressive stresses act at a resultant radius, measured from the neutral line, of $\left(\left(\frac{2}{3}+\frac{1}{2}\right)\div 2 =\right)\frac{7}{12}$ or .5833, taking the half-depth as 1. As .5775 is the geometrical radius of gyration of the semi-rectangle on the neutral axis, when the half-depth = 1 (see page 289), the moment of resistance will, for simplicity, be taken as .5775, when the area of the semi-rectangle and the half-depth are each represented by unity. Then the total moment of either resistance, tensile or compressive, with reference to the neutral axis, is expressed by the product of half the sectional area of the beam by half the depth, and by .5775, and by the extreme tensile or compressive stress per unit of sectional area. That is to say, by,

$$\frac{b d}{2} \times \frac{d}{2} \times .5775 \times s = .1444\, b d^2 s; \quad \ldots\ldots\ldots\ldots\ldots \quad (a)$$

in which b = the breadth, d = the depth, both in inches; and s = the extreme tensile stress per square inch, to which the extreme compressive stress is taken as equal. The sum of the moments of the tensile and the compressive resistances is, therefore, practically twice the moment (a) round the neutral axis; or

$$.1444\, b d^2 s \times 2 = .2888\, b d^2 s \quad \ldots\ldots\ldots\ldots\ldots \quad (b)$$

When the beam is loaded to the point of rupture, the extreme tensile stress in the lower surface of the beam becomes equal to the ultimate strength of the material.[1]

To express the moment of the load or weight W:—Each of the supports *a* and *b* carries half the weight, and presses upwards with a force equal to $\frac{W}{2}$; the leverage of each of these pressures on the section of the beam at the middle, is half the span *l*, equal to *a d* or *d b*, and is $\frac{l}{2}$; therefore the moment of the weight at the middle is expressed by—

$$\frac{W}{2} \times \frac{l}{2} = \frac{W\,l}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (c)$$

The moment of the weight (*c*) is necessarily equal to the moment of resistance (*b*); or, $\frac{W\,l}{4} = .2888\; b\,d^2 s$, and $W\,l = 1.155\; b\,d^2 s$; whence,

$$W = \frac{1.155\; b\,d^2 s}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$s = \frac{W\,l}{1.155\; b\,d^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

That is to say, *when the beam is supported at both ends, and the weight is applied at the centre*, the *breaking weight* is equal to the product of the breadth by the square of the depth, and by the ultimate tensile strength per square inch, and by 1.155; divided by the span.

Also, *the ultimate tensile strength per square inch* is equal to the product of the breaking weight by the span; divided by the product of the breadth by the square of the depth, and by 1.155.

The formula (1) signifies that the breaking weight varies directly as the breadth of the beam, directly as the square of the depth, directly as the tensile strength of the material, and inversely as the span.

When the beam is fixed at both ends and loaded at the middle, the breaking weight is equal to 2 times that of a beam freely supported, or

$$W = \frac{1.733\; b\,d^2 s}{l}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

When the beam is fixed at one end only, and loaded at the other end, the breaking weight is one-fourth of that of a beam freely supported at both ends; or

$$W = \frac{.289\; b\,d^2 s}{l}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

When the load is applied at any other point than the middle of a beam freely suspended at both ends, let *m* and *n* denote the two segments into which the

[1] In the intervention of the diagonal stress in deflected rectangular beams, according to the analysis in the text, is found the solution of the mystery of the "resistance of flexure," which has been so denominated, and has been experimentally demonstrated by Mr. W. H. Barlow. The contrast between the action of the diagonal bracing of lattice-girders and that of the solid web of web-girders, throws a flood of light on the recondite strains in webs and in solid beams.

length is divided by the load. Then the breaking weight is inversely proportional to the product of the segments, $m \times n$. At the middle, $m n = \frac{l}{2} \times \frac{l}{2} = \frac{l^2}{4}$. Whence $W = \frac{1.155\, b d^2 s}{l} \times \frac{l^2}{4} \div m n$; or,

$$W = \frac{.289\, b d^2 l s}{m n} \quad \ldots\ldots\ldots\ldots (5)$$

When the weight is uniformly distributed along the beam, the total breaking weight is equal to twice the breaking weight as applied at the middle of a beam supported at both ends, or at the end of a beam fixed at the other end.

When the beam is supported at both ends and uniformly loaded,

$$W = \frac{2.31\, b d^2 s}{l} \quad \ldots\ldots\ldots\ldots (6)$$

When the beam is fixed at both ends and uniformly loaded,

$$W = \frac{3.466\, b d^2 s}{l}. \quad \ldots\ldots\ldots\ldots (7)$$

When the beam is fixed at one end only, and uniformly loaded,

$$W = \frac{.578\, b d^2 s}{l} \quad \ldots\ldots\ldots\ldots (8)$$

Generalized Formula for the Breaking Weight of Symmetrical Solid Beams.

Let the area of the section, $b d$, be represented in the expression (a), by a, and the radius of gyration by r, the half-depth of the beam being $= 1$; then, by substitution, the moment of resistance of the half-depth on the neutral axis, is expressed by $\frac{a}{2} \times \frac{d}{2} \times r \times s = \frac{a d r s}{4}$; and twice the moment is

$$\frac{a d r s}{4} \times 2 = \frac{a d r s}{2} \quad \ldots\ldots\ldots\ldots (d)$$

Then,

$$\frac{W l}{4} = \frac{a d r s}{2}; \text{ and } W l = 2\, a d r s; \quad \ldots\ldots\ldots\ldots (e)$$

whence the general formulas—

$$W = \frac{2\, a d r s}{l}; \quad \ldots\ldots\ldots\ldots (9)$$

$$s = \frac{l W}{2\, a d r}. \quad \ldots\ldots\ldots\ldots (10)$$

W = the breaking weight at the middle.
a = the sectional area of the beam, in square inches.
d = the extreme depth of the beam, in inches.
r = the radius of gyration, taking the half-depth equal to 1.
l = the length or span of the beam between the supports, in inches.
s = the ultimate tensile strength of the material, per square inch.

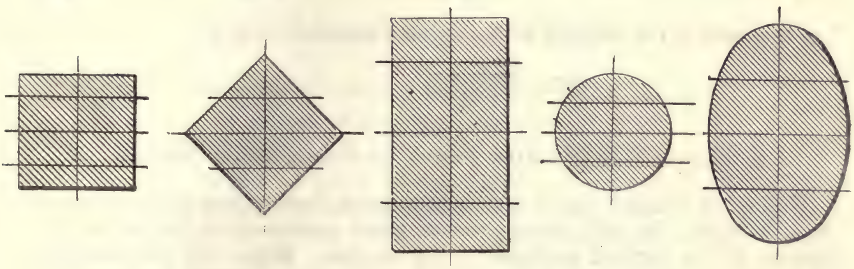
The weight and the tensile strength are to be expressed in terms of the same unit of weight, whether in pounds, hundredweights, or tons.

The formulas (9) and (10) are applicable to solid beams of any symmetrical form of section, no part of which is overhung, and for any weight and maximum tensile stress not exceeding the ultimate weight and stress for the material.

1. Solid Beams (without Overhang) of Symmetrical Section.

The sections of beams which may be noticed are shown by Figs. 137, in which the radius of gyration above and below the neutral axis as a centre, is marked by a horizontal line. The neutral axis passes through the centre of gravity of each section at the level of half the depth.

No. 1. No. 2. No. 3. No. 4. No. 5.



Figs. 137.—Symmetrical Solid Beams, without Overhang. Sections.

1. *Square and Rectangular Sections.*—Nos. 1 and 3. The radius of gyration is .5775 when the half-depth is 1; and substituting this value of r in formula (9), $W = \frac{2 \times .5775 \times a d s}{l}$; or,

$$W = \frac{1.155\, a d s}{l}. \quad \ldots\ldots\ldots\ldots \quad (11)$$

For square beams,

$$W = \frac{1.155\, d^3 s}{l}. \quad \ldots\ldots\ldots\ldots \quad (12)$$

2. *Square Section, with a Diagonal Vertical.*—No. 2. The radius of gyration is .4083 when the half-diagonal is 1; and, by substitution, in formula (9), and reduction,

$$W = \frac{.8166\, a d s}{l}. \quad \ldots\ldots\ldots\ldots \quad (13)$$

The diagonal is equal to 1.414 times the side; and, calling the side d', $d = 1.414\, d'$. Substitute this value in formula (13), and reduce; then,

$$W = \frac{1.155\, a d' s}{l}. \quad \ldots\ldots\ldots\ldots \quad (14)$$

This value of W is the same as that for an upright square section (11), showing that a square beam is equally strong whether placed upright or diagonally.

3. *Circular Section.*—No. 4. The radius of gyration is half the radius of the section, and, by substitution in formula (9), $W = \frac{2 \times .5 \times a d s}{l}$; or,

$$W = \frac{a d s}{l} = \frac{.7854\, d^3 s}{l}, \quad \ldots\ldots\ldots\ldots \quad (15)$$

in which d is the depth or the diameter. The transverse strength of circular sections varies as the cube of the diameter.

4. *Elliptical Section.*—No. 5. The area is .7854 time the product of the breadth by the depth, and the radius of gyration is half the half-depth. Substituting in formula (9), $W = \frac{2 \times .50 \times .7854\, b d \times d \times s}{l}$; or

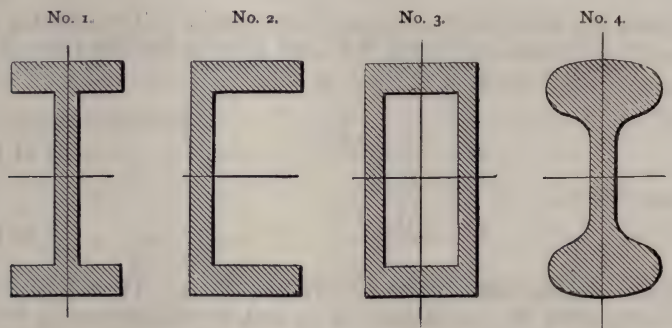
$$W = \frac{.7854\, b d^2 s}{l}. \quad \ldots\ldots\ldots\ldots \quad (16)$$

Let the ratio of the breadth to the depth $= c$, then $b = c d$, and

$$W = \frac{.7854\, c d^3 s}{l}. \quad \ldots\ldots\ldots\ldots \quad (17)$$

2. Flanged or Hollow Beams of Symmetrical Section.

Hollow or flanged beams may be generally described as beams of overhung section. In such beams the diagonal resistance to flexure is only excited in the vertical portions of the section. Figs. 138 are examples of overhung sections; and the neutral axis passes through the centre of gravity.



Figs. 138.—Symmetrical Flanged Beams. Sections.

1. *Hollow-rectangular or Double-flanged Sections.*—Nos. 1, 2, 3.

When the depth is considerable compared with the thickness of the flanges, calculate for the flanges and for the web separately. The separation of the web from the flanges is shown in Fig. 139, for the flanged beam, No. 1, and it is to be done in the same manner for the hollow beam, No. 3. The moment of resistance of one flange is sensibly equal to the product of its sectional area multiplied by the distance d'' between the centres of the flanges, and by the tensile strength per square inch; or to

$$b t d'' s = a d'' s, \quad \ldots\ldots\ldots\ldots \quad (f)$$

in which t is the depth or thickness of a flange, and a its sectional area.

The web is treated as a rectangular beam of the depth d'', the distance between the centres of the flanges. This is greater than the actual depth, as between the flanges; but the excess is compensated by the metal filled in at the angles. Putting t' for the thickness of the web, the moment of resistance is, by (b), page 506,

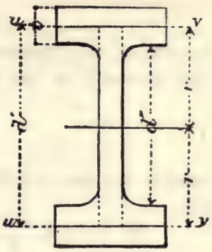


Fig. 139.—Calculation of Strength of Beam.

$$.2888\, t' d''^2 s = .2888\, a'' d'' s, \text{ .... } (g)$$

in which a'' is the sectional area of the web. The sum of the moments (f) and (g) is equal to the moment of the weight; or,

$$\frac{W\, l}{4} = a\, d'' s + .2888\, a'' d'' s; \text{ .... } (18)$$

whence, $W\, l = 4\, a\, d'' s + 1.155\, a'' d'' s = d'' s\,(4\, a + 1.155\, a'')$; and,

$$W = \frac{d'' s\,(4\, a + 1.155\, a'')}{l} \text{ .... } (19)$$

That is to say: *When the depth is considerable compared with the thickness of the flanges*—multiply the sectional area of one flange by 4; and multiply the sectional area of the web by 1.155. Add the products together, and multiply the sum by the reputed depth of the beam, and by the tensile strength per square inch, and divide the product by the span. The quotient is the breaking weight.

Note.—The reputed depth of the beam, and also that of the web, are taken, for calculation, as the total depth minus the thickness of one flange.

2*d Method.*—In some cases the strength of the flanges only is calculated, when the web is comparatively slight. Then, $\frac{W\, l}{4} = a\, d'' s$, or $W\, l = 4\, a\, d'' s$; and

$$W = \frac{4\, a\, d'' s}{l} = \frac{4\, b\, t\, d'' s}{l} \text{ .... } (20)$$

That is to say: *When the strength of the web is neglected*, the breaking weight is equal to four times the sectional area of one flange by the distance apart between the centres of the flanges, and by the ultimate tensile strength per square inch, divided by the length.

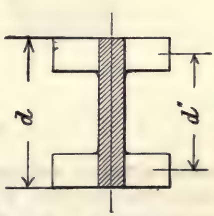


Fig. 140.—Calculation of Strength of Beam.

In applying the formula to the hollow beam, No. 3, Fig. 138, t is taken as the sum of the thicknesses of the sides.

When the thickness of the flanges is considerable compared with the depth of the beam, No. 4, Figs. 138, and Fig. 140.—In the double-flanged section, Fig. 140, calculate the strength of the web for the whole depth, as indicated in sectioning. For the lateral flange portions, the average stress is less than s in the ratio of the total depth d to the reputed depth d'', or it is $s\frac{d''}{d}$; and the net area of one flange, or of

the unshaded parts, Fig. 140, being put equal to a', the moment of resistance of one flange is,

$$a'\frac{d''}{d} \times d'' \times s = a'\frac{d''^2}{d}s \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (h)$$

The sum of the moments of resistance of the flanges and the web is equal to the moment of the weight, or

$$\frac{W\,l}{4} = a'\frac{d''^2}{d}s + .2888\,t'\,d^2\,s \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

in which t' = the thickness of the web, and d = the depth of the section. Then,

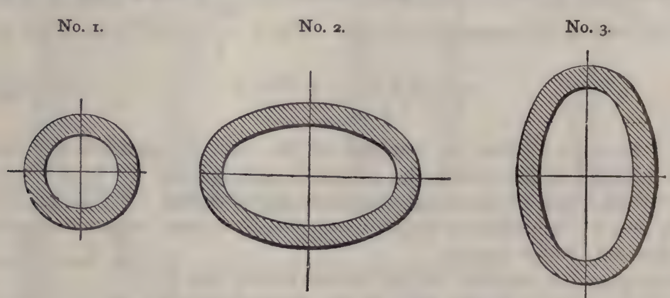
$$W\,l = 4\,a'\frac{d''^2}{d}s + 1.155\,t'\,d^2 s = s\left(4\,a'\frac{d''^2}{d} + 1.155\,t'\,d^2\right); \text{ and}$$

$$W = \frac{s\left(4\,a'\frac{d''^2}{d} + 1.155\,t'\,d^2\right)}{l} \ldots\ldots\ldots\ldots\ldots \quad (22)$$

That is to say: *When the thickness of the flanges is considerable compared with the depth of the beam.*—Multiply the net sectional area of one flange, calculated for its width minus the thickness of the web, by the square of the reputed depth, and by 4, and divide by the total depth. Multiply the thickness of the web by the square of the total depth, and by 1.155. Add together the quotient and the product, and multiply the sum by the tensile strength per square inch, and divide the product by the span. The quotient is the breaking weight.

Note.—The reputed depth is equal to the total depth minus the thickness of one flange. Double-headed rails, as No. 4, Fig. 138, will be specially treated.

2. *Annular Section*, Figs. 141, No. 1.—The hollowness of the section deprives it of a large proportion of the diagonal resistance exerted in the

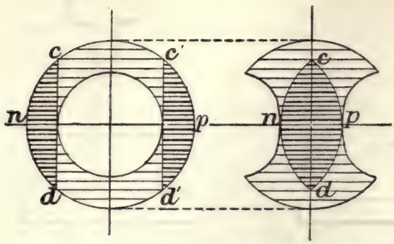


Figs. 141.—Symmetrical Hollow Beams. Sections.

solid circular section; otherwise the strength might have been calculated from the section, Fig. 143, in which the material of the annular section is collected about the vertical centre line.[1] The lateral portions of the

[1] This mode of aggregation of the section is employed in Mr. Edwin Clark's work on the Britannia Bridge, page 111; and also by Mr. Baker in his excellent work on the *Strength of Beams*, page 26. Mr. Baker very properly points out the fallacy of the ordinary mode of calculating the transverse strength of a beam of annular section, which does not take cognizance of the loss of "resistance to flexure" in a hollow beam.

section, *c n d*, *c′ p d′*, Fig 142, are by their position subject to diagonal stress, and they are reproduced in darker shading in Fig. 143. The breaking strength may be approximated to by, in the first place, reducing the breadth of the overhung portions, and calculating the strength of the reduced section, on the principle to be explained in treating of beams of unsymmetrical sections.



Figs. 142, 143.—Annular Section of Beams.

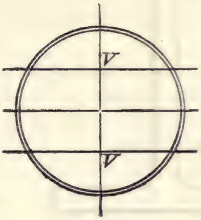


Fig. 144.—Thin Annular Section of Beam.

2*d Method.*—When the section is thin, as in Fig. 144, the matter of the section may be assumed to be collected at the outer circumference, for which the radius of gyration is .7071, when the radius of the section is 1; whence, rating the stress s exerted throughout the section as the maximum stress, the resultant point of resistance, V, of the half-section is $\frac{.7071^2}{1} = .50$, when the radius is 1; or the distance of the centres of resistance, V, V, is half the diameter. The sectional area is equal to the product of the circumference by the thickness, or to 3.14 $d \times t$; and the half-section $= 1.57\ dt$. The moment of the half-section is $1.57\ dt \times \frac{1}{2} d = .785\ d^2 t$, and $\frac{W\,l}{4} = .785\ d^2\ ts$; whence,

$$W = \frac{3.14\ d^2 t s}{l} \qquad (23)$$

Hollow Elliptical Sections.—These sections, Nos. 2 and 3, Fig. 141, page 512, may be treated on the same principle as the annular sections, No. 1, Figs. 141, and Fig. 143.

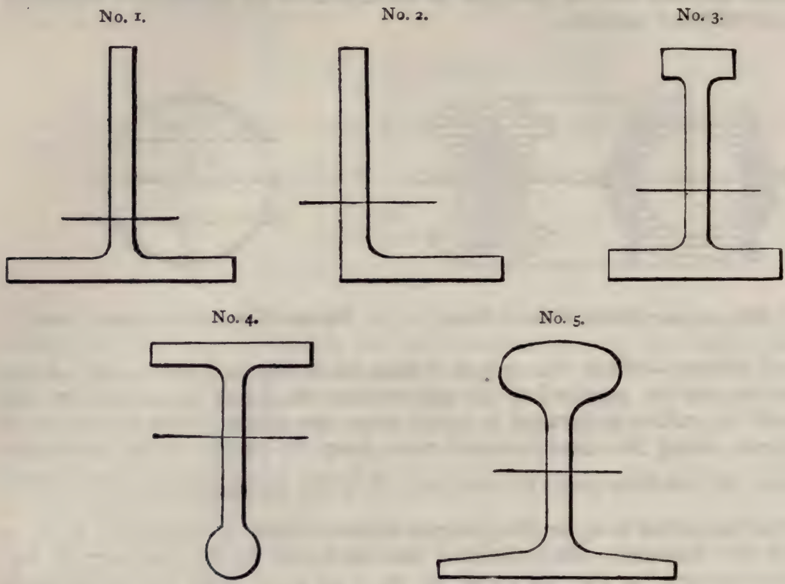
2*d Method.*—When the section is thin the breaking strength is, by adapting formula (23), putting b = the breadth, and d = the depth, $W = 3.14 \frac{b^2 + d^2}{2} ts$; or,

$$W = 1.57\ (b^2 + d^2)\ ts \qquad (24)$$

3. Flanged Beams which are not Symmetrical in Section.

For such beams, of which the sections, Figs. 145, are examples, it is necessary to ascertain the quantity of longitudinal tensile resistance, and the distance apart of the resultant centres of tensile and compressive stress, for a given section; and to multiply these together to obtain the moment of resistance of the section; whence the ultimate transverse strength may be calculated. The first operation is to find the neutral axis

of the section; and as the ultimate longitudinal resistance in the web is greater than that of a flange, the neutral axis does not pass through the centre of gravity of the section. But, if the area of the flange be reduced in proportion to the potential or ultimate unit-resistance in the web to that



Figs. 145.—Sections of Unsymmetrical Beams.

of the flange, or as 1.73 to 1, the neutral axis will pass through the centre of gravity of the reduced section.

RULE.—*To find the neutral axis of a beam of unsymmetrical section.* Divide the section, as reduced, into its simple elements, and assume a datum-line from which the moments of the elements are to be calculated. Multiply the area of each element by the distance of its own centre of gravity from the datum-line, to find its moment. Divide the sum of these moments by the total reduced area; and the quotient is the distance of the centre of gravity of the reduced section, or of the neutral axis of the whole section, from the datum-line.

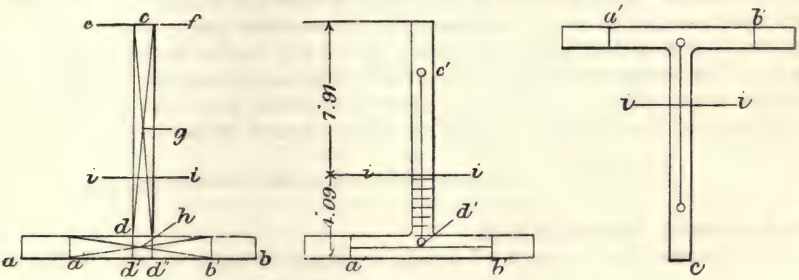
For example, the ⊥ section, No. 1, Figs. 145, and shown in Fig. 146 annexed, is 12 inches deep, 12 inches wide, and 1 inch thick throughout. Extend the web, *c d*, to the lower surface at *d′* and *d″*, leaving 5½ inches of web, *a d′* and *d″ b*, on each side. Reduce this width in the ratio of 1.73 to 1, or to (5.5 ÷ 1.73 =) 3.2 inches, and set off *d′ a′* and *a″ b′* each equal to 3.2 inches. Then the reduced flange *a′ b′* is (6.4 + 1 =) 7.4 inches wide, and the reduced section consists of the two rectangles *a′ b′* and *c d*. Assume any datum-line, as *e f* at the upper edge of the section, and bisect the depths of the rectangles, or take the intersections of their diagonals at *g* and *h*, for their centres of gravity. The distances of these from the datum-line are 5½ and 11½ inches respectively, and the areas of the rectangles are 11 × 1 = 11 square inches, and 7.4 × 1 = 7.4 square inches. By the rule,

Upper rectangle, ...	$11 \times 5\frac{1}{2} =$	60.5
Lower do.	$7.4 \times 11\frac{1}{2} =$	85.1
	$18.4 \times 7.91 =$	145.6

Showing that the centre of gravity of the reduced section, being the neutral axis of the whole section, is 7.91 inches below the upper edge, in the line *i i*. The centre of gravity of the entire section, it may be added, is 8.63 inches below the upper edge, or .72 inch lower than that of the reduced section.

The neutral axes of the other sections, Figs. 145, found by the same process, are marked on the figures. The section of a flange rail, No. 5, which is very various in breadth, may be treated in two ways: either by preparatorily averaging the projections of the head and the flange into rectangular forms; or, by taking it as it is, and dividing it into a considerable number of strips parallel to the base, for each of which the moment with respect to the assumed datum-line is to be found. The first mode of treatment is approximate; the second is more nearly exact.

Ultimate Strength of Beams of Unsymmetrical Section.—Resuming the ⊥ section, Fig. 146, for which the neutral axis has been ascertained, to find the tensile resistance, divide the portion below the neutral axis *i*, Fig. 147. with the reduced width of flange, *a′ b′*, into parallel strips, say ½ inch deep,



Figs. 146, 147, 148.—Beam of Unsymmetrical Section.

as shown, and multiply the area of each strip by its mean distance from the neutral axis for the proportional quantity of resistance at the strip. Divide the sum of the products, amounting to 31.3, by the extreme depth below the neutral axis—in this instance 4.09 inches, and multiply the quotient by 1.73 *s*, the ultimate tensile resistance at the lower surface. The final product is the total tensile resistance of the section; or,

$$\frac{31.3 \times 1.73\, s}{4.09} = 13.24\, s \text{ total tensile resistance.}$$

Again, multiply the area of each strip by the square of its mean distance from the neutral axis, and divide the sum of these new products, amounting to 104.64, by the sum of the first products. The quotient is the distance of the resultant centre of tensile stress, *d′*, from the neutral axis. Or, the resultant centre is,

$$\frac{104.64}{31.3} = 3.34 \text{ inches below the neutral axis.}$$

This process is, in fact, the process for finding the centre of gravity of all the tensile resistances.

By a similar process for the upper portion, in compression, the sum of the first products is found to be the same as for the lower part, and is 31.3. But the maximum compressive stress at the upper surface is greater than the maximum tensile stress at the lower surface, in the ratio of their distances from the neutral axis; or it is $1.73\ s \times \frac{7.91}{4.09} = 3.34\ s$, and

$$\frac{31.3 \times 3.34\ s}{7.91} = 13.24\ s, \text{ total compressive resistance,}$$

which is the same as the total tensile resistance, in conformity to the general law of the equality of tensile and compressive stress in a section. The sum of the products of the areas of the strips, divided by the squares of their distances respectively from the neutral axis, is 164.90, and the resultant centre, c', is

$$\frac{164.90}{31.3} = 5.27 \text{ inches above the neutral axis.}$$

The sum of the distances of the centres of stress or of resistance from the neutral axis, $(3.34 + 5.27 =)\ 8.61$ inches, is the distance apart of these centres, as represented by a central line, $c'd'$, in Fig. 147.

Abbreviated calculation.—As the upper part of the section is a rectangle, detailed calculation is not necessary, for its resultant centre is known to be at two-thirds of the height, or $(7.91 \times {}^2/_3 =)\ 5.27$ inches above the neutral axis. The average resistance, too, is half the maximum stress,—namely, that at the upper edge,—which is, as above explained, $3.34\ s$ per square inch. The area of the rectangle is $(7.91 \times 1 =)\ 7.91$ square inches; and

$$\frac{7.91 \times 3.34\ s}{2} = 13.24\ s \text{ the compressive resistance,}$$

as has already been calculated.

To resume: the moment of tensile resistance is $13.24\ s \times 8.61$ inches $= 114\ s$, and it is equal to $\frac{W\,l}{4}$; whence $W = \frac{114\ s \times 4}{l}$; or, in a general form,

$$W = \frac{4\ S\ d_3}{l} \qquad \text{(25)}$$

W = the breaking weight, in tons.
S = the total tensile resistance of the section, in tons.
d_3 = the vertical distance apart of the centres of tension and compression, in inches.
l = the span, in inches.

Strength of the Beam, Fig. 147, *inverted.*—When inverted, the maximum tensional resistance of the beam at the lower surface c, in Fig. 148, is $1.73\ s$. The area of the rectangle $i\,c$ is 7.91 square inches, and

$$\frac{7.91 \times 1.73\ s}{l} = 6.84\ s, \text{ total tensile resistance;}$$

which is only about half the tensile resistance offered by the beam in its first position, Fig. 147. The breaking weight is, therefore, also only about

a half; and the reason why it is calculated that the beam bears double the breaking weight in the first position that it does in the second is, that the rectangular portion, *c i*, is expected to oppose at least twice as much resistance, when above, to compression as it does, when below, to tension. If the effective resistance to compression were only equal to the resistance to tension, the beam would have the same ultimate strength in both positions.

RULE.—*To find the Ultimate Strength of a Homogeneous Beam of Unsymmetrical Section.* 1. Reduce the section to a section of uniform potential resistance, as explained at page 514. 2. Find the position of the neutral axis of the reduced section by the previous rule. 3. Divide the section into thin strips parallel to the neutral axis; if such division has not already been made, for No. 2. 4. Multiply the areas in square inches of the strips, under tensional stress, by their mean distances respectively (that is, the distances of their centres of gravity) from the neutral axis. 5. Multiply the same areas by the squares of their mean distances respectively from the neutral axis. 6. Divide the sum of the first products by the extreme distance of the surface in tension from the neutral axis, and multiply the quotient by the ultimate tensile resistance per square inch; the product is the total tensile resistance of the section. 7. Divide the sum of the second products (5) by the sum of the first products (4); the quotient is the distance of the resultant centre of tension from the neutral axis. 8. Multiply the ultimate tensile resistance by the distance of the neutral axis from the upper surface, and divide the product by the distance of the neutral axis from the lower surface; the quotient is the maximum compressive stress at the upper surface. 9. Make the calculations 4, 5, and 7 for the strips under compression. 10. The sum of the distances of the resultant centres from the neutral axis is the distance apart of the centres. 11. Find the breaking weight by formula (25); that is, multiply the total tensile resistance of the section in tons by the distance apart of the resultant centres of tension and compression in inches, and by 4; and divide the product by the span in inches. The quotient is the breaking weight in tons at the middle.

Note to Rule.—It is assumed that the ability of the beam to resist compression is sufficient to insure that fracture shall take place by tension. Beams of comparatively slender dimensions laterally are liable to cant under the thrust of compression if not supported laterally, and canting occasionally takes place in beams which are tested experimentally. But beams, when in their destined places, are in general so supported that any liability to canting is removed.

Elastic Strength of Beams of Unsymmetrical Section.—The elastic strength is approximately deducible from the ultimate strength, according to the ordinary ratio of one to the other, ascertained experimentally. The elastic strength and deflection of a homogeneous beam of any section is the same, whether in its normal position or turned upside down.

FORMS OF BEAMS OF UNIFORM STRENGTH.

A beam is said to be of uniform strength when its capability of resistance to transverse stress under a given load, applied in a given manner, is the same at all parts of its length.

SEMI-BEAMS OF UNIFORM STRENGTH LOADED AT THE END.

By a semi-beam is meant a beam fixed at one end and free at the other; as it represents the half of a beam supported at both ends.

The moments of stress due to the weight on the end, at any section of the beam, increase directly as the distance of the section from the end of the beam. Let *c b*, Fig. 149, be a rectangular beam, fixed at the base *c d*, and loaded at the end *b*. Draw the diagonal straight line *b d*, then the ordinates to the triangle *b c d*, represent proportionally the moment of stress at all parts of the length; and the moments of stress vary directly as the length. Now, the ultimate moments of resistance at any section are as the square of the depth, when the breadth is uniform; and it follows, conversely, that the depth of the beam, of uniform strength, must vary as the square root of the distance from the end *b*. Take, for instance, the section *c′ d′* at the half-length of the beam; the moment of stress at *c d* is to that of the stress at *c′ d′*, as 1 to ½; and the required depth at the origin *c d*, is to the depth at *c′ d′*, as $\sqrt{1}$ to $\sqrt{½}$, or as 1 to .707. The depth *c′ c″*, equal to .707, would be the depth of a beam of uniform strength, at that section. The depth for uniform strength at any other section may be calculated in the same way; and the form of the lower side of the beam, of uniform strength, is that of a parabola, *b c″ d*, of which the vertex is at the end *b*. With respect to transverse resistance, then, the semi-beam would be equally strong if the lower portion *b b′ d* were removed.

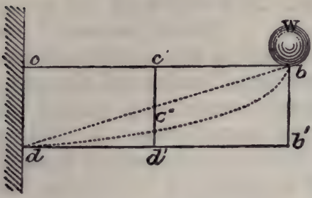
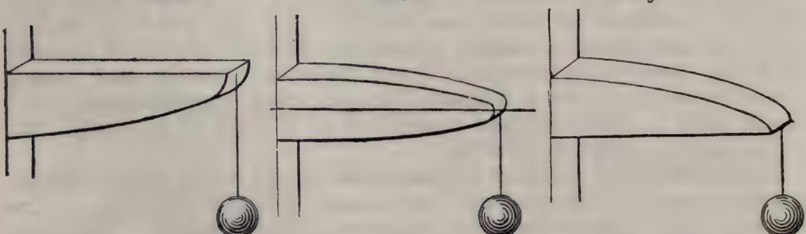


Fig. 149.—Stress in a Semi-beam loaded at the end.

The semi-beam, rectangular in section, of uniform strength, fixed at one end, and loaded at the other end, having the breadth constant, may therefore be moulded in depth to any of the parabolic outlines, Figs. 150.

No. 1. No. 2. No. 3.



Figs. 150.—Semi-beams loaded at one end.

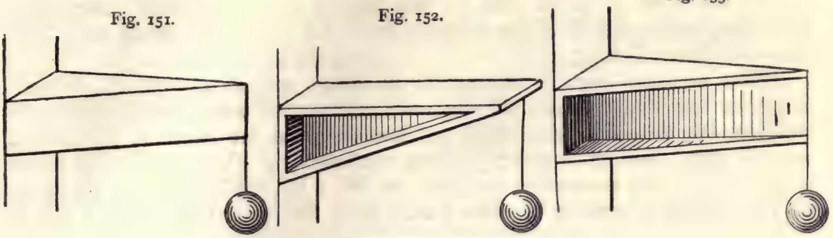
When the depth of the semi-beam, rectangular in section, is constant, the breadth is in simple proportion to the distance from the end of the beam, as in Fig. 151, and the beam is triangular in plan.

When the section of the semi-beam is double-flanged, or is hollow rectangular, and the breadth is constant, the flanges are assumed to be of a constant sectional area. Leaving out of the calculation the strength of the vertical

web, and calculating only for the flanges, the moment of resistance at any section is as the depth, and the form of the beam is triangular, as in Fig. 152, which shows a semi-beam with double flanges.

If the strength of the vertical web be taken into the calculation, the form of the beam is intermediate between the triangular and the parabolic.

Fig. 151. Fig. 152. Fig. 153.



Semi-beams loaded at one end.

When the section of the semi-beam is double-flanged, or is hollow-rectangular, and the depth is constant, calculating only for the flanges, Fig. 153, their sectional area increases uniformly with the distance from the end, and if their thickness be uniform, they are triangular in plan, as shown.

If the web be taken into the calculation, it is calculated as a solid semi-beam rectangular in section, and the thickness should increase as the distance from the end. The web would, therefore, be triangular in plan.

When the section of the semi-beam is circular, the moment of resistance varies as the cube of the diameter, and the cube of the diameter is therefore as the distance from the end; or, inversely, the diameter is as the cube root of the distance, and the outline of the semi-beam may be formed by the revolution of a cubic parabola on its axis, Fig. 154.

When the section of the semi-beam is annular; when the thickness is uniform and small in proportion to the diameter, the square of the diameter varies as the distance from the end, or the diameter varies as the square root of the distance, and the semi-beam is formed by the revolution of a parabola on its axis.

If the thickness varies with the diameter, the diameter varies as the cube root of the distance from the end, and the semi-beam is cubic-parabolic, like Fig. 154.

When the section of the semi-beam is elliptical, the sections being similar at all points of the length, the cube of the depth varies as the distance from the end, or the depth varies as the cube root of the distance, and the elevation of the beam is cubic-parabolic, like Fig. 154.

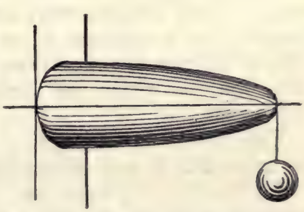


Fig. 154.—Semi-beam loaded at one end.

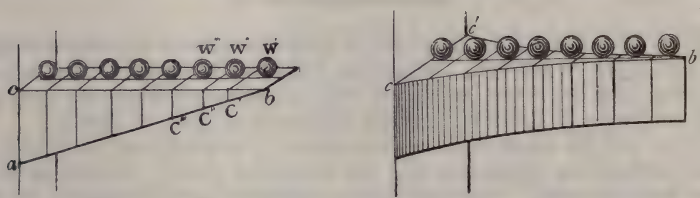
When the section of the semi-beam is hollow-elliptical, the beam being of similar sections throughout. When the thickness is uniform, and is small in proportion to the depth, the square of the depth varies as the distance from the end, or the depth varies as the square root of the distance, and the side elevation of the beam is parabolic.

If the thickness varies with the depth, the depth varies as the cube root of the distance from the end, and the beam is cubic-parabolic in side elevation, like Fig. 154.

Semi-beams of Uniform Strength Uniformly Loaded.

The moment of stress due to the weight when uniformly distributed increases as the square of the distance from the end of the beam, as will be shown in the following case:—

When the semi-beam is rectangular in section, and its breadth is constant. Suppose the load equally divided and distributed as a great number of weights, W′, W″, W‴, &c., Fig. 155; and suppose the beam to be divided into an equal number of corresponding sections at *c*′, *c*″, *c*‴, &c. The loads supported by the successive sections are W′, 2 W′, 3 W′, &c.; the distances of the centres of gravity of these loads, from the respective sections, are as



Figs. 155, 156.—Semi-beams uniformly loaded.

1, 2, 3, &c. Therefore, the moments of stress at the successive intersections, *c*′, *c*″, *c*‴, &c., are as 1^2, 2^2, 3^2, &c., or as the square of the distance from the end. But the moments of resistance at the intersections are as the squares of the depths at *c*′, *c*″, *c*‴, &c.; and so the square of the depth is as the square of the distance, or the depth is as the distance from the end. The beam is therefore triangular in elevation.

When the semi-beam is rectangular in section, and has the depth constant, Fig. 156. As the depth is constant, the breadth must increase as the square of the distance; and it may be, in outline, of the form of two parabolas *b c*, *b c*′, back to back, touching each other at their vertices at *b*; the axes being perpendicular to the length.

When the section of the semi-beam is hollow-rectangular, or is double-flanged; and the breadth is constant. Calculating the strength of the upper and lower members, or flanges, only, and supposing the thickness to be uniform, the moment of resistance is as the depth; the depth is, therefore, as the square of the distance from the end, and is of the form of a parabola, Fig. 157, of which the vertex is at *b*, and the axis is perpendicular to the length.

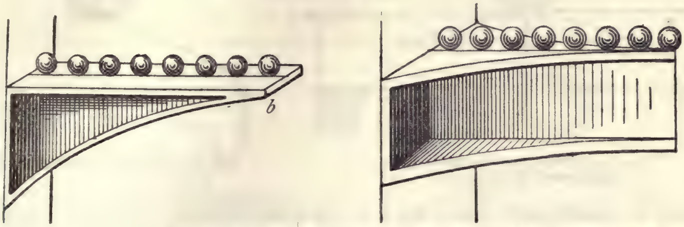
Calculating the strength of the vertical webs or rib only, the beam would be triangular in side elevation.

Combining the webs and the flanges in the calculation, the form of the beam would be intermediate between the parabolic and the triangular.

2d. *When the depth is constant.* Calculating for the flanges only, the thickness being uniform; the breadth of the flanges is as the square of the distance from the end, Fig. 158, the same as in Fig. 156.

If the vertical web or rib, of uniform thickness, be included in the calculation, it does not materially modify the form of the flange.

When the section of the semi-beam is circular. The moment of resistance is as the cube of the diameter, and the moment of stress is as the square of the length; therefore the cube of the diameter is as the square of the



Figs. 157, 158.—Semi-beams uniformly loaded.

length, or the diameter is as the cube root of the square of the length, or as the ⅔ power, or .666 power of the length. The solid is formed by the revolution of a semi-cubic parabola on its axis.

When the section of the semi-beam is annular, the thickness being uniform and small compared to the diameter. The moment of resistance of any section is as the square of the diameter. The square of the diameter is, therefore, as the square of the length, or the diameter is as the length, and the semi-beam is triangular or conical in elevation, Fig. 159.

When the thickness diminishes with the diameter, the moments of resistance of sections are as the cubes of the diameters, and the diameter varies as the ⅔ power, or .666 power of the length, as with a solid circular section, and the form is derived from the revolution of a semi-cubic parabola on its axis.

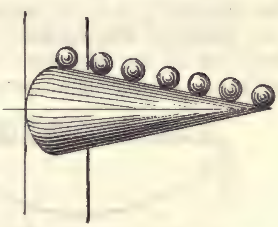


Fig. 159.—Annular Semi-beam uniformly loaded.

When the section of the semi-beam is elliptical. The moment of resistance of a section is as the cube of the depth, and the form is the same as that of a circular beam.

When the section of the semi-beam is hollow-elliptical. The form is the same as that of a beam of annular section.

Beams of Uniform Strength Supported at Both Ends.

The forms of beams supported at both ends, and loaded at the middle, are simply doubles of the forms of semi-beams, or such as are fixed at one end and unsupported at the other end. In the beam of rectangular section, for example, A B, Fig. 160, the diagonal lines, *c a* and *c b*, from the top at the middle to the supports at each end, are simply doubles of the diagonal *b d*, in the semi-beam, Fig. 149, and represent the graduated moment of bending stress from the middle, where it is a maximum, to the ends, where it vanishes; and the parabolic curves *c a* and *c b*, meeting base to base at the middle *c d*, form the outline of the rectangular beam of uniform strength, when the breadth is constant.

The beam, rectangular in section, of uniform strength, loaded at the middle,

and having breadth constant, may therefore be moulded according to any of the parabolic forms, Figs. 161, 162, 163, having the axes horizontal, and the vertices at the points of support.

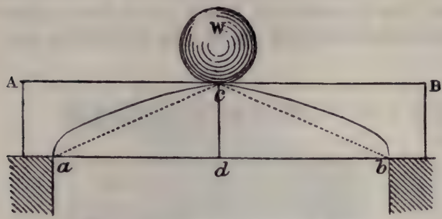


Fig. 160.—Stress in rectangular beam supported at both ends.

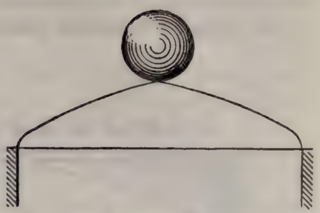
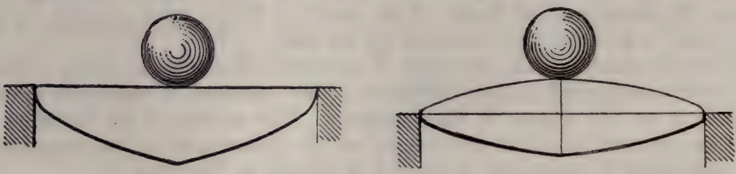


Fig. 161.—Beam loaded at the middle.

When a rectangular beam, with a constant breadth, is loaded uniformly; referring to formula (5), page 508. When the weight is constant, together with the breadth b, and the length l, the square of the depth, d^2, varies as



Figs. 162, 163.—Beams loaded at the middle.

the products, $m\,n$, of the segments, m and n, of the length of the span at any point of the length. Or, the depth varies as the square root of the product of the segments, and the form of the beam, Fig. 164, is a semi-ellipse. It may be a complete ellipse, Fig. 165.

Figs. 164, 165.—Beams uniformly loaded.

For a rectangular beam, with a constant depth, and loaded at the middle, the form of the breadth, Fig. 166, is a double of Fig. 151, page 519; consisting of two triangles, in plan, united at their base.

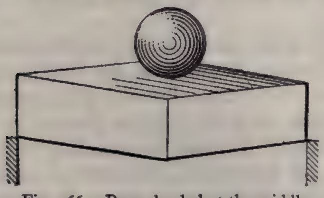


Fig. 166.—Beam loaded at the middle.

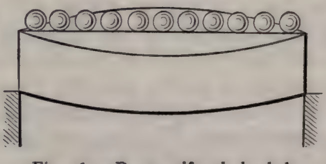
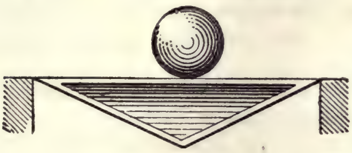


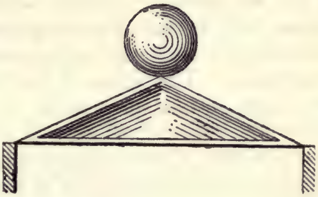
Fig. 167.—Beam uniformly loaded.

When a rectangular beam, with a constant depth, is uniformly loaded; referring to the formula (5) above noticed, the variables are the breadth b and the product $m\,n$, and the breadth varies as the product of the

segments of the length of the span at any point of the length. The form of the breadth in plan is therefore that of two parabolas having their vertices at the middle and meeting at the points of support, Fig. 167.

A hollow-rectangular or double-flanged beam with a constant breadth, and loaded at the middle, consists of the double of Fig. 152, page 519; being two triangles united at their base, at the middle, Figs. 168, 169. In this and the three following cases, the resistance of the flanges only is calculated; and the flanges are supposed to be of uniform thickness.





Figs. 168, 169.—Beams loaded at the middle.

When a hollow-rectangular or double-flanged beam, with a constant breadth, is uniformly loaded; the depth varies as the product of the segments of the beam at any point in the span; and the side of the beam, Fig. 170, is of the form of a parabola, having its axis at the middle. The resistance of the flanges only is here calculated.

Fig. 170.—Beam uniformly loaded.

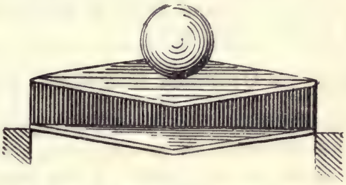


Fig. 171.—Beam loaded at the middle.

A hollow-rectangular or a double-flanged beam, with a constant depth, and loaded at the middle, consists of the double of Fig. 153, page 519; the flanges being of uniform thickness, and forming two triangles in plan, joined at their base at the middle of the beam. Their form, Fig. 171, is the same as that of a rectangular beam, with a constant depth, Fig. 166.

When a hollow-rectangular or a double-flanged beam, with a constant depth, is uniformly loaded, the form of the flanges in plan is the same as that of a rectangular beam (Fig. 167), consisting of two parabolas, having their vertices at the middle of the beam, Fig. 172.

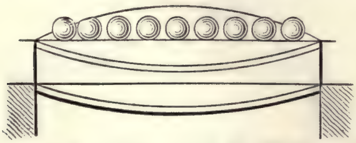


Fig. 172.—Beam uniformly loaded.

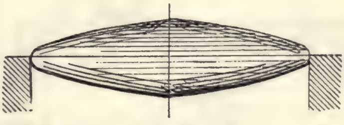


Fig. 173.—Beam loaded at the middle.

When the section of the beam is circular, and the load is at the middle, the form is the double of Fig. 154, page 519, consisting of the revolutions of two cubic parabolas, base to base, at the middle, Fig. 173.

When a beam of circular section is uniformly loaded, the cube of the diameter varies as the product of the segments of the length of the span at any point in the length; and the radius varies as the cube root of the product of the segments.

When the section of the beam is annular, and the load applied at the middle, the form is that produced by the revolution of two parabolas, base to base, with their vertices at the ends of the beam. The thickness is supposed to be inconsiderable.

When the annular beam is uniformly loaded, the square of the diameter varies as the product of the segments of the length of the span at any point in the length; and the radius varies as the square root of the product of the segments. The form of the beam is that produced by the revolution of an ellipse on one of its axes.

When the section of the beam is elliptical, and the load applied at the middle, the form is that of two cubic parabolas joined base to base.

When the beam of elliptical section is uniformly loaded, the form is that of an ellipse.

When the section of the beam is hollow-elliptical, and the load applied at the middle, and the thickness is uniform, and is small in proportion to the depth, the form of the beam is that of two parabolas, united base to base, having their vertices at the points of support.

If the thickness varies with the depth, the forms are cubic parabolas.

When the hollow beam of elliptical section is uniformly loaded, the form of the beam is elliptical.

Beams of Uniform Strength under a Concentrated Rolling Load.

Reverting to formula (5), page 508, it signifies that the breaking weight varies inversely as $m \times n$, or the product of the segments of the length of the span, at any point of the length; but if the weight be constant, the moment of stress at any point is as the product $m \times n$, and therefore, also, the moment of resistance of a beam of uniform strength varies as $m \times n$, at all points of its length.

Hollow-rectangular, or flanged beam, with a constant breadth, under a concentrated rolling load. Calculating the resistance of the booms or flanges only, the depth varies as $m \times n$, and is according to the form of a parabola, of which the axis is vertical, when the upper or lower side is horizontal, Figs. 174, 175; or of two parabolas, on the same axis, meeting at the points of support.

Under these conditions, the sectional area of the flanges is constant.

Figs. 174, 175.—Flanged Beams under a Concentrated Rolling Load.

Hollow-rectangular or flanged beam, with a constant depth, or parallel flanges, under a concentrated rolling load. The breadth varies as $m \times n$, and the flanges, supposed to be of uniform depth, are of the form of two parabolas on the same axis passing through the middle, Fig. 172, page 523.

Stress in the curved flange, Figs. 174, 175. Mr. Stoney gives a simple means of finding the stress diagramatically. Let A B, Fig. 176, represent the horizontal stress, which is uniform throughout the length. Draw A C parallel to the tangent of the curve at the given point, and B C perpendicular to A B. Then A C is the maximum longitudinal stress at the given point, and A B and B C are its horizontal and vertical components. It follows that the section of the curved flange should increase as it approaches the points of support in proportion to A C, or the secant of the angle A.

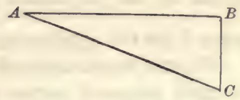


Fig. 176.—Diagram for Stress in Curved Flange.

SHEARING STRESS IN BEAMS AND PLATE-GIRDERS.

Shearing stress in beams is caused by the vertical pressure of the load. A conception of this stress is easily formed on reflecting that the weight of the beam and its load tends to force it downwards at the abutment, whilst the abutment, by its upward pressure, tends to force upwards the part of the beam which rests on it. The stress thus caused tends to a vertical rupture, or slicing off of the loaded end of the beam, called shearing stress. The same kind of stress acts with various intensity in the portion of the beam between the abutments, and it is the duty of the web to resist the shearing stress.

In a beam supported at one end, and loaded at the other end, the vertical shearing stress is equal to the weight, at every point of the length.

In the same beam, uniformly loaded, the shearing stress increases uniformly from the end, where it is nothing, to the abutment, where it is equal to the weight. A diagram indicating the gradations of stress would have the form of a triangle.

In the same beam, uniformly loaded, and also weighted at the end, the shearing stress is represented by a compound diagram, Fig. 177, in which the triangle $a\,b\,c$ represents the graduated shearing stress due to a uniform load, in a beam of the length $a\,b$; and the rectangle $a\,b\,d\,e$, the uniform shearing stress due to a weight at the end. The whole depth $d\,c$ at the abutment represents a total shearing stress equal to the sum of the distributed and end loads; and the total stress at intermediate points is represented by the corresponding ordinates.

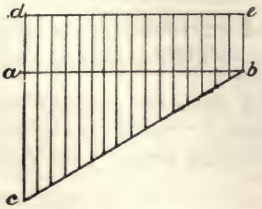


Fig. 177.—Shearing Stress.

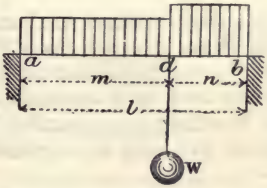


Fig. 178.—Shearing Stress.

In a beam supported at both ends, and loaded at any point, the shearing stress in each segment is equal to the pressure on its abutment. The

pressures at *a* and *b*, Fig. 178, are as the segments *m* and *n*, and the shearing stress in the segments *a d* and *d b* are equal to $W \times \frac{n}{l}$ and $W \times \frac{m}{l}$; represented by the graduated rectangles on *a d* and *d b*.

When a concentrated load is moved over the beam, the shearing stress in each segment varies as the length of the other segment:—from 0 to W, the weight; represented by the two graduated triangles, *a b c*, *a b d*, Fig. 179, in which the verticals *a c* and *b d*, at the ends, represent the weight.

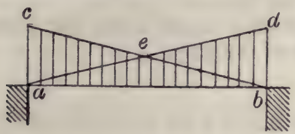


Fig. 179.—Shearing Stress.

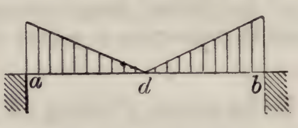


Fig. 180.—Shearing Stress.

When a number of weights are placed irregularly on a beam, the shearing stress of some is neutralized more or less by that of others; and, referring to any given section of the beam, the shearing stress is equal to the difference of the sum of those portions of the weights placed on one side of the section which are conveyed to the abutment on the other side, and the sum of those portions of the weights on the other side which are conveyed to the abutment on the first side.

When a beam, supported at both ends, is loaded uniformly, the shearing stress is 0 at the centre, as in Fig. 180, and increases uniformly towards the abutments, where it is equal to half the weight.

When a load of uniform density, as a railway train, traverses a girder, the shearing stress at the front of the train increases as the square of the length of the loaded segment. Suppose that the train advances from *b* to *a*, Fig. 181, covering the whole length, the curve of increasing shearing stress, *b c*, is parabolic, having its apex at *b*. When the girder is wholly covered, the shearing stress follows the triangular gradations shown by dot-lines.

Fig. 181.—Shearing Stress.

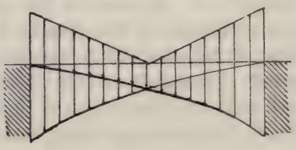


Fig. 182.—Shearing Stress.

When a fixed uniform load and a rolling load are combined, the maximum shearing stress to which the girder is liable at different points of its length is shown by the combined ordinates in Fig. 182.

Sectional area of a continuous web calculated from the shearing stress.— "When the flanges are parallel," says Mr. Stoney,[1] "the theoretic area of a continuous web may be calculated from the shearing stress by the following rule:—

$$\text{Sectional area of web} = \frac{\text{shearing stress}}{\text{unit-stress}},$$

in which the unit-stress is the safe unit-stress for shearing. This gives the

[1] *The Theory of Strains in Girders and Similar Structures.*

minimum thickness, which, however, is often much less than a due regard for durability requires."

"When a girder, with parallel flanges and a continuous web, is loaded in the manner described below, where l = the length, and f = the safe unit-strain for shearing force, the theoretic quantity of material in the web would be as follows:"—

Kind of Load.	Theoretic Quantity of Material in a Continuous Web.	Proportional numbers.
Fixed central load........... = W	$\frac{Wl}{2f}$	12
Concentrated rolling load.. = W	$\frac{3Wl}{4f}$	18
Uniformly distributed load = W	$\frac{Wl}{4f}$	6
Distributed rolling load.... = W	$\frac{7Wl}{24f}$	7

DEFLECTION OF BEAMS AND GIRDERS.

Compressive strain is taken as equal to *tensile strain*, per ton of direct stress on the fibres, and the *strain* is directly proportional to the *stress*, within the elastic limits. When a beam is deflected under a load, the lower side is lengthened and the upper side is shortened in proportion to the direct stress per unit of section of the fibres.

In a beam of uniform strength, the fibres at the surface, on the upper and lower sides, are, by the definition, equally stressed and equally strained throughout the length of the beam; and the form assumed by a straight parallel beam, when deflected under its proper load, is that of a circular arc.

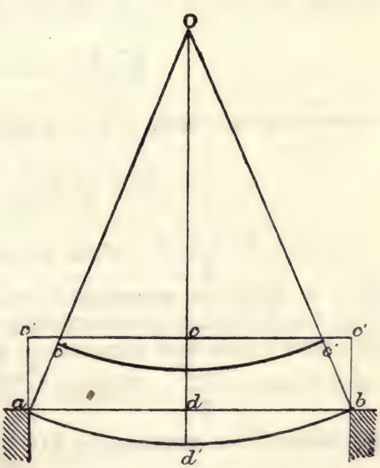


Fig. 183.—Deflection of a Beam.

Let $a\,b\,c'\,c'$, Fig. 183, be a parallel beam, rectangular in section, having a constant depth, and of uniform strength, when loaded at the middle. Let its lower side assume, by deflection, the form of the circular arc $ad'b$, the ends ac', bc', which were upright, in their normal position, are now convergent in the positions ac'', bc''; and when produced, they meet at the centre of the arc, O, in the vertical radius d'O. The deflection at the centre, dd', is the versed sine of the arc. Let,

R = the radius Od',
l = the length of the beam, or the chord, $a\,b$,
b = the breadth of the beam at the middle,
d = the depth of the beam at the middle, $c\,d$.
a = the sectional area of the beam,
D = the deflection of the beam, $d\,d'$,
l' = the difference of length of the upper and lower sides,
E = the coefficient of elasticity, or the denominator of the fraction of the length, by which the beam is extended or compressed, per ton of direct stress per square inch of section,
s' = the direct tensile stress on the extreme outer fibres, in tons per square inch,
s'' = the direct compressive stress on the extreme outer fibres, in tons per square inch,
W = the weight in tons.

Note.—The dimensions are to be all in inches, or all in feet.

By the properties of the circle, the square of half the chord is equal to the product of the versed sine, or deflection, by the diameter minus the deflection; or

$$\left(\frac{l}{2}\right)^2 = \mathrm{D} \times (2\,\mathrm{R} - \mathrm{D}); \text{ or sensibly } \left(\frac{l}{2}\right)^2 = \mathrm{D} \times 2\,\mathrm{R}; \text{ and}$$

$$\mathrm{D} = \frac{l^2}{8\,\mathrm{R}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (a)$$

Again, by similar triangles, in Fig. 183, Oc'' : Oa : : $c''c''$: $a\,b$; or, in symbols, R : d : : l : l', substantially; whence

$$\mathrm{R} = \frac{d\,l}{l'}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (b)$$

Substituting this value of R in equation (a),

$$\mathrm{D} = \frac{l^2}{8} \times \frac{l'}{d\,l} = \frac{l\,l'}{8\,d}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (c)$$

Now, $l' = \frac{(s' + s'')\,l}{\mathrm{E}}$. When the two stresses, s' and s'', are equal to each other, let them be represented by s. When they are not equal to each other, the deflection is nevertheless the same as if they were so, and that the direct tensile and compressive stress were each equal to the mean of s' and s'', or to $\frac{s'+s''}{2}$. Putting $\frac{s'+s''}{2} = s$, then $l' = \frac{2\,s\,l}{\mathrm{E}}$; and, substituting this value of l' in equation (c), $\mathrm{D} = \frac{2\,s\,l^2}{8\,d\,\mathrm{E}}$; or,

$$\mathrm{D} = \frac{s\,l^2}{4\,d\,\mathrm{E}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\text{and, } s = \frac{4\,d\,\mathrm{E}\,\mathrm{D}}{l^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Deflection of Beams of Rectangular Section.

No. 1. *Rectangular beam, of constant depth, of uniform strength, loaded at the middle*, Fig. 166, page 522.—This beam is double-triangular in plan. The value of s, the direct stress on the fibres at the upper and lower surfaces, in terms of the weight, is, by formula 2, page 507,

$$s = \frac{W\,l}{1.155\,b\,d^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Equating this value of s and the above value (2),

$$\frac{W\,l}{1.155\,b\,d^2} = \frac{4\,d\,E\,D}{l^2}; \text{ and, } W\,l^3 = 4.62\,b\,d^3\,E\,D;$$

whence

$$D = \frac{W\,l^3}{4.62\,b\,d^3\,E}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$W = \frac{4.62\,b\,d^3\,E\,D}{l^3}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$E = \frac{W\,l^3}{4.62\,b\,d^3\,D}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

These equations express the relations of the weight, the coefficient of elasticity, and the deflection.

The formula (4), for the value of the deflection, signifies that the deflection varies directly as the weight, and as the cube of the span; and that it varies inversely as the breadth, the cube of the depth, and the coefficient of elasticity.

No. 2. *Rectangular beam, of constant breadth, of uniform strength, loaded at the middle*, Figs. 161, 162, 163, page 522.—The form of the beam in side elevation is parabolic. The average depth is, by the properties of the parabola, two-thirds of the depth at the middle, or of the depth of the circumscribed rectangle; and the beam may be treated, for finding the deflection, as a parallel beam, or beam of constant depth, having two-thirds of the depth of No. 1 beam, and under the same stress on the extreme fibres. As the strain, and the difference of length of the upper and lower sides, of the suppositious beam, are approximately the same as those of the original beam; and as the deflection is inversely as the depth (see equation (c), page 528), or as two to three, in No. 1 and No. 2 beams; then, modifying formula (4), above, $D = \frac{3}{2} \times \frac{W\,l^3}{4.62\,b\,d^3\,E}$; or,—

$$D = \frac{W\,l^3}{3.08\,b\,d^3\,E}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

No. 3. *Rectangular beam, of uniform section, loaded at the middle*, Fig. 160, page 522.—Compared with No. 1 beam, Fig. 166, No. 3 beam is rectangular in plan, and contains twice the surface of No. 1, which is triangular. Under a given weight, therefore, the stress on the extreme, or surface, fibres of No. 3, averages only half the stress on those of No. 1.

The deflection would also, if it followed the same proportion, be just half that of No. 1. But it must be more than half, since it is not according to a circular outline, but follows an outline like that of a hyperbolic section—the curvature being localized mostly at the middle. It appears from experimental results that the deflection may be approximately taken as equal to that of a beam of No. 1 form, and the formula for No. 1 is, therefore, provisionally adopted for No. 3, until more complete data are established:—

$$D = \frac{W l^3}{4.62\, b\, d^3\, E}. \quad \text{(8)}$$

No. 4. *Rectangular beam, of constant depth, of uniform strength, uniformly loaded*, Fig. 167, page 522. The stress in the upper and lower surface fibres of No. 4 is only half the stress in those of No. 1, under equal loads; and therefore also the deflection is only half. Doubling, accordingly, the numerical coefficient of formula (4),—

$$D = \frac{W l^3}{9.24\, b\, d^3\, E}. \quad \text{(9)}$$

No. 5. *Rectangular beam, of constant breadth, of uniform strength, uniformly loaded*, Figs. 164, 165, page 522. This beam is elliptic in elevation, and the area, and, therefore, the average depth, are four-fifths of those of the circumscribed rectangle, or of No. 4 beam. Reasoning on this beam, as on No. 2 beam, the deflection is five-fourths of that of No. 4 beam, and the numerical coefficient is four-fifths; or,

$$D = \frac{W l^3}{7.40\, b\, d^3\, E}. \quad \text{(10)}$$

No. 6. *Rectangular beam, of uniform section, uniformly loaded.*—The deflection under a uniform load is found, by experiments with timber, to be about five-eighths of that of the same beam loaded with an equal weight at the middle. Increase, therefore, the numerical coefficient for No. 3 to eight-fifths, or $(4.62 \times \frac{8}{5} =)$ 7.40:—

$$D = \frac{W l^3}{7.40\, b\, d^3\, E}. \quad \text{(11)}$$

Deflection of Double-Flanged, or Hollow-Rectangular Beams.

No. 7. *Double-flanged beam, of constant depth, of uniform strength, loaded at the middle*, Fig. 171, page 523.

1st. When the strength of both the flanges and the web is calculated. The value of s, the direct stress on the fibres at the upper and lower surfaces, is, by inversion of formula (19), page 511,

$$s = \frac{W l}{d''(4a + 1.155\, a'')}, \quad \text{(12)}$$

in which d'' is the distance apart between the centres of the flanges; a is the sectional area of one flange; and a'' the sectional area of the web,

taking the height of the web $= d''$. Equating this value of s to the value (2), page 528, in terms of the deflection,—

$$\frac{W\,l}{d''(4\,a + 1.155\,a'')} = \frac{4\,d\,E\,D}{l^2};\ \text{and}\ W\,l^3 = 4\,d''^2\,E\,D(4\,a + 1.155\,a'');$$

Here, d is taken as equal to d''; whence,

$$D = \frac{W\,l^3}{4\,d''^2\,E(4\,a + 1.155\,a'')}. \quad \text{.................} \quad (13)$$

From this equation it may be inferred that the deflection varies inversely as a power of the depth greater than the square, and less than the cube.

2*d*. *If the strength of the flange alone be calculated.* By inversion of formula (20), page 511,

$$s = \frac{W\,l}{4\,a\,d''}. \quad \text{....................................} \quad (14)$$

Equating this value to the value (2), page 528,

$$\frac{W\,l}{4\,a\,d''} = \frac{4\,d''\,E\,D}{l^2};\ \text{and}\ W\,l^3 = 16\,a\,d''^2\,E\,D;$$

whence,

$$D = \frac{W\,l^3}{16\,a\,d''^2\,E}. \quad \text{..............................} \quad (15)$$

In this equation, it is seen that the deflection varies inversely as the square of the depth.

No. 8. *Double-flanged beam, of constant breadth, of uniform strength, loaded at the middle,* Figs. 168, 169, page 523. The side of the beam is triangular, and the average depth is half the maximum depth. Reasoning on this beam, as on No. 2 beam, page 529, the deflection is found to be twice that of No. 7 beam. The numerical coefficients for No. 7 are therefore halved, and,

1*st*. *When the strength of both the web and the flanges is calculated:*—

$$D = \frac{W\,l^3}{2\,d''^2\,E\,(4\,a + 1.155\,a'')} \quad \text{.................} \quad (16)$$

2*d*. *When the strength of the flanges only is calculated:*—

$$D = \frac{W\,l^3}{8\,a\,d''^2\,E}. \quad \text{...............................} \quad (17)$$

No. 9. *Double-flanged beam, of uniform section, loaded at the middle.* The superficies of the flanges is double that of the triangular flanges of No. 7; and the average stress is a half. Reasoning on this, as on No. 3 beam, the deflection is taken as equal to that of No. 7 beam, and is determined by the formulas (13) and (15).

No. 10. *Double-flanged beam, of constant depth, of uniform strength, uniformly loaded,* Fig. 172, page 523. The deflection is half of that of No. 7, with equal loads. Doubling the numerical coefficients of the formulas (13) and (15),—

1*st*. *When the strength of both the flanges and the web is calculated:*—

$$D = \frac{W\,l^3}{8\,d''^2\,E(4\,a + 1.155\,a'')}. \quad \text{..............} \quad (18)$$

2*d*. *When the strength of the flanges only is calculated:—*

$$D = \frac{W\,l^3}{32\,a\,d''^2\,E}. \quad\ldots\ldots\ldots\ldots\ldots \quad (19)$$

No. 11. *Double-flanged beam, of constant breadth, of uniform strength, uniformly loaded*, Fig. 170, page 523.—This beam is parabolic in elevation, and has an average depth two-thirds of the maximum. The deflection is three-halves of that of No. 10, and two-thirds of the numerical coefficients of formulas (18) and (19) are to be taken.

1*st*. *When the strength of both the flanges and the web is calculated:—*

$$D = \frac{W\,l^3}{5.33\,d''^2\,E(4a + 1.155\,a'')}. \quad\ldots\ldots\ldots \quad (20)$$

2*d*. *When the strength of the flanges only is calculated:—*

$$D = \frac{W\,l^3}{21.33\,a\,d''^2\,E}. \quad\ldots\ldots\ldots\ldots\ldots \quad (21)$$

No. 12. *Double-flanged beam, of uniform section, uniformly loaded.*—Increase the numerical coefficient for No. 9 to eight-fifths, as was done correspondingly for No. 6; or to $(4 \times \frac{8}{5} =)$ 6.4, and $(16 \times \frac{8}{5} =)$ 25.6.

1*st*. *When the strength of both the flanges and the web is calculated:—*

$$D = \frac{W\,l^3}{6.4\,d''^2\,E(4a + 1.155\,a'')} \quad\ldots\ldots\ldots\ldots \quad (22)$$

2*d*. *When the strength of the flanges only is calculated:—*

$$D = \frac{W\,l^3}{25.6\;d''^2\,E}. \quad\ldots\ldots\ldots\ldots\ldots \quad (23)$$

Note.—As to double-flanged, or hollow-rectangular beams, Nos. 7 to 12. It has been supposed, for convenience of investigation, that the flanges are uniformly thick; and that the variation in their section takes place entirely in the breadth.

Relative Deflections of the six forms of beams, both solid-rectangular, and double-flanged.—The deflections are inversely as the numerical coefficients in the respective formulas, and are as follows, table No. 169:—

Table No. 169.—Relative Deflection of Beams, variously Proportioned and Loaded.

Loaded at the Middle.	Rectangular. ratio.		Double-flanged. ratio.	
1. Constant depth, uniform strength,	1.0	or 1	1.0	or 1
2. Constant breadth, do.	1.5	or 1½	2.0	or 2
3. Uniform section,......................	1.0	or 1	1.0	or 1
Uniformly Loaded.				
4. Constant depth, uniform strength,	.5	or ½	.5	or ½
5. Constant breadth, do.	.625	or ⅝	.75	or ¾
6. Uniform section,......................	.625	or ⅝	.625	or ⅝

No. 13. *Deflection of a Cylindrical Beam of Uniform Diameter.*—By formula (2), page 528, $s = \frac{4\, d\, \mathrm{E\,D}}{l^2}$; and by inverting formula (15), page 510, $s = \frac{\mathrm{W}\, l}{.7854\, d^3}$. Equating these values of s,

$$\frac{\mathrm{W}\, l}{.7854\, d^3} = \frac{4\, d\, \mathrm{E\,D}}{l^2};\ \text{and, } \mathrm{W}\, l^3 = 3.1416\, d^4\, \mathrm{E\,D}.$$

Whence,

$$\mathrm{D} = \frac{\mathrm{W}\, l^3}{3.1416\, d^4\, \mathrm{E}}. \quad \text{(24)}$$

Deflection of Semi-Beams and Semi-Girders.—The deflection of a semi-beam or semi-girder, loaded at one end, is double that of a beam of twice the length, loaded with twice the weight, at the middle:—comparing beams and semi-beams of the same principle of uniform strength, or of uniform section. Therefore, the deflection of a beam loaded at one end is ($2 \times 2 \times 2^3 =$) 32 times that of the same beam supported at both ends and loaded at the middle.

To find the deflection of semi-beams or semi-girders uniformly loaded; ascertain, first, the deflection as found by the ratio just stated, applicable when the load is applied at one point; secondly, multiply the deflection thus ascertained by the respective multipliers subjoined. The product is the deflection for a uniform load:—

	Multipliers for Uniform Loads. Rectangular section.	Double-flanged section.
(Fig. 156) For constant depth, uniform strength,...	.5	.5
(Fig. 155) For constant breadth, do. ...	.67	.76
(Fig. 149) For uniform section, ...	.625	.625

These multipliers have been deduced by the consideration of average stress combined with average depth, already employed for beams and girders of constant depth, and of constant breadth.

UNIFORM BEAMS SUPPORTED AT THREE OR MORE POINTS.

The distribution of weight of a continuous beam uniformly loaded on three or more points of support, at equal spans, is deducible from the laws that regulate the deflection of such a beam between the supports. Let the load per unit of length $= w$, and the length of the span $= l$; then the total load for one span $= w\, l$.

1. Beam of two equal spans, on three supports:—

Weight resting on 1st and 3d supports,			$= \frac{3}{8}\, w\, l.$
Do. do.	2d	do.	$= \frac{10}{8}\, w\, l.$

2. Beam of three equal spans, on four supports:—

Weight resting on 1st and 4th supports,			$= \frac{4}{10}\, w\, l.$
Do. do.	2d and 3d	do.	$= \frac{11}{10}\, w\, l.$

3. Beam of four equal spans, on five supports:—

Weight resting on 1st and 5th supports,			$= \frac{11}{28}\, w\, l.$
Do. do.	2d and 4th	do.	$= \frac{32}{28}\, w\, l.$
Do. do.	3d	do.	$= \frac{26}{28}\, w\, l.$

Deflection of Continuous Beams or Girders.—When a continuous girder, uniformly loaded, is supported at three points, by two equal spans, the middle portion is deflected downwards over the middle pier, and it sustains, by suspension, the extreme portions, which also have a bearing on the outer supports. The middle portion is, by deflection, convex upwards, and the outer portions are concave upwards; and there is a point of "contrary flexure," where the curvature is reversed, being at the junction of the convex and concave curves, at each side of the middle support. This point is distant from the middle pier, on each side, one-fourth of the span. Of the remaining three-fourths of each span, a half is carried by suspension by the middle portion, and a half is supported by the abutment. Hence, the distribution of the load on the supports is easily computed, as given above. The deflection of each span is to that of an independent beam of the same length of span, as 2 to 5.

In a beam of three equal spans, the deflection at the middle of either of the side spans is to that of an independent beam, as 13 to 25.

In a long continuous beam, supported at regular intervals, the deflection of each span is to that of an independent beam of one span, as 1 to 5.

TORSIONAL STRENGTH OF SHAFTS.

Solid Round Shaft.—When a solid round shaft is subjected to torsional stress, the centre is a neutral axis, about which the intensity and the leverage of the resistance each increase as the radius; and the two in combination, or the moment of resistance per square inch, increases as the square of the radius. Again, the ring, or annular area of surface, exposed to stress, increases as the radius; therefore, the moment of resistance for each ring is as the cube of the radius; and the total moment of resistance for shafts of different diameters, is as the cube of the radius, or of the diameter.

The radius of the resultant ring of resistance is the radius of gyration of the section, being the same as that of a circular plate revolving on its axis, namely, .7071 r, the radius being equal to r. (See page 289.) By reasoning analogous to that which was applied to the transverse resistance of beams, it is deducible that, whilst the resultant radius is .7071 r, the intensity of resistance over the whole sectional area of the shaft may be taken as equivalent to that of the resistance at the circumference. The ultimate moment of resistance is, then, expressed by the product of the sectional area of the shaft by the ultimate shearing resistance per square inch, and by the radius, and by .7071; that is to say, by

$$.7854\, d^2 \times \frac{d}{2} \times h \times 0.7071;$$

$$\text{or by } .278\, d^3 h, \quad \dots\dots \quad (a)$$

in which d = the diameter in inches, and h = the ultimate shearing resistance per square inch.

The moment of the load W is the product of the load by the radius R through which it is applied, or W R; and, W R = .278 $d^3 h$; or,

$$W = \frac{.278\, d^3 h}{R}, \quad \dots\dots \quad (1)$$

that is to say, the breaking force is equal to the product of the cube of the diameter by the ultimate shearing strength per square inch, and by .278, divided by the radius of the force. Also,

$$d = 1.534 \sqrt[3]{\frac{W R}{h}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$h = \frac{W R}{.278\, d^3}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Hollow Round Shafts.—The diagonal resistance to torsion is diminished by the hollowing of the shaft; and in the absence of experimental evidence, it will be provisionally assumed that the torsional strength is equal to that of a solid shaft of the same diameter, minus the resistance contributed by the imaginary core; though this assumption can afford but a rough approximation for a rule. The stress h' at the circumference of the core is less than the stress h at the outer circumference in proportion to the diameter, or $h : h' :: d : d'$, and $h' = h\frac{d'}{d}$. The resistance, W′, of the imaginary core, adapting formula (1), is

$$W' = \frac{.278\, d'^3 h'}{R}; \text{ and, by substitution for } h',\ W' = \frac{.278\, d'^4 h}{R\, d} \quad \ldots \quad (b)$$

The strength of the hollow shaft is, therefore, by deduction,

$$W = \frac{.278\, d^3 h}{R} - \frac{.278\, d'^4 h}{R\, d} = \frac{.278\, (d^4 h - d'^4 h)}{R\, d}; \text{ or,}$$

$$W = \frac{.278\, h\, (d^4 - d'^4)}{R\, d} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

That is to say: Multiply the difference of the 4th powers of the outer and inner diameters by the ultimate shearing strength per square inch, and by .278, and divide by the product of the outer diameter and the radius of the force. The quotient is the ultimate torsional strength of the hollow shaft.

2*d Method.*—When the section is comparatively thin, the material may be conceived to be collected at the circumference, for which the radius of gyration is equal to the radius of the shaft. Let t = the thickness, then the sectional area $= 3.14\, d \times t$, and $W = 3.14\, d \times t \times \frac{d}{2} \times h \div R$; or,

$$W = \frac{1.57\, d^2 t\, h}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Square Shafts.—The calculable moment of torsional resistance of a square shaft is greater than that of a round shaft having the same sectional area, since the corners of the square project farther from the centre than any portion of the circle. On the contrary, the material is less favourably disposed for resisting torsional stress, as the corners are comparatively unsupported. It may, therefore, be assumed that practically the torsional strength of a square shaft is equal to that of a round shaft having the same sectional area. The side of the square section is to the diameter of the

round section as 1 to 1.128; and putting b = the breadth of the side, and d = the diameter of the equivalent round shaft, $d = 1.128\ b$. Substitute this value of d in formula (1); then, $W = \frac{.278 \times (1.128\ b)^3 \times h}{R}$; or,

$$W = \frac{.4\ b^3 h}{R} \quad \text{.........................} \quad (6)$$

Inversely, the breadth of a square shaft having the ultimate torsional stress W R is, after reduction,

$$b = 1.36\ \sqrt[3]{\frac{W\,R}{h}} \quad \text{.......................} \quad (7)$$

That is to say, the breaking force (6) is equal to the product of the cube of the breadth of the shaft by the ultimate shearing strength per square inch, and by .4; divided by the radius of the force.

Also, the breadth of the shaft is equal to the cube root of the quotient obtained by dividing the product of the force and its radius by the shearing strength, multiplied by 1.36.

Torsional Deflection.

When a round shaft is twisted by torsional stress, the angular deflection within the elastic limit is approximately proportional to the twisting force, and to the length of the shaft. Let

d = the diameter of the shaft,
l = the length of the shaft subjected to torsion,
h = the shearing stress at the circumference in tons per square inch within the elastic limits,
D = the total angular deflection of the shaft subjected to torsion, expressed in parts of one revolution,
E′ = the coefficient of elasticity, being the denominator of the fraction of the length by which the circumference of the shaft is deflected, or the ratio of the length to the circumferential arc of deflection per ton of shearing stress per square inch at the circumference.
R = the radius of the force.
W = the twisting force in tons.

The total circumferential deflection is equal to

$$l \times \frac{1}{E'} \times h = \frac{l\,h}{E'}; \quad \text{.......................} \quad (c)$$

and if the circumferential arc of deflection be divided by the circumference, equal to $3.1416\ d$, the quotient is the angular deflection, or

$$D = \frac{l\,h}{3.1416\ d\,E'} \quad \text{.........................} \quad (8)$$

and

$$h = \frac{3.1416\ d\,E'\,D}{l} \quad \text{......................} \quad (9)$$

Equating this value of h and the previous value of h, (3).

$$\frac{3.1416\, d\, E'\, D}{l} = \frac{W\, R}{.278\, d^3}; \text{ and } W\, R\, l = .873\, d^4\, E'\, D;$$

whence

$$D = \frac{W\, R\, l}{.873\, d^4\, E'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$W = \frac{.873\, d^4\, E'\, D}{R\, l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$E' = \frac{W\, R\, l}{.873\, d^4\, D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

These equations express the relations of the weight or force, the coefficient of elasticity, and the deflection within the elastic limits. The formula (10) for the deflection signifies that the deflection varies directly as the force, and as the radius of the force, or, jointly, as the moment of the force; and as the length of the shaft; and that it varies inversely as the 4th power of the diameter, and as the coefficient of torsional elasticity.

The torsional deflection of a square shaft may be found by means of the same formula, substituting, for calculation, a round shaft of equivalent strength.

The torsional deflection of round and square shafts varies with the diameter in the same ratio as the transverse deflection; namely, as the 4th power.

Hollow Round Shafts.—Equating the value of h, obtained by inversion of formula (4), and the value in equation (9):—

$$\frac{W\, R\, d}{.278\, (d^4 - d'^4)} = \frac{3.1416\, d\, E'\, D}{l}; \text{ and } W\, R\, l = .873\, (d^4 - d'^4)\, E'\, D;$$

whence

$$D = \frac{W\, R\, l}{.873\, (d^4 - d'^4)\, E'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

STRENGTH OF TIMBER.

A number of delicate experiments, described by M. Morin,[1] were made by various experimentalists, with specimens of wood of different kinds, uniform in texture, of very small scantling, and on very wide spans, loaded at the middle. It was satisfactorily proved by the results of these experiments: 1st, that the deflection was sensibly proportional to the load; 2d, that the compression and extension were nearly the same, though the compression was slightly the less; 3d, that, to produce equal deflections, the load when placed on the middle, was to the load when uniformly distributed, as .638 to 1, or as 5 to 7.84; 4th, that the deflections under equal loads were inversely as the breadths, inversely as the cubes of the depths, and directly as the cubes of the spans.

Thus, the correctness of the principles of the deflection of beams under transverse stress is established by the results of most carefully conducted experiments; though in ordinary practice, no doubt, there are, in individual instances, considerable degrees of divergence from those laws of deflection in the behaviour of timber, which are attributable to the want of uniformity of structure.

[1] *Résistance des Matériaux.*

MM. Chevandier & Wertheim, who have made many experiments on the strength of timber, arrived at the following general conclusions:—1st. That the density of wood varies very little with the age. 2d. That the coefficient of elasticity diminishes after a certain age; and that it depends also on the dryness and the aspect of the ground where the wood is grown. Woods from a northerly aspect, on dry ground, have always a high coefficient, whilst woods from swampy districts have the lowest coefficients. 3d. That the cohesive strength is influenced by the age and the aspect. 4th. The coefficients of elasticity of trees cut down in full vigour, and of those cut down before they arrive at this condition, do not present any sensible difference. 5th. That there is no limit of elasticity, properly so called, in wood. There is a permanent set for every elastic extension.

A condensed table of the results of their experiments on the elastic and absolute strength of timbers, is given in the section on the elastic strength of timber. It may be added that the same woods were tested for tensile strength, in directions at right angles to the length of the trees, in a radial line, and in a line tangential to the annular layers. The average ultimate strengths were as follows:—

Parallel to the axis of the tree,...	3.08	tons per square inch, or as	1
Radially...........................	.305	" " "	$^1/_{10}$
Tangentially......................	.323	" " "	$^1/_{10.5}$

Mr. Laslett's Experiments.

The recently published results of Mr. Laslett's experiments on the strength of timber, afford valuable data for the ultimate strength of timbers.[1] The specimens tested for tensile and transverse strength were 2 inches square. For transverse strength they were 7 feet long, on a 6-foot span, with the load applied at the middle; and for tensile strength they had usually a clear length of 30 inches. The specimens tested for crushing, or compressive strength, consisted of cubes of from 1 to 4 inches, and of pieces 2 inches square and upwards, of various lengths.

English Oak.—Twelve specimens were cut side by side, in a line diametrically across one tree. Six on one side of the centre came out with a long clean straight grain; six on the other side had a wavy and twisted grain with a short fibre. They were tested for transverse strength; the breaking weight varied from 390 lbs. to 740 lbs., and the ultimate deflection from 3.5 to 7 inches.

	Straight Grain.	Wavy Grain.	Together.
Average breaking weight...........	562 lbs.	407 lbs.	484 lbs.
Average ultimate deflection........	5.10 in.	3.95 in.	4.52 in.
Average specific gravity............	.858	.867	.862

The stronger half-dozen specimens were afterwards tested for tensile strength. The results are given in table No. 170, together with a selection of results of specimens from two trees of average quality, fairly seasoned. The tensile strength is shown by the table to increase with the specific gravity; and it ranges from 1 ton to 4 tons per square inch. The trans-

[1] *Timber and Timber Trees, Native and Foreign.* By Thomas Laslett, Timber Inspector to the Admiralty. 1875. The data extracted from Mr. Laslett's work are here published by permission of the proprietors of the work.

verse strength varies proportionally as from 1 to 2.27; and the deflection under 390 lbs., from 1.5 to 4 inches, or as 1 to 2.63.

Table No. 170.—TRANSVERSE AND TENSILE STRENGTH OF ENGLISH OAK.

(Reduced from Mr. Laslett's Experiments.)

First—Specimens cut from one side of a tree.

No. of Specimen, 2 inches square. Span, 6 feet.	Specific Gravity.	Transverse Strength.				Tensile Strength per Square inch.
		Deflection under 390 lbs.	Set for 390 lbs.	Ultimate Deflection.	Breaking Weight.	
		inches.	inches.	inches.	lbs.	lbs. tons.
1 (next centre)......	.900	2.00	—	7.00	740	5,320 or 2.375
2......................	.900	2.00	—	4.50	630	4,400 „ 1.964
3......................	.854	2.25	—	5.00	620	4,200 „ 1.875
4......................	.864	3.50	—	4.50	470	4,340 „ 1.938
5......................	.838	3.75	—	5.00	480	2,520 „ 1.125
6 (outside)............	.791	4.00	—	4.50	430	2,240 „ 1.000
Averages.........	.858	2.916	—	5.10	562	3,837 or 1.713
Second—From two trees, good average quality, moderately seasoned.						
	1.003	1.75	.000	9.25	882	8,890 or 3.969
	1.005	1.625	.125	9.50	977	7,840 „ 3.500
	1.002	1.50	.000	8.75	827	8,400 „ 3.750
	.905	3.50	.200	5.25	590	8,260 „ 3.687
	.720	3.50	.250	7.00	804	6,160 „ 2.750
	.725	3.25	.125	6.50	797	5,880 „ 2.625
Averages.........	.893	2.524	.117	7.71	813	7,571 or 3.380
Total Averages.	.876	2.720	—	6.40	688	5,704 or 2.546

Dantzic Fir.—In a series of six experiments for transverse strength—

The specific gravity varied from	.478 to .673;	average,	.582
The deflection under 390 lbs.	„ 1.25 to 2.25;	„	1.63 inches.
The set under 390 lbs.	„ .000 to .100;	„	.066 „
Ultimate deflection	„ 4.50 to 6.15;	„	5.14 „
Breaking weight	„ 700 to 970;	„	877 lbs.

In experiments for tensile strength—

The specific gravity varied from	.512 to .673;	average	.603
The breaking weight per square inch	1.0 to 2.0;	„	1.5 tons.

The resistance to crushing of 1-inch, 2-inch, 3-inch, and 4-inch cubes of various woods was practically the same per square inch of surface for the different sizes of cube; though there was in general a slight difference in favour of the smaller cubes.

The table No. 171, compiled from the results of Mr. Laslett's experiments, shows the average transverse strength and tensile strength of various woods, hard and soft; and table No. 172 shows their compressive strength, or the resistance of cubes to crushing:—

Table No. 171.—Transverse and Tensile Strength of Timber.

(Reduced from Mr. Laslett's data.)

The specimens for transverse strength were supported at both ends and loaded at the middle.

Name of Timber. Specimens, 2 inches square.	Transverse Strength.					Tensile Strength.		
	Average Specific Gravity.	Deflection. Span, 6 feet. Load, 390 lbs.	Set.	Ultimate Deflection.	Breaking Weight.	Average Specific Gravity.	Breaking Weight per square inch.	
		inches.	inches.	inches.	lbs.		lbs.	tons.
Oak:—								
English, one side of tree	.858	2.92	—	5.10	562	.858	3,837	1.713
English, other side of tree	.867	3.25	—	3.95	407	—	—	—
Do.	.893	2.52	.117	7.71	813	.893	7,571	3.380
French	.976	1.48	.041	6.00	877	.976	8,102	3.617
Do.	1.082	1.58	.125	7.58	831	—	—	—
Tuscan	1.040	3.76	.113	7.66	758	—	—	—
Sardinian	.990	2.61	.125	6.50	758	—	—	—
Dantzic	.836	5.00	.240	6.46	474	.838	4,217	1.882
Spanish	1.042	4.03	.250	6.62	562	—	—	—
American White	.983	1.92	.208	8.83	804	.969	7,021	3.143
Do. Baltimore	.747	1.47	.191	7.13	723	.742	3,832	1.443
African (or teak)	.993	2.25	.050	5.14	1,108	.971	7,052	3.148
Teak, Moulmein	.776	1.65	.083	3.38	913	.777	3,301	1.474
Do.	.809	1.94	.083	6.49	843	—	—	—
Iron Wood, Burmah	1.176	.96	.033	4.25	1,273	1.176	9,656	4.311
Chow, Borneo	1.116	.92	.025	2.83	975	1.134	7,199	3.214
Greenheart, Guiana	1.150	2.15	.066	4.62	1,333	1.141	8,820	3.937
Sabicu, Cuba	.917	.96	.033	3.75	1,293	.917	5,558	2.481
Mahogany, Spanish	.769	1.208	.025	3.45	856	.765	3,791	1.692
Honduras	.659	1.916	.083	4.06	802	.659	2,998	1.338
Mexican	.678	1.125	.058	3.92	783	.655	3,427	1.530
Eucalyptus, Australia:—								
Tewart	1.169	1.27	.108	4.75	1,029	1.169	10,284	4.591
Mahogany	1.010	3.21	.133	4.71	686	.996	2,940	1.312
Iron-Bark	1.142	.94	.000	3.81	1,407	1.150	8,377	3.740
Blue Gum	1.029	1.26	.100	4.21	712	1.049	6,048	2.700
Ash, English	.736	1.62	.050	8.63	862	.750	3,780	1.687
Canadian	.480	2.75	.125	7.37	638	.588	5,495	2.453
Beech	—	—	—	—	—	.705	4,853	2.166
Elm, English	.558	4.90	1.300	5.29	393	.642	5,460	2.437
Rock Elm, Canada	.748	1.75	.290	8.79	920	.748	9,182	4.100
Hornbeam, England	—	—	—	—	—	.819	6,405	2.860
Fir, Dantzic	.582	1.63	.066	5.14	877	.603	3,231	1.442
Riga	.541	1.29	.092	3.63	600	.553	4,051	1.808
Spruce, Canada	.484	1.23	.055	5.19	670	.484	3,934	1.756
Larch, Russia	.646	1.57	.175	4.33	626	.649	4,203	1.876
Cedar, Cuba	.439	2.27	.258	4.37	560	.469	2,870	1.281
Red Pine, Canada	.554	1.67	.133	4.63	653	.553	2,705	1.207
Yellow Pine, Canada	.435	2.12	1.833	4.66	627	—	—	—
Do. do.	.551	1.71	.714	3.39	483	.551	2,759	1.231
Do. do.	.552	2.09	.706	3.45	304	.552	2,259	1.008
Pitch Pine, American	.659	1.12	.075	4.79	1,049	.659	4,666	2.083
Do. do.	.710	1.24	.063	4.67	930	—	—	—
Do. do.	.538	1.42	.104	4.42	744	—	—	—
Kauri Pine, New Zealand	.550	1.39	.125	4.00	719	.544	4,040	1.803

Table No. 172.—CRUSHING RESISTANCE OR COMPRESSIVE STRENGTH OF TIMBER.

(Reduced from Mr. Laslett's data.)

Name of Timber. Specimens—1-inch, 2-inch, 3-inch, and 4-inch cubes.	Average Resistance per square inch.	Name of Timber. Specimens—1-inch, 2-inch, 3-inch, and 4-inch cubes.	Average Resistance per square inch.
	tons.		tons.
Oak, English (unseasoned)	2.194	Eucalyptus, Mahogany......	3.198
Do. (seasoned)...	3.337	Iron-Bark......	4.601
French..................	3.547	Blue Gum......	3.078
Tuscan	2.437	Ash, English..................	3.109
Sardinian	2.604	Canadian.................	2.453
Dantzic..................	3.344	Elm, English..................	2.583
American, White......	2.709	Rock	3.832
Do. Baltimore	2.630	Hornbeam......................	3.711
Teak, Moulmein..............	2.559	Fir, Dantzic....................	3.102
Iron Wood.....................	5.208	Riga	2.342
Chow............................	5.621	Spruce	2.166
Greenheart......................	6.438	Larch.............................	2.596
Sabicu...........................	3.776	Cedar............................	2.000
Mahogany, Spanish.........	2.863	Red Pine........................	2.537
Honduras.......	2.853	Yellow Pine....................	1.877
Mexican.........	2.503	Pitch Pine......................	2.885
Eucalyptus, Tewart..........	4.174	Kauri............................	2.867

CRUSHING RESISTANCE OF COLUMNS OF WOOD.

English Oak, 3 inches square:—

			per square inch.
Unseasoned, 9 specimens,	8 to 16 inch. high,	spec. grav. .922,	1.68 tons.
Seasoned, 2 do.,	17 and 18 „	„ „ .778,	2.52 „
Average of 4 specimens, 6 inches square,	12 to 36 inches high,		3.68 „
Do. 4 do., 9 „ „	12 to 21 „ „		2.85 „
One specimen, ... 9 × 10 inches,	24 „ „		3.24 „
Two specimens,........... 10 × 11 „	18 and 21 „ „		2.72, 2.91 „

Indian Teak:—

6 inches square, specific gravity .795 .... 4.38 „
9 „ „ „ .838 .... 3.81 „

Dantzic Fir, under 30 inches high, average results:—

6 inches square, specific gravity .600 .... 3.897 „
9 × 10 inches, do. .608 .... 2.562 „
10 inches square, do. .660 .... 1.812 „
10 „ „ do. .563 .... 2.446 „

English Oak and Fir of considerable length in proportion to the scantling:—

Length of Specimens.	English Oak, 2 inches square.		English Oak, 4 inches square.		Dantzic Fir, 2 inches square.		Riga Fir, 2 inches square.	
	Specific Gravity.	Crushing Weight per square inch.	Specific Gravity.	Crushing Weight per square inch.	Specific Gravity.	Crushing Weight per square inch	Specific Gravity.	Crushing Weight per square inch.
inches.		tons.		tons.		tons.		tons.
1	.740	3.37	—	—	.756	2.72	—	2.47
2	"	3.41	—	—	.756	3.17	—	2.11
3	"	3.47	—	—	.720	2.97	—	2.88
4	"	3.50	—	—	.756	3.44	—	2.25
5	"	3.94	—	—	.669	3.44	—	2.63
6	"	3.72	—	—	.648	3.25	—	2.81
7	"	3.69	—	—	.617	3.19	—	2.78
8	"	3.63	—	—	.621	3.03	—	2.75
9	"	3.75	—	—	.720	3.03	—	2.50
10	"	slipped.	—	—	.669	3.13	—	2.00
11	"	3.69	—	—	.726	2.91	—	2.44
12	.720	3.44	—	—	.774	3.00	—	2.78
15	—	—	.958	1.60	—	—	—	—
16	—	—	.972	1.58	—	—	—	—
17	—	—	.934	1.69	—	—	—	—
18	.720	2.75	.930	1.72	.636	2.88	—	2.47
19	—	—	.932	1.76	—	—	—	—
20	—	—	.972	1.76	—	—	—	—
21	—	—	.946	1.75	—	—	—	—
22	—	—	.932	1.63	—	—	—	—
23	—	—	.921	1.47	—	—	—	—
24	.720	2.63	1.003	1.88	.684	2.72	—	1.72
30	.734	2.44	—	—	.662	2.63	—	1.84

Mr. Fincham's Experiments on the Transverse Strength of Soft Woods.[1]

Mr. Fincham made many experiments on 3-inch square scantlings, at spans of 4 feet, of wood of three degrees of seasoning:—"green" wood, "dry" wood, and "very dry and particularly good" wood. Table No. 173 contains results of his experiments on very dry wood, to which are added the average results for the same woods "green" and "dry."

These results show that the ultimate strength of the woods is the same whether green or dry, but that the stiffness is materially increased by thorough drying. It seems from the experiments that the elastic limit of strength of dry wood is about a half of the breaking strength, and that the deflection is about ½ inch for a load of .75 ton, or .64 inch per ton.

Transverse Strength of Beams of Large Scantling.

Mr. H. H. Maclure made experiments on the transverse strength of Memel fir, supported at both ends, and loaded at the middle; for the results of which see table No. 174.

[1] Papers on Naval Architecture, vol. i.

Table No. 173.—TRANSVERSE STRENGTH OF SOFT WOODS (VERY DRY).

(Reduced from Mr. Fincham's tables.)

Specimens 3 inches square, 4 feet span; loaded at the middle.

Description of Timber.	Specific Gravity.	Load 1680 lbs., or .75 ton.		Load 2520 lbs., or 1.125 tons.		Load 2520 lbs., or 1.125 tons. After 1 hour's pressure.		Breaking Weight.
		Defl'tion.	Set.	Defl'tion.	Set.	Defl'tion.	Set.	
		inch.	inch.	inch.	inch.	inch.	inch.	lbs. tons.
Riga Fir........	.610	.25	.00	.37	.04	.40	.07	4530, or 2.022
Red Pine.......	.544	.36	.01	.68	.06	.86	.08	3780, or 1.688
Yellow Pine...	.439	.37	.07	.78	.06	1.00	.18	2756, or 1.230
Norway Fir....	.517	.31	.01	.61	.01	.86	.23	3292, or 1.470
Scotch Pine ...	.453	.62	.02	.93	.03	—	—	2520, or 1.125
Kauri............	.579	.29	.00	.46	.02	.50	.05	4110, or 1.835
Average Results for the above Six Woods, under different conditions.								
Green, Top....	.704	.75	.08	.94	.13	1.09	.22	3431, or 1.532
Do., Butt....	.645	.54	.06	.75	.04	1.35	.70	3746, or 1.672
Dry, Top	.466	.58	.04	.92	.03	1.04	.09	3050, or 1.361
Do., Butt	.541	.48	.02	.70	.10	.95	.16	2945, or 1.315
Very dry.......	.524	.37	.02	.64	.04	.72	.12	3498, or 1.561
Total averages	.576	.54	.04	.79	.07	1.03	.26	3334, or 1.488

Table No. 174.—TRANSVERSE STRENGTH OF MEMEL FIR, 1849.

(Reduced from Mr. H. H. Maclure's data.)

Breadth and Depth.	Span.	Breaking Weight.	Ultimate Deflection.	Calculated Tensile Strength per square inch, by formula (2), page 507.
inches. inches.	inches.	pounds. tons.	inches.	tons.
1 × 1	16	483, or .215	.75	2.978
1 × 1	16	450, or .201	.75	2.784
2 × 2	32	1910, or .853	1.00	2.953
2 × 2	32	1311, or .584	1.125	2.023
	feet.			
3 × 3	9	1104, or .493	3.5	1.707
3 × 3	9	1482, or .661	4.5	2.289
6 × 12	12	— 15.5	2.0	2.222
9 × 12	12	— 17.0	2.5	1.635
12 × 12	12	— 27.5	3.25	1.992

Mr. Edwin Clark tested the transverse strength of red pine of large scantling selected from the scaffolding employed in constructing the Britannia Bridge:—two whole balks 17 feet long, and a piece cut from the centre of a balk.

Mr. C. Graham Smith gives the results of tests for transverse strength of

pine timber of large scantling at Liverpool. The pieces were selected as average samples from cargoes.[1]

The table No. 175 contains the leading results of the experiments of Mr. E. Clark and Mr. C. G. Smith.

Table No. 175.—TRANSVERSE STRENGTH OF PINE AND FIR.

(Reduced and arranged from the experiments of Mr. Edwin Clark, and of Mr. C. Graham Smith.)

(Mr. Edwin Clark.)

Breadth and Depth.	Span.	Application of the Load.	Elastic Strength.	Elastic Deflection.	Breaking Weight.	Ultimate Deflection.	Ratio of Elastic to Breaking Weight.
inches.	feet.		tons.	inches.	tons.	inches.	per cent.
American Red Pine							
1. 12 × 12 (Sp. gr., .509)	15	Centre.	9.0	1.00	14.82	4.00	61
2. 12 × 12 (Sp. gr., .543)	15	Do.	9.0	1.25	13.24	3.10	68
3. 6 × 6	7.5	Do.	2.0	.62	3.29	1.68	61
(Mr. C. G. Smith.)							
Memel Fir.							
4. 13.5 × 13.5 (from the butt)	10.5	Distributed.	38.0	.37	61.00	—	62
5. 13.5 × 13.5 (from the top)	10.5	Do.	38.0	.51	61.00	—	62
Baltic Fir.							
6. 6 × 12	12.25	Centre.	6.0	.66	8.50	1.11+	75
7. 6 × 12	12.25	Do.	6.0	.72	10.50	1.93+	57
Pitch Pine.							
8. 6 × 12	12.25	Do.	5.0	.28	10.2	1.31	50
9. 6 × 12	12.25	Do.	8.0	.97	10.5	1.31+	76
10. 14 × 15	10.5	Do.	40.0	.49	60.0	1.14	67
11. 14 × 15	10.5	Do.	35.0	.49	59.2	—	59
Red Pine.							
12. 6 × 12	12.25	Do.	5.0	.70	7.5	—	67
13. 6 × 12	12.25	Do.	5.0	.70	8.5	1.94+	59
Quebec Yellow Pine.							
14. 14 × 15	10.5	Distributed.	35.0	.39	61.0	—	58
15. 14 × 15	10.5	Do.	35.0	.39	61.0	—	58
16. 14 × 15	10.5	Centre.	30.0	.56	38.3	—	78
17. 14 × 15	10.5	Do.	—	—	34.0	—	—

Three beams of oak, mentioned by Mr. Baker,[2] appear to have been broken transversely by the following loads at the middle:—

1. 1 inch square × 2 feet span............................ .212 tons breaking weight.
2. 8½ inches square × 11 feet 9 inches span......... 14.365 " "
3. 10⅔ in. wide × 12¼ in. deep, 24 ft. 6 in. span.. 8.780 " "

[1] See Mr. Smith's paper on *Pine Timber*, read before the students of the Institution of Civil Engineers in 1875, and published in *Engineering*, vol. xix. page 392.

[2] *On the Strengths of Beams, Columns, and Arches.* 1870.

MM. Chevandier and Wertheim tested the transverse strength of rectangular beams of fir and oak from the Vosges.[1]

Table No. 176.—Transverse Strength of Fir and Oak from the Vosges.

(Reduced from MM. Chevandier and Wertheim's data.)

Vosges Timber. Specific Gravity.	Breadth and Depth.	Span.	Breaking Weight at the middle.
Fir.	inches. inches.	feet.	pounds. tons.
.530	11.4 × 12.8	42.64	14,120, or 6.30
.506	10.0 × 11.2	36.08	11,867, or 5.30
.548	8.8 × 9.6	29.52	7,584, or 3.38
.525	6.7 × 7.7	29.52	4,580, or 2.04
.481	3.65 × 4.85	29.52	1,137, or .508
.493	9.7 × 2.16	9.91	2,017, or .900
.479	9.5 × 1.11	9.91	581, or .260
Oak.			
1.008	9.2 × 10.9	18.04	17,356, or 7.75
.958	8.6 × 9.3	18.04	15,816, or 7.06
.922	7.6 × 8.6	18.04	11,495, or 5.23
.928	6.3 × 7.4	18.04	12,155, or 5.43
.985	5.4 × 6.3	18.04	4,895, or 2.19
.636	3.26 × 3.20	9.84	1,188, or .530
.759	3.07 × 3.16	8.20	1,617, or .722
.685	11.5 × 2.15	18.04	957, or .427
.824	5.64 × 1.66	9.84	825, or .368
.712	9.5 × 1.11	9.84	715, or .319

Elastic Strength and Deflection of Timber.

Reverting to the conclusions of MM. Chevandier and Wertheim, on the strength and elasticity of timber, page 538, these experimentalists found that there was no limit of elasticity, properly so called, in wood; though there was a permanent set for every elastic extension. They, nevertheless, adopted empirically, as the limit of elasticity for tensile strength, the point at which a set of $^1/_{20,000}$th of the length is acquired. This is a fanciful distinction, for a set of 1 in 20,000 parts may be simply the effect of a straightening of the fibres. With this explanation, the following table, No. 177, of the tensile strength of timbers, condensed from their tables, is of some value; although the fractions of extension in the second last column are scarcely consistent with the results of the scanty experiments of others.

[1] Morin's *Résistance des Matériaux.*

Table No. 177.—TENSILE STRENGTH OF TIMBER.

(Reduced from the tables of MM. Chevandier and Wertheim.)

	1st SERIES.—Comparative Elastic Strength per ton per square inch, taken when the set is 1 in 20,000.			2d SERIES.—Elastic and Ultimate Strength.			
	Green Wood.	Wood Dried. In closed premises.	Wood Dried. In the air and the sun.	Specific Gravity.	Elastic Strength, when set is 1 in 20,000. Total per square inch.	Elastic Strength, when set is 1 in 20,000. Extension per ton per square inch in parts of the length.	Ultimate Strength per square inch.
	tons.	tons.	tons.		tons.		tons.
Acacia	—	2.016	2.024	.717	2.024	1/801	4.978
Fir	—	1.014	1.367	.493	1.367	1/707	2.654
Hornbeam...	.814	—	—	.756	.814	1/690	1.899
Birch	.483	—	1.027	.812	1.027	1/632	1.369
Beech........	—	1.281	1.471	.822	1.471	1/623	2.267
Oak...........	—	—	—	.808	—	1/621	4.121
Do............	—	1.229	1.491	.872	1.491	1/585	3.594
Pine...........	—	.883	1.037	.559	1.037	1/356	1.575
Elm	.627	—	1.170	.723	1.170	1/740	4.439
Sycamore....	1.046	—	1.462	.692	.723	1/739	3.912
Ash	1.096	—	1.288	.697	.728	1/712	4.305
Alder.........	.920	—	1.149	.601	.712	1/704	2.883
Aspen........	1.462	—	1.957	.602	.657	1/683	4.572
Maple........	—	—	1.724	.674	.678	1/648	2.273
Poplar	—	.762	.942	.477	.639	1/328	1.240

The following are the results of experiments by Mr. Laslett on the elongation of hard woods under tensile stress. The specimens were 2 inches square; length, 36 inches. The elastic limit reached up to, or nearly to, the breaking point:—

WOOD.	Elastic Strength, per square inch.	Breaking Weight, per square inch.	ELASTIC EXTENSION. Total.	ELASTIC EXTENSION. Per ton per square inch.	ELASTIC EXTENSION. Fraction of length.
	tons.	tons.	inch.	inch.	
English oak.....	2.75	—	.25	.091	1/396
Dantzic oak	2.66	2.66	.25	.094	1/383
Indian teak......	1.75	1.92	.18	.108	1/333

The following are the chief results of tests by Mr. Kirkaldy of the compressive resistance of two balks of fir—White Riga and Red Dantzic, about 13 inches square, and 20 feet long, with square ends, in a horizontal position. They were "not very dry." The limits of the elastic strengths are taken at the points where the rate of compression for equal increments of pressure became accelerated.

Compression.	White Riga.	Red Dantzic.
Elastic strength	133.9 tons.	111.6 tons.
Do. per square inch	.792 "	.627 "
Breaking strength	147.9 "	138.0 "
Do. per square inch	.875 "	.775 "
Ratio of elastic to breaking strength	90 per cent.	81 per cent.
Elastic compression	.523 inch.	.414 inch.
Do. per ton per square inch, in parts of the length	$^1/_{364}$	$^1/_{364}$
Final compression before rupture	.642 inch.	.548 inch.
Set under total elastic stress	.022 "	.02 "

Mr. Barlow's Experiments on Transverse Strength, 1837.[1]

Mr. Barlow made a number of experiments to test the transverse deflection and strength of timber of average quality, taken as seasoned, from the stores in Woolwich dockyard. The specimens were 2 inches square, and tested on a span of 7 feet, except a few which were tested on a span of 6 feet. The average ratio of the elastic to the breaking strength is, from the table, 31 per cent.; but Mr. Barlow has not stated the conditions of the elastic limit prescribed by him.

Table No. 178.—Elastic Transverse Strength of Timber.

(Condensed and adapted from Mr. Barlow's experiments.)

Specimens 2 inches square, 7 feet span; loaded at the middle.

Name of Timber.	Specific Gravity.	Elastic Strength.		Breaking Weight.		Ratio of Elastic to Breaking Strength.
		Weight.	Deflection.	Weight.	Deflection.	
		pounds.	inches.	pounds.	inches.	per cent.
Teak	.745	300	1.151	938	4.32	32
Poon	.579	150	.822	846	5.92	17.7
English oak	.969	150	1.590	450	5.92	33
Do.	.934	200	1.280	637	8.10	31.4
Canadian oak	.872	225	1.080	673	6.00	33.4
Dantzic oak	.756	200	1.590	560	4.86	36
Adriatic oak	.993	150	1.430	526	5.73	28.5
Ash	.760	225	1.266	772	8.92	29
Beech	.696	150	1.026	593	5.73	25
Elm	.553	125	1.685	386	6.93	32.4
Pitch pine	.660	150	1.134	622	6.00	24
Red pine	.657	150	.755	511	5.83	29
New England fir	.553	150	.931	420	4.66	36
Riga fir	.753	125	.870	422	6.00	30
Do. (span 6 feet)	.738	150	.883	467	6.00	32
Mar Forest fir	.696	125	1.442	436	6.00	29
Do. (span 6 feet)	.693	150	1.006	561	6.42	27
Do. "	.703	150	1.006	561	6.42	27
Larch	.531	125	1.885	325	8.58	38
Do. (span 6 feet)	.522	125	.812	370	5.00	34
Do. "	.556	150	.831	501	5.00	30
Do. "	.560	150	.831	510	5.00	30
Norway spar (span 6 feet)	.577	200	.800	655	4.00	30

[1] *On the Strength of Materials;* edition of 1845.

RULES FOR THE STRENGTH AND DEFLECTION OF TIMBER.

The results of Mr. Laslett's experiments, tables Nos. 171 and 172, throw some light on the relations of the tensile, compressive, and transverse strength of timber. Employing formula (2), page 507, namely,

$$s = \frac{W\,l}{1.155\,b\,d^2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

to calculate the direct tensile strength of the specimens, the results may be classified as follows:—

Calculated Tensile Strength WITHIN 5 PER CENT. *of the Experimental Strength.*

SIX HARD WOODS.	Transverse Breaking Weight.	Tensile Strength, calculated.	Tensile Strength, experimental.	Compressive Strength, experimental.
	lbs.	tons.	tons.	tons.
English Oak (mean)	687	2.392	2.546	3.337
Iron Wood......................	1,273	4.428	4.311	5.208
Chow	975	3.392	3.214	5.621
Iron Bark........................	1,407	4.000	3.740	4.601
Blue Gum........................	712	2.477	2.700	3.078
Canadian Ash...................	638	2.219	2.453	2.453
Averages (hard woods)...	949	3.151	3.161	3.615

Calculated Tensile Strength MUCH GREATER *than Experimental Strength.*

EIGHT HARD WOODS.—Baltimore oak, African teak, Moulmein teak, greenheart, sabicu, average of American mahoganies, Eucalyptus mahogany, English ash:—

Averages,	967	3.354	2.120	3.493

NINE SOFT WOODS.—Dantzic fir, Riga fir, spruce fir, larch, cedar, red pine, yellow pine, pitch pine, Kauri pine:—

683	2.375	1.597	2.486

Calculated Tensile Strength MUCH LESS *than Experimental Strength.*

SIX HARD WOODS.—French oak, Dantzic oak, American white oak, Eucalyptus Tewart, English elm, Rock elm:—

750	2.607	3.295	3.365

Averages of the Twenty Hard Woods preceding:—

896	3.069	2.785	3.490

Averages of the Nine Soft Woods preceding:—

683	2.375	1.597	2.486

Averages of Twenty-nine Woods, Hard and Soft:—

830	2.853	2.416	3.168

This analysis shows that for only six out of twenty-nine woods does the formula (1) give the experimental tensile strength in terms of the transverse strength; and these are all hard woods. For the remainder of the woods, comprising the soft woods, the formula (1) shows a tensile strength varying extremely, both by excess and by deficiency, from the experimental strength; and for all the soft woods the calculated tensile strength is far in excess of the experimental tensile strength. In every instance the experimental compressive strength is greater than the experimental tensile strength —for the soft woods much greater;—and the calculated tensile strength excepting for six hard woods, lies between these values. It is, therefore, to be inferred that the transverse strength is a function of the compressive strength as well as of the tensile strength; and that it would be safe to calculate the transverse strength in terms of the mean of the tensile and compressive strengths, supposing that these values can be truly averaged for large scantlings.

Calculating likewise the tensile strength of the pieces of soft woods tested for transverse strength by Mr. Fincham, table No. 173, page 543, they are as follows:—

			Calculated Tensile Strength.
Soft woods,	six specimens,	green, top	2.358 tons.
Do.	do.	green, butt	2.573 „
Do.	do.	dry, top	2.095 „
Do.	do.	dry, butt	2.024 „
Do.	do	very dry	2.403 „
Average,			2.290 „
Average from Mr. Laslett's experiments on soft woods,			2.375 „

showing a fair accord between the two calculated tensile strengths; though Mr. Fincham's 3-inch square specimens give a lower value than Mr. Laslett's 2-inch square specimens.

Calculated Tensile Strength of Timber of Large Scantling.

Selecting the experimental results for the transverse strength of beams of larger scantling, from six inches square upwards, the calculated tensile strengths, by formula (1), averaged for each set of specimens, are as follows:—

					Calculated Tensile Strength.
Maclure,	last 3 pieces,	table No. 174,	page 543,	Memel fir	1.950 tons.
Smith,	2 „	„ 175,	„ 544,	Do. „	1.334 „
Smith,	2 „	„ 175,	„ 544,	Baltic „	1.400 „
Chevandier,	4 „	„ 176,	„ 545,	Vosges „	1.483 „
				Average for Fir,	1.542 „
E. Clark,	3 pieces,	table No. 175,	page 544,	Red pine	1.240 tons.
Smith,	2 „	„ 175,	„ 544,	Do.	1.163 „
				Average for Red Pine,	1.202 „
Smith,	4 pieces,	table No. 175,	page 544,	Quebec yellow pine	1.200 tons.
Smith,	2 „	„ 175,	„ 544,	Pitch pine	1.834 „
Baker,	2 „	—	„ 544,	English oak	1.416 „
Chevandier,	5 „	„ 176,	„ 545,	Vosges oak	1.943 „

Formulas for the Transverse Strength of Timber of Large Scantling.

Adopting the foregoing data as the proper values of s, the tensile strength in tons per square inch, in the general formula (1), page 507, as applied to find the breaking weight of timber beams of considerable scantling, the numerical constant, 1.155 s, for each is obtained:—

Ultimate Transverse Strength of Timber of Large Scantling, loaded at the middle.

Fir........................... $W=\frac{1.78\,b\,d^2}{l}$ .... (2)

Red pine..................... $W=\frac{1.39\,b\,d^2}{l}$ .... (3)

Quebec yellow pine........ $W=\frac{1.39\,b\,d^2}{l}$ .... (4)

Pitch pine.................... $W=\frac{2.12\,b\,d^2}{l}$ .... (5)

English oak .... $W=\frac{1.64\,b\,d^2}{l}$ .... (6)

French oak.................. $W=\frac{2.24\,b\,d^2}{l}$ .... (7)

W = the breaking weight, in tons; b the breadth, d the depth, and l the span, all in inches.

For other timbers, in the absence of direct experimental data, formulas may be deduced for transverse strength by substituting for s, in the general formula, the mean of the tensile and crushing resistances of a given wood, reduced in the proportion by which the strength of large scantlings is less than that of small scantlings; which may be taken at two-thirds.

Meantime, Mr. Laslett's data, table No. 171, may be utilized by fixing the value of the coefficient, 1.155 s, directly from the transverse breaking weights of the timbers, taken at two-thirds of the observed values. Inverting the general formula (1), page 507,

$$1.155\,s=\frac{W\,l}{b\,d^2}; \quad \text{........................} \quad (8)$$

By means of this formula, the values of the numerical coefficients to be substituted for the coefficient in any of the formulas (2) to (7), for the ultimate transverse strength of other timbers, are found to be as follows in table No. 179:—

Formulas for the Transverse Deflection of Timber Beams of uniform Rectangular Section.

The deflection of beams of small scantling may aid as a basis for calculating the deflection of large beams, by means of the general formula (4), page 529, in which the value of E, the coefficient of elasticity, may be calculated from the various data already given for such timber by means of the inverted general formula (8), preceding.

Table No. 179.—VALUES OF 1.155 *s*, NUMERICAL COEFFICIENT FOR THE TRANSVERSE STRENGTH OF TIMBER BEAMS; TO BE USED IN ANY OF THE FORMULAS (2) TO (7), page 550. (From Mr. Laslett's data.)

Description of Timber.	Values of 1.155 *s*.	Description of Timber.	Values of 1.155 *s*.
Oak, English (average)	1.63	Iron Bark, Australia	3.87
French	2.41	Blue Gum, do.	1.96
Do.	2.28	English ash	2.37
Tuscan	2.08	Canadian ash	2.30
Sardinian	2.08	Beech (estimated)	2.40
Dantzic	1.30	English elm	1.08
Spanish	1.54	Rock elm, Canada	2.53
American White	2.21	Hornbeam, England	2.53
Baltimore	1.99	Dantzic fir	2.41
African (or teak)	3.05	Riga fir	1.65
Moulmein teak	2.51	Spruce fir	1.84
Do.	2.32	Larch, Russia	1.62
Iron Wood, Burmah	3.50	Cedar, Cuba	1.54
Chow, Borneo	2.68	Red pine, Canada	1.80
Greenheart, Guiana	3.67	Yellow pine, Canada	1.71
Sabicu, Cuba	3.56	Do. do.	1.33
Mahogany, Spanish	2.35	Do. do.	.84
Honduras	2.21	Pitch pine, American	2.88
Mexican	2.15	Do. do.	2.56
Tewart, Australia	2.83	Do. do.	2.05
Mahogany, do.	1.89	Kauri pine, New Zealand	1.98

Transverse Deflection of Rectangular Timber Beams of uniform section:—

$$D = \frac{W l^3}{4.62 \, E \times b d^3} \quad \text{.......................} \quad (9)$$

D = the deflection, *l* the span, *b* the breadth, and *d* the depth, all in inches; W the load at the middle in tons, E the coefficient of elasticity.

The values of E and 4.62 E are given in the annexed table No. 180.[1]

SHEARING STRENGTH OF TIMBER.

Oak treenails, firmly held, of from 1 inch to 1¾ inches in diameter, were found by Mr. Parsons to have a shearing strength of about 2 tons per square inch of section. For the development of so much resistance, Professor Rankine deduces that the planks connected by the treenails should have a thickness of at least three times their diameter. Treenails of 1¾ inches, in 3-inch planks, bore only 1.43 tons per square inch; and in 6-inch planks, 1.73 tons.

[1] It may here be stated, that, whilst the value of E possesses importance as an element in a scientific theory of deflection, it is not necessary, for the purposes of calculation for the deflection of beams, that the value of E should be exactly ascertained, since, in its employment in the formula for deflection, it is merged in the compound coefficient 4.62 E, the value of which can be determined, independently, from practical data.

Table No. 180.—Values of E and 4.62 E in Formula (9), page 551, for the Transverse Deflection of Timber Beams.

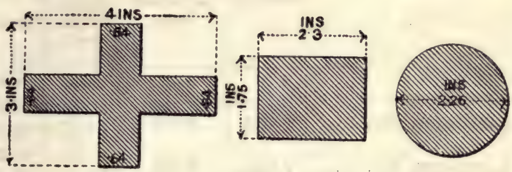
Description of Timber.	By Laslett's Data.		By Barlow's Data.		Various Data.		Averages.	
	E.	4.62 E.	E.	4.62 E.	E.	4.62 E.	E.	4.62 E.
English oak	348	1611	450	2072	—	—	400	1848
French do.	576	2656	—	—	—	—	576	2656
Tuscan do.	234	1080	—	—	—	—	234	1080
Sardinian do.	338	1555	—	—	—	—	338	1555
Dantzic do............	450	2080	—	—	—	—	450	2080
American white do.	458	2114	—	—	—	—	458	2114
Baltimore do.	598	2761	—	—	—	—	598	2761
Canadian do..........	—	—	746	3445	—	—	746	3445
Adriatic do.	—	—	376	1735	—	—	376	1735
African do. (or teak)	390	1804	—	—	—	—	390	1804
Moulmein teak	494	2276	934	4311	—	—	714	3293
Poon.................. ..	—	—	654	3018	—	—	654	3018
Iron wood............	916	4228	—	—	—	—	916	4228
Chow	956	4412	—	—	—	—	956	4412
Greenheart	410	1888	—	—	—	—	410	1888
Sabica	916	4228	—	—	—	—	916	4228
Spanish mahogany..	726	3360	—	—	—	—	726	3360
Honduras do.........	460	2118	—	—	—	—	460	2118
Mexican do............	780	3608	—	—	—	—	780	3608
Tewart	692	3196	—	—	—	—	692	3196
Iron Bark.............	948	4378	—	—	—	—	948	4378
Blue Gum.............	698	2559	—	—	—	—	698	2559
English ash............	542	2506	636	2939	—	—	588	2722
Canadian do.	320	1476	—	—	—	—	320	1476
Beech	—	—	524	2418	—	—	524	2418
Elm	—	—	266	1227	—	—	266	1227
Rock-elm	502	2319	—	—	—	—	502	2319
Memel fir	—	—	—	—	[1]S. 776	3630	776	3630
Dantzic do............	538	2490	—	—	—	—	538	2490
Riga do.................	680	3147	450	2072	F. 756 S. 578	3491 2669	616	2920
Spruce do.............	714	3300	—	—	—	—	714	3300
New England do....	578	2669	—	—	—	—	578	2669
Scotch do.	328	1514	—	—	F. 358	1652	344	1583
Larch	560	2585	350	1615	—	—	456	2100
Red pine..............	526	2430	712	3286	E.C. 460 S. 474 F. 464	2124 2188 2141	528	2434
Pitch do..............	704	3257	474	2187	S. 690	3461	622	2968
Yellow do.	414	1915	—	—	F. 410 S. 530	1891 2445	452	2084
Norway spar.........	—	—	564	2602	F. 504	2328	534	2465
Kauri pine............	632	2920	—	—	F. 616	2847	624	2884

[1] E. C.,—E. Clark; S.,—G. G. Smith; F.,—Fincham.

STRENGTH OF CAST IRON.

Tensile Strength and Compressive Strength.

Mr. Hodgkinson's experiments and investigations form the basis of most of what is known on the strength of cast iron. To ascertain the relative strength of cast iron according to the form of the cross section, he tested specimens of cruciform, rectangular, and circular sections—the first melting of the pigs. The area for each section, Figs. 184, 185, and 186, was intended to be four square inches, but the castings were accurately measured, and the exact area of each was ascertained. The following were the average breaking weights or absolute tensile strengths per square inch for the different sections:—

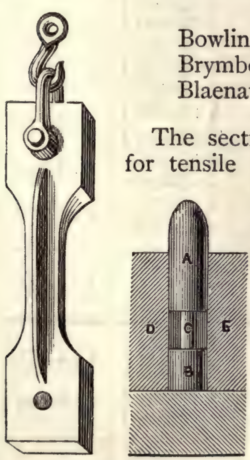


Figs. 184, 185, 186.—Trial Sections for Cast Iron.

	Section.	Tensile strength per square inch.
Bowling iron, No. 2	Cruciform	6.784 tons.
	Rectangular	6.267 „
Brymbo iron, No. 3	Cruciform	6.661 „
	Rectangular	6.115 „
Blaenavon iron, No. 2	Cruciform	6.253 „
	Circular	6.614 „
Total average		6.450 „

From these results it appears, that, taking the strength of the cruciform section as 1, the strengths of the other sections were relatively as follows:—

	Cruciform.	
Bowling iron, No. 2,	as 1 to	.924 rectangular.
Brymbo iron, No. 3,	as 1 to	.918 rectangular.
Blaenavon iron, No. 2,	as 1 to	1.054 circular.



Figs. 187, 188.—Specimens for Testing Tensile Strength and Compressive Strength.

The section of the specimens tested by Mr. Hodgkinson for tensile strength was cruciform, and the specimens were of the form Fig. 187; having a uniform section for one foot of length. For compression, they were cylindrical, ¾ inch in diameter, and were made to two heights, respectively equal to 1 diameter and 2 diameters, and they were placed for testing within a cylinder under a loaded plug, as shown in Fig. 188. He tested the strength of 16 denominations of cast iron, 51 specimens of which were tested for tension and 81 for compression. The results of the tests are condensed from the *Commissioner's Report on the Application of Iron to Railway Structures*, in table No. 181.

Table No. 181.—TENSILE AND COMPRESSIVE STRENGTHS OF CAST IRONS AND STIRLING'S IRON.

(Mr. Hodgkinson.)

Iron.	Mean Specific Gravity.	Mean Tensile Strength per Square Inch.	Mean Compressive Strength per Square Inch.		Ratio of Tensile to Compressive Strength.	
			Height of Specimen.	Strength.		
		tons.	inch.	tons.		mean.
Lowmoor, No. 1 ...	7.074	5.667	¾	28.809	1 : 5.084	1 : 4.765
			1½	25.198	1 : 4.446	
Lowmoor, No. 2 ...	7.043	6.901	¾	44.430	1 : 6.438	1 : 6.205
			1½	41.219	1 : 5.973	
Clyde, No. 1.........	7.051	7.198	¾	41.459	1 : 5.759	1 : 5.631
			1½	39.616	1 : 5.503	
Clyde, No. 2.........	7.093	7.949	¾	49.103	1 : 6.177	1 : 5.953
			1½	45.549	1 : 5.729	
Clyde, No. 3.........	7.101	10.477	¾	47.855	1 : 4.568	1 : 4.518
			1½	46.821	1 : 4.469	
Blaenavon, No. 1...	7.042	6.222	¾	40.562	1 : 6.519	1 : 6.149
			1½	35.964	1 : 5.780	
Blaenavon, No. 2, 1st sample.......	7.113	7.466	¾	52.502	1 : 7.032	1 : 6.577
			1½	45.717	1 : 6.123	
Blaenavon, No. 2, 2d sample........	7.051	6.380	¾	30.606	1 : 4.797	1 : 4.796
			1½	30.594	1 : 4.795	
Calder, No. 1........	7.025	6.131	¾	32.229	1 : 5.256	1 : 5.394
			1½	33.921	1 : 5.532	
Coltness, No. 3.....	7.024	6.820	¾	44.723	1 : 6.557	1 : 6.611
			1½	45.460	1 : 6.665	
Brymbo, No. 1......	7.071	6.440	¾	33.390	1 : 5.186	1 : 5.216
			1½	33.784	1 : 5.246	
Brymbo, No. 3......	7.037	6.923	¾	33.988	1 : 4.909	1 : 4.936
			1½	34.356	1 : 4.963	
Bowling, No. 2......	6.989	6.032	¾	33.987	1 : 5.635	1 : 5.555
			1½	33.028	1 : 5.476	
Ystalifera anthracite, No. 2.......	7.119	6.478	¾	44.610	1 : 6.886	1 : 6.735
			1½	42.660	1 : 6.585	
Yniscedwyn anthracite, No. 1..	7.034	6.228	¾	37.281	1 : 5.985	1 : 5.811
			1½	35.115	1 : 5.638	
Yniscedwyn anthracite, No. 2..	7.013	5.959	¾	34.430	1 : 5.778	1 : 5.712
			1½	33.646	1 : 5.646	
Averages of cast irons	7.055	6.830		38.525		1 : 5.641
Stirling's iron, 2d quality............	7.165	11.502	¾	55.952	1 : 4.865	1 : 4.751
			1½	53.329	1 : 4.637	
Stirling's iron, 3d quality............	7.108	10.474	¾	70.827	1 : 6.762	1 : 6.149
			1½	57.980	1 : 5.536	
Average of Stirling's iron........	7.136	10.988		59.522		1 : 5.417

It appears from the table that the tensile strength of cast iron varied from 5.667 to 10.477 tons, and averaged 6.830 tons per square inch.

That the compressive strength varied from 25.198 tons to 52.502 tons, averaging 38.525 tons per square inch.

That the compressive strength was from 4.518 to 6.735 times the tensile strength; average ratio of tensile to compressive strength, 1 to 5.641.

That the specific gravity varied from 6.989 to 7.113, and averaged 7.055, and that, generally, the strength increased with the specific gravity, though there were many exceptions to such relation.

That the tensile strength of Stirling's metal (a mixture of cast and wrought iron) averaged 10.988 tons per square inch, and the compressive strength 59.522 tons per square inch; ratio, 1 to 5.417.

The average compressive resistances of the pieces one and two diameters high were respectively as 100 to 95.6.

Dr. Anderson tested, at Woolwich Arsenal, 850 specimens of cast iron. The ultimate tensile strength of selected specimens varied from 4.90 tons to 14.5 tons per square inch, averaging 9.45 tons, and of all the 850 specimens, from 4.20 tons to 15.30 tons. He found that the average tensile strength of ordinary irons of commerce was 6 tons per square inch. It is probable that the higher strengths were those of bars of 2d or 3d meltings.

Strength as Affected by the Mass of Metal.

Mr. Hodgkinson, comparing the tensile strength of bars of cast iron, 1 inch, 2 inches, and 3 inches square, found that the relative strengths were approximately as 100, 80, 77.

Captain James found that the tensile strengths of 1-inch, 2-inch, and 3-inch bars were as 100, 66, 60; and that the tensile strength of ¾-inch bars cut out of 2-inch and 3-inch bars had only half the strength of the bar cast 1 inch square.

The ascertained inferiority in strength of massive castings as compared with thinner castings is attributable to the greater proportion of surface or "skin" on the thinner castings. It is known that the skin is harder and stronger than the interior of a casting. Besides, the interior of massive castings becomes more spongy in texture as the thickness is increased.

Strength of Cast Iron as Affected by Cold Blast and Hot Blast.

Mr. Hodgkinson tested several cast irons, made by cold blast and hot blast, with the following results, table No. 182; showing an average tensile strength, of all irons, 7.36 tons per square inch, and average compressive strength, 47.0 tons; ratio, 1 to 6.11. At the same time, it is shown that the hot-blast irons had 9.17 per cent. less tensile strength, but that they had 3.39 per cent. more compressive strength, than the cold-blast irons.

Mr. Robert Stephenson concluded from experiments of more recent date, conducted by him, that the average strength of hot-blast iron was not much less than that of cold-blast iron; but that cold-blast irons, or mixtures of cold-blast irons, were more certain and regular, and that mixtures of cold-blast and hot-blast irons were better than either separately mixed.

Table No. 182.—STRENGTH OF COLD-BLAST AND HOT-BLAST IRON.

(Mr. Hodgkinson.)

Description of Iron.	Tensile Strength per Square Inch.		Compressive Strength per Square Inch.	
	Cold Blast.	Hot Blast.	Cold Blast.	Hot Blast.
	tons.	tons.	tons.	tons.
Carron iron, No. 2	7.45	6.03	47.50	48.50
Carron iron, No. 3	6.43	7.84	51.50	59.50
Devon iron, No. 3	—	9.68	—	64.9
Buffery iron, No. 1	7.80	6.00	41.65	38.50
Coed-Talon iron, No. 3	8.42	7.45	36.50	36.90
Lowmoor iron, No. 3	6.49	—	—	—
Total averages	7.32	7.40	44.30	49.70
	7.36		47.0	
Comparative averages of cold and hot blast	7.52	6.83	44.30	45.80

Sir William Fairbairn, writing in 1870, maintained that the quality of iron had been greatly improved since the introduction of the hot blast, and that nothing, at the time of writing, was said of the difference between hot-blast and cold-blast irons. Dr. Siemens, on the same occasion, stated that the ironmasters had seen the advantage of raising the temperature of the blast, and that, in using the Siemens-Cowper regenerative hot-blast stoves, the temperature had been raised as high as 1400° F., without any deterioration of the quality of the metal having been observed.[1]

STRENGTH OF CAST IRON INCREASED BY REMELTING.

The strength, as well as the density, of cast iron are increased by repeated remeltings. The increase of strength and density appears to be the consequence of the gradual abstraction of the constituent carbon of the iron, and the approximation of the metal in composition to wrought iron.

Mr. Bramwell proved the increase of the tensile strength of Acadian cold-blast iron by remelting it. The tensile strengths of successive samples were as follows:—

ACADIAN IRON.

Sample bars.	Tensile strength per square inch.
	tons.
1st samples	7.5
2d do. after 2 hours longer fusion	8.3
3d do. after 1¾ " "	10.8
4th do. remelted, with fresh pigs	11.0
5th do. after 4 hours longer fusion	18.5
Maximum of 5th samples	19.6

[1] *Proceedings of the Institution of Civil Engineers*, "Regenerative Hot Blast Stoves," by Mr. E. A. Cowper, vol. xxx. p. 321.

Showing that the tensile strength was increased 150 per cent. by 8 hours of continued fusion, and by remelting. The compressive strength averaged 3½ times the tensile strength.[1]

Sir William Fairbairn tested for compressive strength, samples of Eglinton No. 3 hot-blast iron of from 1 to 18 meltings—the resistance was doubled by 18 meltings; but the maximum resistance was attained at the 14th melting, and amounted to 2.2 times the first resistance. The following are the results of these tests:—

EGLINTON NO. 3 HOT-BLAST IRON.

Melting.	Compressive strength. tons.	Melting.	Compressive strength. tons.
1.	44.0	10.	57.7
2.	43.6	11.	69.8
3.	41.1	12.	73.1
4	40.7	13.	66.0 (defective)
5.	41.1	14.	95.9
6.	41.1	15.	76.7
7.	40.9	16.	70.5
8.	41.1	18.	88.0
9.	55.1		

Remelting, or continued fusion, of cast iron is practised in the United States. The pig iron generally used has, in the state of pig, a tensile strength of from 5 to 6½ tons per square inch. When melted, it is kept for some time in a state of fusion, and the first castings have a tensile strength of about 9 tons per square inch. For guns, the metal is melted three or four times in an air-furnace, and at each melting is retained in fusion for from one to three hours before being poured; and, according to the experiments of Major Wade, the strength of iron so treated was successively increased. The following are some of the results obtained by Major Wade:—

AMERICAN IRON.	TENSILE STRENGTH. tons per square inch.
Pigs	5 to 6½
1st melting	9.32
2d do.	11.06
3d do.	11.96
4th do.	12.45
Maximum strength observed	20.5
Samples from 100 gun-heads	14.9
Proof bars (in other trials)	16.23
38 samples from a Rodman gun	15.3 to 19.8
Do. average	16.88
A lot of pig iron, in the crude state	5.66
27 guns cast from this pig iron, 3d melting	15.75

The specific gravity of the metal was increased by successive meltings and protracted fusion, from 6.90, in some instances, to 7.40.

The compressive strength of the irons tested by Major Wade, varied from 37.7 tons to 78 tons per square inch. The specimens were ½ inch

[1] The above particulars are reduced from the *Proceedings of the Institution of Civil Engineers*, vol. xxii. page 559.

in diameter, and $1\frac{1}{4}$ inches high. Some of the mean results were as follows:—

	2d Melting.	3d Melting.
No. 1 cast iron,	44.5 tons	62.5 tons.
Mixtures of Nos. 1, 2, 3,	69.4 „	74.6 „

It may be inferred that the ratio of tensile to compressive strength of the American irons above tested, was about 1 to 4.

Elastic Strength of Cast Iron.

Mr. Hodgkinson made experiments with round cast-iron bars of one square inch sectional area, and 50 feet long, suspended in a lofty building, to find the extension and permanent sets. These experiments were made

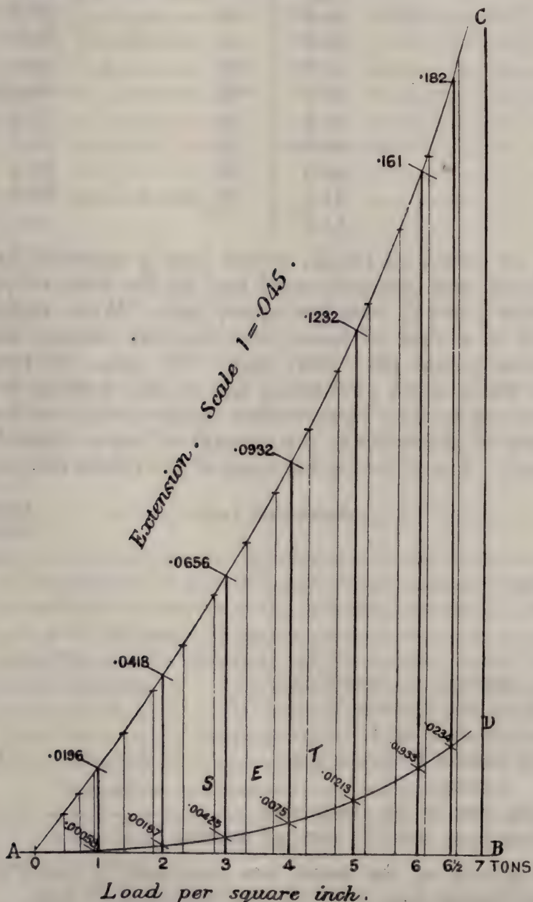


Fig. 189.—Diagram to show Rate of Extension and Set of 10-feet bars of cast iron. Table No. 183.

with the object of insuring exceptional accuracy of results. But, by much the greater number of Mr. Hodgkinson's experiments, both for extension and for compression, were made with bars limited to 10 feet in length, 1 inch square. The results of the observations on the extension and com-

pression of 10 feet bars, are plotted in Figs. 189 and 190, in which the base-line A B represents the loads, and the verticals in light lines are the observed extensions, compressions, and sets, for the given loads. The curves A C and A D are traced through the ends of these verticals, and the vertical black lines show the extensions and sets, for integral tons of load.

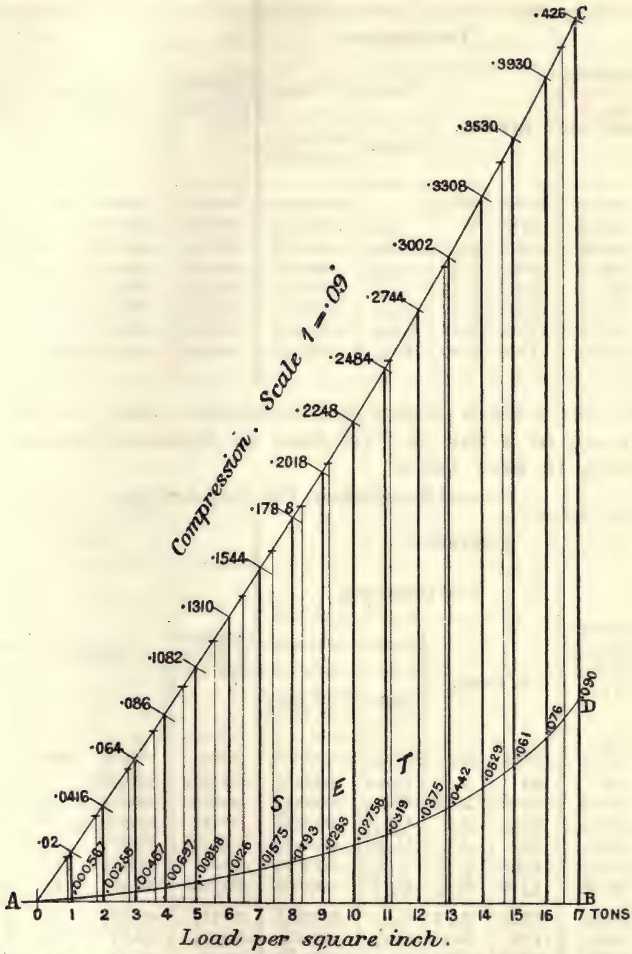


Fig. 190.—Diagram to show Rate of Compression and Set of 10-feet bars of cast iron. Table No. 184.

The following table, No. 183, is constructed from the diagram of extension and set, Fig. 189, and it shows the mean extension and set for given stresses in integral tons up to 6½ tons on a 1-inch square bar of average quality 10 feet long.

The table No. 184 is likewise constructed from the diagram of compression and set, Fig. 190, and it shows the mean compression and set for given stresses in integral tons up to 17 tons on a 1-inch square bar of average quality 10 feet long.

Table No. 183.—Mean Reduced Extension and Set for given Stresses, of a Bar of Cast Iron of Average Quality, 1 Inch Square, 10 Feet Long.

Deduced from diagram, Fig. 189, page 558.

Stress.	Extension.					Set.			
	Increment of Extension for each Ton.	Total Extension.				Increment of Set for each Ton.	Total Set in Inches.		Ratio of Total Set to Extension.
		In Inches.		Fraction of Length.					
				Total.	Per Ton per Square Inch				
tons.	inches.	inches.	ratio.		length = 1.	inches.	inches.	ratio.	ratio.
1	.0196	.0196	1	1/6122	.000163	.00058	.00058	1	1 to 34
2	.0222	.0418	2.13	1/2871	.000174	.00139	.00197	3.4	1 to 21
3	.0238	.0656	3.35	1/1829	.000183	.00228	.00425	7.3	1 to 15.4
4	.0276	.0932	4.75	1/1287	.000194	.00325	.0075	13	1 to 12.4
5	.0300	.1232	6.29	1/974	.000205	.00463	.01213	21	1 to 10.2
6	.0378	.1610	8.21	1/745	.000223	.00720	.01933	33	1 to 8.3
6.5	.0210	.1820	9.29	1/659	.000257	.00407	.0234	40	1 to 7.8

Table No. 184.—Mean Reduced Compression and Set for given Stresses, of a Bar of Cast Iron of Average Quality, 1 Inch Square, 10 Feet Long.

Deduced from diagram, Fig. 190, page 559.

Stress.	Compression.					Set.			
	Increment of Compression for each Ton.	Total Compression.				Increment of Set for each Ton.	Total Set.		Ratio of Total Set to Compression.
		In Inches.		Fraction of Length.					
				Total.	Per Ton per Square Inch				
tons.	inches.	inches.	ratio.	fraction.	length = 1.	inches.	inches.	ratio.	ratio.
1	.02	.02	1	1/6000	.000167	.000567	.000567	1	1 to 35.3
2	.0216	.0416	2.08	1/2884	.000173	.00198	.00255	4.5	1 to 17.6
3	.0224	.064	3.2	1/1875	.000178	.00212	.00467	8.2	1 to 13.7
4	.022	.086	4.3	1/1395	.000179	.00230	.00697	12.2	1 to 12.3
5	.0222	.1082	5.41	1/1109	.000180	.00261	.00958	17	1 to 11.3
6	.0228	.1310	6.55	1/916	.000182	.00302	.0126	22	1 to 10.4
7	.0234	.1544	7.72	1/777	.000184	.00315	.01575	28	1 to 9.8
8	.0236	.178	8.9	1/674	.000185	.00355	.0193	34	1 to 9.2
9	.0238	.2018	10.09	1/595	.000187	.0040	.0233	41	1 to 8.7
10	.023	.2248	11.24	1/534	.000188	.0043	.0276	48	1 to 8.1
11	.0236	.2484	12.42	1/483	.000188	.0043	.0319	56	1 to 7.8
12	.026	.2744	13.72	1/437	.000191	.0056	.0375	66	1 to 7.3
13	.0258	.3002	15.01	1/400	.000193	.0077	.0442	78	1 to 6.8
14	.0306	.3308	16.54	1/363	.000197	.0087	.0529	93	1 to 6.3
15	.0222	.353	17.65	1/340	.000196	.0081	.061	108	1 to 5.8
16	.04	.393	19.65	1/305	.000205	.0193	.0803	142	1 to 4.9
17	.033	.426	21.3	1/282	.000209	.0060	.0863	152	1 to 4.9

It is clear from the diagrams, Figs. 189 and 190, and the tables Nos. 183 and 184, that both the extension and the compression of cast iron, with the respective sets, begin at the beginning of the loading; and, strictly interpreted according to the definition of elasticity, the evidence is to the effect that there is no such thing as perfect elasticity in ordinary cast iron. The progression of extension, compression, and set, moreover, is regular, and it is gradually accelerated whilst the stress is increased in arithmetical proportion. There is no sudden change in the rate of progression anywhere, no "yielding point" for cast iron, and no indication of a permanent elastic limit before rupture takes place.

In this respect cast iron radically differs from wrought iron and steel, for in the behaviour of these metals the "yielding point" is a clearly defined characteristic.

Shearing Strength of Cast Iron.

Professor Rankine states that the shearing strength of cast iron is 12.37 tons per square inch. But Mr. Stoney found by experiment that it was from 8 to 9 tons per square inch. Both of these data may be correct: it has been seen that cast iron varies very much in tensile strength, according to the character of the specimens operated upon.

It is very probable that the shearing resistance of cast iron is, by reason of its comparative incompressibility, equal to its direct tensile resistance.

Malleable Cast Iron.

The tensile strength of annealed malleable cast iron is guaranteed by manufacturers to 25 tons per square inch. It is capable of supporting 10 tons per square inch, tensile strength, without permanent distortion.

Transverse Strength of Cast Iron.

Cast-Iron Bars of Rectangular Section.—Mr. Barlow found, by experiment, that for 1-inch square bars of cast iron, the breaking weight in tons, applied at the middle, was expressed by the formula,

$$W = \frac{bd^2}{l} \times 13.6, \qquad (1)$$

in which b, the breadth, d the depth, and l the span, are in inches. Mr. Robert Stephenson arrived, by experiment, at exactly the same coefficient. If the coefficient be taken as only 12, the breaking weight of a 1-inch square bar, at 12 inches span, is, by the formula, just 1 ton; and if the span, l, be expressed in feet, the formula (1), with a coefficient of 12, is resolved into the form,

$$W = \frac{bd^2}{l} \qquad (2)$$

If cast-iron bars were homogeneous, and of uniform density for all dimensions, the formulas 1 and 2 would give the breaking strengths correctly for all sizes of bars; but so great is the diminution of strength in thicker castings, due to the comparatively open or spongy structure of cast iron in thick masses, that 3-inch square bars, relatively, have scarcely two-thirds of the transverse strength of 1-inch bars, and the proper coefficient for

formula (1), is only 8.6, as applicable to 3-inch bars. It is obvious that no constant coefficient can be employed, even in iron of the same denomination, for the transverse strength of cast-iron bars, when the thickness is various.

To apply the general formula (1), page 507, for the transverse strength of rectangular bars of cast iron, in terms of the tensile strength; namely,

$$W = 1.155 \frac{b d^2 s}{l}; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

in which b, d, and l are in inches, s, the tensile strength, in tons per square inch, and W, the breaking weight, in tons at the middle; the mean tensile strength of 1-inch square bars of ten different irons was found by Mr. Hodgkinson to be 16,502 lbs., or 7.36 tons, and their transverse strength, at 54 inches of span, was 464 lbs. By the formula (3), taking the forces in pounds, the transverse strength is,

$$\frac{1.155 \times 1^3 \times 16{,}502}{54} = 353 \text{ lbs.};$$

or seven-ninths of the actual strength. The excess of actual strength, 31 per cent., results from the distribution of the stronger portion of the section of the bar at the outside,—the skin, in fact,—where its moment, or power of resistance, is, by reason of the leverage of the resistance, much more effective than that of the interior and weaker portions. By such tubular distribution, a greater total strength transversely is exerted than would have been exerted if the material had been of uniform tensile strength throughout the section, as was assumed in the construction of the formula. In bars of greater section, the influence of the skin on the strength is comparatively less. Accordingly, in the following examples selected for comparison, from data supplied by Mr. Edwin Clark, the excess of strength diminishes generally as the scantling of the specimen is increased. A tensile strength of 7 tons per square inch is assumed in the calculation of the strength, by the formula (3)—

	Bars.			Transverse Strength.		
	Width.	Depth.	Span.	Calculated.	Actual.	Excess, Actual.
(1.)	1 inch ×	1 inch ×	4.5 feet,	.158 ton,	.252 ton,	60 per cent.
(2.)	1 " ×	3 " ×	18 "	.337 "	.429 "	27 "
(3.)	3 " ×	1 " ×	2.25 "	.898 "	1.376 "	53 "
(4.)	2 " ×	2 " ×	13.5 "	.399 "	.475 "	19 "
(5.)	2 " ×	3 " ×	9 "	1.347 "	1.800 "	35 "
(6.)	3 " ×	2 " ×	4.5 "	1.800 "	2.410 "	34 "
(7.)	3 " ×	3 " ×	13.5 "	1.347 "	1.436 "	6.5 "

The 1-inch square bar has 60 per cent. excess of strength. The 2d bar has only 1 inch of bottom skin for three times the depth of the 1st, and so has only 27 per cent. excess. The 3d bar, of the same section as the 2d, was tried on its side, and has three times as much bottom skin as the 2d; and so has nearly double the excess of the 2d, but not so much as that of the 1st, which has comparatively more side skin. The 4th bar, 2 inches square, has less skin in proportion to its bulk, and has only 19 per cent. excess of strength; whilst the 5th bar, of the same thickness, but deeper, has a greater excess, for its bottom skin has more leverage. The

6th bar has the same section as the 5th, but is tested on its side, and has the same excess of strength; whilst the 7th bar, 3 inches square, has only 6½ per cent. excess of strength above that calculated for it by formula (3). The strength of the 7th bar is, on the contrary, 34 per cent. less than what would be calculated for it by the formula (1).

Diminishing differences with increasing sections, are also exemplified by experimental observations of Mr. Hodgkinson, selected from one of his tables,[1] with bars of Carron No. 2, averaged for hot and cold blast, of three sizes, comprising two or more bars of each size. The sizes are here given in round numbers:—

	Bars.			Transverse Strength.		
	Width.	Depth.	Span.	Calculated.	Actual.	Excess.
(1.)	1 inch ×	1 inch ×	54 inches,	.158 ton,	.219 ton,	40 per cent.
(2.)	1 „ ×	3 „ ×	54 „	1.381 „	1.736 „	26 „
(3.)	1 „ ×	4 „ ×	54 „	3.794 „	4.600 „	21 „

For the calculation of the transverse strength of cast-iron bars of rectangular sections, and of the larger scantlings, even of the commonest quality, formula (3) may, it appears, be safely employed, allowing a wide margin, with a minimum factor of 7 tons per square inch tensile strength. This gives a numerical coefficient of $(1.155 \times 7 =)$ 8.08; say, 8, in formula (3).

Transverse Strength of Rectangular Bars of ordinary Cast Iron, of the first melting. Tensile strength, 7 tons per square inch:—

Loaded at the middle, ... $W = \frac{8\,b\,d^2}{l}$; .... (4)

Loaded at one end, ... $W = \frac{2\,b\,d^2}{l}$; .... (5)

Round Cast-Iron Bars.—The strength of round cast-iron bars, taking a tensile strength of 7 tons, is found from the general formula (15), page 510; in which $.7854 \times s = .7854 \times 7 = 5.50$.

Transverse Strength of Round Bars of ordinary Cast Iron:—

Loaded at the middle, ... $W = \frac{5.50\,d^3}{l}$; .... (6)

Loaded at one end, ... $W = \frac{1.375\,d^3}{l}$; .... (7)

in which b, d, and l are in inches, and W is the breaking weight in tons. With tensile strengths greater than 7 tons, the constants to be used in these formulas are as follows:—

Tensile strength per square inch.	Constant in formula (4).	Constant in formula (5).	Constant in formula (6).	Constant in formula (7).
8 tons,	9.2	2.3	6.3	1.6
9 „	10.4	2.6	7.1	1.8
10 „	11.5	2.9	7.9	2.0
11 „	12.7	3.2	8.6	2.2
12 „	13.8	3.4	9.4	2.4

[1] *On the Strength and Properties of Cast Iron*, 1846, pp. 398, 399.

Test Bars.—It is usual, in specifications for cast-iron work, to require that sample bars of cast iron, say 1 inch thick, 2 inches deep, and on bearings 36 inches apart, shall support a given weight applied at the centre. Ten years ago, a weight of 25 cwt. was considered sufficient as a test load; but the load has since been increased to 28 cwt. and 30 cwt. This is not very severe, for, by formula (1), based on the strength of 1-inch square bars, the modern test-bar, of ordinary iron, should support 27.2 cwt. before breaking.

Transverse Deflection and Elastic Strength of Cast Iron.

Cast-Iron Rectangular Bars.—Mr. Hodgkinson gives particulars of the deflection of rectangular bars under various loads. The deflections under medium loads are here averaged as follows; and the coefficients of elasticity E, are calculated by means of the formula (8), page 530:—

	Span. inches.	Load. tons.	Deflection. inches.	E.
Average of 8 bars, 1 inch square,..........	54	.100	.561	6076
1 bar, 1 inch × 3 inches deep,	54	1.066	.216	6230
1 bar, 1 inch × 5 inches deep,	54	3.348	.153	5966
Average value of coefficient of elasticity..........				6090

This value of E agrees with that which was found for direct tensile strength, table No. 183, page 560, and the resulting numerical coefficient, 4.62 E = 4.62 × 6090 = 28,136, say 28,000; which may be substituted for 4.62 E in the formula (8), page 530, to give the deflection of cast-iron bars, within ordinary elastic limits, thus:—

Deflection of Cast-Iron Rectangular Bars of Uniform Section.

Loaded at the middle, $D = \frac{W l^3}{28,000\, b\, d^3}$ .... (8)

Loaded at one end,... $D = \frac{W l^3}{875\, b\, d^3}$ .... (9)

D = the deflection, b = the breadth, d = the depth, l = the span, all in inches; W = the load in tons.

Cast-Iron Round Bars.—For round bars of uniform diameter, substitute the above-found average value of E, in the general formula (26), page 533. Then, 3.1416 × E = 3.1416 × 6090 = 19,132, say 19,000.

Deflection of Cast-Iron Round Bars.

Loaded at the middle, $D = \frac{W l^3}{19,000\, d^4}$ .... (10)

Loaded at one end,... $D = \frac{W l^3}{594\, d^4}$ .... (11)

Torsional Strength of Cast Iron.

The only direct experiments recorded, worth notice, on the torsional resistance of cast-iron, are those of Mr. Dunlop at Glasgow, in 1819.[1] They were made to ascertain the torsional strength of shafts as usually cast in Glasgow at the time. Two old bars of cast iron, about 5 feet long each, one of them 3 inches and the other 4 inches square, were turned down in the lathe at five different places, to ten different diameters, of from 2 to 4¼ inches. The load was applied at the end of a lever 14 feet 2 inches long. Particulars of the experiments are given in table No. 185; the values of h, the shearing resistance, calculated by the general formula (3), page 535, are added.

Table No. 185.—Torsional Strength of Cast Iron. 1819.

(Reduced from Mr. Dunlop's data.)

Diameters.	Cubes of Diameters.	Ratio of the Cubes.	Breaking Weight.	Ratio of the Breaking Weights.	Shearing Stress per square inch.
inches.			tons.		tons.
2	8	1	.1116	1	8.530
2¼	11.4	1.4	.1714	1.5	9.201
2½	15.6	2.0	.182	1.6	7.123
2¾	20.8	2.6	.312	2.8	8.761
3	Failed	—	—	—	—
3¼	34.3	4.3	.522	4.7	9.299
3½	42.9	5.4	.554	5.0	7.902
3¾	52.7	6.6	.742	6.6	8.604
4	64	8.0	.865	7.7	8.265
4¼	76.8	9.6	.963	8.6	7.691
Average,..					8.375

It seems that the ultimate torsional strength increased very nearly as the cube of the diameter, and that the average torsional resistance per square inch of section was 8.375 tons. Assuming, as explained at page 561, that the shearing resistance of cast iron is equal to its direct tensile resistance, the general formulas for torsional strength (1), page 534, and (6), page 536, become, by substitution,

For cast-iron round shafts, $$W = \frac{.278\, d^3 s}{R} = \frac{d^3 s}{3.6\ R} \quad \text{............} \quad (12)$$

For cast-iron square shafts, $$W = \frac{.40\, b^3 s}{R} = \frac{b^3 s}{2.5\ R} \quad \text{............} \quad (13)$$

W = the force, in tons.
R = the radius of the force, in inches.
WR = the moment of the force, in statical inch-tons.
d = the diameter of the round shaft, in inches.
b = the side of the square shaft, in inches.
s = the ultimate tensile strength, in tons per square inch.

[1] *Annals of Philosophy*, vol. xiii. 1819.

If the tensile strength, s, be taken at 7.2 tons, for iron of average quality, then, by substitution and reduction:—

Ultimate Torsional Strength and Sizes of Cast-Iron Shafts of average quality.

Round shaft,................ $W = \dfrac{2\,d^3}{R}$ .... (14)

$$d = \sqrt[3]{\frac{W\,R}{2}} = .79\sqrt[3]{W\,R} \quad \ldots \; (15)$$

Square shaft,................ $W = \dfrac{2.88\,b^3}{R}$ .... (16)

$$b = \sqrt[3]{\frac{W\,R}{2.88}} = .70\sqrt[3]{W\,R} \quad \ldots \; (17)$$

Torsional Deflection of Cast-Iron Bars.

In the absence of direct data for the torsional deflection of cast-iron bars, it is assumed that it is 1⅔ times that of wrought-iron shafts—the same proportion as that of the transverse deflections of cast-iron and wrought-iron shafts, as indicated by a comparison of the formulas (8), page 564, and (5), page 590. Multiply, therefore, the second member of the formula (14), page 592, by 1⅔; the coefficient 1070, or exactly 1073, becomes ($1073 \times 3/5 =$) 644:—

Torsional Deflection of Round Cast-Iron Bars.

$$D = \frac{W\,R\,l}{644\,d^4} \quad \ldots\ldots\ldots\ldots \; (18)$$

STRENGTH OF WROUGHT IRON.

Tensile Strength.

Mr. Telford deduced from his experiments, an average tensile strength of 29.25 tons per square inch for wrought-iron bars.

Mr. Barlow deduced from the results of eight bars of wrought iron—Swedish, Russian, and Welsh, from 1¼ inches square to 2 inches in diameter—an average tensile strength of 25 tons per square inch.

Mr. Barlow also deduced from experiments on bars of from 1 inch in diameter to 2 inches square, that the elastic tensile strength of good medium wrought iron was 10 tons per square inch; and that the extension was at the rate of $^1/_{10,000}$th part of the length per ton per square inch; and that, therefore, the elasticity was fully excited when the bar was stretched $^1/_{1000}$th part of its length.

Sir William Fairbairn published, in 1861, results of experiments on the tensile strength of wrought iron, which are rendered, slightly adapted, in tables Nos. 186 and 187:—

Table No. 186.—Tensile Strength of Wrought Iron. 1861.

(Sir William Fairbairn.)

Description of Iron.	Mean breaking Weight per square inch. With Fibre.	Mean breaking Weight per square inch. Across Fibre.	Ultimate Elongation.
	tons.	tons.	fraction of length.
Lowmoor iron (specific gravity 7.6885)….	28.66	23.43	—
Lancashire boiler plates (9 specimens)……	21.82	20.10	$^1/_{23}$ and $^1/_{36}$
Staffordshire iron (two ¼-inch plates rivetted together)…………	21.36		—
Charcoal bar iron …………	28.40	—	$^1/_5$
Best best Staffordshire charcoal plate (4 experiments)…………	20.10	18.49	$^1/_{20}$ and $^1/_{22}$
Best best Staffordshire plates (4 experiments)	22.30	20.75	$^1/_{20}$ and $^1/_{26}$
Best best Staffordshire plate …………	26.71	24.47	$^1/_{15}$ and $^1/_{25}$
Best Staffordshire…………	27.36	24.03	$^1/_{13}$ and $^1/_{22}$
Common Staffordshire…………	22.69	23.58	$^1/_{20}$ and $^1/_{23}$
Lowmoor rivet iron (2 experiments)………	26.80	—	$^1/_4$
Staffordshire rivet iron…………	26.56	—	$^1/_4$
Staffordshire rivet iron…………	26.65	—	$^1/_5$
Bar of the same, cold-rolled…………	37.96	—	$^1/_{12}$
Staffordshire bridge iron…………	21.25	19.82	$^1/_{25}$ and $^1/_{35}$
Yorkshire bridge iron…………	22.29	19.62	$^1/_{25}$ and $^1/_{28}$

Table No. 187.—TENSILE STRENGTH OF IRON AND STEEL PLATES THAT HAD BEEN SUBJECTED TO EXPERIMENT WITH ORDNANCE AT SHOEBURYNESS. 1861.

(Sir William Fairbairn.)

DESCRIPTION.	THICKNESS OF PLATES, IN INCHES.							Averages of plates of one make.
	¼	½	¾	1½	2	2½	3	
IRON.	tons.	tons.	tons.	tons.	tons.	tons.	tons.	tons.
Lowmoor.........	24.34	25.75	—	24.16	25.35	24.11	25.04	24.79
Thames Co.'s....	24.17	23.22	29.43	22.30	23.66	23.92	23.54	24.32
Beale & Co.'s....	?	?	26.47	25.16	24.63	22.73	24.16	24.63
Averages of plates of the same thickness	24.26	24.49	27.95	23.87	24.55	23.59	24.25	—
Average specific gravity ...	—	—	—	7.70	7.72	7.72	7.72	—
STEEL.								
Howell & Co.'s homogeneous metal..........	30.70	33.69	30.91	26.20	27.04	27.51	27.39	29.06
Specific gravity...	—	—	—	7.89	7.91	7.91	7.91	—
Elongations before rupture of specimens, in part of the length (2½ to 3 inches).								
IRON.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.
Lowmoor.........	6.2	7.6	—	17.6	30.5	28.8	32.0	20.5
Thames Co.......	3.0	4.0	10.0	14.6	25.3	32.0	26.5	16.5
Beale & Co......	—	—	4.0	19.3	17.9	16.0	23.3	16.1
Averages..........	4.6	5.8	7.0	17.2	24.6	25.6	27.3	—
Homogeneous metal.........	25.6	10.0	20.8	19.3	34.5	29.5	25.8	—

From the first table, No. 186, it appears that the strength of iron plates in the direction of the fibre, varied from 28.66 tons for Lowmoor iron, to 20.10 tons for Staffordshire charcoal-iron; and that there is no tensile strength in the direction of the fibre so low as 20 tons per square inch.

Also, that the averages of nine irons show, for the breaking weight,—

With the fibre...............23.68 tons per square inch.
Across the fibre.............21.59 „ „

Difference.......... 2.09 tons, or about 9 per cent.

From the second table, No. 187, it appears that the thicker plates have less tensile strength than the thinner plates; whilst the elongation before rupture is greater; thus:—

	¼ to ¾ inch thick. tons.		per cent.	1½ to 3 inches thick. tons.		per cent.
Iron	25.57	elongation	5.8	24.06	elongation	23.7
Homogeneous metal	31.77	„	18.8	27.03	„	27.3

Sir William Fairbairn tested the resistance of iron plates to a bulging stress. He stretched two ¼-inch plates and two ½-inch plates over a cast-iron frame 12 inches square inside, as in Fig. 191, and subjected them to pressure from an iron bolt 3 inches in diameter, with a hemispherical end, which was applied to the plate.

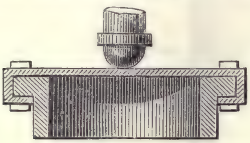


Fig. 191.—Specimen Plate to resist Bulging Stress. Sir W. Fairbairn.

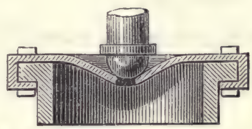


Fig. 192.—Effects of Bulging Stress.

The ¼-inch plates were indented ¼ inch and ½ inch respectively, when they commenced to fail by cracking on the convex side, under a pressure of 4.7 tons applied at the centre. Under 7 tons of pressure they were cracked through.

The ½-inch plates were indented .33 inch when they commenced to crack, under a pressure of 9 tons; and they were cracked through under a pressure of 17 tons. See Fig. 192.

The resistance to bulging in these experiments was in proportion to the thickness of the plates.

To test the effect of cold-rolling on iron bars, Sir William Fairbairn tested three bars, and obtained the following results:—

	Tensile Strength per square inch.	Elongation.
Black bar from the rolls	26.0 tons.	20 per cent.
„ turned to 1 inch in diameter	27.1 „	22 „
„ cold-rolled to 1 inch in diameter	39.4 „	8 „

showing that the tensile strength was increased by one-half; but that the elongation was reduced to less than a half.

With respect to the influence of temperature, Sir William Fairbairn, in 1857, found that the strength of ordinary Staffordshire iron plates, either with or across the grain, remained the same for temperatures varying from 0° F. to 400° F. At higher temperatures the strength declined, until, at a red heat, it fell from an ordinary average of 20 tons to 15¼ tons per square inch.

Mr. Thomas Lloyd tested the tensile strength of Staffordshire S. C. Crown bars, 1⅜ inches in diameter, of one kind. The same bars were broken four times in succession, and the successive breaking weights were for the

	Tensile Strength.
1st Breakage (10 trials)	23.94 tons per square inch.
2d „ (10 „)	25.86 „ „
3d „ (7 „)	27.06 „ „
4th „ (6 „)	29.20 „ „

showing a variation in the same bars of from 23.94 to 29.20 tons, or 18 per cent. of the maximum tensile strength.

The tensile strength of 1⅜-inch round Staffordshire bars of various lengths was found by Mr. Lloyd as follows:—

	Breaking Weight.
6 Bars, 10 feet long	32.21 tons.
6 Do. 3.5 " "	32.12 "
6 Do. 3 " "	32.35 "
6 Do. 2 " "	32.00 "
6 Do. 10 inches long	32.29 "

showing that the strength was not affected by the length.

Mr. Edwin Clark found the average tensile strength of iron per square inch as follows:—

Staffordshire boiler plates, ½ to $^{11}/_{16}$ inch thick, with fibre,	19.6	tons.
Do. special trials ... with fibre,	19.93	"
Do. special trials ... across fibre,	16.82	"
Difference, 15½ per cent. less	3.11	"
Best scrap rivet iron, ⅞ inch diameter,	24.0	"

He also found that the ultimate resistance to compression in a wrought-iron bar was 16 tons per square inch, at which pressure the metal began to ooze away. Under 12 tons per square inch, the set was so great that the form began to change; and this pressure was taken as the average ultimate resistance to compression that may be recognized in practice. The elastic limit of tensile strength was also taken as 12 tons per inch; and Mr. Edwin Clark concluded thus:—

"It is very nearly true, and very convenient in practice, to assume both the extension and compression to take place at the rate of one ten-thousandth ($\frac{1}{10,000}$) of the length for every ton of direct strain per square inch of section"—agreeing in this respect with Mr. Barlow.

Mr. Edwin Clark found that the resistance of ⅞-inch rivet iron to shearing was as follows:—

	Tons per Square Inch of Section.
Resistance by single shearing,	24.15
Do. by double shearing,	22.1
Do. in two ⅝-inch plates rivetted together (one section),	20.4
Do. in three ⅝-inch plates rivetted together (two sections),	22.3
Tensile strength,	24.0

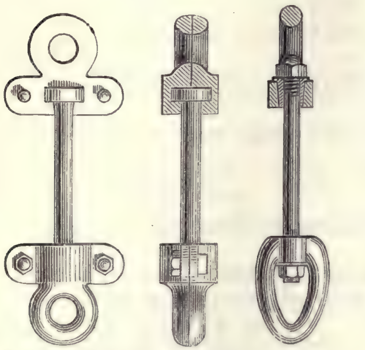
When three ⅝-inch plates were rivetted together with a ⅞-inch rivet, the frictional resistance to displacement of the middle plate, the hole through which was larger than the rivet, was from 6 to 8 tons.

Experiments on the Tensile Strength of Wrought Iron and Steel.

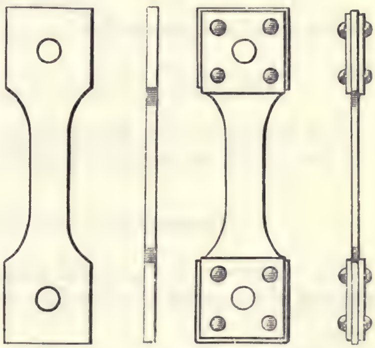
By Mr. Kirkaldy, 1858–61.

Mr. David Kirkaldy conducted, for Messrs. Robert Napier & Sons, an extensive series of trials of the tensile strength of iron and steel bars and plates, the results of which threw much light on the properties of these metals.[1]

The specimens of bars were formed with a head at each end, united to the body of the bar by taper necks, to receive the shackles, as shown in Figs. 193 and 194. Screw bolts and nuts were shackled as in Fig. 195.



Figs. 193, 194, 195.—Mr. Kirkaldy's Test Specimens of Bars.



Figs. 196 to 199.—Mr. Kirkaldy's Test Specimens of Plates.

The clear length of bar was about 7 inches. The plate specimens were formed as in Figs. 196 to 199, the ends of the thinner plates being fortified by flitches rivetted on both sides. The increments of load were applied slowly and gradually.

The specimens of bar iron varied from ⅝ inch to 1¼ inch in diameter; but they were, for the most part, from ¾ inch to 1 inch in diameter. There did not appear to exist any material difference of strength that could be ascribed to difference of diameter.

Tensile Strength and Elongation of Iron Bars.

The average ultimate tensile strengths of iron bars, and their total elongations or stretching when fractured, were as follows:—

[1] The results of Mr. Kirkaldy's important investigations are published in his work, *Experiments on Wrought Iron and Steel,*—a mine of experiment and research. The data in the text are reduced from this work.

It is right to explain that Mr. Kirkaldy expresses the resistance, in all cases, in pounds; and that they are, in the text, converted into tons. For, though the pound-unit commends itself as a basic unit of great simplicity, yet engineers are accustomed to think in terms of tons, and will continue to do so, until some universal decimal system is adopted, by which the ordinary ton may be superseded. It must be admitted that the ton of 2240 lbs. is a barbarous unit, and that the New York ton of 2000 lbs. is in every sense superior to the old British relic.

	Tensile strength per square inch.	Elongation.
Yorkshire rolled bars,..................	27.39 tons.	25.2 per cent.
Staffordshire do.	25.90 „	23.5 „
Lanarkshire do.	26.55 „	19.4 „
Rivet iron, do.	26.00 „	20.5 „
	——— „	——— „
Averages,..................	26.46 „	22.2 „
Hammered scrap, forged down, ...	23.85 „	24.8 „
Bushelled iron (turnings), forged down,	24.95 „	16.8 „
Crank-shaft, scrap iron, with fibre,...	20.37 „	21.8 „
Do. do. across fibre,	18.55 „	12.5 „
Armour plate, across fibre,............	16.92 „	9.0 „
	———	———
Averages,......................	20.93 „	17.0 „

The lowest tensile strengths of the better kinds were not more than 1½ ton below the average for each brand.

Contraction of the Sectional Area in Fracture.

The reduction, in size, of a piece subjected to tensile strain is practically uniform throughout the portion that is strained, except near to the point of rupture, where the piece is locally much more contracted in size, as illustrated by Fig. 200, which shows the contraction of a 1-inch round bar of soft Bowling iron, with the line of fracture. The diameter, in this instance, was reduced $^3/_{32}$ inch, and the sectional area was reduced at the fracture to 43.56 per cent. of the area of the original section. In the following selection of examples of fractured sectional areas, the percentage ranges from 29.5 per cent. to 87.8 per cent.

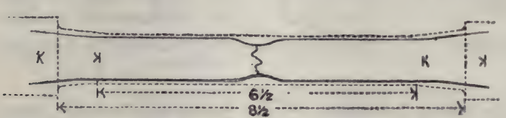


Fig. 200.—Contraction of Specimen Bars.

Iron Bars—Fractured Sectional Areas.

	Per cent. of original area.
Swedish R. F. charcoal,......................................	29.5
Staffordshire, charcoal,......................................	38.4
Yorkshire, Lowmoor,......................................	46.3
Staffordshire B. B. scrap,......................................	47.6
Do. S. C. crown,......................................	53.4
Scotch extra best best,......................................	58.5
Do. best best,......................................	68.9
Do. common,......................................	71.6
Do. common,......................................	85.2
Russian C. C. N. D. (for steel),......................................	89.8
	———
Average,......................................	59.0

Strength as the Diameter is Reduced by Rolling.

Four pieces were cut off a 1½-inch bar, reheated and rolled down to different sizes. They had the following tensile strengths:—

Diameter.	Tensile strength per square inch.	Elongation.
inch.	tons.	per cent.
1¼	22.38	28.3
1	25.60	26.7
¾	25.97	25.2
½	26.65	23.8

showing an increase of tensile strength, and less elongation.

Strength as Affected by Turning, or Removing the Skin.

Rolled bars 1¼ inch diameter were turned down to 1 inch, and tested. The average results of four irons, so treated, were as follows:—

	Tensile strength. tons per square inch.	Elongation. per cent.
Rough bars,	24.38	17.2
Turned bars,	25.00	19.3

showing that the turned bars were at least as strong as the rough bars.

Strength as Affected by Forging.

1¼-inch round bars of four kinds of iron were reduced by forging to 1 inch and ¾ inch in diameter.

	Tons per inch.	Elongation.
Rough bars,	25.13	24.5 per cent.
Forged bars,	26.10	17.3 ,,

showing an increase of strength equal to 1 ton per square inch by the forging down, and a reduction of elongation.

Strength as Affected by Reheating only.

Five different irons, 1 inch in diameter, of which the collars had failed at the first trial, had the collars replaced. The effects of the reheating to which the five bolts were subjected in the operation are thus shown:—

	Tons per inch.	Elongation.
1st Trial,	25.86	10.1 per cent.
2d Trial,	24.88	32.6 ,,

showing a reduction of 1 ton per square inch of tensile strength, whilst the elongation was trebled.

On the contrary, two pieces of a ¾-inch bar of good iron were tested,—one in its ordinary condition, and the other after it had been brought to a welding heat, and cooled slowly.

	Tons per inch.	Elongation.
In ordinary condition,	25.27	22.3 per cent.
Heated, and cooled slowly,	25.21	17.7 ,,

Strength as Affected by Intense Cold.

Three pieces of a ¾-inch bar were tested, one at 64° F.; the others, after having been exposed over night to intense frost, were broken at 23° F.:—

	Tons per inch.	Elongation.
At 64°	24.87	24.9 per cent.
At 23°	24.28	23.0 „

showing that at the lower temperature the strength was a little less by 0.59 ton.

Strength as Affected by Notching the Bar.

The two ends of 1 inch round specimen bars were screwed, and the screw at one end was divided by turning out a square notch or groove ⅛ inch wide, as at Fig. 201, leaving a diameter of .70 inch at the bottom of the groove. After having been broken at the notch, the body of the bar was turned down to the same diameter as the notch had originally, and the bar broken a second time, through the body. The following are selections from the results of such tests applied to bar irons; the strength of the rough bar being added for comparison. The elongations are not recorded; but the contracted sectional areas of fracture are here given:—

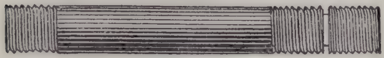


Fig. 201.—Notched Specimen Bar.

	Tensile Strengths.			Contracted Sectional Area.		
	Notched.	Turned Down.	Rough Bar.	Notched.	Turned Down.	Rough Bar.
	tons.	tons.	tons.	per cent.	per cent.	per cent.
Lowmoor (hardest bar),...	40.95	31.57	29.10	92.0	50.8	49.0
Bowling (softest bar),......	29.87	25.01	26.20	72.2	44.4	43.6
Govan Diamond,...........	30.96	25.70	25.71	75.9	46.4	50.8
Dundyvan, common,......	29.87	28.17	23.15	100.0	92.0	96.1
	32.91	27.61	26.04	85.0	58.4	59.9

showing a remarkable excess of resistance, more than 5 tons, at the notch, due apparently to the shortness of the notched portion, which was partially sustained by the thicker parts on each side, whilst the contraction of area was in a measure prevented.

Strength of Bars or Bolts as Affected by Screwing.

Screws with rounded threads were cut in three modes—by means of old blunt dies, new sharp dies, and chasers; and tested for tensile strength. The following selection of results has been made for the purpose of fair comparison; premising that the diameter at the base of the thread was the same for all the screwed bolts of one diameter:—

	Not Screwed.	Screwed.	Difference.
	tons.	tons.	tons
1¼-inch bolts, screwed,...........	24.47	18.22	6.25, or 25 per cent. less.
„ „ chased, ...	24.45	17.20	7.25, or 29 „ „
1-inch bolts, screwed (old dies),	27.60	24.62	2.98, or 11 „ „
„ „ screwed (new dies),	27.10	19.04	8.06, or 30 „ „
„ „ chased, ...	27.95	20.02	7.93, or 28 „ „
⅝-inch bolts, screwed (old dies),	26.47	22.77	3.70, or 14 „ „
„ „ screwed (new dies),	26.47	19.47	7.00, or 26 „ „
„ „ chased, ...	26.47	18.70	8.77, or 33 „ „

A decided variation of strength is due to the mode of cutting the screw. The blunt dies, compressing the metal, as Mr. Kirkaldy argues, harden it, and increase its tensile resistance. The sharp dies, cutting more readily, do not compress the metal so much as the blunt dies. The chaser does not compress it at all. Hence, the reduction of strength is greater in screwing with sharp dies than with blunt dies, and is greatest when the chaser is used.

Strength of Bars as Affected by Welding.

The pieces of bar iron to be tested were cut through the middle, and scarfed and welded in the ordinary manner. The tensile strength at the weld was in all cases less than that of the original bar, as well as the elongation before fracture. The following are the averaged results for each class of irons:—

	Tensile Strength. per cent. less.	Elongation. per cent.
Glasgow B. Best, 1¼ inch diameter,	17.6	6.7
Farnley, 1 ,,	30.2	7.3
Govan B. Best, ¾ ,,	14.5	5.9
Govan Extra B. B. ¾ ,,	15.1	10.5
Average,	19.4	7.6

These averages conceal the excessive fluctuations of strength, which varied from 2.6 to 43.8 per cent. below the normal strength of the bars.

Strength of Bar Iron in Resisting Stress Suddenly Applied.

In a special series of trials, when the steelyard was duly loaded, and all taut, it was suddenly released by means of a trigger; so that the stress was delivered upon the specimen suddenly, but without any blow or jerk. Mr. Kirkaldy ascertained the value of the rupturing stress for each iron by taking the mean between the lowest stress that caused rupture and the highest that did not do so. The specimens consisted of 1-inch round bars. The following are the comparative tensile strengths:—

	Load Applied.		Difference.	Elongation.	
	Gradually.	Suddenly.		Gradually.	Suddenly.
	tons.	tons.	tons. per cent. less.	per cent.	per cent.
Bradley charcoal,	25.45	22.05	3.40, or 13.4	30.2	40.1
Bradley Crown S.C.,	27.82	22.10	5.72, or 20.6	25.3	22.5
Lowmoor,	26.48	21.37	5.11, or 19.0	—	25.0
Lowmoor,	26.81	20.91	5.90, or 22.0	27.0	23.6
Glasgow B. Best,	26.70	20.45	6.25, or 24.8	—	25.3
Glasgow B. Best,	26.70	21.07	5.63, or 19.1	—	20.1
Glasgow B. Best (¾-inch bars),	24.87	22.50	2.37, or 9.6	24.9	17.3
Crank-shaft,	19.54	15.82	3.72, or 19.0	21.1	16.9
	25.54	20.78	4.76, or 18.6	24.6	20.1 (5 specimens.)

These results show that the tensile resistance to fracture by suddenly-applied stress is from 2 to 6 tons per square inch, or from 10 to 25 per cent. less than when it is gradually applied. The average elongation is also less,—decidedly more so, if the exceptional specimen, Bradley charcoal bar, be omitted from the average.

The *Influence of Frost*, at 23° F., on the tensile resistance of bar iron to strains suddenly applied was tried on seven specimens cut from a ¾-inch bar of Glasgow B. Best iron. The average results showed 3.6 per cent. diminution of resistance; and a reduction of elongation from 25 per cent. to 20½ per cent.

Influence of Additional Hammering on the Iron in a large Crank-shaft.

Three pieces 1¾ inches square were cut out, and forged down to 1⅛ inch, and turned to 1 inch in diameter. Compared with two pieces which were simply cut out, and turned down to 1 inch, the results were as follows:—

	Tons per inch.	Elongation.
Cut out and turned down,...............	19.90	16.8 per cent.
Cut out, forged down, and turned,....	23.70	11.7 ,,

showing 20 per cent. increase of strength with reduced elongation.

Strength of Hammered Iron as Affected by Removing the Skin.

Two 1½-inch square bars of Govan hammered iron were turned down to 1 inch in diameter. Compared with 1 inch square Govan hammered bars, in their skins, they gave better results, thus:—

	Tons per inch.	Elongation.
1-inch square bars,......................	28.60	20.6 per cent.
1½-inch square bars turned down,....	30.35	23.5 ,,

Hardening Iron Bars.

A 1¼-inch round bar of Bowling iron was cut into several pieces, which were turned and forged down, and hardened:—

Diameter.	Tons per inch.	Elongation.
Turned to 1 inch,........................	27.15	28.3 per cent.
Forged to .87 inch, hardened in water,	32.79	19.6 ,,
Do. .78 ,, ,, oil,...	28.85	19.8 ,,
Do. .70 ,, ,, tar,...	28.06	22.4 ,,

showing that hardening in water increases the strength more than in oil or tar.

The tensile strength of the second piece, above noted, namely, 32.79 tons per square inch, was the greatest strength of iron observed by Mr. Kirkaldy.

Experiments on pieces cut from the large crank-shaft already mentioned, and from an armour-plate, and hardened, show that there was no increase of tensile strength by hardening, and that the elongation was reduced.

Case-hardening Iron Bars.

By case-hardening specimens of several irons, and cooling them in oil, or in water, or slowly, the loss of tensile strength averaged 2.21 tons per

square inch; whilst three-fourths of the elongation was gone. The averages may be placed together for easy comparison:—

Case-hardened.	Tensile Strength.	Elongation.	In Ordinary Condition.	
	tons.	per cent.	tons.	per cent.
A. Forged, and cooled in oil,	25.39	6.2	26.50	26.6
B. Forged, and cooled in water,	23.70	2.9	26.50	26.6
C. Forged, and cooled slowly,	25.36	11.6	27.07	23.8
D. Turned down, and cooled slowly,	22.11	4.9	25.32	20.7
Averages,	24.14	6.4	26.35	24.4

Cold-rolled Iron Bars.

Five pieces of ¾-inch Blochairn bar-iron were treated as follows:—

	Tons per inch.	Elongation.
Cold-rolled (2 pieces)	31.86	11.8 per cent.
Cold-rolled and annealed (2 pieces)	26.50	25.6 „
In ordinary condition (1 piece)	27.06	22.8 „

showing that cold-rolling added nearly 5 tons to the strength, which was lost when the bars were subsequently annealed.

Strength of Angle-iron, Ship-strap, and Beam-iron.

The tensile strength of angle-irons, about ⅝-inch thick, is generally less by from 1 to 2 tons per square inch than that of bar-iron. The tensile strength of ship-strap and beam-irons, from ⅜ to 1 inch thick, is 2 tons less than that of angle-irons. The elongations, correspondingly, are also less.

Tensile Strength of Iron Plates.

Iron plates, of thicknesses varying for the most part from ⅜ inch to ¾ inch thick, cut into specimens 1½ and 2 inches wide, were tested:—

Plates.	Tons per Inch.		Elongation.	
	With Fibre.	Across Fibre.	With Fibre.	Across Fibre.
	tons.	tons.	per cent.	per cent.
Yorkshire	24.75	22.64	13.4	8.0
Staffordshire	23.01	21.40	9.3	5.3
Durham	22.89	21.39	9.5	5.2
Shropshire	23.37	19.22	9.6	2.8
Lanarkshire	21.96	19.56	7.0	3.2
General averages	23.20	20.84	9.8	4.9

The greatest difference of the lowest tensile strength in any group was 3 tons per inch below the average of the group. In the Yorkshire plates it did not exceed 2 tons.

The tensile strength across the fibre is from 1½ tons to 4 tons per inch less than that with the fibre. The average difference is 10 per cent.

Fractured Sectional Area of Iron Plates.

	With Fibre.	Across Fibre.
Yorkshire	63.5 per cent.	79.7 per cent. of original area.
„	76.5 „	83.7 „ „
Staffordshire crown S. C.	78.5 „	89.9 „ „
„ Bradley	84.3 „	92.0 „ „
Scotch best boiler	87.3 „	93.6 „ „
Staffordshire best best	90.9 „	94.6 „ „
Scotch ship	95.4 „	97.5 „ „
Scotch common	94.4 „	98.5 „ „

Cold-rolled Iron Plates.

Pieces of Blochairn plate .345 inch thick were reduced by cold-rolling to .238 inch thick, or to two-thirds:—

	Tons per Inch.		Elongation.	
	With Fibre.	Across Fibre.	With Fibre.	Across Fibre.
			per cent.	per cent.
In ordinary condition	20.45	19.20	4.4	2.6
Cold-rolled	39.73	36.00	0.1	0.0
Cold-rolled and annealed	22.75	21.72	8.0	6.0

Cold-rolling nearly doubled the strength, but annihilated the elongation. By annealing, all but 2½ tons per inch of the extra strength was lost; but the original elongation was doubled.

Strength of Iron Plates as Affected by Galvanizing.

Fourteen specimens of Glasgow best boiler plate, from 3/16 to ⅜ inch thick, were prepared for trial, half the number having been galvanized. There was no perceptible difference in any respect between the galvanized and the ungalvanized plates.

Specific Gravity of the Irons Tested.

Yorkshire rolled bars	7.7600	Armour-plate	7.6134
Staffordshire rolled bars	7.6178	Angle-iron	7.6006
Lanarkshire rolled bars	7.6280	Iron plates	7.6287
Crank-shaft	7.6307		

The specific gravity was diminished by cold-rolling, though the tensile strength was increased; as follows:—

	Ordinary.	Cold-rolled.
Bar iron, specific gravity	7.636	7.582
Boiler-plate, „	7.566	7.539

The specific gravity of iron was also diminished by stretching under tensile stress:—

	Specific Gravity. Before Stretching.	Specific Gravity. After Stretching.
Three 1-inch Yorkshire bars, stretched to .90 inch	7.752	7.674
Two .83-inch Blochairn bars, „ .76 „	7.636	7.569
Average for five bars	7.760	7.632

showing an average reduction of .128, or 1.65 per cent., in the specific gravity.

EXPERIMENTS OF THE STEEL COMMITTEE OF CIVIL ENGINEERS. 1870.

The Steel Committee, who will be again noticed in treating of the strength of steel, tested the strength of a number of wrought-iron bars 1½ inches diameter, consisting of twelve bars of Lowmoor iron, six bars of best Yorkshire iron, and six bars of usual S. C. Crown, or Staffordshire iron. Table No. 189 gives condensed results of the experiments for the tensile strength of wrought-iron bars, in 10 feet of length; and table No. 188, the same for their compressive strengths.

Note to Tables Nos. 188, 189.—The lowest elastic strength in any group of bars did not exceed 1 ton per square inch less than the average elastic strength; say, not more than 10 per cent. less than the average for iron bars.

A chemical analysis of these irons is given with that of the steels tested by the committee, in table No. 203, page 603.

Table No. 188.—COMPRESSIVE STRENGTH OF WROUGHT-IRON BARS. 1870.

1½ inches in diameter. Observations made on 10-feet lengths.

(Reduced from results of experiments made by the Steel Committee.)

Mark and Description.	Elastic Strength (Compressive) in Tons per Square Inch.	Elastic Compression.	Elastic Compression per Ton per Square Inch, in parts of the Length.
	tons.	per cent.	length = 1.
L S 3 Lowmoor.............	13.5	.101	
L S 5 ,, ...	13.5	.106	
L S 6 ,, ...	13.0	.101	
Averages..............	13.3	.102, or 1 in 977	.000077, or 1/12,987
L 1 Lowmoor................	12.5	.093	
L 2 ,, ...	10.5	.081	
L 3 ,, ...	11.5	.090	
Averages..............	11.5	.089, or 1 in 1130	.000077, or 1/12,987
K C 1 Yorkshire............	13.0	.100	
K C 2 ,, ...	13.0	.103	
K C 3 ,, ...	—	—	
Averages..............	13.0	.101, or 1 in 987	.000078, or 1/12,821
F R 1, usual S. C. Crown	11.5	.093	
F R 2, ,, ,,	11.5	.095	
F R 3, ,, ,,	12.0	.103	
Averages..............	11.7	.097, or 1 in 1030	.000080, or 1/12,500
Summary Averages.			
Yorkshire.....................	12.6	.097, or 1 in 1030	.000077, or 1/12,987
S. C. Crown..................	11.7	.097, or 1 in 1030	.000083, or 1/12,048
Total averages......	12.1	.097, or 1 in 1030	.000080, or 1/12,500

Table No. 189.—TENSILE STRENGTH OF WROUGHT-IRON BARS. 1870.

1½ inches in diameter. Observations made on 10-feet lengths.

(Reduced from results of experiments made by the Steel Committee.)

MARK AND DESCRIPTION.	Elastic Strength (tensile) in tons per square inch.	Elastic Extension, in parts of the length.	Elastic Extension per ton per square inch, in parts of the length.	Breaking Weight in tons per square inch.	Permanent Extension.	Ratio of Elastic to Breaking Strength.	Sectional Area of Fracture.
	tons.	per cent.	Length = 1.	tons.	per cent.	per cent.	per cent.
L S 1 Lowmoor..	14.0	.110		27.8	4.9		
L S 2 ,, ...	14.0	.107		29.5	7.1		
L S 4 ,, ...	14.0	.114		29.3	9.0		
Averages..............	14.0	.109 or 1 in 916	.000078 or 1/12,821	28.9	7.0	48.4	94.1
L 4 Lowmoor....................	11.0	.086		24.6	14.4		
L 5 ,, ...	12.5	.098		25.7	11.6		
L 6 ,, ...	12.0	.096		24.1	12.0		
Averages	11.8	.093 or 1 in 1072	.000079 or 1/12,658	24.8	12.6	47.7	51.2
K C 4 Yorkshire...............	13.0	.102		23.6	16.7		
K C 5 ,, ...	13.0	.101		24.2	17.6		
K C 6 ,, ...	13.5	.111		23.3	19.4		
Averages..............	13.2	.104 or 1 in 959	.000079 or 1/12,658	23.7	17.9	55.6	48.6
F R 1, usual S. C. Crown....	11.5	.096		22.5	17.4		
F R 2, ,, ,, ...	11.5	.095		22.5	15.7		
F R 3, ,, ,, ...	12.5	.096		22.9	19.4		
Averages..............	11.8	.096 or 1 in 1046	.000081 or 1/12,346	22.6	17.5	52.2	52.3
Summary Averages.							
Yorkshire	13.0	.103 or 1 in 974	.000079 or 1/12,658	25.8	12.5	50.6	64.6
S. C. Crown	11.8	.096 or 1 in 1046	.000081 or 1/12,346	22.6	17.5	52.2	52.3
Total averages..................	12.4	.100 or 1 in 1000	.000080 or 1/12,500	24.2	15.0	51.4	58.4

HAMMERED IRON BARS (SWEDISH).

Table No. 190 contains a selection of results of trials made by Mr. Kirkaldy of the tensile and compressive strength of hammered bar-iron manufactured by Messrs. Gammelbo & Co., Nericia, Sweden.

Table No. 190.—STRENGTH OF SWEDISH HAMMERED IRON BARS. 1866.

TENSILE STRENGTH.

Size of Specimens (2, 3, or 4 of each scantling).	Length for Elongation.	Elastic Strength per Square Inch.			Absolute Strength per Square Inch.	Elongation in parts of Length.
		Lowest.	Highest.	Average.		
		tons.	tons.	tons.	tons.	per cent.
1½-inch round, turned down from bars 2 inches	15 inches	10.75	12.00	11.05	18.80	24.6
1-inch square					20.35	22.9
3-inch round	10 ,,				18.85	33.1
2-inch round	10 ,,				18.87	32.5
1-inch round	10 ,,				18.92	25.9
½-inch round	10 ,,				23.90	5.9
Flat, 3 × ½ inch	15 ,,				23.00	12.1
Flat, 2 × ½ inch	15 ,,				20.62	21.6
Flat, 1½ × ½ inch ...	15 ,,				20.55	16.4
1-inch square iron converted into blistered steel					12.77	1.2
2-inch round, case hardened					22.50	—
1-inch round, case hardened					19.35	—
½-inch round, case hardened					23.25	—

COMPRESSIVE STRENGTH.

Size of Specimens (2, 3, or 4 of each scantling).	Length for Compression.	Elastic Strength per Square Inch.			Absolute Strength per Square Inch.	Compression in parts of Length.
		Lowest.	Highest.	Average.		
		tons.	tons.	tons.	tons.	per cent.
1½-inch round.........	1½ inches	10.10	12.64	10.74	66.45	45.4
1½-inch round.........	15 ,,	8.94	10.75	9.45	12.53	3.7
1½-inch round.........	3 ,,	8.94	9.84	10.42	37.90	33.1
1-inch square...........	1 inch				82.20	53.3
1-inch square converted into blistered steel	1 inch				83.40	48.3

SHEARING STRENGTH.

1½-inch turned........					15.20	

For transverse strength, four 2-inch square bars were tested, on a span of 25 inches, with the following results:—

	Breadth.	Depth.	Elastic Stress.	Deflection.	Ultimate Stress.	Deflection.
	inches.	inches.	pounds.	inches.	pounds.	inches.
1	2.04	× 2.02	7,500	.089	15,888	5.18
2	2.02	× 2.04	7,000	.088	14,875	4.98
3	1.95	× 2.02	6,000	.072	13,965	5.85
4	2.00	× 2.00	6,000	.078	13,338	5.38
Averages..........			6,625 or 2.96 tons	.082	14,516 or 6.48 tons	5.35

The bars remained uncracked under the ultimate stress.

For torsional strength, the averages for four bars turned to 1½ inches in diameter, with a length of 7 diameters, the stress being applied at the end of a 12-inch lever, were as follows:—

Elastic stress	1062 pounds, or .474 ton.
Deflection in parts of a revolution = 1...	.011 turn.
Ultimate stress..............................	2677 pounds, or 1.195 tons.
Ultimate deflection..........................	4.70 turns.

One-inch square bars of the same manufacture were tested. For tensile strength, they broke with an average of 20.34 tons per square inch. Under bending stress, the average results of four bars, 1.04 inches wide by 1.05 inches deep, showed that they bore an elastic stress of 1250 pounds, with .216 inch deflection. The ultimate stress was 1978 pounds, with 6.60 inches of deflection.

Mr. J. Tangye's Experiments on the Compressive Resistance of Wrought Iron.

A 1-inch round bar of soft Lowmoor iron, 8 or 9 inches long, was planed on two opposite sides to a thickness of ¾ inch, and was subjected to pressure on one side under a steel die ½ inch square, having an area of ¼ square inch.

The following are the results of the tests; and they prove clearly that a unit of iron has a much greater power of resistance when it forms a portion of a larger mass, than when it is isolated in the manner customary in making experiments on resistance to compression:—

Load per square inch.			
12 tons	no impression.		
16 „	„		
20 „	„		
24 „	slightest indentation, sensible to the finger-nail.		
28 „	distinctly visible, edge followed by finger-nail.		
32 „	„	„	„
36 „	„	„	„
40 „	indented about $^1/_{64}$th inch.		

Krupp and Yorkshire Iron Plates. 1875.

Mr. Kirkaldy made an experimental inquiry into the relative properties of wrought-iron plates manufactured by Herr Krupp, Essen, and plates manufactured in Yorkshire. The results are detailed in a valuable report by Mr. Kirkaldy, from which the following particulars have been extracted. Twenty-seven plates in all, not less than 4 feet by 3 feet, of three thicknesses, ⅜ inch, ½ inch, and ⅝ inch, were obtained from Mr. Krupp and from six Yorkshire manufacturers; from which the specimens were cut out. The Yorkshire brands were:—Lowmoor, Bowling, Farnley, Taylor's, Cooper & Co., and Monkbridge.

Fig. 202.—Test Specimens for Tensile Strength.

Tensile Strength.—Table No. 191 gives condensed results of the tests for tensile strength. Each entry for Krupp iron is an average for nine specimens; and for Yorkshire iron, for eighteen specimens. The results for the three thicknesses are nearly alike. The form of the specimens is shown in Fig. 202.

Table No. 191.—Krupp and Yorkshire Iron Plates—Tensile Strength. 1875.

Thicknesses ⅜, ½, and ⅝ inch. Breadth, 2 inches; length for extension, 10 inches.

(Reduced from Mr. Kirkaldy's Reports.)

Description.	Elastic Strength per square inch.	Ultimate Strength per square inch.	Ratio of Elastic to Ultimate Strength.	Extension. At 30,000 lbs. per square inch.	Extension. Ultimate.	Sectional Area of Fracture.
	tons.	tons.	per cent.	per cent.	per cent.	per cent.
LENGTHWAY.						
Unannealed—						
Krupp	11.6	22.7	51.3	1.30	25.4	60.4
Yorkshire	12.4	21.3	58.4	.65	16.7	79.4
Annealed—						
Krupp	11.0	21.0	52.5	2.72	28.2	56.3
Yorkshire	12.0	20.1	59.6	1.42	18.4	77.8
CROSSWAY.						
Unannealed—						
Krupp	11.4	21.7	52.6	1.35	17.4	75.2
Yorkshire	12.4	20.3	61.4	.51	11.2	85.3
Annealed—						
Krupp	10.8	20.4	52.7	2.37	19.7	73.0
Yorkshire	12.1	19.2	62.8	.81	12.8	83.1
AVERAGES.						
Krupp	11.2	21.5	52.1	1.94	22.6	66.2
Yorkshire	12.2	20.2	60.4	.85	14.8	81.4

Effect of Drilled Holes and Punched Holes on the Tensile Strength.—Specimens 8 inches wide were prepared according to Fig. 203, with two rows of rivet holes, .85 inch in diameter, in the central portion, 2½ inches apart, and the holes were at 2 inches pitch. The punched holes were conical, as usual. The reduction of width of solid metal by the holes

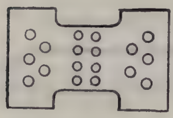
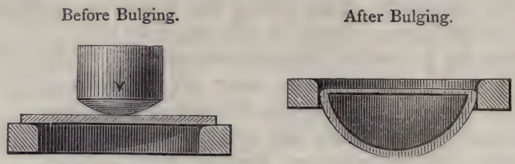


Fig. 203.—Specimen Plate to test Effect of Drilling and Punching.



Figs. 204, 205.—Test Specimen Plates for Bulging Stress.

amounted to $(.85 \times 4 =)$ 3.40 inches, and the net section was $(8 - 3.4 =)$ 4.6 inches wide, or 57.5 per cent. of the total width. Four Krupp specimens and nine Yorkshire specimens of varying thickness, were tested for each result. Table No. 192 gives some deductions from the elaborate results reported by Mr. Kirkaldy.

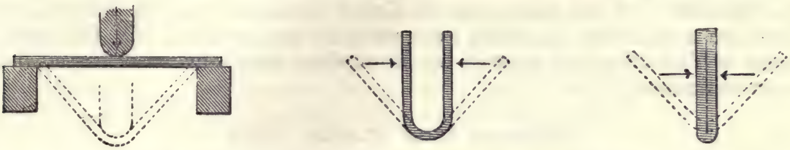
Table No. 192.—Krupp and Yorkshire Iron Plates—Tensile Strength of Drilled and Punched Plates. 1875.

Thickness ⅜, ½, and ⅝ inch. Holes, .85 inch in diameter.

(Deduced from Mr. Kirkaldy's Reports.)

Description.	Reduced Section, in parts of Total Section.	Reduced Strength, in parts of Total Strength.	Tensile Strength per square inch of Net Section, in parts of that of Entire Section.	Total Elongation of Holes.
	per cent.	per cent.	per cent.	per cent.
DRILLED HOLES.				
Lengthway—				
Krupp	57.5	62.8	109.2	30.7
Yorkshire	57.5	56.9	99.0	18.2
Crossway—				
Krupp	57.5	61.1	106.2	22.8
Yorkshire	57.5	57.2	100.0	13.4
PUNCHED HOLES.				
Lengthway—				
Krupp	57.5	51.5	89.6	13.7
Yorkshire	57.5	50.0	87.0	8.3
Crossway—				
Krupp	57.5	50.0	87.0	11.1
Yorkshire	57.5	47.6	82.8	7.0
AVERAGES.				
Krupp	57.5	56.3	98.0	19.6
Yorkshire	57.5	52.9	92.0	11.7

Resistance to Bulging Stress.—Discs 12 inches in diameter, cut in the lathe out of plates, were pressed into an aperture 10 inches in diameter, by a bulger-ram about 5 inches in diameter, of which the end was turned to a radius of 5 inches. The preparation for the trial, and the object after



Figs. 206, 207, 208.—Specimens for Resistance to Bending Stress.

having been bulged, are shown in Figs. 204 and 205. A selection of the results is given in table No. 193. Each result for Krupp iron is an average from four specimens; and for Yorkshire iron an average from six specimens. The stress was gradually increased until the specimen was pushed through the aperture, or until the specimen gave way either by cracking or bursting.

Table No. 193.—Krupp and Yorkshire Iron Plates—Resistance to Bulging Stress. 1875.

Discs 12 inches in diameter, pressed into 10-inch apertures.

(Selected from Mr. Kirkaldy's Table.)

Thickness of Plate.	Stress—Bulging in Inches.			Ultimate.		Effects.
	Lbs. 25,000.	Lbs., 100,000.	Lbs., 200,000.	Bulge.	Stress.	
inches.	inches.	inches.	inches.	inches.	tons.	
Unannealed.						
Krupp,.... .440	.82	2.14	—	3.27	62.10	uncracked
„ .... .533	.64	1.86	—	3.40	73.20	uncracked
„ .... .653	.50	1.59	2.82	3.36	97.06	1 burst
Yorkshire, .390	.83	{4 plates 2.50}	—	2.65	41.00	3 burst, 1 cracked
„ .510	.61	{5 plates 1.85}	—	2.72	61.03	5 burst
„ .625	.35	{4 plates 1.57}	{3 plates 2.83}	2.52	73.83	burst
Annealed.						
Krupp,.... .440	.83	2.25	—	3.27	55.40	uncracked
„ .... .533	.72	1.96	—	3.39	71.28	uncracked
„ .... .653	.56	1.75	{1 plate 3.18}	3.45	88.62	1 burst
Yorkshire, .390	.93	2.73	—	3.19	47.35	1 burst, 1 cracked
„ .510	.72	{4 plates 2.05}	—	2.88	55.85	2 burst, 1 cracked
„ .625	.43	1.65	{3 plates 3.14}	2.82	77.30	3 burst, 1 cracked

Resistance to Bending Stress.—Specimens, 2½ inches wide, of plates of the three thicknesses, were bent double both hot and cold. First, by bending them between supports 10 inches apart to a right angle, as in Fig. 206; then the cold specimens were doubled, as in Fig. 207, to a distance apart of four times the thickness; whilst the hot specimens were doubled flat, as in Fig. 208. Of the specimens of Krupp iron, thirty-six in number, all bore the test, except six which were more or less cracked. Of the specimens of Yorkshire iron, seventy-two in number, twenty-five only passed the tests uncracked.

Prussian Iron Plates. 1874.

Two large iron plates, .64 inch thick, manufactured by Mr. Borsig, of Berlin, were tested for tensile strength by Mr. Kirkaldy in 1874. The following abstract contains the averages of four experiments to each result:—

	Unannealed.		Annealed.	
	With Fibre.	Across Fibre.	With Fibre.	Across Fibre.
Elastic strength	13.00 tons	12.60 tons	12.73 tons	12.04 tons
Ultimate strength.....	23.40 „	22.70 „	22.53 „	21.70 „
Ratio	54.5 %	55.5 %	56.5 %	55.4 %
Elongation	23.8 %	14.6 %	24.7 %	15.1 %

The greatest deviation from the average elastic tensile strength was half a ton below it.

The plates were tested for bulging strength. Discs, 12 inches in diameter—two annealed and two unannealed—were cut out of the plates and pressed through an aperture 10 inches in diameter, the same as shown in Figs. 204 and 205, page 584.

	Pressure.	Bulging. Unannealed.	Bulging. Annealed.	
	22.32 tons	.90 inch	.95 inch.	
	44.64 „	1.49 „	1.57 „	
	66.96 „	1.95 „	2.07 „	
	89.28 „	2.39 „	2.60 „	
(ultimate)	104.18 „	2.69 „	—	burst.
	104.20 „	—	3.26 „	one burst.

Tensile Strength of Iron Wire.

Mr. Barlow deduced from experiments by Mr. Telford on the strength of iron wire from $^1/_{10}$ inch to $^1/_{20}$ inch in diameter, that the ultimate tensile strength was equivalent to 36 tons per square inch of section.

The tensile strength of Warrington iron wire is given at page 247:—Unannealed, about 36 tons per square inch; annealed, about 24 tons per square inch.

American Wire.—Mr. Roebling states that bar iron of from 1 inch to 1½ inches square, fit to make the best quality of wire, should have a tensile strength of 60,000 lbs., or 27 tons per square inch. The same iron, reduced to No. 9 wire, bears 100,000 lbs., or 44½ tons per square inch; and, if drawn to No. 20 wire, it will bear from 20,000 lbs. to 30,000 lbs., or 9 to 13 tons, more. From these data it would appear that wire made of the best qualities of iron has about the same strength as some qualities of steel wire.

The tensile strength of American iron wire, together with that of wire from the cables of a suspension bridge after having been 32 years in use, according to Professor Thurston, are as in table No. 194.

Table No. 194.—Tensile Strength of American Iron Wire. 1875.

Diameter.	Breaking Weight.		Fractured Area.
	Actual.	Per Square Inch.	
inch.	pounds.	tons.	per cent.
.029	75	56.90	100
.0535	238	47.26	98.9
.071	368	40.35	91.7
.08	474	42.10	98.7
.1205	963	37.70	96.7
.134	1310	41.47	98.5
Wire from suspension bridge, .1236 diameter. Average of 12 tests.....	1081	40.17	56

Shearing and Punching Strength of Wrought Iron.

Swedish bar iron bore an average shearing stress of 15.20 tons per square inch; the ultimate tensile strength was 18.80 tons (page 581).

The shearing resistance of bars 3 inches by ½ inch and 1 inch thick, flatwise, with parallel cutters, and to punching 1-inch and 2-inch holes through bars ½ inch, 1 inch, and 1½ inches thick—the power being applied through a hydraulic shearing press,—was found by Mr. C. Little[1] to be:—

Bars.	Per square inch of area cut. tons.
½ inch thick, shearing, and punching 1-inch holes.........	22.35
1 " " " 1 "	21.83
½ " punching 2-inch holes.........................	19.00
1 " " 2 "	19.90
1½ " " 2 "	19.50

The shearing resistance of "ordinary round bar iron of commerce," by direct pull, was ascertained by Chief Engineer W. H. Shock, of the United

[1] *Proceedings of the Institution of Mechanical Engineers*, 1858; page 73.

States Navy. The following are averages of the results from 12 specimens to each average. The diameters were exactly measured; the attachments were slightly rounded at the edges, and hardened:[1]—

Diameter of Bolts.	Resistance in tons per square inch cut.	
inch.	single shear.	double shear.
½	19.68	18.32
⅝	17.41	17.23
¾	17.61	17.76
⅞	18.50	16.88
1	17.90	16.78
Averages	18.22	17.40
Mean of the averages	17.81 tons.	

The averaged results of experiments on the strength of rivetted joints,[2] showed that whilst the plates broke with a load of 19.44 tons per square inch, the rivets were sheared by a stress of 17.45 tons per square inch of section.

The shearing strength of wrought iron, in view of the foregoing data, is taken at 80 per cent., or four-fifths of the ultimate tensile strength.

Transverse Strength of Wrought Iron.

Rectangular Bars of Wrought Iron.—Wrought-iron bars are not readily ruptured by transverse stress. Their transverse elastic strength, therefore, naturally constitutes the chief matter of investigation. Actual data are extremely scarce. Mr. Barlow gives the approximate elastic tensile and transverse strengths of four bars of iron; of which the elastic tensile strength of the first bar was 9.5 tons per square inch; and of the others, 10 tons per square inch. The elastic transverse strengths of these bars are here given, as approximately observed, and as calculated from the observed tensile strength by formula (1), page 507.

Bars.			Elastic Transverse Strength. Calculated.	Observed.
2 inches	× 2 in. deep.	33 in. span.	2.66 tons.	2.50 + tons.
1.5 "	× 3 "	33 "	4.72 "	4.25 + "
1.5 "	× 3 "	33 "	4.72 "	4.25 + "
1.5 "	× 2.5 "	33 "	3.28 "	3.00 + "

The limit of elastic strength was not closely ascertained, but it was known to be greater than the observed strengths here noted. The calculated strengths appear, therefore, to be substantially correct. The following is the form of the calculation, as exemplified for the first of these bars:—

$$\frac{1.155 \times 2^3 \times 9.5}{33} = 2.661 \text{ tons.}$$

Mr. Edwin Clark tested three bars of wrought-iron, one 1 inch square, and two 1½ inches square, for transverse strength, as follows:—

[1] *Journal of the Franklin Institute*, 1874.
[2] *Transactions of the North of England Mining Institute.*

Bars.			Elastic Transverse Strength. Calculated.	Actual.
1 inch × 1 in. deep.		12 in. span.	1.155 tons.	1.117 tons.
1.5 „ × 1.5 „		36 „	1.299 „	1.275 „

The Swedish bars, noticed at page 581, 2 inches square, had an ultimate tensile strength of 18.8 tons per square inch, and the 1-inch square bars, 20.34 tons. By the formula (1), page 507,

Swedish Bars.		Ultimate Transverse Strength. Calculated.	Observed.
2.04 in. × 2.02 in. deep.	25 in. span.	7.230 tons.	7.093 tons, uncracked.
2.02 „ × 2.04 „	25 „	7.302 „	6.646 „ „
1.95 „ × 2.02 „	25 „	6.911 „	6.234 „ „
2.00 „ × 2.00 „	25 „	6.948 „	5.955 „ „
Average for 2-inch bars,.........		7.098 „	6.482 „ „

Average for 1-inch bars—

1.04 in. × 1.05 in. deep.	25 in. span.	1.077 tons.	0.883 tons, uncracked.

The calculated strength of the 2-inch bars averaged 9.5 per cent. in excess of the observed strength; and that of the 1-inch bars, 22 per cent. But the bars were not broken, nor even cracked, and they would of course have borne a greater load before breaking.

There is a regular and close correspondence between the calculated and the observed transverse strengths of wrought-iron bars above; contrasting with the diversity observed with cast-iron bars. The regularity results from the more nearly uniform texture and strength of wrought iron.

Formula (1), page 507, in its general form, may be adapted for wrought iron by assuming an average tensile strength of 22.5 tons per square inch for the value of s. Then $1.155\ s = 1.155 \times 22.5 = 26$.

Transverse Strength of Rectangular Bars of Wrought Iron, average quality.

Loaded at the middle,... $W = \frac{26\ b\ d^2}{l}$ .... (1)

Loaded at one end,...... $W = \frac{6.5\ bd^2}{l}$ .... (2)

Round Iron.—The strength of round wrought-iron bars, taking the same tensile strength, 22.5 tons, is found from the general formula (15), page 510; in which $.7854 \times s = .7854 \times 22.5 = 17.7$.

Transverse Strength of Round Wrought-Iron Bars, average quality.

Loaded at the middle,... $W = \frac{17.7\ d^3}{l}$ .... (3)

Loaded at one end,...... $W = \frac{4.4\ d^3}{l}$ .... (4)

W = the breaking weight in tons; b the breadth, d the depth, and l the span, all in inches.

Transverse Deflection and Elastic Strength of Wrought Iron.

Wrought-Iron Rectangular Bars.—The deflections of a few bars under given weights, applied at the middle, were observed by Mr. Barlow and Mr. Edwin Clark. To collate with these the observations of Mr. Kirkaldy on bars of Swedish iron, page 582, the results are here grouped together, and the value of E, the coefficient of elasticity, calculated by formula (6), page 529, are added:—

Bars.	Span.	Load.	Deflection.	E.
	inches.	tons.	inches.	
Barlow, 2 inches square, ...	33	2.50	.100	12,154
„ 2 „ „	33	2.00	.077	16,616
„ 1.5 × 3 inches deep,.........	33	4.00	.088	8,730
„ 1.5 × 3 „ „	33	4.00	.102	7,532
„ 1.5 × 2.5 „ „	33	4.00	.104	12,764
E. Clark, 1 inch square (3 bars),....	12	.711	.026	10,228
„ 1.5 „ „	36	1.275	.320	7,948
„ 1.5 „ „	36	1.145	.290	8,220
„ 1.5 „ „	36	1.695	.400	8,454
„ 1.5 „ „	36	1.695	.350	9,638
Swedish, 2.04 × 2.02 deep,.............	25	3.348	.089	7,566
„ 2.02 × 2.02 „	25	3.125	.088	7,004
„ 1.95 × 2.02 „	25	2.679	.072	7,830
„ 2.00 × 2.00 „	25	2.679	.072	7,260
Average coefficient of elasticity, E, for the first 10 bars (long span),				10,228

Adapting the general formula (8), page 530, the numerical constant becomes $4.62 \times 10{,}228 = 47{,}253$, say 47,000.

Elastic Deflection of Uniform Rectangular Bars of Wrought Iron, loaded at the middle.

$$D = \frac{W\,l^3}{47{,}000\,b\,d^3} \qquad (5)$$

D = the deflection, b the breadth, d the depth, l the span, all in inches; W the weight in tons.

Round Iron.—For round bars, substitute the above-found value of E, in the general formula (24), page 533. Then, $3.1416 \times E = 3.1416 \times 10{,}228 = 32{,}116$, say 32,000.

Elastic Deflection of Round Wrought-Iron Bars, loaded at the middle.

$$D = \frac{W\,l^3}{32{,}000\,d^4} \qquad (6)$$

Torsional Strength of Wrought Iron.

Data are extremely scarce. The Swedish iron, page 581, gave a shearing resistance of 15.2 tons per square inch. By the general formula

(1), page 534, the breaking force at the end of a 12-inch lever, applied to a $1\frac{1}{2}$-inch round bar, is

$$W = \frac{.278 \times 1.5^3 \times 15.2}{12} = 1.188 \text{ tons.}$$

The actual force was found to be (page 582) = 1.195 tons.

Taking the shearing strength of wrought iron at 80 per cent. of the tensile strength, as decided at page 588, put s = the ultimate tensile strength, then $h = .80\ s$, and, by substitution in the general formulas for torsional strength (1), page 534, and (6), page 536,

$$\text{For wrought iron round shafts,} \ldots W = \frac{.2224 d^3 s}{R} = \frac{d^3 s}{4.5\ R} \ldots\ldots\ldots \quad (7)$$

$$\text{For wrought iron square shafts,} \ldots W = \frac{.32 b^3 s}{R} = \frac{b^3 s}{3.12\ R} \ldots\ldots\ldots\ldots \quad (8)$$

W = the force in tons.
R = the radius of the force in inches.
W R = the moment of the force in statical inch-tons.
d = the diameter of the round shaft in inches.
b = the side of the square shaft in inches.
s = the ultimate tensile strength in tons per square inch.

Take the tensile strength s equal to 22.5 tons per square inch as an average value; then, by substitution and reduction:—

Torsional Strength and Sizes of Wrought-Iron Shafts of average quality.

$$\text{Round shafts:—} W = \frac{5 d^3}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$d = \sqrt[3]{\frac{WR}{5}} = .58 \sqrt[3]{W R.} \ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$\text{Square shafts:—} W = \frac{7.2\ b^3}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$b = \sqrt[3]{\frac{W R}{7.2}} = .52 \sqrt[3]{W R.} \ldots\ldots\ldots\ldots \quad (12)$$

Elastic Torsional Strength and Deflection of Wrought-Iron Bars.

The results of experiments with Swedish bars, page 582, show that the elastic torsional strength was 40 per cent. of the ultimate torsional strength.

The elastic shearing stress is found by formula (3), page 535,

$$h = \frac{W R}{.278\ d^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

For Swedish hammered bars, page 582, W R = .474 ton × 12 inches = 5.688, the moment of the force; and $h = \frac{5.688}{.278 \times 1.5^3} = 6.06$ tons per square inch, the elastic limit of shearing stress.

The value of E′, the coefficient of torsional elasticity, as defined at page 536, is found for the Swedish bar, by the general formula (12), page 537:—

$$E' = \frac{W R l}{.873\, d^4 D} = \frac{.474 \times 12 \times 10.5}{.873 \times 1.5^4 \times .011} = 1229.$$

By inversion and reduction, the equation for torsional deflection is obtained:—

Elastic Torsional Deflection of Round Wrought-Iron Bars.

$$D = \frac{W R l}{1070\, d^4} \quad \text{(14)}$$

D = the total angular deflection in parts of a revolution.
W = the twisting force in tons.
R = the radius of the force in inches.
$W R$ = the moment of the force.
l = the length of the shaft in inches.
d = the diameter of the shaft in inches.

STRENGTH OF STEEL.

Mr. Kirkaldy's Early Experiments.

In the course of the experiments already noticed,[1] page 571, Mr. Kirkaldy tested a great number of bars and plates of steel, the general results of which are given in a condensed form in tables Nos. 195 and 196. The bars were from ½ inch to 1 inch in diameter, and possessed an average tensile strength of from 60 tons per square inch for tool-steel, to 28 tons per square inch for puddled steel. The greatest observed strength was 66.2 tons.

The steel plates were from 3/16 to 5/16 inch thick. Their tensile strength ranged from 45½ to 32 tons per square inch, with the fibre. The average tensile strength was 40 tons with the fibre; and across the fibre the tensile strength was 36½ tons, or 91 per cent. of the tensile strength in the direction of the fibre.

Table No. 195.—Tensile Strength of Round Steel Bars. 1861.

(Mr. David Kirkaldy.)

Name.	Condition.	Size.	Breaking Weight per square inch. Lowest.	Highest.	Average.	Elongation in parts of length.
		inch.	tons.	tons.	tons.	per cent.
Turton's cast steel, for tools	Forged, reheated, and cooled gradually.	.53 to .59	50.10	64.90	59.32	5.4
Jowitt's do. do.		.56 to .58	52.55	66.20	59.10	5.2
Do. do. chisels		.56 to .60	50.15	61.60	55.75	7.1
Do. double shear steel...		.56 & .57	47.65	55.95	52.87	13.5
Do. cast steel for drifts...		.57	43.15	58.25	51.76	13.3
Bessemer tool steel..........		.65 to .75	46.10	54.97	49.75	5.5
Moss & Gamble, cast steel for rivets..................	Rolled.	.75	45.27	52.12	47.90	12.4
Naylor, Vickers, & Co., cast steel for rivets...........	"	.75	45.27	50.12	47.60	8.7
Wilkinson, blister steel........	Forged.	.57 to .60	39.97	51.87	46.56	9.7
Jowitt's cast steel, taps........	"	.57 & .59	37.51	49.42	45.15	10.8
Krupp's cast steel, bolts......	Rolled.	.91 to .93	38.42	42.92	41.08	15.3
Homogeneous metal..........	"	.56	36.70	44.45	40.47	13.7
Do. do.	Forged.	.75	38.30	42.30	40.05	11.9
Jowitt's spring steel...........	"	.55 to .57	29.07	36.92	32.37	18.0
Mersey Co.'s puddled steel...	"	.75	29.94	33.62	31.91	19.1
Blochairn do. ...	Rolled.	.75 to 1.0	24.55	33.54	31.32	11.3
Do. do. ...	Forged.	.75	19.00	31.93	28.24	12.0
Do. do. ...	"	.77	20.50	31.40	28.05	9.1
Averages....................................			38.50	46.80	42.66	11.2

[1] *Experiments on Wrought Iron and Steel.*

Table No. 196.—TENSILE STRENGTH OF STEEL PLATES. 1861.

(Mr. David Kirkaldy.)

Description of Steel.	Thickness of Plate.	Breaking Weight per square inch.		Elongation in parts of length.	
		With Fibre.	Across Fibre.	With Fibre.	Across Fibre.
	inch.	tons.	tons.	per cent.	per cent.
Turton & Sons, cast steel............	1/4	42.10	43.00	5.7	9.6
Shortridge & Co., do.	3/16	42.97	43.37	8.6	8.9
Naylor, Vickers, & Co., cast steel...	1/4	36.48	38.90	17.5	17.3
Moss & Gamble, do. ...	3/16 & 1/4	33.75	30.84	19.8	19.6
Shortridge & Co., do. ...	3/8	—	43.30	—	14.4
Mersey Co., puddled steel............	1/8 & 3/16	45.28	37.93	2.8	1.3
Mersey Co., "hard" puddled steel..	1/4	45.80	38.11	4.9	3.3
Blochairn, puddled steel..............	3/16	45.64	37.67	3.6	2.7
Blochairn, do.	5/16	43.00	32.90	8.2	4.1
Shortridge & Co., do.	1/4	32.32	32.85	5.9	3.2
Mersey Co., "mild" puddled steel..	1/4	34.40	30.22	6.2	5.7
Mersey Co., do. do. ..	9/32	31.93	—	3.6	—
Total averages..............................		39.42	37.17	7.8	8.2
Averages for comparison of strengths lengthwise and across...............		40.17	36.56	8.2	7.6

Mr. Kirkaldy discovered that the strength of steel was materially increased by hardening the metal in oil; and that it was materially reduced by hardening in water. Three pieces from a bar of chisel-steel were so treated, with the following results:—

	Tensile Strength.
Soft steel..	54¼ tons.
,, cooled in water........................	40¼ ,,
,, cooled in oil..............................	96⅛ ,,

Coal-tar and tallow were used for cooling steel, and with good effect; but they were not so efficacious as oil.

Steel plates, similarly treated in oil, acquired a gain of strength varying from 56.4 per cent. for the highest temperature at which they were cooled to 12.8 per cent. for the lowest.

The shearing strength of steel rivet-iron was found, from seventeen tests, to average 74 per cent. of the ultimate tensile strength of the same bar.

STRENGTH OF HEMATITE STEEL. 1866.

Mr. Kirkaldy tested the strength of bar-steel manufactured by the Barrow Hematite Steel Company. Four samples were tested for each kind of stress:—For tensile stress, cast steel, forged and turned to 1¼ inches diameter; length, 14 inches. For compressive stress, hammered cast steel, forged and turned to 1¼ inches diameter; length, 14 inches. For shearing stress, hammered cast steel, forged and turned to 1¼ inches

diameter. For transverse stress, hammered cast steel, 1¾ inches square; span, 25 inches. For torsional stress, Bessemer cast steel, forged and turned to 1¼ inches diameter; length, 8 diameters.

	Elastic, per square inch.	Ultimate, per square inch.	Elongation.
Tensile strength	18.63 tons.	32.27 tons.	19.2 per cent.
Compressive strength	23.21 „	71.24 „	—
Shearing strength	—	25.21 „	—

Transverse strength	Elastic load	3.80 tons.	Deflection .122 inch.
	Ultimate load (uncracked)	7.35 „	„ 6.64 ins.
Torsional strength (radius of lever, 12 inches)	Elastic load	.428 „	„ .008 turn.
	Ultimate load	1.03 „	„ 1.54 „

Strength of Krupp Steel. 1867-68.

Blocks of Krupp's cast steel from the heads of broken crank-shafts of the "Jeddo" and the "Sultan," were cut up into numerous specimens by

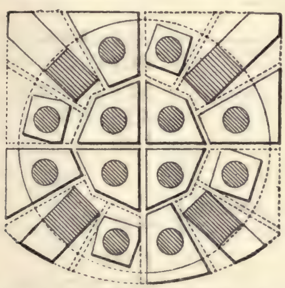


Fig. 209.—Krupp Steel Crank-shaft, "Sultan."

Mr. Kirkaldy, and tested for strength. The annexed Fig. 209 shows how the broken crank of the "Sultan" was divided and cut up.[1]

Specimens.

		Jeddo.	Sultan.
For tensile strength,	Diameter	1.25 inches.	1.128 inches.
	Length	8.5 „	10.0 „
For compressive strength,	Diameter	—	1.128 „
	Length	—	1.128 „
For transverse strength,	Breadth	1.37 „	1.50 „
	Depth	1.76 „	1.91 „
	Span	10 „	10 „
For torsional strength,	Diameter	1.25 „	1.128 „
	Length	2 diameters.	2 diameters.

[1] The author is indebted to Mr. Longsdon for copies of the "Results of Experiments," from which the above particulars have been reduced.

Average Results.

	Jeddo.	Sultan.
For tensile strength:—		
Elastic strength	18.53 tons.	19.10 tons.
Do. extension	.541 per cent.	.586 per cent.
Do. do.	1 in 185	1 in 171.
Elastic extension per ton per square inch; length = 1	$^1/_{3428}$, or .000292.	$^1/_{3266}$, or .000306.
Breaking weight	41.18 tons.	42.07 tons.
Ratio of elastic to breaking weight	45 per cent.	45.4 per cent.
Permanent extension	12.6 "	7.9 "
Sectional area of fracture	77.4 "	76.9 "
For compressive strength:—		
Elastic strength	—	21.13 tons.
Do. compression	—	.798 per cent.
Do. do.	—	1 in 125.
Elastic strength, per ton per square inch; length = 1	—	$^1/_{2641}$, or .000377.
Breaking weight	—	89.30 tons.
For transverse strength:—		
Elastic stress	7.94 tons.	10.74 tons.
Ultimate stress	21.31 "	27.14 "
Ratio	37.2 per cent.	39.6 per cent.
Elastic deflection	.055 inch.	.082 inch.
Ultimate deflection	1.49 "	1.19 "
For torsional strength:—		
Elastic stress, at the end of a 12-inch lever	.491 ton.	.497 ton.
Ultimate stress	1.280 "	1.068 "
Ratio	38.4 per cent.	47.3 per cent.
Elastic torsion	.005 turn.	.011 turn.
Ultimate torsion	.441 "	.339 "

The lowest ultimate tensile strength of the steel of the "Jeddo" was nearly 10 tons per square inch below the average; of that of the "Sultan" it was 2¼ tons below the average. The strength of the specimens cut from the interior of the blocks averaged very little less than that of those from the exterior.

The crank-shaft of the "Jeddo" was supplied to replace a broken shaft of wrought iron, noticed at page 576, of which the tensile strength averaged about 20 tons, as against 41.2 tons for the steel shaft.

Experiments of the Steel Committee.

A Committee of Civil Engineers[1] instituted and completed a series of experiments on the strength of steel bars, in 1868–70. They were con-

[1] The Committee consisted of Messrs. W. H. Barlow, George Berkley, John Fowler, Douglas Galton, C.B., and J. Scott Russel. Mr. Berkley, Secretary; Mr. W. Parsey, Assistant-Secretary.

ducted with every provision for insuring accuracy; and the results were printed in two reports, from which the following particulars are derived.

First Series of Experiments.

The first series of experiments, 203 in number, were conducted by Mr. Kirkaldy, under the instructions of the Committee, with his testing machine, in which the amounts of extension, compression, and deflection were read off a dial. The experiments were directed to test the resistance of steel bars to tension, compression, transverse strain, and torsion. Twenty-nine samples of steel bars, 2 inches square and 15 feet long, of the best marketable quality ordinarily made, were obtained from ten manufacturers; of these, 18 were of Bessemer steel, and 11 of crucible steel. Each bar was parted into lengths by a shaping machine, for bending, twisting, pulling, and thrusting, as shown by Fig. 210. For pulling, the specimen was prepared as in Fig. 211, and divided into inches of length; for bending, as in Fig. 212; for twisting, as in Fig. 213; and for thrusting, or compression, as in Fig. 214. For tensile,

Fig. 210.—Specimen Bars—how divided.

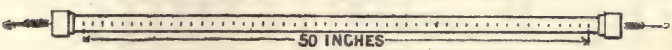


Fig. 211.—Graduation of Bars for Pulling Stress.

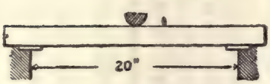


Fig. 212.—For Bending Stress.

Fig. 213.—For Twisting Stress.

Fig. 214.—For Thrusting or Compressive Stress—how divided.

SPECIMEN BARS OF THE STEEL COMMITTEE. 1st Series.

compressive, and torsional tests, the bars were turned down to 1.382 inches in diameter, having a sectional area of 1.5 square inches, and highly polished. For bending, or transverse tests, they were planed to a section of 1.9 inches square. The final results have been condensed from the report, and are worked out in the following tables:—

Table No. 197 shows the tensile strength of the steel bars.

Table No. 198 shows the elastic compressive strength of the steel bars. The ultimate compressive strength of short specimens, which is always an indefinite quantity, is not given here; but it may be stated that the short specimens required a great deal to crush them; and that the long specimens, 36 diameters in length, failed by buckling, when the elastic limit of stress was arrived at.

Tables Nos. 199 and 200 show the transverse strength and the torsional strength of steel bars,—distinguishing the elastic from the ultimate stress.

Table No. 197.—Tensile Strength of Steel Bars. 1868.

Two-inch square bars turned to 1.382 inches in diameter (1.5 square inches of area). Length, 50 inches.

(Reduced from the Experiments of the Steel Committee, 1st Series.)

Description, with respective number of samples.	Elastic Strength (tensile) in tons per square inch.	Elastic Extension, in parts of the length.	Elastic Extension per ton per square inch, in parts of the length.	Ultimate or Breaking Weight in tons per sq. inch.	Permanent Extension.	Ratio of Elastic to Breaking Strength.	Sectional Area of Fracture.
Bessemer Steel.							
Averages.	tons.	per cent.	length=1.	tons.	per cent.	per cent.	per cent.
5, Hammered, tyres,…………	23.30	.290, or 1 in 345	.000124, or $^1/_{8038}$	35.09	11.1	66.4	55.5
5, Hammered, axles,…………	21.87	.254, or 1 in 394	.000116, or $^1/_{8617}$	33.47	12.1	65.5	51.4
4, Hammered, rails,…………	21.43	.268, or 1 in 373	.000125, or $^1/_{7993}$	33.24	12.8	64.4	52.3
4, Rolled; tyres, axles, rails,…	19.30	.250, or 1 in 400	.000130, or $^1/_{7720}$	31.91	17.5	60.4	65.0
Crucible Steel.							
5, Hammered, tyres,…………	20.62	.258, or 1 in 388	.000125, or $^1/_{8000}$	35.51	9.17	58.3	62.5
4, Hammered, axles,…………	25.56	.298, or 1 in 336	.000116, or $^1/_{8588}$	40.93	8.72	62.8	72.1
1, Hammered, rails,…………	19.64	.250, or 1 in 400	.000127, or $^1/_{7856}$	38.14	2.96	51.5	96.9
1, Rolled, axles,………………	18.75	.260, or 1 in 385	.000138, or $^1/_{7219}$	30.62	10.56	61.2	89.9
Summary Averages.							
18, Bessemer steels,…………	21.48	.265, or 1 in 377	.000123, or $^1/_{8098}$	33.43	13.4	64.2	56.0
11, Crucible steels,…………	21.14	.267, or 1 in 375	.000126, or $^1/_{7927}$	36.30	7.85	58.4	80.3
29, Steels,…………………	21.31	.266, or 1 in 376	.000125, or $^1/_{8012}$	34.87	10.62	61.4	68.2

Table No. 198.—COMPRESSIVE STRENGTH OF STEEL BARS. 1868.

Two-inch square bars turned to 1.382 inches in diameter (1.5 square inches of area). Lengths, various.

(Reduced from the Experiments of the Steel Committee, 1st Series.)

BESSEMER STEEL.

DESCRIPTION, with respective number of samples.	Elastic Strength (compressive) in tons per square inch.				Elastic Compression per ton per square inch, in parts of the length of 36 diameters.
	Length, 1 diam.; 1.38 in.	Length, 2 diams.; 2.76 in.	Length, 4 diams.; 5.53 in.	Length, 36 diams.; 50 in.	
	tons.	tons.	tons.	tons.	Length=1.
5, Hammered, tyres,.	23.03	22.32	22.23	19.15	.000065, or 1/15,385
5, Hammered, axles,.	23.84	22.76	21.34	18.51	.000062, or 1/16,260
4, Hammered, rails,..	23.88	23.32	21.77	18.95	.000065, or 1/15,385
4, Rolled, ...	18.98	18.30	18.07	17.20	.000065, or 1/15,385
CRUCIBLE STEEL.					
5, Hammered, tyres,.	24.02	23.93	21.74	21.11	.000065, or 1/15,385
4, Hammered, axles,.	26.90	26.11	23.99	22.30	.000065, or 1/15,385
1, Hammered, rails,..	26.78	26.34	20.55	19.54	.000065, or 1/15,385
1, Rolled, axles, ...	21.87	19.64	18.75	18.77	.000069, or 1/14,493
SUMMARY AVERAGES.					
18, Bessemer steels,...	22.43	21.67	20.85	18.45	.000064, or 1/15,625
11, Crucible steels,.....	24.89	24.01	22.26	20.43	.000066, or 1/15,152
29, Steels,.................	23.66	22.84	21.55	19.44	.000065, or 1/15,385

Table No. 199.—TRANSVERSE STRENGTH OF STEEL BARS. 1868.

Two-inch square bars planed to 1.9 inches square. Distance of supports, 20 inches.

BESSEMER STEEL.

DESCRIPTION, with respective numbers of samples.	Elastic Stress.	Ultimate Stress.	Ratio of Elastic to Ultimate Stress.	Ultimate Deflection.	REMARKS.
AVERAGES.	tons.	tons.	per cent.	inches.	
5, Hammered, tyres,..............	7.59	13.17	57.3	3.82	Bent to 6 inches; uncracked.
5, Hammered, axles, ...	8.11	13.20	61.5	4.08	
4, Hammered, rails, ...	7.99	12.85	61.2	3.94	
4, Rolled; tyres, axles, rails, ...	6.61	11.75	56.3	4.03	
CRUCIBLE STEEL.					
5, Hammered, tyres,..............	8.36	14.65	57.4	3.32	In most cases bent to 6 inches; uncracked.
4, Hammered, axles,..............	8.38	15.52	53.9	3.35	
1, Hammered, rails,................	7.81	17.91	43.6	3.65	
1, Rolled, axles,.....................	6.89	12.06	53.8	3.84	
SUMMARY AVERAGES.					
18, Bessemer steels,..................	7.57	12.74	59.5	3.97	
11, Crucible steels,...................	7.86	15.04	52.2	3.54	
29, Steels,................................	7.74	13.89	55.7	3.76	

Table No. 200.—TORSIONAL STRENGTH OF STEEL BARS. 1868.

Two-inch square bars, turned to 1.382 inches in diameter (1.5 square inch of section). Length for torsion, 8 diameters = 11 inches.

(Reduced from the Experiments of the Steel Committee, 1st Series.)

DESCRIPTION, with respective number of samples.	Elastic Stress at the end of a 12-inch lever.	Elastic Torsion.	Ultimate Stress at the end of a 12-inch lever.	Ratio of Elastic to Breaking Stress.	Ultimate Torsion, * uncracked, for 3.75 turns. Least.	Greatest.	Average.
BESSEMER STEEL.							
AVERAGES.	tons.	1 turn=1.	tons.	per cent.	1 turn=1.	1 turn=1.	1 turn=1.
5, Hammered, tyres,	.701	.014	1.54	45.4	1.50	2.73	2.21
5, Hammered, axles,	.667	.011	1.47	44.9	2.33	3.75*	3.07
4, Hammered, rails,	.688	.012	1.45	46.8	2.10	3.75*	2.73
4, Rolled; tyres, axles, rails, ...	.569	.008	1.44	39.5	2.61	3.75*	3.11
CRUCIBLE STEEL.							
5, Hammered, tyres,	.736	.014	1.59	46.6	1.77	3.39	2.33
4, Hammered, axles,	.731	.013	1.69	43.4	1.07	2.32	1.79
1, Hammered, rails,	.714	.016	1.81	40.0	.86	.86	.86
1, Rolled, rails......	.554	.012	1.34	42.7	1.73	2.14	1.94
SUMMARY AVERAGES.							
18, Bessemer steels,...	.656	.011	1.47	44.6	—	—	2.78
11, Crucible steels,....	.684	.014	1.61	42.5	—	—	1.73
29, Steels, ...	.670	.013	1.54	43.6	—	—	2.26

The lowest tensile and compressive elastic strengths, ranged, for each group in the first series, about 5 tons per square inch below the averages given in the tables;—say 20 per cent. below the averages. The same proportionate range is found in the elastic resistances to torsional and transverse stress.

Second Series of Experiments (made at Woolwich Dockyard).

The object of the second series of experiments by the Steel Committee, was to make experiments on the tension and compression of long steel and iron bars, measuring the changes of length directly from the bars. For this purpose, 91 bars of steel and iron, each 14 feet long and 1½ inches in diameter, were obtained, consisting of 33 bars of Crucible steel, 34 bars of Bessemer steel, 12 bars of Lowmoor iron, 6 bars of best Yorkshire iron, and 6 bars of usual S. C. Crown, or Staffordshire iron.

The extensions were measured on 10 feet length of each bar; and for compressive tests, the bars were cut to a length of 12 feet, and the measurements made on a length of 10 feet.

The bars were tested in their natural skins. Before they were tested, they were thoroughly examined and straightened, and the diameters checked by means of vernier callipers, capable of showing a variation of a 1000th part of an inch. The results for iron bars have been given at page 579.

Table No. 201 gives the condensed results of the experiments for the tensile strength of steel bars, and table No. 202 gives the same for their compressive strength. The bars are here distinguished by letters.

Table No. 201.—Tensile Strength of Steel Bars. 1870.

1½ inches in diameter; 10-feet lengths.

(Reduced from the Experiments of the Steel Committee, 2d Series.)

Description, and Reference Letter.	Elastic Strength (Tensile) in tons per square inch.	Elastic Extension, in parts of the length.	Elastic Extension per ton per square inch, in parts of the length.	Breaking Weight, in tons per square inch.	Permanent Extension.	Ratio of Elastic to Breaking Strength.	Sectional Area of Fracture.
	tons.	per cent.	length = 1.	tons.	per cent.	per cent.	per cent.
Crucible Steel.							
a Chisel; 3 samples	26.0	.207, or 1 in 483	.000078, or 1/12,830	52.76	5.3	49.3	94.3
b 3 samples.	25.5	.194, or 1 in 516	.000076, or 1/13,158	51.01	7.3	49.9	80.0
c Tyres	26.0	.198, or 1 in 506	.000076, or 1/13,158	43.48	4.7	59.8	94.8
d Rods	27.0	.211, or 1 in 475	.000078, or 1/12,821	41.85	1.1	64.5	100.0
e 2 samples	20.5	.166, or 1 in 602	.000081, or 1/12,346	40.54	4.1	50.5	95.9
f Gun-barrels; 3 samples	16.8	.136, or 1 in 735	.000081, or 1/12,346	38.51	8.0	43.7	95.7
g Hammered	25.0	.200, or 1 in 500	.000080, or 1/12,500	37.05	8.0	69.4	98.5
h Hammered	20.0	.150, or 1 in 667	.000075, or 1/13,333	—	—	56.3	—
i Rods; 2 samples	26.75	.201, or 1 in 498	.000075, or 1/13,333	33.65	0.9	79.5	98.7
j Rolled; 2 samples	20.5	.160, or 1 in 625	.000078, or 1/12,821	34.43	2.0	59.5	97.1
Bessemer Steel.							
k Fagotted, hammered, and rolled; 3 samples	19.5	.148, or 1 in 675	.000076, or 1/13,158	35.40	11.1	55.0	55.8
l 3 samples	20.0	.150, or 1 in 667	.000075, or 1/13,333	34.19	11.9	58.4	54.4
m 2 samples	17.5	.140, or 1 in 715	.000080, or 1/12,500	33.63	11.5	52.0	80.8
n Tyres and axles; 3 samples	16.5	.135, or 1 in 740	.000082, or 1/12,200	33.66	13.6	49.0	58.8
Summary Averages.							
Crucible steels	23.4	.182, or 1 in 550	.000078, or 1/12,850	40.88	5.1	58.0	90.6
Bessemer steels	18.4	.144, or 1 in 695	.000078, or 1/12,780	34.22	12.0	53.8	62.5
All steels	20.9	.163, or 1 in 613	.000078, or 1/12,820	37.55	8.5	55.9	76.6

Table No. 202.—Compressive Strength of Steel Bars. 1870.

1½ inches in diameter; 10-feet lengths.

(Reduced from the Experiments of the Steel Committee, 2d Series.)

Crucible Steel.

Description, and Reference Letter.	Elastic Strength (compressive) in tons per square inch.	Elastic Compression.	Elastic Compression per ton per square inch, in parts of the length.
	tons.	per cent.	length=1.
a Chisel; 3 samples......	26.33	.198, or 1 in 506	.000075, or 1/13,333
b Tyre; 3 samples........	26.2	.202, or 1 in 496	.000077, or 1/12,987
c 2 samples..................	25.5	.186, or 1 in 537	.000073, or 1/13,699
d Rods; 2 samples	26.0	.198, or 1 in 506	.000076, or 1/13,158
e	18.0	.137, or 1 in 731	.000076, or 1/13,158
f Gun-barrels; 3 samples	16.2	.126, or 1 in 791	.000080, or 1/12,500
g Hammered; 2 samples	24.0	.180, or 1 in 555	.000075, or 1/13,333
h 2 samples..................	19.5	.138, or 1 in 722	.000071, or 1/14,085
i Rods........................	27.0	.204, or 1 in 490	.000075, or 1/13,333
j Rolled	24.0	.185, or 1 in 541	.000077, or 1/12,987
Bessemer Steel.			
k Fagotted, hammered and rolled; 3 samp.	18.0	.133, or 1 in 751	.000074, or 1/13,514
l 3 samples..................	21.2	.163, or 1 in 612	.000077, or 1/12,987
m 2 samples	16.0	.125, or 1 in 801	.000078, or 1/12,821
n Tyres, axles; 3 samp.	16.0	.125, or 1 in 801	.000078, or 1/12,821
Summary Averages.			
Crucible steels...........	23.3	.175, or 1 in 570	.000076, or 1/13,250
Bessemer steels.........	17.8	.137, or 1 in 732	.000077, or 1/13,040
All steels..................	20.5	.156, or 1 in 641	.000076, or 1/13,140
Bars Tested—for Compression, but not for Extension.			
o Crucible steel; axles, rails, tyres (3 samp.)	23.0	.172, or 1 in 581	.000073, or 1/13,700
p Bessemer steel; axles, rails, tyres (3 samp.)	24.0	.182, or 1 in 550	.000074, or 1/13,514

The lowest elastic strength in any group of bars, in the second series, did not exceed one ton per square inch less than the average elastic strength of the group—say not more than 5 per cent. less than the average for steel bars.

Table No. 203 shows the chemical composition and specific gravity of fourteen of the bars subjected to tests.

Tensile Strength of Tempered Steel.

The Steel Committee publish the results of experiments made at H. M. Gun Factory, Woolwich, on the comparative strength of untempered and tempered steel. A summary of the results is given in table No. 204. The specimens were .50 and .53 inch in diameter, and from 1 inch to two inches in length.

Table No. 203.—Chemical Analysis and Specific Gravity of Steel and Iron Bars.

(Tested by the Steel Committee, 1870.)

Crucible Steel.

Reference.	Chemical Constituents. Iron.	Carbon.	Silicon.	Manganese.	Sulphur.	Phosphorus.	Specific Gravity.	Ultimate Tensile Strength per square inch.
	p. cent.	p. cent.	p. cent.	p. cent.	p. cent.	p. cent.		tons.
a	98.86	.79	.115	.19	trace	.01	7.839	52.76
b	98.67	.67	.20	.44	trace	.02	7.831	51.01
c	98.87	.57	.14	.37	.01	.04	7.851	43.48
d	98.63	.90	—	.39	.02	.06	7.844	41.85
e	98.87	.58	.22	.30	.02	.01	7.825	40.50
f	98.88	.47	—	.61	.02	.02	7.845	38.51
o	99.22	.59	.03	.14	trace	.02	7.859	—
h	99.16	.44	.14	.23	.01	.02	7.850	35.47
i	99.16	.52	.10	.19	trace	.03	7.850	33.65
Averages,	98.89	.62	.114	.34	.01	.026	7.842	42.15
Bessemer Steel.								
p	99.24	.34	trace	.35	.04	.03	7.857	—
l	99.21	.32	.01	.40	.04	.02	7.857	34.19
m	99.22	.31	.05	.38	.02	.02	7.853	33.68
n	99.13	.35	.03	.44	.04	.01	7.852	33.66
Averages,	99.20	.33	.022	.39	.035	.02	7.855	33.84
Yorkshire Iron.								
	99.49	.23	.10	.08	.02	.08	7.758	23.69

Table No. 204.—Tensile Strength of Tempered Steel.

Manufacturer, or Contractor.	Material.	Number of Specimens.	Average Breaking Weight per Square Inch. As Received.	After being Tempered in Oil at High Heat.	Medium Heat.	Low Heat.	?	Tempered in Water.
			tons.	tons.	tons.	tons.	tons.	tons.
Krupp,.....	Cast Steel,....	9	32.1	65.4	54.4	56.4		
Firth,.......	Steel,...........	3	34.4					
Firth,.......	Homog. Steel,	217	31.6	47.7	48.0	47.0	44.4	
Cammell,..	Steel, ...	2	26.6					
Cammell,..	Homog. Steel,	61	29.5	54.6	45.7	51.5	49.0	51.1
—	Steel, ...	4	31.7				36.6	
—	Homog. Steel,	36	29.0		37.2		39.3	53.9
—	Styrian Steel,.	8	56.2		82.2			
Moser	Steel, ...	7	33.5	54.6	55.8			
Whitworth,	Steel, ...	2	38.2		48.2			

Strength of Fagersta Steel. 1873.

Mr. Kirkaldy made a comprehensive set of experiments on the strength of steel manufactured at the Fagersta Works, Sweden.

First Series of Experiments.

Twelve hammered bars, 2 inches square, in four groups of different degrees of hardness, here distinguished as *a*, *b*, *c*, *d*, were tested for tensile, compressive, shearing, torsional, and transverse strength—three samples for each test. For the transverse tests, the specimens were planed to 1.9 inches

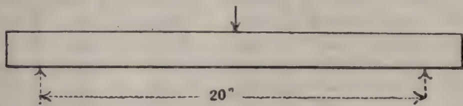


Fig. 215.—Fagersta Steel—Test Specimen for Bending or Transverse Stress.

square; for the other tests, they were turned to a diameter of 1.128 inches, having 1 square inch of sectional area. The forms of the specimens are shown in Figs. 215, 216, 217. The condensed results are given in tables Nos. 205 to 207.

Table No. 205.—Fagersta Steel Bars—Transverse Strength. 1873.

1.9 inches square; span, 20 inches. Load applied at the middle.

Bars.	Elastic Stress.	Ultimate Stress.	Ratio of Elastic to Ultimate Stress.	Ultimate Deflection.	Remarks.
	tons.	tons.	per cent.	inches.	
a	9.43	14.55	66.0	.78	fractured
b	9.69	19.57	49.6	1.49	fractured
c	8.18	17.03	48.0	3.31	uncracked
d	7.04	11.28	62.3	5.11	uncracked
Averages......	8.58	15.61	56.5	2.67	

Table No. 206.—Fagersta Steel Bars—Torsional Strength. 1873.

Diameter 1.128 inches (1 square inch section). Length for torsion, 8 diameters.
Stress applied at the end of a 12-inch lever.

Bars.	Elastic Stress.	Breaking Stress.	Ratio of Elastic to Breaking Stress.	Ultimate Angular Torsion.		
				Least.	Greatest.	Average.
	tons.	tons.	per cent.	1 turn=1.	1 turn=1.	1 turn=1.
a	.507	.946	53.9	.207	.410	.291
b	.502	1.043	48.2	.625	.938	.793
c	.484	1.009	48.3	.897	1.255	1.021
d	.341	.679	50.2	3.053	3.725	3.219
Averages......	.458	.919	50.2	1.195	1.528	1.331

Table No. 207.—Fagersta Steel Hammered Bars—Tensile, Compressive, and Shearing Strength. 1873.

2-inch square bars turned to 1.128 inches in diameter (1 square inch of section).

Tensile Strength.

Bars. 10.15 inches long. (9 diameters.)	Elastic Strength in tons per square inch.	Breaking Weight in tons per square inch.	Ratio of Elastic to Breaking Strength.	Permanent Extension.	Sectional Area of Fracture.
	tons.	tons.	per cent.	per cent.	per cent.
a	27.70	38.04	73.1	1.8	97.7
b	28.15	47.60	59.4	5.1	93.9
c	25.94	45.82	56.6	6.6	85.6
d	19.24	27.37	70.3	16.5	38.5
Averages.....	25.25	39.16	64.8	7.5	78.9

Compressive Strength (per square inch).

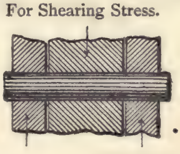
Bars.	Length, 1 diameter, 1.128 ins.	Length, 2 diameters, 2.25 inches.		Length, 4 diameters, 4.51 inches.		Length, 8 diameters, 9.02 inches.	
	Elastic Strength.	Elastic Strength.	Destroying Weight.	Elastic Strength.	Destroying Weight.	Elastic Strength.	Destroying Weight.
	tons.	tons.	tons.	tons.	tons.	tons.	tons.
a	28.57	28.27	75.85	28.27	59.92	27.53	45.62
b	27.98	26.19	77.37	26.19	52.50	25.89	42.50
c	26.78	25.13	69.64	23.81	47.01	23.51	37.87
d	17.41	18.75	54.15	18.30	36.50	18.16	21.05
Averages	25.18	24.70	69.24	24.03	48.88	23.77	36.76

Shearing Strength (per square inch).

Bars.	Ultimate Shearing Strength.		Detrusion before Rupture, as a measure of hardness inversely.	
	Per square inch.	Per cent. of Ultimate Tensile Strength.	Actual.	In parts of the diameter.
	tons.	per cent.	inch.	per cent.
a	27.42	73.3	.193	17
b	35.60	75.2	.249	21
c	31.99	69.5	.281	25
d	20.28	74.0	.323	29
Averages.........	28.82	73.5	.261	23

For Pulling or Tensile Stress.

For Shearing Stress.



Figs. 216, 217.—Fagersta Steel—Test Specimens.

Second Series of Experiments on Fagersta Steel.

To test the influence of hammering, and of annealing steel bars. Four ingots 6 inches square, differing in hardness, *e, f, g, h*, were cut into specimens alike, in duplicate, and hammered down to four sizes, namely, 5, 4, 3, and 2 inches square.

In table No. 208, are given results showing the comparative tensile strength of each ingot, and of the 2-inch hammered bars formed from them. They prove that the steel was made considerably stronger by hammering. In the original Report, it is made apparent that the strength was proportionally increased as the bars were reduced in size.

Table No. 208.—Fagersta Steel Ingots and Hammered Bars—Comparative Tensile Strength. 1873.

Unannealed.

Ingots and Hammered Bars of Various Degrees of Hardness.	Elastic Strength (Tensile), in tons per square inch.	Breaking Weight in tons per square inch.	Ratio of Elastic to Breaking Weight.	Permanent Extension.	Sectional Area of Fracture.
Ingots, 6 inches square.	tons.	tons.	per cent.	per cent.	per cent.
e	21.27	29.98	71.0	1.1	98.5
f	17.21	30.48	58.3	2.0	97.5
g	12.64	24.65	51.2	3.5	95.8
h	9.68	12.46	42.1	11.6	88.1
Averages,..................	15.20	24.39	55.6	4.5	95.0
Hammered Bars, 2 inches square.					
e	29.69	44.03	67.4	2.2	96.8
f	21.30	43.70	48.7	10.2	71.6
g	17.50	33.48	52.3	17.9	47.5
h	15.71	26.76	58.7	22.5	38.7
Averages,..................	21.05	37.00	56.8	13.2	63.6
Annealed.					
Ingots, 6 inches square.					
e	17.34	28.37	61.2	1.7	97.8
f	16.55	33.05	51.3	7.2	85.0
g	11.68	23.66	49.5	4.2	94.8
h	9.00	23.57	38.2	18.2	72.8
Averages,..................	13.64	27.16	50.0	15.6	87.6
Hammered Bars, 2 inches square.					
e	21.20	38.42	55.2	5.5	91.9
f	20.67	40.99	50.4	12.7	54.0
g	16.30	31.60	51.6	19.1	42.4
h	14.78	25.15	58.7	22.2	35.9
Averages,..................	18.24	34.04	54.0	14.9	56.0

Table No. 209.—Fagersta Steel Hammered and Rolled Bars—Comparative Tensile Strength. 1873.

Bars of Various Degrees of Hardness.	Elastic Strength in tons per square inch.	Breaking Weight in tons per square inch.	Ratio of Elastic to Breaking Weight.	Permanent Extension.
Hammered—Unannealed.				
	tons.	tons.	per cent.	per cent.
3-inch square bars.				
i	27.45	39.49	87.5	.4
k	17.32	31.29	55.4	2.3
l	12.68	25.28	50.2	25.2
Averages,	19.15	32.02	64.4	9.3
½-inch square bars.				
i	42.05	60.59	69.4	5.7
k	34.96	42.84	51.6	16.0
l	25.67	32.21	79.7	10.1
Averages,	34.23	45.21	66.9	10.6
Hammered—Annealed.				
3-inch square bars.				
i	20.85	31.66	65.8	1.7
k	13.30	31.09	42.8	7.7
l	12.68	25.28	50.2	25.2
Averages,	15.61	29.34	52.9	11.5
½-inch square bars.				
i	31.12	54.92	56.8	8.3
k	21.34	36.66	58.2	7.7
l	14.24	25.39	56.1	12.6
Averages,	22.23	38.99	57.0	9.5
Rolled—Unannealed.				
3-inch square bars.				
i	26.87	33.75	79.6	.6
k	13.57	27.85	48.7	2.5
l	10.22	23.64	43.2	31.1
Averages,	16.89	28.41	57.2	11.4
½-inch square bars.				
i	35.09	59.82	56.2	7.3
k	20.89	40.50	51.6	16.0
l	15.09	27.13	55.6	22.2
Averages,	23.69	42.48	54.4	15.2

Table No. 209 (*continued*).

ROLLED—ANNEALED.

Bars of Various Degrees of Hardness.	Elastic Strength in tons per square inch.	Breaking Weight in tons per square inch.	Ratio of Elastic to Breaking Weight.	Permanent Extension.
3-inch square bars.	tons.	tons.	per cent.	per cent.
i	19.78	32.55	60.7	2.0
k	12.32	26.87	45.8	3.8
l	10.22	23.64	47.7	26.0
Averages, ...	14.11	27.69	51.4	10.6
½-inch square bars.				
i	28.93	57.14	50.6	8.5
k	18.39	35.81	51.4	9.8
l	12.45	23.50	53.0	23.1
Averages, ...	20.00	38.82	51.7	13.8

Third Series of Experiments on Fagersta Steel.

To compare the tensile strength of steel bars, reduced by hammering and by rolling. Bars of three degrees of hardness were tested, say *i*, *k*, *l*. Of each degree, six 3-inch square hammered bars were tested, five of which were reduced by hammering to 2½, 2, 1½, 1, and ½ inch square, and then turned to given diameters.

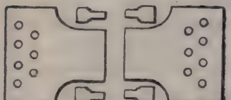
In table No. 209 are given comparative results for the 3-inch and ½-inch square bars, hammered and rolled. The original table shows that the strength was proportionally increased as the bars were reduced in size.

Fourth Series of Experiments on Fagersta Steel.

To test the tensile and compressive strength of Fagersta steel plates, of ⅛, ¼, ⅜, ½, and ⅜ inch thickness, cut into strips 2¼ inches wide. Table No. 210 gives the comparative results of the trials.

Fifth Series of Experiments on Fagersta Steel.

To show the variations in results for tensile strength, arising from differences in the form and proportions of specimens. Two sets of specimens were prepared according to Figs. 218; one set was 10 inches wide and 10 inches long at the parallel middle portion; and the smaller set 1½ inches wide and 4½ inches long at the middle. Condensed results are given in table No. 211; and the results of the 100-inch bars, from table No. 210, are added, for comparison.



Figs. 218.—Fagersta Steel.—Test Specimens of Plates.

Table No. 210.—Fagersta Steel Plates—Tensile and Compressive Strength. 1873.

Specimens 2¼ inches wide, 100 inches long.

Tensile Strength—Unannealed.

Plates. Thickness.	Elastic Strength (Tensile) in tons per square inch.	Elastic Extension.	Breaking Weight in tons per square inch.	Permanent Extension.	Ratio of Elastic to Breaking Strength.	Sectional Area of Fracture.
inch.	tons.	per cent.	tons.	per cent.	per cent.	per cent.
⅛	17.37	.136	24.61	5.21	70.6	62.1
¼	15.89	.124	24.17	10.17	65.7	46.3
⅜	11.34	.091	21.84	20.64	51.9	29.0
½	12.28	.082	22.39	16.30	54.8	38.8
⅝	11.65	.078	22.00	17.95	52.9	39.3
Averages	13.71	.102 or 1 in 980	23.00	14.03	59.2	43.1
Annealed.						
⅛	11.92	.096	20.30	10.98	58.7	36.4
¼	13.30	.098	22.14	16.88	60.1	32.5
⅜	11.56	.096	20.86	18.19	55.4	30.4
½	12.19	.088	22.09	19.15	55.2	35.7
⅝	11.25	.088	21.18	17.45	53.1	36.9
Averages	12.04	.093 or 1 in 1020	21.31	16.53	56.5	34.4

Elastic extension per ton per square inch,...... { Unannealed............ .0000744, or $^1/_{13,435}$.
Annealed................ .0000772, or $^1/_{12,281}$

Compressive Strength—Unannealed.

Plates. Thickness.	Elastic Strength (Compressive) in tons per square inch.	Elastic Compression.	Elastic Compression per ton per square inch, in parts of the length.
inch.	tons.	per cent.	Length=1.
⅛	17.81	.106	
¼	16.20	.115	
⅜	11.83	.089	
½	13.30	.099	
⅝	11.38	.088	
Averages.........	14.10	.100, or 1 in 1000	.000071, or $^1/_{14,100}$
Annealed.			
⅛	10.40	.063	
¼	12.28	.088	
⅜	11.25	.083	
½	10.13	.074	
⅝	8.98	.066	
Averages.........	10.61	.075, or 1 in 1333	.0000707, or $^1/_{14,143}$

Table No. 211.—FAGERSTA STEEL PLATES—TENSILE STRENGTH AS AFFECTED BY THE FORM AND PROPORTIONS OF THE SPECIMENS. 1873.

Averaged results of specimens from ⅛ to ⅝ inch thick.

UNANNEALED.

SPECIMENS.	Elastic Strength (Tensile) in tons per square inch.	Breaking Weight in tons per square inch.	Ratio of Elastic to Breaking Weight.	Permanent Extension.	Sectional Area of Fracture.
	tons.	tons.	per cent.	per cent.	per cent.
Length = breadth....Figs. 218	16.05	26.39	60.0	29.7	48.8
Length = 3 breadths „	15.56	25.56	60.3	35.0	43.2
Length = 44 breadths...........	13.71	23.00	59.2	14.0	43.1
ANNEALED.					
Length = breadth....Figs. 218	13.53	23.65	57.0	33.1	39.1
Length = 3 breadths „	12.98	23.16	56.0	39.3	36.5
Length = 44 breadths...........	12.04	21.31	56.5	16.5	34.4

Sixth Series of Experiments on Fagersta Steel.

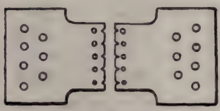


Fig. 219.—Fagersta Steel—Plates Drilled and Punched.

To test the influence of holes drilled and holes punched in steel plates. Specimens were formed 12½ inches wide, and otherwise like the broad specimen, Fig. 219, for comparison. There were three rows of rivet holes, 3 inches apart; and five holes in each row at 2½-inch centres. The holes were .77 inch in diameter, and made $.77 \times 5 = 3.85$ inches of blank; the net section was $(12.5 - 3.85 =)$ 8.65 inches wide, or 69.2 per cent. of the total width. Table No. 212 gives some deductions from the reported results.

Table No. 212.—FAGERSTA STEEL PLATES—TENSILE STRENGTH, WITH RIVET HOLES WHEN DRILLED AND WHEN PUNCHED.

Specimens 12½ inches wide. Three rows of holes .77 inch in diameter.

UNANNEALED.

PLATES. — Thickness.	Reduced Section in parts of the Total Section.	DRILLED HOLES. Reduced Strength in parts of Total Strength.	DRILLED HOLES. Tensile Strength per square inch of Net Section, in parts of that of Unreduced Section.	PUNCHED HOLES. Reduced Strength in parts of Total Strength.	PUNCHED HOLES. Tensile Strength per square inch of Net Section, in parts of that of Unreduced Section.
inch.	per cent.	per cent.	per cent.	per cent.	per cent.
⅛	69.2	74.9	108.5	67.0	97.1
¼	69.2	76.6	111.0	68.5	99.2
⅜	69.2	77.0	111.6	69.5	100.4
½	69.2	78.3	113.5	54.8	79.4
⅝	69.2	77.2	112.0	51.0	74.0
Averages...	69.2	76.8	111.3	62.2	90.0

Table No. 212 (*continued*).

ANNEALED.

PLATES. Thickness.	Reduced Section in parts of the Total Section.	DRILLED HOLES. Reduced Strength in parts of Total Strength.	DRILLED HOLES. Tensile Strength per square inch of Net Section, in parts of that of Unreduced Section.	PUNCHED HOLES. Reduced Strength in parts of Total Strength.	PUNCHED HOLES. Tensile Strength per square inch of Net Section, in parts of that of Unreduced Section.
inch.	per cent.	per cent.	per cent.	per cent.	per cent.
⅛	69.2	72.9	105.7	67.1	97.1
¼	69.2	74.5	108.0	67.8	98.0
⅜	69.2	75.1	108.8	66.4	96.1
½	69.2	77.0	111.6	69.9	105.6
⅝	69.2	75.9	110.0	68.7	99.2
Averages...	69.2	75.1	108.8	68.0	98.6

	Unannealed. per cent.	Annealed. per cent.
Note.—The average elongation with drilled holes, ...	14.9	18.0
Do. do. punched holes, ...	6.3	16.6
Do. do. solid plate,........	29.7	33.1

Seventh Series of Experiments on Fagersta Steel.

To test rolled steel plates under bulging stress. The specimens were discs, 12 inches in diameter, cut out in the lathe, and pressed through an aperture 10 inches in diameter. The bulger or ram was cylindrical, about 5 inches in diameter; and the preparation for the trial is shown in Fig. 204, page 584, and the finished article, after bulging, in Fig. 205.

Table No. 213.—FAGERSTA STEEL PLATES—RESISTANCE TO BULGING STRESS.

Discs 12 inches in diameter; aperture 10 inches in diameter.

UNANNEALED.

DISC. Thickness.	Stress—Bulging in inches. Lbs., 25,000.	Lbs., 100,000.	Lbs., 200,000.	Ultimate. Bulge.	Ultimate. Stress.	Effect.
inch.	inches.	inches.	inches.	inches.	tons.	
⅛	1.86	—	—	3.00	14.50	buckled
¼	1.09	—	—	3.11	31.94	uncracked
⅜	.89	2.68	—	3.22	46.92	„
½	.68	1.93	—	3.33	71.83	„
⅝	.44	1.61	2.77	3.44	97.90	„
ANNEALED.						
⅛	2.25	—	—	3.04	11.53	buckled
¼	1.32	—	—	3.12	26.77	uncracked
⅜	.94	—	—	3.23	43.67	„
½	.73	2.06	—	3.34	67.28	„
⅝	.52	1.72	3.14	3.45	90.09	„

SIEMENS-STEEL PLATES AND TYRES. 1875.

A number of steel plates manufactured to the specification of the Admiralty, by the Landore Siemens-Steel Company, were tested for the Company by Mr. Kirkaldy.

By the terms of the specification, it was required that the ultimate tensile strength should be not less than 26 tons, nor more than 30 tons, per square inch, with an extension of 20 per cent. in a length of 8 inches. Strips cut lengthwise of the plate, 1½ inches wide, heated uniformly to a low cherry-red heat, and cooled in water at 82° F., were to sustain bending double in a press, to a curve of which the inner radius was to be one-and-a-half times the thickness of the plate. Abstracts of the results of the tests for tensile strength are given in table No. 215, together with tests for the tensile strength of steel tyres. Twelve specimens of the plates of the 2d series in the table, were tested for bending, lengthwise and crosswise, between supports at 10 inches apart. All the specimens bore the test uncracked.

Plates of various thicknesses were tested for resistance to bulging stress, 12-inch discs having been forced through 10-inch apertures, in the manner before described, page 584. All the plates bore the test without cracking. Particulars are given in table No. 214.

Two steel tyres, of which the tensile strengths were tested (3d series, table No. 215), were respectively 43 and 37 inches in diameter, and 2.32 and 2.10 inches in thickness. They were collapsed under transverse pressures of 42.22 and 52.16 tons; so that opposite sides of the hoop were pressed into contact with each other. The larger tyre burst at one of the bends; the smaller remained unbroken.

Table No. 214.—SIEMENS-STEEL PLATES—RESISTANCE TO BULGING STRESS. 1875.

Discs 12 inches in diameter, pressed into 10-inch apertures.

(Reduced from Mr. Kirkaldy's Reports.)

Thickness of Plates.	Stress. Bulging in inches.			Ultimate.		EFFECTS.
	lb. 25,000.	lb. 100,000.	lb. 200,000.	Bulge.	Stress.	
inch.	inch.	inches.	inches.	inches.	tons.	
Unannealed.						
.37	.42	1.71	—	3.15	63.750	uncracked.
.71	.05	1.09	1.96	3.48	145.500	do.
Annealed.						
.37	.67	2.02	—	3.17	60.357	uncracked.
.41	.56	1.84	—	3.22	68.191	do.
.41	.59	1.89	—	3.23	68.080	do.
.50	.29	1.45	2.79	3.31	101.920	do.
.62	.15	1.40	2.51	3.38	115.111	do.
.70	.10	1.26	2.18	3.42	123.260	do.

Table No. 215.—SIEMENS-STEEL PLATES—TENSILE STRENGTH. 1875.

From .37 to .71 inch in thickness.

(Reduced from Mr. Kirkaldy's Reports.)

SERIES 1. PLATES OF DIFFERENT THICKNESSES.

Treatment, and Thickness of Plates.	Elastic Strength per square inch.	Ultimate Strength per square inch.	Ratio of Elastic to Ultimate Strength.	EXTENSION. At 60,000 lbs. per sq. inch.	EXTENSION. Ultimate.	Sectional Area of Fracture.
LENGTHWAY. inch.	tons.	tons.	per cent.	per cent.	per cent.	per cent.
Unannealed.... .37	15.446	32.535	47.4	4.50	22.3	62.5
Do. .... .71	13.572	29.870	45.4	6.75	24.5	55.3
Means............	14.509	31.202	46.4	5.62	23.4	58.9
Annealed ...... .37	14.062	30.143	46.6	8.00	24.8	56.9
Do. ...... .40	13.929	29.647	46.9	8.08	21.1	55.3
Do. ...... .40	13.303	29.491	45.1	8.50	24.8	61.5
Do. ...... .50	13.125	29.388	44.6	8.66	26.4	55.5
Do. ...... .62	11.741	27.595	42.5	13.80	25.5	56.7
Do. ...... .70	10.937	26.821	40.7	17.72	25.0	54.5
Averages..........	12.848	28.848	44.4	10.79	24.6	56.8
CROSSWAY.						
Unannealed.... .37	15.314	32.442	47.2	4.52	22.4	62.5
Do. .... .71	13.571	30.062	45.1	7.07	24.7	56.4
Means	14.442	31.250	46.1	5.79	23.5	59.5
Annealed ...... .37	13.928	29.856	46.6	9.39	26.4	53.4
Do. ...... .40	13.840	29.875	46.3	9.07	26.3	50.4
Do. ...... .42	13.393	29.366	45.6	7.81	20.4	61.0
Do. ...... .52	13.303	29.705	44.8	8.50	20.2	53.3
Do. ...... .62	11.741	27.040	43.4	16.61	22.7	64.7
Do. ...... .70	10.937	26.885	40.6	17.30	26.0	49.3
Averages..........	12.856	28.788	44.5	11.44	23.6	59.1
SERIES 2. PLATES ANNEALED, AND HARDENED.						
Annealed ...... .64	—	25.483	—	—	24.1	47.5
Do. ...... .62	—	26.996	—	—	20.2	51.3
Means		26.240			22.2	49.4
Hardened:— Cherry-red, and cooled in water at 82° F......... .64	—	28.867	—	—	22.4	50.7
Do. .... .62	—	29.036	—	—	18.0	54.5
Means		28.951			20.2	52.1

Table No. 215 (*continued*).

SERIES 3. TYRES.

Diameter of Specimens.	Elastic Strength per square inch.	Ultimate Strength per square inch.	Ratio of Elastic to Ultimate Strength.	Extension. At 60,000 lbs. per sq. inch.	Extension. Ultimate.	Sectional Area of Fracture.
inches.	tons.	tons.	per cent.	per cent.	per cent.	per cent.
1st Tyre, specimen.						
1.511	17.098	29.853	57.2	6.58	18.8	55.8
1.511	17.321	30.800	56.2	6.20	23.6	51.9
2d Tyre, specimen.						
1.511	18.482	30.083	61.4	6.48	17.7	58.8
1.511	18.840	31.075	60.6	5.59	16.9	70.4
Averages.........	17.935	30.453	58.8	6.21	19.2	59.2

WHITWORTH'S FLUID-COMPRESSED STEEL.[1]

On Sir Joseph Whitworth's system of treatment, a pressure of 6 tons per square inch is applied as quickly as possible to melted steel, after it is taken from the furnace. A column 8 feet high is reduced 1 foot in height in the course of five minutes.

Specimens for testing tensile resistance are cylindrical, formed as in Fig. 220; the central portion has a sectional area of ½ square inch, being

Figs. 220, 221.—Whitworth's Fluid-Compressed Steel—Test Specimens.

.798 inch in diameter, and has length of 2 inches, or 2½ diameters. The upper and lower portions are screwed, and are seized by nuts. The usual appearance of broken specimens is shown at Fig. 221.

Table No. 216 gives results of tests for the tensile resistance of fluid-compressed steel, and of the purest and best irons made in England.

Sir Joseph Whitworth states that he can produce, with certainty, by compression, steel having 40 tons ultimate strength, with 30 per cent. ductility. In relation to this, Mr. F. W. Webb says that he has no difficulty in producing a mild cast steel having 30 to 32 tons ultimate strength, and 33 or 34 per cent. ductility.

Sir Joseph Whitworth considers that there is no need for more than 30 per cent. of ductility; with this proportion, steel tears when ruptured, and does not fly to pieces.

He expresses the value of steel by the sum of the tensile strength in tons per square inch, and the ductility in percentage of the length, found by fracturing specimens of the form, Fig. 220. Thus, for steel of 40 tons strength, and 30 per cent. ductility, the resultant value is (40 + 30 =) 70.

[1] The materials for this notice are derived from the *Proceedings of the Institution of Mechanical Engineers*, 1875, page 268.

Table No. 216.—Whitworth's Fluid-compressed Steel, and Best Irons—Tensile Strength.

1. Fluid-compressed Steel.

Arbitrary Distinguishing Colours for Groups.	Ultimate Tensile Strength per square inch.	Ductility, or Elongation.	Uses to which the Steel is applicable.
	tons.	per cent.	
Red, Nos. 1, 2, 3....	40	32	Axles, boilers, connecting rods, cross-heads, crank-pins, hydraulic cylinders, cranks, propeller shafts, rivets, tyres, &c.
Blue, Nos. 1, 2, 3...	48	24	Cylinder linings, slide-bars for locomotives, shafting, couplings, drill-spindles, eccentric-shafts for punching machines, large swages, hammers, &c.
Brown, Nos. 1, 2, 3,	58	17	Large planing and lathe tools, large shears, drills, smiths' punches, dies and taps, small swages, &c.
Yellow, Nos. 1, 2, 3,	68	10	Boring tools, finishing tools for planing and turning.
Special alloy with Tungsten........	72	14	For particular purposes.

Note.—In each group No. 1 is most ductile, No. 3 least ductile.

2. Iron.

Description.	Ultimate Tensile Strength.		Elongation.	
	Several Specimens.	Averages.	Several Specimens.	Averages.
Wrought Iron.	tons per square inch.	tons.	per cent.	per cent.
Yorkshire...............	31, 30, 29, 27, 27, 26.8, 26.8	28.3	23, 22, 31, 41, 22, 43, 42	32
Lowmoor...............	27, 24.8	25.9	39, 40	39.5
Northamptonshire...	27, 26.8, 26, 25	26.2	39, 40, 41, 38	39.5
Staffordshire...........	25, 24, 24, 24, 20	23.4	35, 39, 34, 33, 15	31.2
Do. (Dudley Ward).	26, 24, 24	24.7	30, 35, 28	31
Averages...........		25.7		34.6
Cast Iron.	13, 12, 11, 11, 10, 9.5, 7	10.5	.90, 1.10, 1.00, .65, .75, .12, .50	.72

CHERNOFF'S EXPERIMENTS ON STEEL.[1]

Steel, when cast and allowed to cool quietly, assumes a crystalline structure. The higher the temperature to which it is heated, the softer it becomes, and the greater is the liberty its particles possess to group themselves into crystals.

Steel, however hard it may be, will not harden if heated to a temperature lower than what may be distinguished as dark cherry-red (temperature *a*), however quickly it is cooled; on the contrary, it will become sensibly softer, and more easily worked with the file.

Steel heated to a temperature lower than, say, red but not sparkling (temperature *b*), does not change its structure whether cooled quickly or slowly. When the temperature, in rising, has reached *b*, the substance of steel quickly passes from the granular or crystalline condition, to the amorphous, or wax-like structure, which it retains up to its melting point (temperature *c*).

The points *a*, *b*, and *c*, have no permanent place in the scale of temperature, but their positions vary with the quality of the steel; in pure steel, they depend directly on the quantity of constituent carbon. The harder the steel, the lower the temperatures. The tints above specified have reference only to hard and medium qualities of steel; in the very soft kinds of steel, nearly approaching to wrought iron, the points *a* and *b* range very high, and in wrought iron the point *b* rises to a white heat.

The assumption of the crystalline structure takes place entirely in cooling between the temperatures *c* and *b*; when the temperature sinks below *b* there is no change of structure. For successful forging, therefore, the heated ingot, after it is taken out of the furnace, must be forged as quickly as possible, so as not to leave any spot untouched by the hammer, where the steel might crystallize quietly, but that the formation of crystals should be hindered, and that the steel should be kept in the amorphous condition until the temperature sinks below the point *b*.

Below this temperature, if the piece be left to cool in quiet, the mass will no longer have a disposition to crystallize, but will possess great tenacity and homogeneity of structure.

When steel is forged at temperatures lower than *b*, its crystals or grains, being driven against each other, change their shapes, becoming elongated in one direction and contracted in another; whilst the density and the tensile strength are considerably increased. But the available hammer-power is only sufficient for the treatment of small steel forgings; and the object of preventing the coarse crystalline structure in large forgings is more easily and more certainly effected, if, after having given the forging the desired shape, its structure be altered to the homogeneous amorphous condition by heating it to a temperature somewhat higher than *b*, and the condition be fixed by rapid cooling to a temperature lower than *b*. The piece should then be allowed to finish cooling gradually, so as to prevent, as far as possible, internal strains due to sudden and unequal contraction.

[1] Abstracted from *Remarks on the Manufacture of Steel, and the Mode of Working it*, by D. Chernoff, 1868; translated by Mr. William Anderson, C.E., 1876. Mr. Anderson has conferred a substantial favour upon the steel-manufacturing and steel-consuming community by the translation and circulation of this valuable document.

Strength of Steel Wire.

Dr. Pole states that music-wire has a resistance equal to 90 tons per square inch.

Mr. Roebling states that steel wire has been manufactured which would resist a tensile stress of 300,000 lbs., or 134 tons, per square inch; but not in large quantity.

Steel wire, No. 14 W.G., or .085 inch, about $^1/_{12}$ inch, in diameter, made for purposes of steam-ploughing, has a tensile resistance of from 2000 lbs. to 2240 lbs., equivalent to from 160 to 175 tons per square inch.

Shearing Strength of Steel.

The ultimate resistance of steel to shearing stress varies from 69 to 78 per cent. of the ultimate tensile strength per square inch of section. Mr. Kirkaldy found, for 16 specimens from a bar of rivet steel, an average of 73.5 per cent.; and the same for 12 specimens of Fagersta steel. Mr. J. T. Smith, in an article hereafter noticed, states that the force required to punch a hole ⅞ inch in diameter through the ¾-inch webs of Bessemer steel rails varied from 46¼ tons to 82½ tons, according to the hardness of the rail. When a taper of $^1/_{16}$ inch was allowed in the hole, the shearing resistance to punching, per square inch of surface cut through, was such as to average 70.14 per cent. of the tensile strength for the softer steels, and 72.5 per cent. for the harder steels.

Upon the whole, an average of 72 per cent. of the tensile strength may be accepted as the shearing resistance of steel.

Transverse Strength of Steel Bars.

The instances of tests for the transverse strength of steel, detailed in previous pages, are resumed below, showing the dimensions of specimens, with their average ultimate tensile and transverse strengths. The transverse strengths, also, are calculated from the tensile strength by formula (1), page 507, and entered in the second last column of the table. The formula is,

$$W = \frac{1.155\, b d^2 s}{l} \quad \text{.................................}(1)$$

W = the breaking weight at the middle, in tons.
b, d, l = the breadth, depth, and span, in inches.
s = the ultimate tensile strength, in tons per square inch.

Take the first example in the table:—1.75 inches square, 25-inch span, 32.27 tons tensile strength.

$$W = \frac{1.155 \times 1.75^3 \times 32.27}{25} = 7.99 \text{ tons.}$$

Actual weight applied = 7.35 tons (uncracked).

These steels show a still closer correspondence of the calculated to the actual strengths than was shown by the wrought irons, page 589. Naturally, the transverse strength for uncracked specimens, as calculated above, is somewhat greater than the observed strengths, since the strength was not

exhausted by actual fracture. The averages are practically identical, and the identity of the calculated with the experimental strengths, is a natural consequence of the homogeneity of the material.

Number and Description of Specimens.	Section. Breadth.	Section. Depth	Span.	Ultimate Strength. Tensile, per sq. inch.	Ultimate Strength. Transverse. Calculated.	Ultimate Strength. Transverse. Actual.	Effects.
	inch.	inch.	inch.	tons.	tons.	tons.	
4, Hematite, page 595,	1.75	1.75	25	32.27	7.99	7.35	uncracked.
8, Krupp, "Jeddo," page 596,.......	1.50	1.91	10	42.07	26.59	27.14	fractured.
4, Krupp, "Sultan," page 596,.......	1.37	1.76	10	41.18	20.19	21.31	fractured.
18, Bessemer, p. 599,...	1.90	1.90	20	33.34	13.24	12.74	uncracked.
11, Crucible, p. 599,....	1.90	1.90	20	36.30	14.38	15.04	uncracked in most instances.
4, Fagersta, p. 604, ...	1.90	1.90	20	39.16	15.51	15.61	half fractured, half uncracked.
Averages,..................................					16.32	16.53	

The general formula (1) may be adapted for steels of a particular tensile strength, by substituting for $(1.155\,s)$ its numerical value. Thus, for steel of 30 tons tensile strength, $1.155\,s = 1.155 \times 30 = 34.6$; and

$$W = \frac{34.6\,bd^2}{l} \qquad (2)$$

Ultimate Tensile Strength. tons.	Coefficient $(1.155\,s)$.
30	34.6
32	37.0
34	39.3
35	40.3
36	41.6
38	43.9
40	46.2
42	48.5
44	50.8
45	52.0
46	53.1
48	55.4
50	57.8

RULE.—*To find the Ultimate Transverse Strength of Rectangular Steel Bars.*—Multiply the breadth by the square of the depth, and by the coefficient $(1.155\,s)$, corresponding to the ultimate tensile strength, and divide by the span. The quotient is the breaking weight in tons, applied at the middle.

Note.—To find the coefficient for any other tensile strength, not given above, multiply the given tensile strength by 1.155.

Transverse Deflection of Steel Bars.

For want of data, it is assumed that the deflection of steel bars is to that of iron bars of the same dimensions, in the ratio of their extensibilities, or inversely as their coefficients of elasticity. From the results of experiments on iron and on steel rails, it appears that the coefficients are practi-

cally as 11 to 13. Increasing, therefore, the numerical coefficients for wrought-iron bars, in formulas (5) and (6), page 590, in this ratio, the following formulas are deduced:—

Elastic Deflection of Uniform Bars of Steel, loaded at the middle.

Square bars,.............. $D = \dfrac{W l^3}{56{,}000\, b d^3}$ .... (3)

Round bars,.............. $D = \dfrac{W l^3}{38{,}000\, d^4}$ .... (4)

D = the deflection, b the breadth, d the depth, l the span, all in inches; W = the weight, in tons.

Torsional Strength of Steel Bars.

The torsional resistances of steel, already recorded, are, with the ultimate tensile and shearing strengths, resumed below, and the calculated resistances, by formula (1), page 534, are added in the second last column. The shearing strength is taken at 72 per cent. of the tensile strength, as was settled, page 617. The torsional stress was applied, in the following experiments, at the end of a 12-inch lever. The formula is,—

$$W = \frac{.278\, d^3 h}{R}. \qquad (5)$$

W = the breaking stress in tons.
h = the shearing strength in tons per square inch.
d = the diameter in inches.
R = the radius of the force in inches.
W R = the moment of the force, in statical inch-tons.

Take the first example in the table below:—1¼ inches in diameter, 25.21 tons shearing strength, and a 12-inch radius:—

$$\text{Breaking force, by formula} = \frac{.278 \times 1.25^3 \times 25.21}{12} = 1.141 \text{ tons.}$$

Description, and Number of Specimens.	Diameter.	Ultimate Strength.		Ultimate Torsional Force.	
		Tensile.	Shearing.	Calculated.	Actual.
	inches.	tons.	tons.	tons.	tons.
4, Hematite,..........page 595,	1.25	32.27	25.21	1.141	1.030
4, Krupp, "Jeddo," „ 596,	1.25	41.18	say 72%	1.342	1.280
4, Krupp, "Sultan," „ 596,	1.128	42.07	„ „	1.007	1.068
18, Bessemer,.......... „ 600,	1.382	33.43	„ „	1.472	1.470
11, Crucible,............ „ 600,	1.382	36.30	„ „	1.599	1.610
3, Fagersta, *a*,........ „ 604,	1.128	38.04	27.42	.912	.946
3, Do. *b*,........ „ 604,	1.128	47.60	35.60	1.184	1.043
3, Do. *c*,........ „ 604,	1.128	45.82	31.99	1.064	1.009
3, Do. *d*,........ „ 604,	1.128	27.37	20.28	.674	.679
12, Do. average,...........	1.128	39.16	28.82	.958	.919
Averages of all,..................................				1.135	1.105

The results of experiment and of calculation show a close correspondence.

When the shearing strength is not known experimentally, substitute .72 s, or 72 per cent. of the tensile strength s, for h in the formula; and $W = \frac{.278 \times .72\, d^3 s}{R}$, or

Ultimate Torsional Strength of Steel Bars.

$$W = \frac{.20\, d^3 s}{R} = \frac{d^3 s}{5\, R} \quad \ldots \ldots \quad (6)$$

When the tensile strength, s, is 30 tons, then $\frac{s}{5} = 6$, and

$$W = \frac{6\, d^3}{R}; \quad \ldots \ldots \quad (7)$$

$$d = \sqrt[3]{\frac{W\,R}{6}} \quad \ldots \ldots \quad (8)$$

Generally, for tensile strengths of from 30 to 50 tons, the values of the numerical coefficients in formulas (7) and (8), are as follows:—

Tensile Strength. tons.	Coefficient. (.20 s.)	Tensile Strength. tons.	Coefficient. (.20 s).
30	6	42	8.4
32	6.4	44	8.8
34	6.8	45	9
35	7	46	9.2
36	7.2	48	9.6
38	7.6	50	10
40	8		

Elastic Torsional Strength of Steel Shafts.

Hematite steel	41.5	per cent. of ultimate strength.
Krupp, "Jeddo"	38.4	" "
Krupp, "Sultan"	47.3	" "
Bessemer	44.6	" "
Crucible	42.5	" "
Fagersta (average)	50.2	" "
Average	44.1	per cent.

Elastic Torsional Shearing Stress and Deflection of Steel Bars.

The elastic shearing stress h, is found by formula (3), page 535.

$$h = \frac{W\,R}{.278\, d^3} \quad \ldots \ldots \quad (9)$$

For Hematite steel, for example, page 595, W R = .428 ton × 12 inches = 5.136, the moment of the force, and $h = \frac{5.136}{.278 \times 1.25^3} = 9.46$ tons per square inch, the elastic limit of shearing stress. The coefficient of torsional elasticity, E′, as defined at page 536, is found by formula (12), page 537:—

$$E' = \frac{W\,R\,l}{.873\, d^4\, D} = \frac{.428 \times 12 \times 10}{.873 \times 1.25^4 \times .008} = 3012, \text{ for Hematite steel.}$$

For the several steels, the elastic shearing stress and coefficient of elasticity, calculated in the same way, are as follows:—

Steels.		Specimens. Diameter and Length for Observation.	Elastic Shearing Stress per square inch.	Coefficient of Elasticity.
		inches. inches.	tons.	E′
Hematite,............	page 595	1.25 × 10	9.46	3012
Krupp, "Jeddo,".....	" 596	1.25 × 2.5	10.85	1382
Krupp, "Sultan,"....	" 596	1.128 × 2.256	13.95	865
Bessemer,............	" 600	1.382 × 11	10.73	2472
Crucible,...............	" 600	1.382 × 11	11.19	2025
Fagersta *a*, ...	" 604	1.128 × 9	15.25	—
Do. *b*, ...	" 604	1.128 × 9	15.10	—
Do. *c*, ...	" 604	1.128 × 9	14.56	—
Do. *d*, ...	" 604	1.128 × 9	10.25	—
Do. average,.....	" 604	1.128 × 9	13.77	—

Omitting the coefficients of elasticity for the "Jeddo" and the "Sultan," as the specimens were very short, the average of the remaining three coefficients is 2503; and the value of .873 E′ in formula (10), page 537, is (.873 × 2503 =) 2185; say 2200. Whence, by substitution:—

Elastic Torsional Deflection of Steel Bars.

$$D = \frac{W R l}{2200 \, d^4} \quad \text{..........................} \quad (10)$$

D = the total angular deflection in parts of a revolution.
W = the twisting force in tons.
R = the radius of the force in inches.
W R = the moment of the force in statical inch-tons.
l = the length of the shaft under torsional stress in inches.
d = the diameter of the shaft in inches.

Strength of Steel Relatively to the Proportion of Constituent Carbon.

Mr. F. W. Webb produces steel for boiler plates having a tensile strength of 28 tons per square inch, and containing $^1/_5$th per cent. of constituent carbon.

Mr. T. E. Vickers tested the tensile strength of steel of various degrees of carbonization, ranging from No. 2, having 0.33 per cent., to No. 20, having 1.25 per cent. of carbon.[1] The specimens were turned to 1 inch in diameter, and to a length, for observation, of 14 inches. The results of the tests are given in table No. 217.

The table shows that the tensile strength of steel is increased by the addition of carbon, until, with 1¼ per cent., it amounts to 69 tons per square inch. The elongation is, at the same time, reduced. But, beyond

[1] See Mr. Vickers' paper on the "Strength of Steel" (*Proceedings of the Institution of Mechanical Engineers*, 1861, page 158).

the last degree of carbonization, 1¼ per cent., the steel becomes gradually weaker, until it reaches the form and strength of cast iron.

Table No. 217.—TENSILE STRENGTH OF STEEL CONTAINING DIFFERENT PROPORTIONS OF CARBON.

Mr. T. Edward Vickers.

Description of Steel.	Proportion of Carbon[1] (approximate).	Breaking Weight per square inch.	ELONGATION.
	per cent.	tons.	inches.
No. 2	.33	30.4	1.37, or 9.8 per cent.
No. 4	.43	34.0	1.37, or 9.8 "
No. 5	.48	37.5	1.25, or 8.9 "
No. 6	.53	42.5	1.12, or 8.0 "
No. 8	.63	45.0	1.00, or 7.1 "
No. 10	.74	45.5	.69, or 5.0 "
No. 12	.84	55.0	1.12, or 8.0 "
No. 15	1.00	60.0	1.00, or 5.0 "
No. 20	1.25	69.0	.62, or 4.4 "

A specimen bar was turned down to a diameter of ¾ inch at the middle, so as to form a circular notch. On being tested, it broke with 79½ tons per square inch, whilst the ordinary specimen bar of the same steel broke with 60 tons per square inch.

Mr. Webb's datum above given is in harmony with Mr. Vickers' data.

See also on this subject RAILWAY RAILS, at page 664.

RESISTANCE OF STEEL AND IRON TO EXPLOSIVE FORCE.

Sir Joseph Whitworth tested iron and steel by the explosive force of gunpowder. The specimens were cylinders having a bore of ¾ inch, a diameter outside of 1¼ inches, and a length of 4 inches. They were made open at the ends, and were closed for the purpose of the experiments.

Table No. 218.—RESISTANCE OF IRON AND STEEL TO EXPLOSIVE FORCE.

METAL.	Charge of Powder.		Expansion in diameter at middle before bursting.	Number of pieces when burst.
	grains.	ratio.	inch.	pieces.
Cast iron	15	1	.0000	36
Wrought iron, Staffordshire, coiled	95	6.3	.0997	5
Fluid compressed steel, No. 3, red	275	18.3	.1659	2
Do. do. No. 3, brown	325	21.7	.0950	4

[1] The intermediate percentages of carbon in column 2, from No. 4 to No. 15 inclusive, are merely approximate, having been interpolated in proportion to the Nos. of the steel.

RECAPITULATION OF DATA ON THE DIRECT STRENGTH OF IRON AND STEEL.

Cast Iron, pp. 553 to 561.—The ultimate tensile strength ranges from 5 to 7½ tons per square inch: first meltings, specimens under 1 inch in thickness. For thicker castings the strength diminishes. The compressive strength is from four and a half to about seven times the tensile strength. For general calculations, say, tensile strength 7 tons, compressive strength 49 tons.

The ultimate tensile strength is increased by repeated remeltings to from 15 to 20 tons per square inch; and the compressive strength to from 70 to 80 tons.

The elastic strength practically is equal to the ultimate tensile strength.

Wrought Iron, pp. 567 to 591.—The ultimate tensile strength of rolled bar iron varies from 22½ to 30 tons; rivet-iron from 24 to 27 tons. Plates from 20 to 23 tons; about 1 ton less crossway than lengthway of the fibre. The strength is reduced more than 1 ton by annealing. The resistance to compression is an indefinite quantity.

The elastic tensile strength of iron bars averages not less than 50 per cent. of the ultimate strength; and that of iron plates is generally from 55 to 60 per cent. of the ultimate strength.

The elastic strength of bars and plates, both tensile and compressive, may be taken at 12 tons.

The elongation of wrought-iron bars, within the elastic limit, is at the rate of $^1/_{10,000}$ to $^1/_{13,000}$ part of the length—say, an average of $^1/_{12,000}$ part—per ton per square inch; or a total of $^1/_{1000}$ part of the length. The same fraction may be taken for compression within the elastic limit.

Approximate Strength of Wrought-iron Bars in Terms of the Circular Inch (Mr. E. Clark).—"A strength of 20 tons per square inch is nearly equivalent to one of 16 tons per circular inch. An ordinary 1-inch round rod bears tensilely 16 tons, and weighs 8 lbs. per yard.

"For a round rod of any diameter, the square of the diameter, in quarter-inches, is the breaking weight in tons.

"Half this quantity is the weight in pounds per yard.

"A rod will be perceptibly damaged by half this stress, which can never be safely exceeded; one-third being sufficient in practice."

Steel, pp. 593 to 615.—By Mr. Kirkaldy's earliest experiments, it was found that the average tensile strength of bar steel varied from 60 tons for tool-steel, to 28 tons for puddled steel; and that of steel plates from $^3/_{16}$ to $^5/_{16}$ inch thick, from 32 to 45½ tons.

From subsequent experiments, it appears that the ultimate tensile strength of rolled bar steel varies from 30 to 50 tons per square inch. The average tensile strength may be taken at 35 tons, and the elastic strength, tensile and compressive, at 20 tons. The tensile and the elastic compressive strength of hammered steel bars is from 4 to 5 tons more than that of rolled bars.

By annealing, the elastic strength of rolled steel bars is reduced 3 tons, and that of hammered bars 5 tons.

Steel plates have elastic tensile and compressive strengths averaging about 14 tons; the ultimate tensile strength is from 22 to 32 tons, according to

the proportion of constituent carbon. The strength is the same lengthwise and crosswise.

Annealing reduces the tensile strength of steel plates, elastic and ultimate, by 1½ or 2 tons; and the elastic compressive strength by twice as much.

The elongation and the compression of steel bars within the elastic limit may be taken at $^1/_{13,000}$ part of the length per ton per square inch; or a total of $^1/_{1000}$ part of the length.

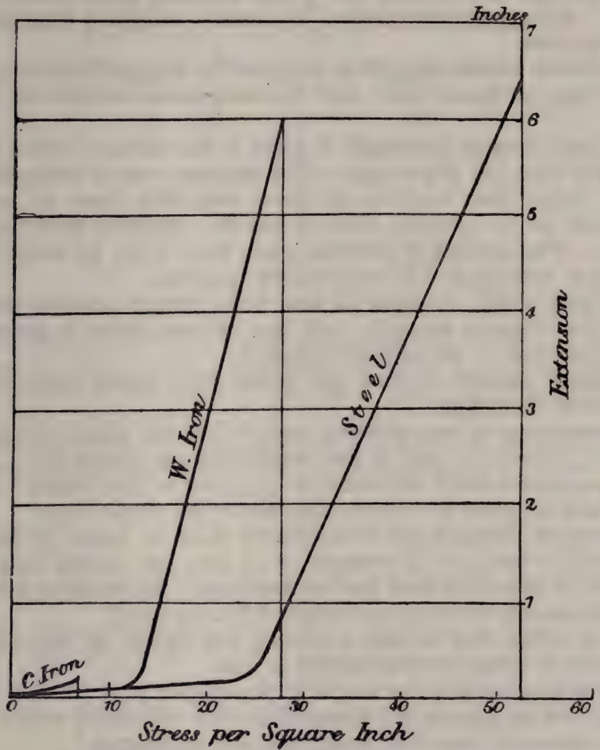


Fig. 222.—Relative Elongation of 10-feet Bars of Cast Iron, Wrought Iron, and Steel.

The comparative behaviours of bars of cast-iron, wrought-iron, and steel, 10 feet in length, under tensile stress, as previously recorded, is shown diagrammatically in Fig. 222, annexed. It is seen that the extension of cast iron, which breaks under a tensile stress of 7 tons per square inch, is very limited, and that the rate of extension is nearly uniform concurrently with the stress; whilst the wrought-iron and the steel bars, which were broken at 28 tons and 52½ tons per square inch respectively, suddenly acquire a greatly increased rate of elongation at the yielding points, which are arrived at when about half the breaking stress has been applied. It is not necessary to enter into more detail, as the diagram is only intended to show the leading characteristics of the three metals under tensile stress.

WORKING STRENGTH OF MATERIALS—FACTORS OF SAFETY.

The elastic strength of materials, cast-iron excepted, is, in general terms, half of their ultimate or breaking strength. For cast-iron, though there is no clearly defined elastic limit, the same measure may be adopted. If a working load of half the elastic strength, or one-fourth of the ultimate strength, be accepted, equal range for fluctuation within the elastic limit is provided. But, as bodies of the same material are not uniform in strength, it is necessary to observe a lower limit than a fourth where the material is exposed to great or to sudden variations of load.

Cast-iron.—Mr. Stoney recommends one-fourth of the ultimate tensile strength, for dead weights; one-sixth for cast-iron bridge girders; and one-eighth for crane-posts and machinery. In compression, free from flexure, according to Mr. Stoney, cast-iron will bear 8 tons per square inch; for cast-iron arches, 3 tons per square inch; for cast-iron pillars, supporting dead loads, one-sixth of the ultimate tensile strength; for pillars subject to vibration from machinery, one-eighth; and for pillars subject to shocks from heavy-loaded waggons and the like, one-tenth, or even less where the strength is exerted in resistance to flexure.

Wrought-iron.—For bars and plates, 5 tons per square inch of net section is taken as the safe working tensile stress; for bar iron of extra quality, 6 tons. In compression, where flexure is prevented, 4 tons is the safe limit; in small sizes, 3 tons. For wrought-iron columns, subject to shocks, Mr. Stoney allows a sixth of the calculated breaking weight; with quiescent loads, one-fourth. For machinery, an eighth to a tenth is usually practised; and for steam-boilers, a fourth to an eighth.

Mr. Roebling says, "Long experience has proved, beyond the shadow of a doubt, that good iron, exposed to a tensile strain not above one-fifth of the ultimate strength, and not subject to strong vibration or torsion, may be depended upon for a thousand years."[1]

Steel.—A committee appointed by the Board of Trade recommended that a stress of 6½ tons per square inch should not be exceeded in bridge work for railways. Mr. Stoney recommends, for mild steel, a fourth of the ultimate tensile strength, or 8 tons per square inch. The limit for compression must be regulated very much by the nature of the steel, and whether it be unannealed or annealed. Probably a limit of 8 tons per square inch, the same as the limit for tension, would be the safe maximum for general purposes. In the absence of experience, Mr. Stoney recommends that, for steel pillars, an addition not exceeding 50 per cent. should be made to the safe load for wrought-iron pillars of the same dimensions.

Timber.—One-tenth of the ultimate stress is an accepted limit. Timber piles have, in some situations, borne permanently one-fifth of their ultimate compressive strength.

Foundations.—According to Professor Rankine, the maximum pressure on foundations in firm earth is from 17 lbs. to 23 lbs. per square inch; and he says that, on rock, it should not exceed one-eighth of the crushing load.

Masonwork.—Mr. Stoney says that the working load on rubble masonry,

[1] *Engineering*, August 16, 1867.

brickwork, or concrete rarely exceeds one-sixth of the crushing weight of the aggregate mass; and that this seems to be a safe limit. In an arch, the calculated pressure should not exceed one-twentieth of the crushing pressure of the stone.

Ropes.—For round ropes, the working load should not exceed a seventh of the ultimate strength; and for flat ropes, one-ninth.

Dr. Rankine[1] gives the following data as factors of strength:—

	Dead Load.		Live Load.
Factors of safety for perfect materials and workmanship	2	....	4
For good ordinary materials and workmanship:—			
Metals	3	....	6
Timber	4 to 5	....	8 to 10
Masonry	4	....	8

A *dead load* on a structure is one that is put on by imperceptible degrees, and that remains steady; such as the weight of the structure itself.

A *live load* is one that is put on suddenly, or is accompanied with vibration; such as a swift train travelling over a railway bridge, or a force exerted in a moving machine.

STRENGTH OF COPPER AND OTHER METALS.

Table No. 219.—Ultimate Tensile Strength of Copper and its Alloys, and other Metals.

Description of Metal.	Specific Gravity	Ultimate Tensile Strength per square inch.	Experimentalist.
		tons.	
Copper, wrought	—	15.00	Dr. Anderson
Do. cast	—	8.48 to 11.67	„
Do. ordinary bolts	—	16.00	„
Do. bolts, with 1 per cent. phosphorus	8.202	7.56	„
Do. do. 1 „	8.592	16.47	„
Do. do. 1.5 „	8.876	17.13	„
Do. do. 1 „	—	19.20	„
Do. do. 2 „	8.614	20.25	„
Do. do. 1 „	—	20.34	„
Do. do. 2 „	8.580	20.41	„
Do. do. 2 „	8.615	20.27	„
Do. do. 3 „	8.422	21.38	„
Do. do. 4 „	—	22.32	„
Gun metal, 12 copper, 1 tin	—	12.94	„
Do. 11 „ 1 „	—	13.71	„
Do. 10 „ 1 „	—	14.73	„
Do. 9 „ 1 „	—	17.00	„

[1] *Useful Rules and Tables*, page 205.

Table No. 219 (*continued*).

Description of Metal.	Specific Gravity	Ultimate Tensile Strength per square inch.	Experimentalist.
		tons.	
Gun metal, average strength of good bronze	—	14.73	Dr. Anderson
Gun metal, average results of tests of specimens from bronze guns—elastic strength, 6.56 tons	—	12.19	„
Gun metal, American guns—			
Gun-heads	8.523	13.24	Wade
Breach-squares	8.765	20.76	„
Small bars cast in same moulds with guns	8.584	18.76	„
Small bars cast separately in iron moulds	8.953	16.82	„
Do. do. in clay moulds	8.313	11.51	„
Finished guns	—	10.3 to 23.3	„
Alloys of copper and tin, unwrought—			
Equivalents. By weight.			
10 Cu + Sn, 84.29 copper + 15.71 tin, gun metal...	8.561	16.1	Mallet
9 Cu + Sn, 82.81 „ + 17.19 „ „ ...	8.462	15.2	„
8 Cu + Sn, 81.10 „ + 18.90 „ „ ...	8.459	17.7	„
7 Cu + Sn, 78.97 „ + 21.03 „ brasses	8.728	13.6	„
Cu + Sn, 34.92 „ + 65.08 „ small bells..	8.056	1.4	„
Cu + 3 Sn, 15.17 „ + 84.83 „ speculum met.	7.447	3.1	„
Sn, 0 „ + 100. „ tin	7.291	2.5	„
Aluminium bronze—90 copper, 1 aluminium	—	32.67	Dr. Anderson
Do. maximum	—	43.00	„
Tin, cast	—	2.11	Rennie
Do. Banco	7.297	.95	Wade
Lead, cast	—	.81	Rennie
Do. sheet	—	.86	Navier
Lead pipe	—	1.00	Jardine
Zinc, cast	—	1.336	Stoney
Soft solder—2 tin, 1 lead	—	3.35	Rankine
Brass, fine or yellow	—	8.02	Rennie
Brass, fine or yellow, 2 copper, 1 zinc	—	12.90	Dr. Anderson
Brass tube, 62 copper, 38 zinc	—	46.00	Everitt
Do. „ 70 „ 30 „	—	36.00	„
Do. wire	—	40.77	Dufour
Muntz's metal—3 copper, 2 zinc	—	22.00	Dr. Anderson
Alloys of copper, zinc, iron, and tin—"Sterro-metal"—			
Copper 10, iron 10, zinc 80	7.000	3.17	Dr. Anderson
Do. 60, „ 3, „ 39, tin 1.5	—	24.00	„
Do. 60, „ 4, „ 44, „ 2:—			
Cast in sand	—	19.25	„
Cast in iron, annealed	—	24.25	„
Cast in iron, forged red hot	—	31.00	„
Copper 60, iron 2, zinc 37, tin 1	—	34.0	„
Do. 60, „ 2, „ 35, „ 2	—	38.0	„
Do. 55.0, „ 1.77, „ 42.36, „ 0.83:—			
Cast	—	27.0	„
Forged red hot	—	34.0	„
Drawn cold	—	38.0	„

Table No. 220.—PHOSPHOR-BRONZE, BRONZE, AND BRASS. FROM LIÈGE. TENSILE STRENGTH.

(Reduced from Mr. Kirkaldy's Report.) DESCRIPTION.	Elastic Strength per square inch.	Ultimate Strength per square inch.	Ratio of Elastic to Ultimate Strength.	Set. At 20,000 lbs. per square inch.	Set. Ultimate.	Sectional Area of Fracture.
	tons.	tons.	per cent.	per cent.	per cent.	per cent.
PHOSPHOR-BRONZE.						
Lowest values	4.777	9.712	31.5	.09	3.6	96.1
Highest values...............	10.625	22.730	68.5	5.13	33.4	68.1
Averages of 12 specimens	7.482	15.386	48.6	1.59	11.4	87.4
ORDINARY BRONZE.						
Lowest values	7.321	9.061	66.7	.18	1.2	98.5
Highest values...............	8.794	13.184	85.9	1.10	4.0	91.6
Averages of 6 specimens	8.095	10.582	76.5	.56	2.23	95.6
BRASS.............	4.410	12.284	36.7	5.80	16.1	81.7

TENSILE STRENGTH OF WIRE OF VARIOUS METALS.

M. Baudrimont, in 1835, tested the strength of annealed metallic wires at various temperatures, from 32° F. to 392° F.[1] The wires of gold, platinum, copper, silver, and palladium were about $^1/_{60}$th inch in diameter; the iron wires were $^1/_{145}$th inch in diameter. The results of the tests are

Table No. 221.—TENACITY OF METALLIC WIRES AT VARIOUS TEMPERATURES. 1835.

METAL.	Diameter of wire at 61° F.	Sectional Area.	Ultimate Tensile Strength. At 32° F.	At 212° F.	At 392° F.	Tensile Strength per square inch. At 32° F.	At 212° F.	At 392° F.	
	inch.	sq. inch.	lbs.	lbs.	lbs.	tons.	tons.	tons.	
Gold	.0162	.000207	5.61	4.64	3.86	12.1	10.0	8.3	Maximum
			5.42	4.49	3.80	11.7	9.7	8.2	Minimum
Platinum	.0161	.000205	6.70	5.94	5.27	14.6	13.0	11.5	Maximum
			6.59	5.61	5.03	14.4	12.2	11.0	Minimum
Copper ...	.0177	.000247	10.11	8.80	7.92	18.3	16.0	14.4	Maximum
			10.02	8.73	7.27	18.1	15.8	13.1	Minimum
Silver	.0157	.000193	7.86	6.74	5.13	18.1	15.6	11.9	Maximum
			7.78	6.39	5.10	18.0	14.8	11.8	Minimum
Palladium	.0156	.000192	10.12	9.00	7.99	23.5	20.9	18.5	Maximum
			9.98	8.89	7.41	23.1	20.6	17.2	Minimum
Iron	.0069	.000373	11.12	10.66	11.31	133.2	127.6	135.4	Maximum
			10.89	10.16	11.15	130.3	121.7	133.5	Minimum

[1] *Annales de Chimie*, 1850.

arranged in table No. 221, and it is shown that, 1st, the tenacity varies with the temperature; 2d, it decreases as the temperature rises, except for iron; 3d, that iron presents a peculiar case. At 212° F. its tenacity is less than at 32° F., but at 392° F. it is greater.

Table No. 222.—TENSILE STRENGTH OF WIRE—PHOSPHOR-BRONZE, COPPER, BRASS, STEEL, AND IRON.

(Reduced from Mr. Kirkaldy's Report.) DESCRIPTION OF WIRE.	Unannealed. Diameter.	Unannealed. Ultimate Tensile Strength. Total.	Unannealed. Ultimate Tensile Strength. Per square inch.	Annealed. Diameter.	Annealed. Ultimate Tensile Strength. Total.	Annealed. Ultimate Tensile Strength. Per square inch.	Annealed. Ultimate Extension.
	inch.	lbs.	tons.	inch.	lbs.	tons.	p. c'nt.
Phosphor-Bronze—							
Lowest values.......	.0585 or 1/17.1	—	43.59	.1070 or 1/9.3	—	22.58	33.0
Highest values......	.0665 or 1/15	—	71.21	.1125 or 1/9	—	28.82	46.6
Averages of 20 specimens......	.063 or 1/15.1	394	56.28	.1108 or 1/9	527	24.42	39.0
Copper..................	.0640 or 1/15.6	203	28.18	.0640 or 1/15.6	119	16.52	34.1
Brass....................	.0605 or 1/16.5	233	36.23	.0605 or 1/16.5	148	23.01	36.5
Steel.....................	.0600 or 1/16.7	342	54.07	.0600 or 1/16.7	211	33.32	10.9
Iron, galvanized, BBC	.0580 or 1/17.2	170	28.71	.0580 or 1/17.2	162	27.36	17.1
Do. do. BCE	.0580 or 1/17.2	174	29.40	.0580 or 1/17.2	122	20.61	28.0

STRENGTH OF STONE, BRICKS, &c.

Table No. 223.—TENSILE STRENGTH OF STONE, BRICKS, AND CEMENT.

DESCRIPTION OF MATERIAL.	Weight per cubic foot.	Ultimate Tensile Strength per sq. inch.	Experimentalist.
	lbs.	tons.	
Sandstone....................................	—	.150	Buchanan
Whinstone....................................	—	.655	"
Arbroath pavement..........................	—	.563	"
Caithness do.	—	.471	"
White Marble	—	.322	"
Do.	—	.246	Hodgkinson
Flint-glass rod, annealed..................	—	1.07	Fairbairn
Green glass rod	—	1.29	"
White crown-glass rod......................	—	1.14	"
Thin glass globes, cohesion	—	2.23	"
		lbs.	
Plaster of Paris..............................	—	71	Rondelet
Mortar of quartzose sand and hydraulic lime..............................	—	136 to 85	Vicat
Mortar of quartzose sand and ordinary lime..................................	—	21 to 51	"

Table No. 223.—(*continued*).

Description of Material.	Weight per cubic foot.	Ultimate Tensile Strength per sq. inch.	Experimentalist.
	lbs.	lbs.	
Adhesion of Plaster of Paris to brick or stone—average	—	50	Rondelet
Adhesion of bricks cemented with Portland cement, 12 months old, and 1 cement to 1 sand	neat	1 to 1	
Gault-clay bricks, pressed; in air	45	44	Grant[1]
Do. do. in water	46	46	"
Gault-clay bricks, wire cut; in air	68	43	"
Do. do. in water	47	39	"
Gault-clay bricks, perforated; in air	108	83	"
Do. do. in water	84	75	"
Stock bricks, in air	78	63	"
Do. in water	96	70	"
Staffordshire blue brick, pressed with frog; in air	74	56	"
Do. in water	76	37	"
Do. rough, without frog; in air	48	47	"
Do. do. do. in water	40	29	"
Fareham red bricks; in air	126	83	"
Do. do. in water	123	62	"
Portland cement:—			
Seven-day tests	—	862 to 408	"
Average of do. per bush. 115.2 lbs	90	358.5	"
Portland cement, 123 lbs. per bushel, mixed with equal weight of Thames sand:—	neat cement.	cement & sand	
Age in water, 7 days	363	157	"
Do. 1 month	416	201	"
Do. 6 do.	523	284	"
Do. 12 do.	547	319	"
Do. 2 years	600	351	"
Do. 4 do.	583	363	"
Do. 7 do.	590	384	"
Portland cement, 112 lbs. per bushel, mixed with various proportions of sand; 12 months old:—			
3 sand, 1 cement	—	241	"
5 do. 1 do.	—	214	"
7 do. 1 do.	—	163	"
Roman cement—averages:—			
Age in water, 7 days	—	90	"
Do. 1 month	—	115	"
Do. 6 do.	—	210	"
Do. 12 do.	—	286	"
Do. 2 years	—	243	"
Do. 4 do.	—	281	"
Do. 7 do.	—	315	"

[1] *Proceedings of the Institution of Civil Engineers*, vols. xxv. and xxxii.

Table No. 224.—Crushing Strength of Stones and Bricks.

Description of Material.	Specific Gravity.	Tons per square inch.	Experimentalist.
Granite :—			
Aberdeen, blue	2.625	4.87	Rennie
Peterhead	—	3.70	„
Cornish	2.662	2.83	„
Dublin	—	4.66	Wilkinson
Wicklow	—	1.52	„
Newry	—	5.86	„
Mount Sorrel	2.675	5.74	Fairbairn
Whinstone, Scotch	—	3.70	Buchanan
Greenstone, Irish	—	9.30	Wilkinson
Sandstones and Grits :—			
Arbroath pavement	—	3.52	Buchanan
Craigleith freestone	2.452	2.61	Rennie
Derby grit, friable sandstone	2.316	1.40	„
Yorkshire paving	2.507	2.55	„
Red sandstone, Runcorn	—	.97	L. Clark
Quartz rock, Holyhead, across lamination	—	11.40	Mallet
Do., parallel to lamination	—	6.25	„
Marble :—			
Statuary	—	1.44	Rennie
Italian, white, veined	—	4.32	„
Irish	—	6.75 to 9.00	Wilkinson
Limestone :—			
Compact	2.584	3.44	Rennie
Purbeck	2.599	4.09	„
Magnesian	—	1.36	Fairbairn
Anglesea	2.720	3.38	L. Clark
Irish	—	5.06 to 7.56	Wilkinson
Chalk	—	.224	Rennie
Slates :—			
Irish, on bed of strata	—	10.60	Wilkinson
Do. on edge of strata	—	6.23	„
Bricks :—			
Red	—	.358	Rennie
Yellow-faced, baked	—	.446	„
Do. burnt	—	.643	„
Gault-clay, pressed	—	1.111	Grant
Do. wire-cut	—	.884	„
Do. perforated	—	1.180	„
Stock	—	1.044	„
Fareham red	—	2.500	„
Staffordshire blue, pressed with frogs	—	3.100	„
Do. rough, without frogs	—	3.275	„
Stourbridge fire-clay	—	.766	L. Clark
Do. do.	—	.670	J. R. Walker
Tividale blue	—	.620	„
Brickwork in cement, not hard	—	.232	E. Clark

Table No. 224 (*continued*).

Description of Material.	Specific Gravity.	Tons per square inch.	Experimentalist.
Portland cement, 3 months old	—	1.70	Grant
1 do. to 1 sand, "	—	1.11	"
1 do. to 5 sand, "	—	.43	"
Portland cement, 9 months old	—	2.67	"
1 do. to 1 sand, "	—	2.04	"
1 do. to 5 sand, "	—	.75	"
Portland cement, concrete blocks—12-inch cubes compressed, 12 months old:—		Total Crushing Weight. tons.	
1 cement to 1 sand and gravel	—	170.5	"
1 " to 3 "	—	115.5	"
1 " to 6 "	—	91.0	"
Mortar:—		per square inch.	
Lime and river sand	—	.194	Rondelet
Do. do. beaten	—	.266	"
Lime and pit sand	—	.258	"
Do. do. beaten	—	.357	"
Glass	—	12.31 to 14.23	Fairbairn

STRENGTH OF ELEMENTARY CONSTRUCTIONS.

RIVET-JOINTS

IN IRON PLATES.

There are two elements by which the strength of rivetted joints is determined:—the tensile strength of the perforated plate, and the grip and shearing strength of the rivets.

Strength of Perforated Iron Plates.—The usual effect of perforation by punching, is a weakening of the metal about the holes; so that the tensile strength per unit of section between the holes is less than that of the unpierced plate. Yorkshire iron (table No. 192, page 584) loses from 13 to 17 per cent. of its tensile strength, and Krupp iron from 10 to 13 per cent., by punching. It is generally assumed, according to Mr. Wilson, that hard plates of fair quality lose from 20 to 24 per cent. of their tensile strength by punching for steam-joints, but that many soft plates do not lose more than 8 per cent. When plates are drilled, on the contrary, it is considered that the tensile strength remains unimpaired.

But, Mr. J. Cochrane found from experiments with bar iron that there was no loss of tensile strength by punching holes in the bars.[1] Lowmoor and Staffordshire bars were planed down to a nominal thickness of ½ inch, and shaped to a width of 2 inches. One-inch holes were made through the bars in three ways:—1st, by drilling; 2d, by punching ⅛ inch too small and rimering out to the size; and 3d, by punching at once to the full size. The tensile resistances per square inch were as follows:—

Formation of Holes.	Lowmoor Iron.	Staffordshire Iron.
Drilling	24.72 tons.	23.15 tons.
Punching and rimering	25.51 „	23.15 „
Punching	24.53 „	23.69 „

No doubt the holes were punched with a wide clearance in the die—a provision which very much facilitates the separation of the metal, eases the punch, and eases the stress on the metal.

Strength of Rivetted Joints.—Sir William Fairbairn, in 1838, deduced from experiment with small specimens of ¼-inch iron plate, that double-rivetted lap-joints were stronger than single-rivetted joints, and that their relative values were as follows:—

Tensile strength of the solid plate, as		100
Do.	double-rivetted lap-joint, as	70
Do.	single-rivetted lap-joint, as	56

[1] *Proceedings of the Institution of Civil Engineers*, vol. xxx. 1869-70, p. 265.

Mr. Bertram's Experiments.—At Woolwich Dockyard, Mr. W. Bertram tested various plate-joints. The results were reported, and they were investigated by the author, in 1860,[1] in connection with Mr. Bertram's method of welding joints—the scarf-weld and lap-weld, Figs. 224 and 225, in which the lap is 1¼ inches. Staffordshire plates of good quality were selected for

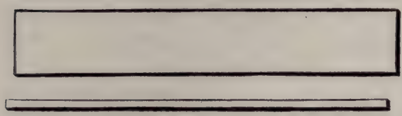


Fig. 223.—Entire plate.


Fig. 224.—Scarf-welded joint.

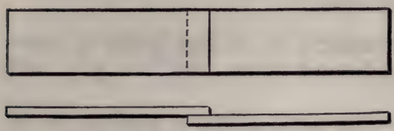


Fig. 225.—Lap-welded joint.

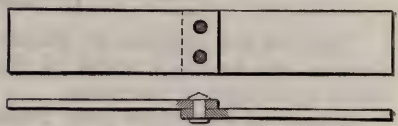


Fig. 226.—Single-rivetted joint, by hand.

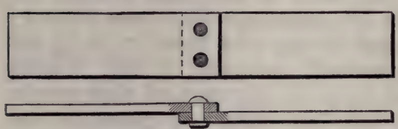


Fig. 227.—Single-rivetted joint, by hand, snap-headed.

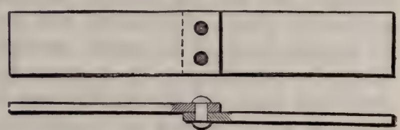


Fig. 228.—Single-rivetted joint, by machine.

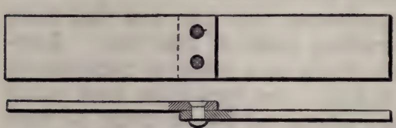


Fig. 229.—Single-rivetted joint, with countersunk head.

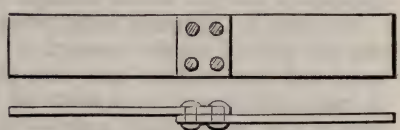


Fig. 230.—Double-rivetted joint, snap-headed.

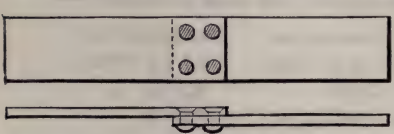


Fig. 231.—Double-rivetted joint, countersunk and snap-headed.

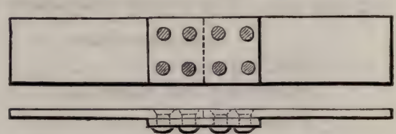


Fig. 232.—Double-rivetted joint, with single welt, countersunk and snap-headed.

Boiler-Plate Joints, tested by Mr. Bertram.

the trials: of three thicknesses, ½-inch, 7/16-inch, and ⅜-inch; and made up into ten varieties of specimens, 4 inches broad and 24 inches long, in which the rivet-joints were made with ¾-inch rivets at a pitch of 2 inches. Three specimens of each variety of joint, for each thickness of plate, were tested, and the results averaged for each set of three specimens. These joints are illustrated and described at Figs. 223 to 232. The net sectional area of

[1] *Recent Practice in the Locomotive Engine*, 1858–59; also *Railway Locomotives*, 1860, by D. K. Clark. Blackie & Son. See these works for an extended notice of plate-joints.

plate, in the line of the rivets, was 62.5 per cent. of the solid section. The sectional area of a ¾-inch rivet is .4417 square inch, giving for two rivets a shearing section of .8834 square inch.

	Net Sectional Area.	Shearing Section of Rivets.	
⅜-inch plate,	.94 sq. ins.	.8834 sq. in.,	or 94 per cent. of net section.
7/16 „	1.094 „	.8834 „	or 80.8 „ „
½ „	1.25 „	.8834 „	or 70.7 „ „

The fractures took place, in nearly all cases, in one of the plates, in the line of the rivet-holes; but, in a few cases, the rivets were shorn across. The normal strength for the solid plates was nearly uniform, and averaged, for all thicknesses, 20 tons per square inch.

Table No. 225.—Ultimate Tensile Strength of Welded and Rivetted Joints of Boiler-Plate.

Tensile strength of the entire plate, 20 tons per square inch.

(Reduced, in 1860, from Mr. Bertram's experiments.)

Description of Joint.	Form of Joint.	Net Ultimate Tensile Strength of Joint; that of the entire plate=100.			
		½-inch Plate.	7/16-inch Plate.	⅜-inch Plate.	Average for three thicknesses.
		per cent.	per cent.	per cent.	per cent.
1. Entire plate................	Fig. 223	100	100	100	100
2. Scarf-welded joint........	Fig. 224	faulty	106	102	104
3. Lap-welded joint..........	Fig. 225	50	69	66	62
4. Single-rivetted joint, by hand................	Fig. 226	40	50	60	50
5. Single-rivetted joint, by hand, snap-headed..	Fig. 227	50	52	56	53
6. Single-rivetted joint, by machine............	Fig. 228	40	54	52	49
7. Single-rivetted joint, with countersunk head....................	Fig. 229	44	50	52	49
8. Double-rivetted joint, snap-headed	Fig. 230	59	70	72	67
9. Double-rivetted joint, countersunk and snap-headed..........	Fig. 231	53	72	69	65
10. Double-rivetted joint, with single welt, countersunk and snap-headed..........	Fig. 232	52	60	65	59

From these data, it appears that the scarf-welded joint is as strong as the entire plate, and that the strength of the lap-welded joint averages only

five-eighths, or 62 per cent. of that of the entire plate. The varieties of single-rivetted joints average nearly equally strong for each variety; and they have only half the strength of the entire plate, excepting the snap-headed, which has rather more than half the strength. Of the double-rivetted joints the ordinary lap is the strongest, having two-thirds of the entire strength; the welt-joint is weakest.

Comparing the different thicknesses of plate, the averages of all the lap-joints, at the foot of the table, show that the ⅜-inch is the strongest, that the 7/16-inch is nearly as strong, and that they are about one-fourth stronger than ½-inch lap-joints, relatively to the thickness of plate.

Leaving the averages, the drift of the evidence is, that the thinner the plate the more efficient the joint. The single-rivetted joints, No. 4, have successively 40, 50, and 60 per cent. of the strength of the entire plates, and the double-rivetted joints, 59, 70, and 72 per cent.; insomuch that the ⅜-inch single-rivetted joint is absolutely stronger than the thicker joints—the actual breaking weights being successively 16, 17½, and 18 tons for the ½-inch, 7/16-inch, and ⅜-inch joints. For the double-rivetted joints, the actual breaking weights are about 23.5, 24.5, and 21.5 tons; showing that the 7/16-joint is absolutely stronger than the ½-inch, and that the ⅜-inch joint has only one-twelfth less absolute strength than the ½-inch joint. The double-rivetted welt-joint, similarly, is more efficient for the thinner plates, and its absolute strength is practically the same for them all.

It appears, then, that ⅜-inch rivetted plates are practically stronger than 7/16-inch and ½-inch rivetted plates; and that, of the ⅜-inch joints, the order of strength is as follows:—

	Tensile Strength.
Entire plate, ⅜-inch thick	100
Double-rivetted lap-joint, average	71
Double-rivetted single welt-joint	65
Single-rivetted lap-joint, average................................	55

These proportions do not differ widely from those that were given by Sir William Fairbairn.

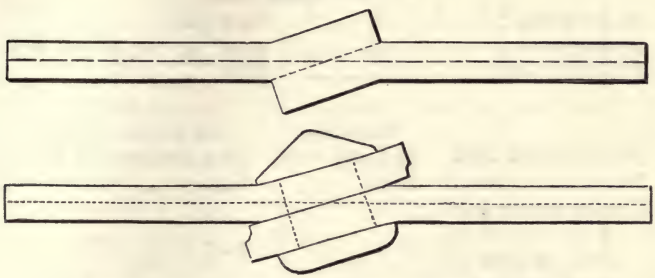
It appears that countersunk rivetting does not impair the strength of the joint, as compared with external heads.

To bring out the comparative weakness of the joints of the thicker plates, the fourth line of the following tablet, which is obtained by dividing the third by the second line, shows that the tensile strength per square inch of net section of the ⅜-inch single-rivetted joint, was nearly nine-tenths of that of the entire plate; whilst that of the 7/16-inch joint was just over eight-tenths; and of the ½-inch joint, seven-tenths.

Single-rivetted lap-joint, thickness of plate,....	½-inch	7/16-inch	⅜-inch.	
Net section,.................	62.5 %	62.5 %	62.5 %	of that of entire plate.
Net tensile strength, av.,...	43.5 "	51.5 "	55 "	" "
Do. per square inch of net section, in parts of that of entire plate,...	70 "	82 "	88 "	
Net tensile strength of lap-welded joint,......	50 "	69 "	66 "	of that of entire plate.

The lap-weld joint is strikingly weaker than the body of the plate, though there is no reduction of section. The weakness arises from the indirectness of the lap, for the joint, though solid, is not straight. The experiment proves that the lap is essentially an element of weakness, irrespective of the loss of strength by rivet-holes: the thicker the plate, the greater is the distorting leverage, insomuch that the absolute strength of the ½-inch lap-welded joint was not greater than that of the ⅜-inch joint. The annexed Figs. 233 and 234, show the ultimate distortion by the oblique stress on lap-joints.

Scale, One-half.



Figs. 233, 234.—Ultimate effects of Oblique Stress on Lap-Joints.

On the principle here noticed, one may account for the practically equal strength of the joints made with countersunk rivets, compared with those having external rivet-heads, notwithstanding the greater reduction of solid section by countersinking:—the leverage is shortened, and it may be measured from the centre of the cylindrical part of the rivet in the line *a a*, Fig. 235, or thereabouts, towards the inner side of the plate. On the same principle, the conical form of punched holes reduces the leverage and the obliquity of the pulling stress.[1]

Scale, One-half.

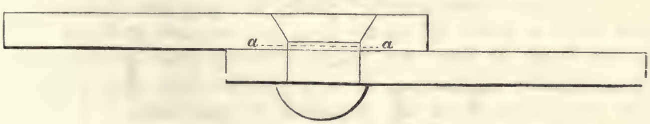


Fig. 235.—Diagram to show Stress on Countersunk Rivets.

As the double-rivetted joints, No. 8 of the series, exhibited respectively 59, 70, and 72 per cent. of the tensile strength of the entire plate, it appears that its resistance per square inch of net section, was 94, 112, and 115 per cent. of that of the entire plate. There is an apparent anomaly here:—it may be supposed that the normal strength of the particular plates exceeded 20 tons per square inch, aided, perhaps, by the frictional grip of rivets, first pointed out by Mr. Edwin Clark (see page 570).

Mr. J. G. Wright's Experiments.[2]—Mr. Wright gives the strength of two

[1] The author believes he was the first to publish the rationale of the strength and the weakness of rivetted joints, as the cause of the grooving of plates at such joints.

[2] Discussion upon Mr. W. R. Browne's paper, "On the Strength and Proportions of Rivetted Joints," in the *Proceedings of the Institution of Mechanical Engineers*, 1872.

specimens of single-rivetted square lap-joints, and two of diagonal joints, at angle of 45°, which were tested by Mr. Kirkaldy. They were made with ⅜-inch Staffordshire plate, exactly .38 inch thick, 12 inches wide, with 2¼-inch lap, punched holes, and six $\frac{13}{16}$-inch rivets in the square joint, at 2 inches pitch. The diagonal joint was made with eight rivets of the same size and pitch. The ultimate tensile strength of the solid plate was 19.69 tons per square inch with the fibre, and 16.80 tons across. The section of the entire plate was $(12 \times .38 =)$ 4.56 square inches, and the total ultimate strength with fibre was $(4.56 \times 19.69 =)$ 89.8 tons.

	Ultimate Tensile Strength.	
Entire plate	89.8 tons.	
Square joint	43.0 „	or 48 per cent.
Diagonal joint	58.0 „	or 64 „

	Square Joint.	Diagonal Joint.
Net sectional area,	59.4 per cent.	91.7 per cent. of entire plate.
Net tensile strength,	48.0 „	64.0 „ „
Do. per square inch of net section, in parts of that of entire section...	81.2 „	70.5 „ „

The diagonal joint was one-third stronger than the square joint; although, per square inch of net section, it opposed less resistance, because its resistance, which was necessarily exerted in an oblique direction, was a resultant compound of shearing resistance with the lengthwise resistance of the plates.

The net sectional area of the square joint was 2.71 square inches; and the shearing section of the rivets, 3.11 square inches, or 115 per cent. of the net section.

Mr. L. E. Fletcher's Experiments.—In these experiments, to be afterwards noticed, a double-rivetted lap-joint, made with punched holes, zigzag, of 7/16-inch Staffordshire plate, in a 7-feet Lancashire boiler, was burst with a force of 20.01 tons per square inch of net section between the rivets in line, the fracture taking place in the plate. With so high a tensile resistance, it is probable that the strength of the plate was very little, if at all, impaired by punching. The rivets were placed at 2.44 inches pitch in line, and had an average diameter of 13/16 inch. The ultimate strength of the joint may be taken as two-thirds of that of the solid plate—being in the ratio of the net sectional area to the section of the entire plate.

Messrs. John Elder & Co.'s Experiments.—A double-rivetted lap-joint of ¾-inch iron plate failed with a force of 15.06 tons per square inch of the net section between the rivets, the strength of the solid plate being 20.5 tons; also, a similiar joint of 9/16-inch plate failed with a force of 14.28 tons per square inch of net section, whilst the strength of the solid plate was 20.2 tons. Here, it was found that the net tensile resistance of the plates between the holes was less by one-fourth than the direct strength:—confirmatory of the deductions on the comparative weakness of the rivet-joints of the thicker plates.

Mr. Brunel's Experiments.—Mr. Brunel made experiments on double-rivetted double-welted plate-joints, of which the author published an

analysis in 1858–59.[1] The specimens were of ½-inch best Staffordshire plates, 20 inches wide, butt-jointed, with a covering or fishing plate on each side, 10 inches deep, put together with punched holes and rivets, as in Figs. 236-238, showing chain rivetting and zigzag rivetting. See tablet, p. 640.

The 1st specimen failed with 153 tons, shearing 10 rivets; and the 2d specimen failed with 164 tons, breaking a plate through the rivets; mean strength, 158.5 tons, = 15.85 tons per square inch of the entire section. Fig. 236.

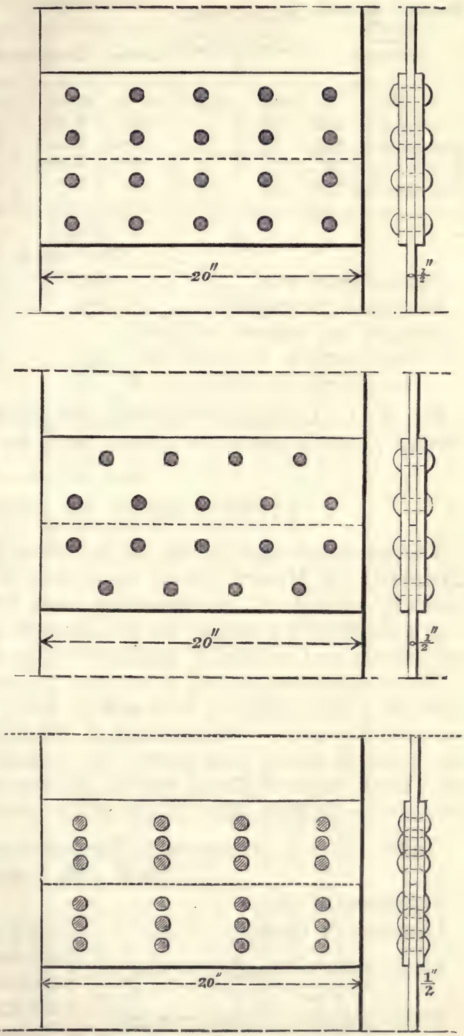
The 3d and 4th specimens failed with 167 and 147 tons respectively, through a line of rivet-holes; mean strength, 157 tons, = 15.7 tons per square inch of the entire section. Fig. 236.

The 5th, 6th, 7th, and 8th specimens broke with 158, 160, 161, and 168 tons respectively; mean strength, 162 tons, = 16.2 tons per square inch of the entire section. The fractures took place in the plates, following, in one case, the zigzag course of the rivets. In two cases, the rivets partly failed. Fig. 237.

The 9th and 10th specimens broke through the plate with 171 and 176 tons respectively; mean strength, 173½ tons, = 17.35 tons per square inch. Fig. 238.

Five solid ½-inch plates, from 12 to 16 inches in width, of the same quality as the specimens, were broken by from 19.4 to 22 tons per square inch; mean strength, 20.6 tons.

Scale, One-twelfth.



Figs. 236–238.—Rivetted Plate-Joints. Tested by Mr. Brunel.

The third line following the tablet, p. 640, shows that the strength of the plate per square inch was impaired by from 1 to 7 per cent. by punching.

The average efficiency, or actual strength, of the double-welt double-

[1] *Recent Practice in the Locomotive Engine*, 1858–59; also *Railway Locomotives*, 1860, page 2*.

rivetted joint may be taken as 80 per cent. of that of the entire plate, when the net sectional area is 83½ per cent.

BRUNEL'S EXPERIMENTS. — SPECIMEN.	Rivets, diameter.	Pitch, transversely.	Sectional Area of Plate.			Total Shearing Section of Rivets.
			Entire.	Net, transversely.		
Nos.	inch.	inches.	sq. in.	sq. inch.	per cent.	square inches.
1 and 2, Fig. 236,	$\frac{11}{16}$	4	10	8.28	82.8	7.42, or 90 % of net sect.
3 and 4, Fig. 236,	¾	4	10	8.125	81.25	8.84, or 110 ,, ,,
5 to 8, Fig. 237,..	¾	4	10	8.125	81.25	8.84, or 110 ,, ,,
9 and 10, Fig. 238,	¾	5	10	8.5	85	10.61, or 125 ,, ,,

	Chain. Nos. 1 and 2.	Chain. Nos. 3 and 4.	Zigzag. Nos. 5 to 8.	Chain. Nos. 9 and 10.
Net sectional area...............	82.8 %	81.25 %	85 %	85 %
Net tensile strength............	77	76	78.6	84
Strength per square inch of net section, in terms of that of entire section......	93	93.5	92.5	99

Mr. R. B. Longridge reported the results of tests for the strength of rivetted joints in iron boiler plates, made for him by Mr. Kirkaldy.

RIVET-JOINTS IN STEEL PLATES.

The results of experiments on the strength of rivet-joints in steel plates, conducted by Messrs. David Greig and Max Eyth, Professor A. B. W. Kennedy, and Mr. C. H. Moberley, have, with those on rivet-joints in iron, been exhaustively analysed in the *Strength of Materials*, in preparation by the author; and noticed in summary in his work on the *Steam Engine*.

The conclusions arrived at on the proportions and strength of rivetted joints in boiler plates of iron and of steel, ⅜-inch thick, are collected in the table No. 227. The relations of thickness of plate, diameter of rivets, and pitch of rivets, here shown for ⅜-inch plate-joints, are applicable to other thicknesses of plates, and are generalized as follows. The "spacing" denotes the distance apart of the two rows of rivets in double-rivetting.

Table No. 226.—STANDARD PROPORTIONS OF RIVETTED JOINTS IN IRON AND STEEL.

Thickness of plates..	unity or	1
Diameter of rivets...........................	thickness of plate, ×	2
Pitch of rivets (single-rivetting)....	thickness of plate, ×	5⅓
	diameter of rivets, ×	2⅔
Pitch of rivets (double-rivetting)...	thickness of plate, ×	8
	diameter of rivets, ×	4
Diagonal pitch (double-rivetting)..	longitudinal pitch, ×	¾
	diameter of rivets, ×	3
Spacing (double-rivetting).............	longitudinal pitch, ×	.56, or 9/16
Lap (single-rivetting)................	thickness of plate, ×	6
	diameter of rivets, ×	3
Lap (double-rivetting)...............	thickness of plate, ×	10.48, or 10½
	diameter of rivets, ×	5.24, or 5¼

Table No. 227.—RIVETTED JOINTS IN BOILER PLATES OF IRON OR STEEL, ⅜-INCH THICK, OF EQUAL RESISTANCE AND MAXIMUM STRENGTH:—STANDARD DIMENSIONS AND PROPORTIONS.

JOINT IN ⅜-INCH PLATE.	Thickness of Plate.	Rivets.		Net Section of Plate, in parts of that of the whole Plate.	Shearing Section.		Breaking Strength, in parts of that of the whole Plate.		Breaking Strength per Square Inch, in parts of that of the whole Plate per Square Inch.			
		Diameter.	Pitch Longitudinally.		In parts of Net Section of Plate.	In parts of whole Plate.			Of Net Plate Section.		Of Shearing Section.	
							Iron.	Steel.	Iron.	Steel.	Iron.	Steel.
	inch.	inch.	inches.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.
Single-rivetted lap............	⅜	¾	2	62.5	94.0	58.75	56	60	89.6	96.0	95.3	102.1
Do. single welt......	⅜	¾	2	62.5	94.0	58.75	50	58	80.0	92.8	85.1	98.7
Do. double welt.....	⅜	¾	2	62.5	188.0	117.50	60	65	96.0	104.0	51.1	55.3
Double-rivetted lap............	⅜	¾	3	75	104.7	78.50	70	80	93.3	106.7	89.1	102.0
Do. single welt....	⅜	¾	3	75	104.7	78.50	65	78	86.7	104.0	82.8	99.3
Do. double welt...	⅜	¾	3	75	209.4	157.00	72	80	96.0	106.7	45.9	51.0

NOTES TO TABLE.—1. The diagonal pitch for the double-rivetted joints is 2¼ inches, or 75 per cent. of the longitudinal pitch. It is equal to the length of interspace between rivets longitudinally.

2. The distance apart, or "spacing," of the two rows of rivets is 1.68 inches,—say $1^{11}/_{16}$ inches bare,—or 56 per cent. of the longitudinal pitch.

On these proportions, the diameter and pitch of rivets suitable for plates of from ⅛ inch to $^{11}/_{16}$ inch in thickness, are as given in the following table, No. 227*a*. The calculation is not extended for thicknesses greater than $^{11}/_{16}$ inch, for which the corresponding rivet has a diameter of 1⅜ inches, as this size of rivet is assumed to be the maximum properly available in practical operations.

Table No. 227*a*.—Standard Rivetted Joints of Maximum Strength, in Iron and Steel Plates, of various thicknesses.

(Net section for single-riveting, 62.5 per cent. of whole plate-section; for double-riveting, 75 per cent.)

Thickness of Plate.	Diameter of Rivets.	Pitch of Rivets.				Lap.	
		Single Rivetting	Double Rivetting.			Single Rivetting.	Double Rivetting.
			Longitudinal.	Diagonal.	Spacing.		
inch.	inches.	inches.	inches.	inches.	inches.	inches.	inches.
⅛	¼	$^2/_3$	1	¾	$^9/_{16}$	¾	1 $^5/_{16}$
$^3/_{16}$	⅜	1	1½	1⅛	$^{27}/_{32}$	1⅛	2
¼	½	1$^1/_3$	2	1½	1⅛	1½	2⅝
$^5/_{16}$	⅝	1$^2/_3$	2½	1⅞	1$^{13}/_{32}$	1⅞	3¼
⅜	¾	2	3	2¼	1$^{11}/_{16}$	2¼	3$^{15}/_{16}$
$^7/_{16}$	⅞	2$^1/_3$	3½	2⅝	2	2⅝	4⅝
½	1	2$^2/_3$	4	3	2¼	3	5¼
$^9/_{16}$	1⅛	3	4½	3⅜	2½	3⅜	5⅞
⅝	1¼	3$^1/_3$	5	3¾	2$^{13}/_{16}$	3¾	6½
$^{11}/_{16}$	1⅜	3$^2/_3$	5½	4⅛	3$^1/_{16}$	4⅛	7¼

Other proportions than those given in table No. 227*a* may, of course, be adopted; and, in fact, must be adopted for plates of greater thickness than $^{11}/_{16}$ inch. By means of the following general formulas the pitches of rivets, and their diameters, producing joints of equal resistance, may be found for plates thicker than $^{11}/_{16}$ inch.

Pitch of Rivets for Equal Resistance.

$$p = \frac{.7854}{rt} d^2 + d \quad \text{(1)}$$

$$d = \sqrt{\frac{rt}{.7854} p + \left(\frac{rt}{1.57}\right)^2} - \frac{rt}{1.57} \quad \text{(2)}$$

t = thickness of plates, in inches.
d = diameter of rivets, in inches.
p = pitch of rivets, in inches.
r = ratio of shearing section of rivets to net section of plates.

STRENGTH OF PILLARS OR COLUMNS.

Mr. Stoney lucidly develops the leading principles of the resistance of columns supporting incumbent loads,[1] which are, no doubt, strictly applicable to columns of perfectly homogeneous material. He shows that the strength of very long square or round pillars varies directly as the 4th power of the diameter, and inversely as the square of the length; that it depends not on the direct strength of the material, but on the coefficient of elasticity, which represents the stiffness and capability of resisting flexure; that the strengths of similar long columns are as the squares of any linear dimension, or as the sectional areas, whilst their weights are as the cubes of the dimension; and that, if the strengths of long pillars of similar section are the same, while the length varies, the sectional areas vary as the lengths, and the weights vary as the squares of the lengths. Finally, that the weight which produces moderate flexure in a very long pillar is also very near the breaking weight, as a trifling additional load bends the pillar very much more, and strains the fibres beyond what they can bear—a conclusion of great practical importance, which has been corroborated by experience.

Mr. Hodgkinson's Investigations.

The following is an abstract of Mr. Hodgkinson's conclusions on the resistance of cast-iron columns, under loads. The mode of fracture of cast-iron struts or columns under compression, is the same when the height is greater than the diameter of the specimen, and not greater than four or five times the diameter. When the height is greater, the specimen bends. Fracture usually takes place by the two ends of the specimen forming cones or pyramids, splitting the sides, and throwing them out. Sometimes the end slides off as a wedge, the height of which is somewhat less than 1.5 diameters. The tensile and compressive resistances average as 1 to 6.6; or, as the specimens were of unequal quality, the ratio should be 1 to 7 or 8, giving 49 tons per square inch for the ultimate compressive resistance.

Long Columns.—Experiments were made on castings of Lowmoor No. 3 iron. Of three cylindrical columns having the same diameter and length, the first had the ends rounded; the second, one flat end and one round end; the third, both ends flat. The strengths were as 1, 2, 3 nearly.

A long flat-end column has the same strength as a round-end pillar of half the length. The same properties apply to pillars of steel, wrought-iron, and wood. Swelling a pillar at the middle adds not more than one-seventh to the strength. The power of resistance is as the 3.6 power of the diameter, and as the 1.7 power of the length.

These remarks apply to all pillars not less than 30 diameters in length, up to 120 diameters; and the following are the formulas for the breaking load of flat-ended columns:—

$$\text{Solid columns,} \quad W = 44 \frac{D^{3.6}}{L^{1.7}} \qquad (4)$$

$$\text{Hollow columns,} \quad W = 44 \frac{D^{3.6} - d^{3.6}}{L^{1.7}}, \qquad (5)$$

in which D is the external diameter in inches, d the internal diameter in inches, L the length in feet, W the breaking load in tons.

[1] *The Theory of Strains*, 1873, page 244.

Short Flexible Columns.—The resistance is compounded of compressive resistance and transverse resistance. Let W' = the breaking load, and c = the direct compressive resistance of the column (say 49 tons × sectional area in square inches), then, having first calculated the breaking weight W by the above formula, (4) or (5), the strength is found by the formula,

$$W' = \frac{W c}{W + .75 c} \quad \text{(6)}$$

This formula is inferred from the nature of the compound strain; the results given by it are nearly correct, but rather excessive.

The strength of long similar columns is nearly as the sectional area, or nearly as the square of the diameter; it is as the 1.865 power.

The strength of taper columns is to that of cylindrical columns as $D^2 D'^2$ to D^4, the extreme diameters of the taper column being D and D′. If the two columns be of the same length and solid content, the cylindrical one is the stronger.

The strength of a column of a double-flanged section is only three-fourths of that of a uniform hollow cylinder of equal weight; and that of a column of cruciform section is less than half.

The strength of a solid square cast-iron column is 50 per cent. more than that of a round column of the same diameter.

A column irregularly fixed, so that the pressure is taken diagonally, has only a third of the strength when squarely fixed.

Cast-iron pillars, with discs on the ends, are somewhat stronger than those with simply flat ends.

Solid square cast-iron pillars bend or break in the direction of a diagonal.

A slight inequality in the thickness of hollow cast-iron pillars does not reduce the strength materially.

Square is the strongest section for timber rectangular in form.

COMPARATIVE STRENGTH OF LONG COLUMNS.

Cast iron	1000
Wrought iron	1745
Cast steel	2518
Dantzic oak	109
Red deal	78.5

3.6 POWERS OF DIAMETERS.

Diameter.	Power.	Diameter.	Power.	Diameter.	Power.	Diameter.	Power.
1	1	3	52.196	6	632.91	10	3981.07
1.5	4.3	3.5	90.917	7	1102.4	11	5610.7
2	12.125	4	147.03	8	1782.9	12	7674.5
2.5	27.076	5	328.32	9	2724.4		

1.7 POWERS OF LENGTHS.

1	1	7	27.33	13	78.3	21	176.92
2	3.25	8	34.29	14	88.8	22	191.48
3	6.47	9	41.9	15	99.85	24	222.0
4	10.55	10	50.12	16	111.43	26	254.3
5	15.42	11	58.93	18	136.13	28	288.5
6	21.03	12	68.33	20	162.84	30	324.4

Mr. F. W. Shields gives the safe load on hollow cast-iron columns of good construction, with flat ends, and with base plates.[1]

Thickness.	Load per square inch of Sectional Area.	
	Length 20 to 24 Diameters.	25 to 30 Diameters.
inches.	tons.	tons.
¾ and upwards,	2	1¾
⅝,	1¾	1½
½,	1½	1¼
⅜,	1¼	1

The reduction of the load per square inch with the thickness, is devised to allow for liability to weakness from inequalities of the casting.

Mr. Gordon's Rules.

The first and second formulas were deduced by Mr. Lewis D. B. Gordon from the results of Mr. Hodgkinson's experiments.

As here given, they show the total breaking weight of a cast-iron column with flat ends. The succeeding formulas for the strength of columns of wrought-iron and steel have been constructed on the basis of Mr. Gordon's formulas.

1. *For solid or hollow round cast-iron columns:—*

$$W = \frac{36\,a}{1 + \dfrac{r^2}{400}} \qquad (7)$$

2. *For solid or hollow rectangular cast-iron columns:—*

$$W = \frac{36\,a}{1 + \dfrac{r^2}{500}} \qquad (8)$$

3. *For solid rectangular wrought-iron columns (Mr. Stoney):—*

$$W = \frac{16\,a}{1 + \dfrac{r^2}{3000}} \qquad (9)$$

4. *For columns of angle, tee, channel, or cruciform iron (Mr. Unwin):—*

$$W = \frac{19\,a}{1 + \dfrac{r^2}{900}} \qquad (10)$$

5. *Solid round columns, of mild steel (Mr. Baker):—*

$$W = \frac{30\,a}{1 + \dfrac{r^2}{1400}} \qquad (11)$$

[1] *Transactions of the British Association*, 1861.

6. *Solid round columns, strong steel (Mr. Baker):—*

$$W = \frac{51\,a}{1 + \dfrac{r^2}{900}} \quad \text{.......} \quad (12)$$

7. *Solid rectangular columns, mild steel (Mr. Baker):—*

$$W = \frac{30\,a}{1 + \dfrac{r^2}{2480}} \quad \text{.......} \quad (13)$$

8. *Solid rectangular columns, strong steel (Mr. Baker):—*

$$W = \frac{51\,a}{1 + \dfrac{r^2}{1600}} \quad \text{.......} \quad (14)$$

W = the breaking weight in tons.
a = the sectional area of the material in inches.
r = the ratio of the length to the diameter. The diameter for calculation is the least diameter of the section, or that in the direction of which the piece is most flexible.

Mr. Hodgkinson's Rules for Timber Columns.

When both ends are flat and well-bedded, and the length exceeds 30 diameters:—

Long square columns of Dantzic oak (dry):—

$$W = 10.95\frac{d^4}{l^2} \quad \text{.......} \quad (15)$$

Long square columns of red deal (dry):—

$$W = 7.80\frac{d^4}{l^2} \quad \text{.......} \quad (16)$$

Long square columns of French oak (dry):—

$$W = 6.90\frac{d^4}{l^2} \quad \text{.......} \quad (17)$$

W = the breaking weight in tons.
d = the breadth in inches.
l = the length in feet.

When timber columns are less than 30 diameters in length, their strength is calculated by formula (6), page 644, for which the value of W is to be calculated by one of the above formulas.

When the column is oblong in section, multiply the result as found for the shorter dimension of the section by the ratio of the longer to the shorter dimension.

Mr. Brereton's Experiments on Timber Piles.

Mr. R. P. Brereton gives the loads that could be borne, as found from experiments, by large fir or pine timber 12 inches square, of various lengths: 10 feet long bore 120 tons; 20 feet long, 115 tons; 30 feet long, 90 tons; 40 feet long, 80 tons. Mr. Stoney plotted these results, and constructed the following table from the curve:—

Ratio of length to least breadth..........	10	15	20	25	30	35	40	45	50
Weight that can be borne in tons per square foot of section....................	120	118	115	100	90	84	80	77	75

Checking this table by Mr. Kirkaldy's experiments (page 547) on balks of Riga and Dantzic timber, having a length of 20 feet, which was 18½ times the width, the actual breaking weights were—

	Total.	Per square foot of Section.	By Mr. Stoney's Curve.
Riga...............	148 tons,	or 126 tons, ...	116 tons.
Dantzic..........	138 „	or 111 „	116 „
Mean........................		119 „	116 „

showing a close correspondence between the results of experiments conducted independently. Mr. Hodgkinson's rule gives results which are rather less than those that are given in Mr. Stoney's table.

Mr. Laslett's Experiments on Columns of Wood.

A notice of Mr. Laslett's experiments is given at page 541. He deduces from his experiments, repeated for many kinds of wood: that the maximum resistance of square pieces to compression is exerted when the sectional area in square inches is to the length in inches approximately as 4 to 5, for equal seasoning and equal specific gravities. According to this deduction, the maximum resistance of 12-inch square balks on end, would be exerted when they are 15 feet in length.

CAST-IRON FLANGED BEAMS.

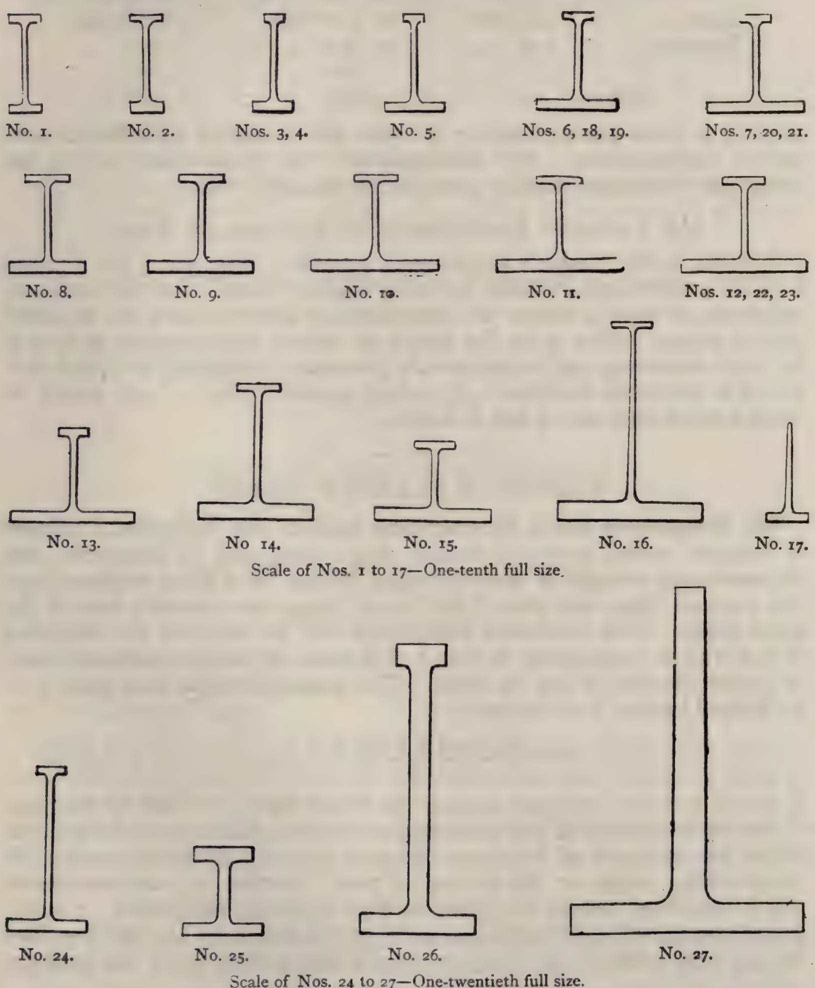
Mr. Hodgkinson tested, for transverse strength and deflection, a number of cast-iron model beams of various proportions, and he discovered that the maximum strength of double-flanged beams, for a given sectional area, was realized when the area of the upper flange was one-sixth that of the lower flange. This conclusion harmonizes with the fact that the resistance of cast iron to compression is from 5 to 6 times the tensile resistance; and, as a scientific fact, it has its value. The general formula (19), page 511, for flanged beams, is as follows:—

$$W = \frac{d'' s(4a + 1.155 a'')}{l}, \qquad (1)$$

in which a is the sectional area of the lower flange; a'' that of the web, d'' the reputed depth of the beam and of the web, taken as the total depth minus the thickness of the lower flange, l the span, all in inches, and W the breaking weight at the middle, in tons. Sections of cast-iron beams which have been tested, are given in Figs. 239–258, comprising 17 model beams tested by Mr. Hodgkinson, and 6 model beams by Mr. Berkley; likewise 11 large beams.[1] Mr. Berkley's model beams are in pairs, and have the

[1] See Hodgkinson on the *Strength and other Properties of Cast Iron*, 1846; and the *Proceedings of the Institution of Civil Engineers*, vol. xxx. page 252 (Mr. Berkley's paper on the "Strength of Iron and Steel").

same dimensions as Nos. 6, 7, and 12 respectively. Table No. 228 contains, in part 1, the needful particulars of those beams, and of the ultimate breaking weight, both actual and as calculated by formula (1). The deflection and the elastic strength of the beams are given in part 2 of the table. A tensile strength of 7 tons per square inch has been adopted in the calculation of the ultimate strength of Mr. Hodgkinson's model beams, 6½ tons for the large beams, and 10½ tons for Mr. Berkley's model beams; for, though his test-castings bore a greater tensile load than 10½ tons, they were too short in the tested portion, which was 1 inch square and only 1½ inches long, for the action of simple tensile resistance.



Figs. 239–258.—Sections of Cast-Iron Beams tested for Transverse Strength by Mr. Hodgkinson, Mr. Berkley, and others. Table No. 228.

Table No. 228.—Strength of Cast-iron Flanged Beams.

Part 1.—Ultimate Transverse Strength.

1. Model Beams.

Reference Number.	Span. l	Depth at middle. d	Thickness. Web.	Thickness. Lower flange.	Sectional Area of Flanges. Lower flange. a	Sectional Area of Flanges. Upper flange, ratio to lower.	Breaking Weight at the middle. Calculated. W	Breaking Weight at the middle. Actual. W
	feet.	inches.	inches.	inches.	sq. inches.	ratio.	tons.	tons.
Mr. Hodgkinson's Beams. Assumed tensile strength, 7 tons per square inch.								
1	4.5	5⅛	.29	.39	.69	1 to 1	2.47	2.98
2	,,	,,	.30	.55	.98	1 to 2	3.27	3.29
3	,,	,,	.32	.57	1.20	1 to 4	3.83	3.69
4	,,	,,	.33	.56	1.20	1 to 4	3.87	3.64
5	,,	,,	.305	.51	1.57	1 to 4½	4.68	4.79
6	,,	,,	.38	.53	2.20	1 to 4	6.45	6.46
7	,,	,,	.34	.56	2.89	1 to 5½	7.85	7.47
8	,,	,,	.33	.57	2.31	1 to 3.2	6.49	6.71
9	,,	,,	.35	.537	2.92	1 to 4.3	8.04	7.54
10	,,	,,	.34	.54	3.57	1 to 5.6	9.56	8.68
11	,,	,,	.266	.66	4.40	1 to 6	10.98	11.65
12	,,	,,	.335	.65	4.31	1 to 7	11.00	10.40
13	,,	,,	.34	.51	3.32	1 to 6.7	9.02	9.40
14	7.0	6.93	.38	.75	4.54	1 to 6	10.26	9.90
15	,,	4.10	.40	.74	4.44	1 to 6	5.41	6.05
16	9.0	10¼	.25	.77	4.72	1 to 8.3	13.28	12.80
17	4.5	5⅛	.40	.46	1.04	none	3.83	3.93
Averages for 17 beams........................						1 to 4.6	7.07	7.01
Mr. Berkley's Beams. Assumed tensile strength, 10.5 tons per square inch.								
18	4.5	5⅛	.38	.53	2.20	1 to 4	9.67	10.00
19	,,	,,	,,	,,	,,	,,	9.67	10.00
20	,,	,,	.34	.56	2.89	1 to 5½	11.85	11.75
21	,,	,,	,,	,,	,,	,,	11.85	11.85
22	,,	,,	.335	.65	4.31	1 to 7	16.47	14.25
23	,,	,,	,,	,,	,,	,,	17.08	18.00
Averages for 6 beams						1 to 5.5	12.76	12.64
2. Large Beams.—Assumed tensile strength, 6.5 tons per square inch. (Reported by Mr. Hodgkinson.)								
24 3 beams	18.0	17	.625	1.25	10.31	1 to 4.6	24.93	25.0
25	11.67	9	1.5	1.5	12.00	1 to 1.33	21.24	20.0
26[1]	27.4	30.5	2.08	2.07	25.05	1 to 2.1	94.64	76.6+
27[2]	23.1	36.1	3.36	3.12	74.60	none	330.0	153.0+
Mr. Cubitt's Beams.								
28	15.0	7.15	1.04	1.59	7.98	1 to 3.6	7.75	7.00
29	,,	7.17	1.10	1.59	8.11	1 to 3.6	7.96	7.13
30	,,	10.75	.93	1.04	5.25	1 to 2.3	11.02	11.50
31	,,	10.75	1.05	1.05	5.42	1 to 2.3	11.71	12.00
32[3]	,,	12.75	.73	.88	4.47	1 to 2.7	11.95	10.25
33	,,	12.8	.95	1.09	5.59	1 to 2.25	14.89	15.75
34[4]	,,	14.0	.91	1.00	6.50	none	18.39	12.38
35[5]	,,	17.25	.68	.84	4.96	1 to 2.2	19.39	16.00
36	7.5	7.15	1.06	1.59	8.03	1 to 3.4	15.63	15.63
37[6]	,,	10.75	.92	1.02	5.16	1 to 2.25	21.76	23.87
Averages for 11 of these large beams......						1 to 3.17	17.00	17.08
Total averages for 34 beams.................						1 to 4.30	11.31	11.26

Table No. 228 (*continued*).

Part 2.—Deflection and Elastic Strength.

1. Model Beams.

Reference Number.	Form of Beams.	Limits of Elastic Strength.			Ratio of Elastic Strength to Breaking Weight.	Coefficient of Elasticity.
		Deflection at middle.	Load.	Inches per ton of load.		
	tons.	inches.	tons.	inches.	per cent.	E.
	Mr. Hodgkinson's Beams.					
1						
2						
3						
4						
5						
6	Elliptical.	.45	5.670	.079	88	5232
7	Do.	.49	7.244	.068	91	4980
8						
9	Uniform depth.	.33	6.872	.048	91	4372
10	Do.	.36	7.454	.048	86	3420
11						
12	Do.	.48	10.254	.047	99	3324
13	Elliptical.	.46	8.503	.054	90	5438
14	Uniform depth.	.60	9.204	.065	93	4284
15	Do.	.70	3.787	.185	63	5520
16	Do.	.55	11.450	.048	89	5208
17	Segmental.	.42	3.700	.114	94	5600
	Averages for 10 beams..				88	4758
	Mr. Berkley's Beams.					
18	Uniform depth.	.245	7.00	.035	70	7400
19	Do.	.271	8.00	.034	80	7634
20	Do.	.387	10.00	.039	85	5486
21	Do.	.332	9.00	.037	76	5760
22	Do.	.264	8.00	.033	56	5274
23	Do.	.303	10.00	.030	55	5154
	Averages for 6 beams..				70	6118
	2. Large Beams. (Reported by Mr. Hodgkinson.)					
24 3 beams	} Segmental.	1.00	20.0	.050	80	4632
25	—	.33	10.0	.033	50	?
26	Segmental.	1.29	76.6	.017	?	4236
27	—	.68	153.0	.0044	?	?
	Mr. Cubitt's Beams.					
28	Uniform section.	1.54	6.0	.257	86	4760
29	Do.	1.215	5.0	.243	70	4706
30	Do.	.645	7.0	.092	61	5340
31	Do.	1.04	11.0	.095	92	4900
32	Do.	.60	9.0	.067	88	5566
33	Do.	.90	15.0	.060	95	5032
34	Uniform depth.	.41	6.0	.070	49	4840
35	Do.	.76	16.0	.048	100	5226
36	Uniform section.	.31	10.0	.031	64	4884
37	Do.	.261	20.0	.013	84	4762
	Averages for 12 beams..				77	4906
	Total averages for 28 beams..............................				79	5113

Notes to Table No. 228.

[1] No. 26—"broke apparently in consequence of an accidental shake."
[2] No. 27—"with this load, 153 tons in the middle, the experiment was discontinued, as the apparatus was overstrained."
[3] No. 32—"bottom flange unsound."
[4] No. 34—"bottom flange unsound."
[5] No. 35—"bottom flange unsound."
[6] No. 37—"nearly but not quite sound."

The contents of the table exhibit a surprisingly close conformity throughout between the calculated and the actual ultimate strengths of such beams as were sound, and were fairly tested and broken. The total averages for 34 cast-iron beams show that, with a ratio of upper to lower flange of 1 to 4.30, the breaking weight, as calculated, was 11.31 tons, and as tested, 11.26 tons.

It further appears that the ultimate strength of a cast-iron beam is scarcely affected by the proportionate size of the upper flange; and that the formula (1), page 647, may be adopted for the calculation of the strength of flanged cast-iron beams of any ordinary section. It is sufficient to employ the factor, 7 tons, for castings of less than ¾ inch in thickness, and 6.5 tons for those which have a thickness of 1 inch and upwards. Taking the span in feet:—

Ultimate Transverse Strength of Cast-iron Flanged Beams.

For the thinner castings...... $W = \dfrac{d'' (7a + 2a'')}{3l}$ .... (2)

For the thicker castings...... $W = \dfrac{d''(6.5a + 1.9a'')}{3l}$ .... (3)

W = the breaking weight in tons at the middle; a the sectional area of the lower flange, and a'' the sectional area of the web, taken at the reputed depth, both in square inches; l the span in feet. The reputed depth is the total depth minus the thickness of the lower flange.

The following are the constants for other factors of tensile strength:—

Tensile strength per square inch.	Constants in formula 2 or 3. for a.	for a''.
6 tons	6	1.7
6¾ "	6.75	2.0
7½ "	7.5	2.2
8 "	8	2.3
9 "	9	2.6
10 "	10	2.9

Approximate Rule for the Strength of Cast-iron Flanged Beams.

Referring to formula (2) for a tensile strength of 7 tons:—To the sectional area of the lower flange add a fourth of the sectional area of the web, calculated on the total depth, both in inches; multiply the sum by the total depth in inches, and by 2⅓; and divide the product by the span in feet. The quotient is the breaking weight at the middle, in tons.

For any other tensile strength, use it as the multiplier instead of 2⅓, and divide the product by 3, and by the span. The quotient is the breaking weight.

Elastic Strength and Deflection of Cast-iron Flanged Beams.

From the observations of the experimentalists on the deflection of the beams noted in the second part of table No. 228, it is shown that the elastic strength—that is, the limit of load for uniform increments of deflection, irrespective of set—as given in the table, averages about 80 per cent. of the ultimate strengths. The value of the coefficients of elasticity, E, were calculated tentatively by means of the formula (13), page 531, for beams of constant depth and uniform strength loaded at middle; but, since all the beams excepting six of Cubitt's beams were proportioned for carrying uniform loads, the tentative values found for these beams were modified according to their special forms, on the principles already adopted in the general section on the deflection of beams, pages 529, 530, to give the proper values of E. The general average value for E is 5113, and the numerical coefficient of the formula (13), page 531, being multiplied into this value, gives the resultant coefficient for beams of cast iron. The general formula is as follows; c being the coefficient:—

Deflection of Cast-iron Flanged Beams.

$$D = \frac{W\,l^3}{(c\,E)\,d''^2(4\,a + 1.155\,a''^2)} \qquad (4)$$

Values of (c E), *to be employed in this formula.*

				Length in inches.		Length in feet.
1.	Constant depth, uniform strength,		load at middle....	20,700	...	12
2.	Do.	do.	uniform load......	41,400	...	24
3.	Constant breadth,	do.	load at middle....	10,350	...	6
4.	Do.	do.	uniform load......	27,600	...	16
5.	Uniform section,		load at middle....	20,700	...	12
6.	Do.		uniform load......	33,120	...	19

D = the deflection, b = the breadth, d'' = the reputed depth, or the extreme depth minus the thickness of the lower flange, all in inches; l = the span, in inches or feet; a = the sectional area of the lower flange at the middle, and a'' = that of the web at the middle, reckoned on the reputed depth; W = the load in tons.

Approximate Rule.

To the sectional area of the lower flange, add one-fourth of the sectional area of the web, calculated on the whole depth, both in inches; multiply the sum by the square of the depth in inches, and by the proper coefficient in the following list; making a product A. Multiply the load at the middle in tons, by the cube of the span in feet; and divide this product by the product A. The quotient is the deflection in inches.

No. 1. Coefficient.........	48	No. 4. Coefficient.........	64
No. 2. "	96	No. 5. "	48
No. 3. "	24	No. 6. "	76

WROUGHT-IRON FLANGED BEAMS OR JOISTS.

SOLID-ROLLED WROUGHT-IRON JOISTS.

The usual section of solid-rolled wrought-iron beams or joists is shown by Figs. 259, 260, 261, and 262, page 654. For practical facility of rolling, the flanges rarely ever exceed 6 inches in breadth; and the breadth of flange varies from a third of, to an equality with, the depth,—the latter ratio, of course, only occurring for small sizes. The flanges, also, have a taper section on each side of the web.

In joists of ordinary proportions, the thickness of the web is from $^1/_{26}$th to $^1/_{13}$th of the depth of the beam, being thicker as the relative breadth of the flanges is increased; but, by setting the rolls wider apart, the thickness may be increased a half or two-thirds more. The mean thickness of the flanges is from $^1/_{19}$th to $^1/_{10}$th of the depth. The slope or taper of the flanges in section is usually about 1 in 7, on each side of the web. The beams can be rolled to lengths of 30 feet; but they cost less per ton when the lengths do not exceed 20 feet.

Table No. 229 shows the average proportions of the thickness of the web, and the mean thickness of the flange, for various proportional breadths of flanges, the depth being taken as 1.

Table No. 229.—PROPORTIONAL DIMENSIONS OF SOLID WROUGHT-IRON JOISTS.

Depth of Joist.	Breadth of Flange.	Thickness of Web.	Mean Thickness of Flanges.	Depth of Joist.	Breadth of Flange.	Thickness of Web.	Mean Thickness of Flanges.
1	.3	1/26	1/19	1	.75	1/16.3	1/11.5
1	.35	1/24.5	1/18	1	.8	1/15.3	1/11
1	.4	1/23	1/17	1	.85	1/14.6	1/10.5
1	.45	1/22	1/16.2	1	.9	1/14	1/10
1	.5	1/21	1/15.3	1	.95	1/13.7	1/9.7
1	.55	1/20	1/14.3	1	1.00	1/13.5	1/9.5
1	.6	1/19	1/13.5				
1	.65	1/18	1/12.8				
1	.7	1/17	1/12				

The dimensions of a variety of solid-rolled joists actually manufactured, having the minimum, or what may be called the normal, thickness of web, are given in table No. 230, next page. The reputed weights per lineal foot are given in the fifth column. The ultimate strengths for a span of 10 feet, in the sixth column, are calculated by the first approximate rule with formula (7), page 656; and the safe distributed load in the last column is taken as one-third of the breaking weight at the middle, according to a factor of 6.

To find, from table No. 230, the ultimate strength of, or the safe permanent load for, a joist, for any other span than 10 feet, multiply the tabular weight for the beam of the given section by 10, and divide the product by the given span.

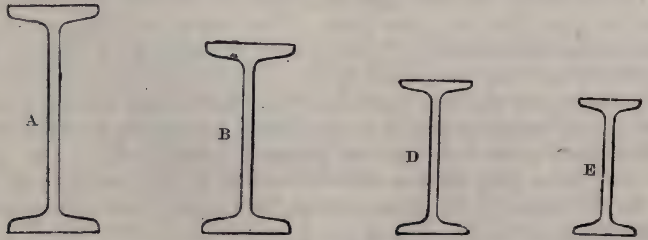
Inversely, to find the span for a joist of a given section, with a given weight, multiply the tabular weight by 10, and divide the product by the given weight.

Table No 230.—Solid-rolled Wrought-iron Joists:—Dimensions, Weight, and Strength. Span, 10 Feet.

Depth of Beam.	Breadth of Flanges.	Thickness. Of Web.	Thickness. Of Flanges.	Reputed Weight per lineal foot.	Ultimate Strength, Loaded at the Middle.	Safe Permanent Load, Uniformly Distributed.
inches.	inches.	inch.	inch.	lbs.	tons.	cwts.
16	5⅝	3/4	13/16	62	84	560
14	6	9/16	13/16	60	68	453
14	5½	9/16	7/8	60	67	447
12	6	9/16	15/16	56	61	407
12	5	7/16	13/16	42	45	300
10	4¾	3/4	5/8	36	34	227
10	4¾	7/16	9/16	32	26	173
10	4½	7/16	9/16	32	25	167
9½	4½	3/8	11/16	30	26.5	177
9¼	3¾	7/16	1/2	24	18.6	124
8	5⅛	7/16	9/16	29	21	140
8	5	3/8	9/16	29	20	133
8	4	3/8	1/2	21	15.4	103
8	2½	3/8	3/8	15	9.5	63
8	2⅜	5/16	7/16	15	9.3	62
7	3⅝	5/16	1/2	19	11.6	77
7	3⅝	5/16	7/16	19	10.4	69
7	2¼	5/16	7/16	14	7.6	51
7	2¼	9/32	3/8	14	6.6	44
6¼	3¼	5/16	13/32	—	7.8	52
6¼	2¼	5/16	3/8	18	5.8	39
6¼	2	5/16	7/16	11	6	40
6	5	7/16	9/16	30	15.1	101
5½	2	3/8	7/16	10	5.3	35
5	4½	3/8	1/2	23	8.6	57
5	3	5/16	7/16	13	6	40
4¾	2	1/4	5/16	8	3.1	21
4	3	1/4	3/8	12	3.8	25
4	2	1/4	5/16	8	2.45	16
3⅛	1⅝	3/16	7/32	5½	1.11	7.4
3	3	1/4	5/16	10	2.5	17
3	2	3/16	7/32	5½	1.22	8.1

Transverse Strength of Wrought-iron Joists.

A number of solid-rolled joists of uniform symmetrical section were



Figs. 259–262.—Sections of Solid Wrought-iron Flanged Beams, or Joists.

tested by Mr. Kirkaldy for Mr. Moser, the sections of four of which have

been ascertained approximately, and are here annexed, Figs. 259–262, with the following particulars:—

Joists.	Weight per foot.	Depth.	Breadth.	Web. — Thickness.	Flange. — Mean thickness.	Sectional Area. Web at reputed depth.	Sectional Area. One Flange.	Sectional Area. Total.
	lbs.	inches.	inches.	inch.	inch.	sq. ins.	sq. ins.	sq. ins.
A	43.56	11.75	5.70	.60	.643	6.537	3.665	13.34
B	37.54	9.85	4.60	.50	.804	4.523	3.700	11.50
D	26.22	7.90	3.76	.50	.619	3.640	2.329	8.033
E	19.16	7.07	3.00	.50	.485	3.293	1.455	5.870

The elastic and ultimate transverse strengths of these beams are reduced from the observations of Mr. Kirkaldy, and are given in the table No. 231. In the last column, the probable real ultimate strengths of the beams are added; they are computed at twice the elastic strength, in correspondence with the well established ratio of the tensile elastic and ultimate strengths of wrought iron. The ultimate strength, column 4, was calculated by formula (19), page 511, in which the ultimate tensile strength, s, is taken as 20 tons:—

Table No. 231.—Transverse Strength of Solid-rolled Wrought-iron Joists.

(Results of Experiment.)

Joists.	Span.	Elastic Strength. Observed.	Ultimate, or Breaking Weight. Calculated.	Ultimate, or Breaking Weight. Observed.	Cause of Failure.	Probable Real Breaking Weight.
	feet.	tons.	tons.	tons.		tons.
A	20	10.714	20.390	14.310	Buckling	21.428
	10	21.428	40.780	32.450	"	42.856
B	20	8.482	15.060	11.445	"	16.964
	10	17.857	30.120	26.530	"	35.714
D	20	4.018	8.203	6.371	"	8.036
	10	8.705	16.406	15.112	"	17.410
E	10	5.402	8.150	8.150	"	10.804
	5	11.607	21.124	19.520	"	23.214
Averages		11.027	20.331	16.736	—	22.053

As the cause of failure was buckling, it is clear that the beams would have supported greater weights if they had been supported laterally. That the want of such support was the cause of the weakness, is evidenced by the fact that the observed breaking weights more nearly approach the calculated weights for the shorter than for the longer spans. The probable real breaking weights average more than the weights, as calculated from the experimental data. Adapting the formula (19), page 511, by assuming the value of $s = 20$ tons, then—

Ultimate Transverse Strength of Solid Wrought-iron Joists of Uniform Symmetrical Section.

$$\text{In tons, } W = \frac{d''(4a + 1.155a'')}{0.6l} \quad \ldots\ldots (5)$$

$$\text{In cwts., } W = \frac{d''(4a + 1.155a'')}{0.03l} \quad \ldots\ldots (6)$$

W = the breaking weight at the middle.
a = the sectional area of the lower flange, in square inches.
a'' = the sectional area of the web, taken at the reputed depth, in square inches.
d'' = the reputed depth in inches; that is, the total depth minus the average thickness of one flange.
l = the span in feet.

Approximate Rules for the Strength of Solid Wrought-iron Joists of Ordinary Proportions.

Reduce the second coefficient in the numerator of the above formulas, to 1, and increase the depth to the total depth, d, of the beam.

1*st Approximate Rule. In Tons.*—To the sectional area of one flange add one-fourth of the sectional area of the web, calculated on the total depth, both in inches; multiply the sum by the depth in inches and by 7, and divide by the span in feet. The quotient is the breaking weight at the middle.

In Hundredweights.—Substitute 133 for the multiplier 7 in the preceding calculation.

The second last column in table No. 230 was calculated by this rule. The formulas are—

$$\text{In tons, } W = \frac{7d(a + \frac{1}{4}a'')}{l} \quad \ldots\ldots (7)$$

$$\text{In cwts., } W = \frac{133d(a + \frac{1}{4}a'')}{l} \quad \ldots\ldots (8)$$

2*d Approximate Rule. In Tons.*—Multiply the breadth of the joist by the square of the depth in inches, and by 0.6; and divide the product by the span in feet. The quotient, plus 1, is the breaking weight at the middle.

In Hundredweights.—Substitute 12 for the multiplier 0.6 in the preceding calculation. The quotient, plus 20, is the breaking weight.

The formulas are:—

$$\text{In tons, } W = \frac{0.6bd^2}{l} + 1 \quad \ldots\ldots (9)$$

$$\text{In cwts., } W = \frac{12bd^2}{l} + 20 \quad \ldots\ldots (10)$$

W = the breaking weight at the middle, b = the breadth, and d = the depth in inches; l = the span in feet.

Note.—The 1st approximate rule is better than the 2d rule.

Deflection and Elastic Strength of Solid Wrought-iron Flanged Beams or Joists of Uniform Symmetrical Section.

With the particulars already given for the beams A, B, D, and E, page 655, and the subjoined deflections under given loads at the middle, within the elastic limits, the values of the coefficient of elasticity, E, calculated from the general formula (13), page 531, by inversion, are as follows:—

Beam.	Span.	Load at the Middle.	Deflection.	E.
	feet.	tons.	inches.	
A	20	8.929	.848	13,196
	10	8.929	.132	10,588
B	20	7.143	1.150	13,100
	10	14.286	.330	11,414
D	20	3.571	1.420	12,120
	10	8.929	.440	12,232
E	10	5.357	.405	13,692
	5	8.929	.158	7,314
Average coefficient of elasticity, E, for 7 beams, excluding the last, as exceptional...........				12,334

To adapt the general formula just named, the constant becomes $4 \times 12{,}334 = 49{,}336$; or $(49336 \div 12^3 =)$ 28.5, when the span is in feet:—

Deflection of Solid Rolled Wrought-iron Flanged Beams of Uniform Symmetrical Section, loaded at the Middle.

$$D = \frac{W l^3}{28.5\, d''^2 (4a + 1.155\, a'')} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

D = the deflection in inches, W = the load in tons, l the span in feet, d'' the reputed depth in inches, being the whole depth minus the thickness of the lower flange; a the sectional area of the lower flange, and a'' the sectional area of the web reckoned on the reputed depth, both in square inches.

Note.—If the same weight be uniformly distributed, the divisor 45.7 is to be used.

Approximate Rule.

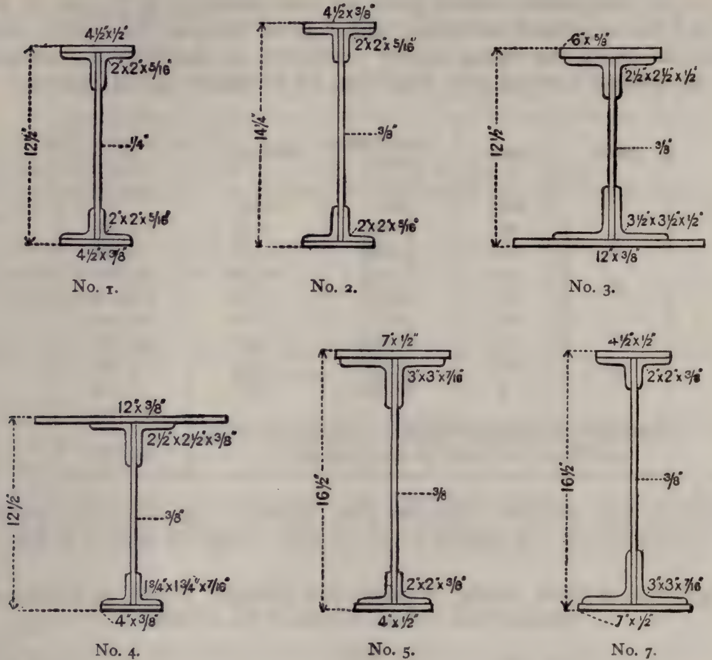
Load at the Middle.—To the sectional area of one flange add one-fourth of the sectional area of the web, calculated on the total depth, both in inches; multiply the sum by the square of the depth in inches, and by 114, making a product A. Multiply the load in tons by the cube of the span in feet; and divide this product by the product A. The quotient is the deflection in inches.

Load Uniformly Distributed.—Use the multiplier 183 in the calculation, instead of 114.

Strength of Rivetted Wrought-iron Joists.

Compared with solid-rolled joists, the strength of rivetted joists is less, and the deflection is greater. A series of rivetted plate-joists of uniform

section were constructed and tested for deflection by the late Mr. Thomas Davies, in 1856.[1] The sections of these beams are shown by Figs. 263–268;



Figs. 263–268.—Sections of Rivetted Wrought-iron Joists.

they consist of plate-webs and flanges, united by angle-irons, of which the scantlings are given on the figures,—

No. of joist,.......	1.	2.	3.	4.	5.	7.
Weight of joist,...	4.25 ...	6.5 ...	20.5 ...	14.75 ...	13.62 ...	13.62 cwts.
Span for trial,.....	11.66 ...	16.5 ...	28.5 ...	28.5 ...	22.5 ...	22.5 feet.

The loads rested on spaces at the middle of the beams, 21 inches wide.

The elastic strengths and deflections of the joists, as deduced from the record of the results, were as follows:—

Elastic strength,....	11¾ ...	11½ ...	10¾ ...	7 ...	13 ...	13 tons.
Deflection,...........	.437 ...	.625 ...	2.000 ...	1.620 ...	.875 ...	.875 inches.

All the beams except No. 2 were unsymmetrical, and an approximate rule for strength and deflection may be constructed, by making a calculation for each beam in the position in which it was tested, and in an inverted position, in terms of the flange and angle-irons which are undermost, in each position respectively; and finding the mean results. In this way the

[1] See a paper read at a meeting of the Edinburgh Architectural Institute, February 18, 1856.

breaking weights, columns 2 and 3 in the following tablet, were arrived at for each joist, applying the formula (5), page 656, for solid-rolled joists. Two-thirds of these mean calculated breaking weights are given in the 4th column; and they are probably the real breaking weights, since they average exactly twice the observed elastic strength given in the last column.

Joists.	Calculated Breaking Weight.	Mean Breaking Weight.	Two-thirds of the Mean.	Observed Elastic Strength.
	tons.	tons.	tons.	tons.
No. 1,	34.52			
Do. inverted,	38.65	36.59	24.4	11¾
No. 2,		31.80	21.2	11½
No. 3,	37.36			
Do. inverted,	28.92	33.14	22.1	10¾
No. 4,	13.74			
Do. inverted,	27.03	20.40	13.6	7
No. 5,	33.47	42.53	28.4	13
No. 7 (No. 5 inverted),	51.59			13
Averages,	—	—	22.0	11.0

By an appropriate alteration of the coefficient in the rule for solid-rolled joists, therefore, the following rule is obtained:—

Approximate Rules for the Strength of Rivetted Wrought-iron Joists.

In tons.—Find the sectional areas of the upper and the lower flanges with their angle-irons respectively; to half the sum of these areas add one-fourth of the sectional area of the web, calculated on the total depth, all in inches; multiply this last sum by the depth in inches, and by 4⅔; and divide by the span in feet. The quotient is the breaking weight at the middle.

In hundredweights.—Substitute 94 for the multiplier 4⅔ in the preceding operation.

The formulas are:—

In tons, ... $$W = \frac{4\tfrac{2}{3}\, d(a' + \tfrac{1}{4}a'')}{l} \qquad (12)$$

In cwts., ... $$W = \frac{94\, d(a' + \tfrac{1}{4}a'')}{l} \qquad (13)$$

in which a' is half the sum of the upper and lower sectional areas, a'' the sectional area of the web, d the depth, l the span, and W the load at the middle.

Note to the rule.—If the beam is symmetrical in section, the section for one flange only is calculated.

Similarly, let the deflections for the elastic strengths, for each beam in its first position, and as inverted, be calculated by the formula (11), page 657, for solid-rolled joists. They are given in the 2d column of the

following tablet, and the mean for each is given in the 3d column. The actual deflections, in the 4th column, are greater than those in the 3d column, in the ratios indicated in the last column.

Joists.	Calculated Deflections.	Mean Calculated Deflections.	Actual Deflection.	Ratio of Actual to Calculated Deflections.
	inches.	inches.	inches.	ratio.
No. 1,	.227			
Do. inverted,	.203	.215	.437	1 to 2.033
No. 2,		.418	.625	1 to 1.495
No. 3,	1.148			
Do. inverted,	1.483	1.316	2.000	1 to 1.520
No. 4,	2.130			
Do. inverted,	1.083	1.606	1.620	1 to 1.009
No. 5,	.730	.654	.875	1 to 1.338
No. 7 (No. 5 inverted),	.578		.875	1 to 1.338
Average ratio, not including No. 7,				1 to 1.479

There is considerable variation in the ratio of the calculated to the actual deflections; the average is 1 to 1½. Modify accordingly the coefficient in the approximate rule for solid-rolled joists, page 657:—

Approximate Rule for the Deflection of Rivetted Wrought-Iron Joists.

Load at the middle.—Find the sectional areas of the upper and the lower flanges with their angle-irons respectively; to half the sum of these areas add one-fourth of the sectional area of the web, calculated on the total depth, all in inches; multiply this last sum by the square of the depth in inches, and by 75, making a product A. Multiply the load in tons by the cube of the span in feet; and divide this product by the product A. The quotient is the deflection, in inches.

Load uniformly distributed.—Use the multiplier 120 in the calculation, instead of 75.

Note to the Rule.—If there be no flanges, the angle-irons alone are to be taken as representing flange-area.

Remarks.—1. The experimental elastic strengths, as well as the deflections, of Nos. 5 and 7 beams, which were in fact the same beam in reverse positions, are identical. The identity here observed is confirmatory of the general principle of the elasticity of beams, enunciated at page 517.

2. It follows, from experimental tests, that the strength of solid-rolled joists is to that of rivetted joists, of equal weights, as 1½ to 1; and that their deflections, under equal loads, are as 1 to 1½.

BUCKLED IRON PLATES.

Buckled plates, so named by Mr. Mallett, the inventor, are bulged plates, which are curved or arched, with a very small rise or curvature, springing from the edges of the plate, a narrow strip of which, all round,

is retained in the original plane of the plate. Buckled plates are very rigid, and are capable of sustaining heavy loads. When bolted down, or rivetted all round the edges, they offer twice the resistance that they do if simply supported; and if two opposite sides be wholly unsupported, the resistance is only 5/8 ths. Less than 2 inches of rise is sufficient for a ¼-inch plate, 4 feet square. A ¼-inch Staffordshire plate, 3 feet square, with a 2-inch flat border, buckled with a rise of 1¾ inches, is crippled with a load of 9 tons distributed over half the surface; if rivetted down, 18 tons are required to cripple it. Plates of soft puddled steel bear twice these loads before being crippled. The strength appears to increase as the square of the thickness. The factor of safety adopted by Mr. Mallet is 4 for steady loads, and 6 for moving loads.

RAILWAY RAILS.

Rails of Symmetrical Section.

These are beams of limited depth and considerable flange-area, and the strength should be calculated by formula (22), page 512, repeated below; for the application of which the section of a double-headed rail is to be divided for the calculation, according to the annexed diagram, Fig. 269. *a*, *a*, *a*, *a*, are the flange portions, *c d* the web, *d* the depth of the rail, and d'' the vertical distance apart of the centres of the flanges. That the sectional area of the flange may be correctly ascertained,

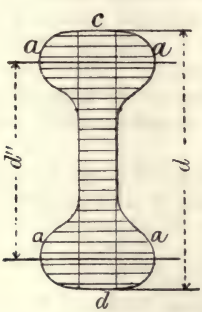


Fig. 269.—For Transverse Strength of Rail of Symmetrical Section.

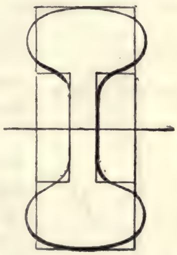


Fig. 270.—Squaring the Section of a Double-headed Rail.

the surface should be divided into thin strips parallel to the neutral axis, as in the diagram, the area of each of which should be calculated. If the outer contour of the flange is circular, as is usually the case, the resultant centre of the flange *a a* may be taken as passing through the centre of the circle. If the flanges are otherwise formed, the position of their centre of gravity, ascertained by the rule, page 514, may be taken. Approximate results may be obtained by squaring the section of the flanges, by the eye, and calculating from the centres of the rectangles, as in Fig. 270.

$$W = \frac{s\left(4\,a'\,\dfrac{d''^2}{d} + 1.155\,t'd^2\right)}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$s = \frac{W\,l}{\left(4\,a'\,\dfrac{d''^2}{d} + 1.155\,t'\,d^2\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

W = the breaking weight at the middle, in tons.
a' = the net sectional area of one flange, in inches (excluding the central portion pertaining to the web).
d = the total depth of the rail, in inches.
d'' = the vertical distance apart of the centres of the flanges.
t' = the thickness of the web.
l = the length of span between the supports, in inches.
s = the ultimate tensile strength, in tons per square inch.

RULE 1.—*To find the Ultimate Transverse Strength of a rail of symmetrical section.* Multiply the sectional area of the flange portion of one head by the square of the vertical distance apart of the centres of the heads, and by 4; and divide by the depth of the rail [B]. Multiply the thickness of the web by the square of the depth of the rail, and by 1.155 [C]. Multiply the sum of the quotient B and the product C by the ultimate tensile strength, and divide by the span. The last quotient is the breaking weight at the middle.

RULE 2.—*To find the Ultimate Tensile Strength of a rail of symmetrical section.* Multiply the breaking weight at the middle by the span, and divide the product by the sum of the quotient B and the product C described in Rule 1. The last quotient is the ultimate tensile strength.

Mr. R. Price Williams, in a paper of exceptional value,[1] gives a number of tests of the ultimate transverse strength of iron and steel rails, made for him by Mr. Kirkaldy. From this paper, the following data, collected in table No. 232, are derived for several double-headed rails of symmetrical section.

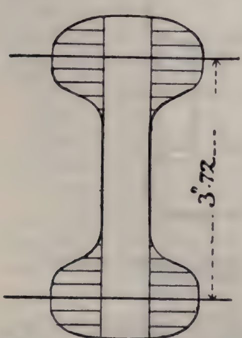


Fig. 271.—Section of Steel Rail for the Great Indian Peninsular Railway. Scale, one-third.

The tensile strengths of these rails, in the last column, are calculated by Rule 2 above. There is no information as to the observed tensile strength of the rails; but, in the course of discussion, Mr. Berkley gives the tensile strength of steel rails tested by him, varying from 40 to 50 tons per square inch; and the mean of these strengths is the same as the average of the strengths for steel rails calculated in the last column.

Mr. Baker gives a full-size section, with the breaking weight, for double-headed steel rails, manufactured by Sir John Brown & Co. for the Great Indian Peninsula Railway, weighing 68 lbs. per yard.[2] The section, Fig. 271, has an area of 6.88 square inches, of

[1] "On the Maintenance and Renewal of Permanent Way," in the *Proceedings of the Institution of Civil Engineers*, 1865–66, vol. xxv., page 353.
[2] *The Strength of Beams*, page 86.

which the flange-area at the bottom is 1.667 inches. The web is .70 inch thick, the total depth 5 inches, and the vertical distance of the centres of gravity of the heads 3.72 inches. The ultimate tensile strength varied from

Table No. 232.—TRANSVERSE STRENGTH OF IRON AND STEEL DOUBLE-HEADED RAILS. 1866.

Span 60 inches. Load applied at the middle.

(Deduced from Mr. Price Williams' data).

DESCRIPTION.	Weight per Yard (estimated).	DEPTH.		Thickness of Web.	SECTIONAL AREA.	
		Total.	Centres of Flanges.		One Flange.	Total.
IRON RAILS.	lbs.	inches.	inches.	inches.	sq. inches.	sq. inches.
L L. & N. W. Ry.	82	5.40	4.20	.82	1.911	8.25
N Do.	82	5.40	4.00	.82	1.931	8.29
M Ebbw Vale Co.	82	5.25	3.90	.78	2.037	8.17
P Do.	68	5.04	3.75	.68	1.70	6.83
STEEL RAILS.						
A Crewe	78	5.40	4.20	.75	1.81	7.67
B Brown & Co.	79	5.22	3.72	.75	1.902	7.72
C Bessemer	74	5.20	3.90	.74	1.701	7.25
D Cammell	72.5	5.02	3.82	.73	1.722	7.11

DESCRIPTION.	Breaking Weight.	ELASTIC STRENGTH.		Elastic Deflection.	Nature of Failure.	Calculated Tensile Strength per square inch.
IRON RAILS.	tons.	tons.	per cent.	inches.		tons.
L L. & N. W. R.	23.000	11.604	50.4	.198	cracked, & canted	26.24
N Do.	21.107	9.823	46.5	.184	canted	25.07
M Ebbw Vale.....	22.371	10.716	47.9	.235	snapped	27.71
P Do.	15.500	6.698	43.2	.148	torn	24.06
STEEL RAILS.						
A Crewe	35.793	16.070	44.9	.252	snapped	43.91
B Brown & Co....	35.432	15.181	42.9	.264	cracked, & canted	45.75
C Bessemer........	33.446	16.967	50.7	.300	canted	46.67
D Cammell........	31.935	16.070	50.3	.322	snapped	46.43

37 to 50 tons in nine specimens, averaging 45 tons per square inch. The breaking weight at the middle, on a span of 43.5 inches, averaged, in forty-five experiments, 38.7 tons. Applying Rule 1 above, the calculated breaking weight is,

$$\left[\left(4 \times \frac{3.72^2}{5} \times 1.667\right) + \left(1.155 \times .70 \times 5^2\right)\right] \times \frac{45}{43.5} =$$

$$(18.45 + 20.21) \times 45 \div 43.5 = 40 \text{ tons.}$$

This weight is 1.3 tons more than the average given by experiment; and it is as near as can reasonably be expected, considering the variable elements of the data. But, even this small excess may be explained away; for a sectional area of 6.88 square inches, at 10.2 lbs. per square inch, gives a weight of 70 lbs. per yard, whilst the actual weight was 68 lbs. It is, therefore, probable that the section was less than 6.88 square inches, and that the strength, if it were calculated from the exact section, would be less than 40 tons in the ratio of 70 to 68, or about $(40 \times \frac{68}{70} =)$ 38.8 tons, which is virtually identical with the experimental breaking weight.[1]

Strength of Double-headed Bessemer Steel Rails Relatively to the Proportion of Constituent Carbon.

Mr. Price Williams gives, in an appendix to his paper, already mentioned, a table showing the strength and deflection and set of double-headed steel rails, of 86 lbs. per yard, 5½ inches deep, manufactured by Sir John Brown & Co. for the Great Indian Peninsula Railway. The sectional area was probably 8.43 square inches. The experiments were made, under Mr. Berkley's direction, with rails containing various proportions of carbon. The following table is reduced from Mr. Price Williams' table, and it shows a notable correspondence between the percentages of constituent carbon and the breaking load applied at the middle; for whilst the carbons increase from .40 to .55 per cent., the ultimate loads increase from 40 to 52½ tons.

Table No. 233.—Transverse Strength of Double-headed Bessemer Steel Rails. 1864.

Span 43.5 inches. Load applied at the middle.

(To show the influence of the constituent carbon on the strength.)

Constituent Carbon.	Ultimate Strength.			Elastic Strength. (Deduced from the experimental data.)			
	Load.	Deflection.	Set.	Load.		Deflection.	Set.
per cent.	tons.	inches.	inches.	tons.	per cent.	inches.	inches.
.40	40	3.94	3.74	15	37.5	.10	.01
.46	40	2.64	2.34	20	50	.14	.05
.49	50	4.18	3.77	22.5	45	.165	.03
.50	52.5	4.68	4.28	22.5	43	.130	.01
.55	52.5	4.40	4.02	25	48	.165	.04

Thirty Bessemer-steel rails, manufactured at Barrow-in-Furness, were analyzed and tested in different ways for strength. The tensile strength increased generally with the proportion of carbon in the steel, as may be seen from the following abstract for thirty rails, from a table given by Mr. J. T. Smith:[2]—

[1] Mr. Baker arrives at a breaking weight of 38.9 tons by means of the ingenious diagrammatic reduction already noticed.

[2] "On Bessemer Steel Rails," in the *Proceedings of the Institution of Civil Engineers*, 1874–75, vol. xlii., page 74.

Soft Rails. Carbon. per cent.	Tensile Strength per square inch. tons.	Hard Rails. Carbon. per cent.	Tensile Strength per square inch. tons.
.28	30.90	.36	37.01
.29	32.60	.39	41.41
.30	32.94	.40	37.68
.31	32.67	.43	39.10
.32	33.04	.44	41.02
		.45	44.00
		.50	45.79
		.57	50.42
Averages .30	32.43	.44	42.05

Rails of Unsymmetrical Section.

The general rule at page 517, is applicable for the calculation of the transverse strength of bridge-rails and flange-rails, which are the only varieties of rails that are not symmetrical in section. That rule embodies the final calculation formulated in equation (25), page 516, in terms of the total tensile resistance of the section, and the vertical distance apart of the centres of tension and compression. From that equation, it follows that $W\,l = 4\,S\,d_3$; and, as S is, by the sixth step of the rule, page 517, equal to $1.73\,s \times$ (sum of 1st products, tensional) $\div h_1$, in which h_1 is the height of the neutral axis above the base, by substitution, $W\,l = \dfrac{4 \times 1.73\,s \times (\text{sum of 1st products}) \times d_3}{h_1}$; and, putting A = the sum of the 1st products,—

$$W = \frac{6.92\,s\,d_3\,A}{l\,h_1} \quad \text{.........................} \quad (3)$$

$$s = \frac{W\,l\,h_1}{6.92\,d_3\,A} \quad \text{.........................} \quad (4)$$

W = the breaking weight at the centre, in tons.

s = the ultimate tensile strength of the metal, in tons per square inch.

d_3 = the vertical distance apart of the centres of tension and compression, in inches.

h_1 = the height of the neutral axis above the base of the section, in inches.

l = the span, in inches.

A = the sum of the products obtained by multiplying the areas of the strips of the reduced section under tensional stress, by their mean distances respectively from the neutral axis, in inches, as described in step 4 of the rule, page 517.

Rule 3.—*To find the Ultimate Transverse Strength of a Rail of Unsymmetrical Section.* 1. Divide the section into strips, which may be of equal thickness, parallel to the base. 2. Reduce the width of the flange portions in the ratio of 1.73 to 1. 3. Find the position of the centre of gravity of the section as thus reduced (by the rule, page 514); it is that of the neutral axis of the total section. 4. Multiply the areas of the strips of the reduced section, below the neutral axis, by their respective mean distances from it; and also by the squares of these distances; and divide the sum of these second products by the sum [A] of the first products; the quotient

is the distance of the position of the resultant centre of resistance below the neutral axis. 5. Make the same calculation (4) to find the position of the resultant centre above the neutral axis. 6. The sum of the two distances thus found, is the vertical distance apart of the centres of tension and compression. 7. Multiply the sum A by the distance apart of the centres of stress, and by the ultimate tensile strength in tons per square inch, and by 6.92; and divide the product by the height of the neutral axis above the base of the section, and by the span. The quotient is the breaking weight in tons at the middle.

RULE 4.—*To find the Ultimate Tensile Strength of a Rail of Unsymmetrical Section, when the Transverse Strength is given.* Perform the same preliminary operations as for Rule 3—Nos. 1, 2, 3, 4, 5, and 6; then, 7, multiply the breaking weight in tons at the middle by the length of the span, and by the height of the neutral axis above the base of the section; and divide the product by the sum A (referred to in Rule 3), and by the distance apart of the centres of stress, and by 6.92. The quotient is the ultimate tensile strength in tons per square inch.

Note.—The dimensions are in inches, and the pressures and weights in tons.

Steel Flange-Rails.—The steel rails designed by Mr. John Fowler, and manufactured by the Dowlais Iron and Steel Company, for the Metropolitan Railway (Fig. 272), are 4½ inches high and 6⅜ inches wide at the base; they have a sectional area of 8.24 square inches, and weigh 84 lbs. per yard. Several of these rails were tested by Mr. Kirkaldy: in which the web was .65 inch thick, the head 2.5 inches wide, and the flange 6.4 inches wide. The thickness of the flanges was .65 inch near the web, and .37 inch near the edge. The ultimate tensile strength is said by Mr. Baker to be 35 tons per square inch.

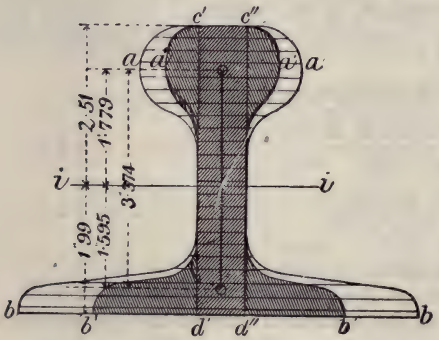


Fig. 272.—Section of Steel Flange-Rail for the Metropolitan Railway.

To apply the rule for the transverse strength, produce the sides of the web to the top and the bottom of the section, at c' c'' and d' d''; and reduce the width of the flange portions, $a\,a$ and $b\,b$, in the ratio of 1.73 to 1, following the contour-lines a' a' and b' b'. Divide the section into strips, say .20 inch in width, and find the centre of gravity of the reduced section; the line $i\,i$, passing through it, is the neutral axis, 2.51 inches below the top, and 1.99 inches above the bottom. The resultant centres of resistance are 1.595 inches above and 1.779 inches below the neutral axis; and their distance apart is (1.595 + 1.779 =) 3.374 inches.

To find the total stress in tension, in the lower part of the section, the sum of the first products, 4.324 (which is the same for tension and compression), is multiplied by 1.73 times 35 tons, the ultimate tensile strength, and divided by 1.99 inches, the distance of the neutral axis from the base:—

$$\frac{4.324 \times (35 \times 1.73)}{1.99} = 131.56 \text{ tons, total tensile resistance.}$$

The breaking weight at the middle, on a span of 60 inches, is, then,

$$W = \frac{131.56 \times 3.374 \times 4}{60} = 29.59 \text{ tons.}$$

This, it may be noted, is an application of formula (25), page 516.

Let the Metropolitan rail be calculated for its transverse strength when upside down: the product $4.324 \times (35 \times 1.73)$ is divided by 2.51 inches, the distance of the upper surface, now downwards, of the head of the rail from the neutral axis:—

$$\frac{4.324 \times (35 \times 1.73)}{2.51} = 104.30 \text{ tons, total tensile resistance upside down;}$$

and the breaking weight at the middle, on a span of 60 inches, is

$$W = \frac{104.30 \times 3.374 \times 4}{60} = 23.46 \text{ tons.}$$

To compare these calculated results with the results of Mr. Kirkaldy's experimental tests, these are, with Mr. Fowler's permission, here abstracted:—

Table No. 234.—TRANSVERSE STRENGTH OF STEEL FLANGE-RAILS FOR THE METROPOLITAN RAILWAY. 1867.

Span 60 inches. Load applied at the middle.

(Reduced from Mr. Kirkaldy's Reports.)

	Ultimate Strength.		Elastic Strength.				Elastic Deflection per ton.
	Load.	Deflection.	Load.		Deflection.	Set.	
	tons.	inches.	tons.	per cnt.	inches.	inches.	inches.
Solid rail, normal position—							
1st specimen	30.393	9.42	12.500		.255	.022	
2d do.	29.632	8.69	11.607		.232	.028	
3d do.	29.043	10.54	11.607		.238	.030	
4th do.	28.457	9.18	12.500		.250	.021	
5th do.	28.733	14.78	11.607		.232	.030	
Averages...............	29.270	10.52	11.964	41	.2414	.026	.0202
Solid rail, inverted..........	22.014	5.42	15.180	69	.290	.040	.0191
Normal position, holes punched in the flanges. Average of six specimens	15.618	.669	11.904	76	.235	.023	.0197
Normal position, holes drilled. Mean of two specimens	23.934	4.51	12.500	52	.267	.025	.0214

Note.—The holes, punched or drilled, were 1.10 inches in diameter, .75 inch from the edge of the flange.

The breaking weight of the rail varied, in six specimens, from 28.46 tons to 30.39 tons; average, 29.27 tons. The calculated breaking weight is 29.59 tons, or about ⅓ ton in excess of the average.

Inverted, one specimen broke with 22.01 tons; the calculated breaking weight is 23.46 tons, or nearly 1½ tons more. The elastic strength in this position was greater than in the normal position.

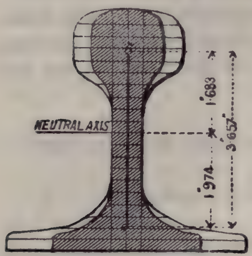


Fig. 273.—Section of Iron Flange-Rail. Scale, ¼th.

Influence of Holes in the Flange.—When punched, the effect was to reduce the breaking weight nearly a half. When drilled, the ultimate strength was only reduced about a sixth. But the elastic strength remained, in both cases, unimpaired; and the elastic deflection per ton was practically identical in all cases—averaging about .20 inch per ton.

Wrought-Iron Flange-Rails.—The annexed section, Fig. 273, shows a wrought-iron flange-rail, 5 inches high, weighing 75 lbs. per yard. Ten specimens of rail of this section, of Cleveland manufacture, were tested for transverse, tensile, and compressive strength by Mr. Kirkaldy. The samples for the tensile and compressive tests were cut from the middle of the head and of the flange.

		Tensile.	Compressive.
HEAD:—	Elastic strength per square inch......	13.21 tons.	18.13 tons.
	Ultimate strength " "	20.93 "	67.00 "
FLANGE:—	Elastic strength per square inch....	13.62 "	—
	Ultimate strength " "	21.83 "	—

Ultimate transverse strength, span 3 feet......33.60 tons.

The centre of gravity of the reduced section, that is, the neutral axis of the entire section, shown in the Fig. 273, is 2½ inches above the base of the section, or the height is one-half the total height of the rail. The resultant centre of tensile stress is 1.974 inches below the neutral axis, and that of compressive stress is 1.683 inches above it, as indicated. The vertical distance apart of these resultant centres is (1.683 + 1.974 =) 3.657 inches, and by Rule 4,

$$\frac{33.6 \times 36 \times 2.5}{6.92 \times 3.657 \times 5.058} = 23.62 \text{ tons per square inch,}$$

the ultimate tensile strength of the wrought-iron flange-rail, in its lower or flange portion.

DEFLECTION OF RAILS.

Double-headed Rails.—Formulas for the deflection of double-headed rails are deduced by equating the values of s, the tensile strength per square inch, given by formula (2), page 662, and by formula (2), page 528; thus:—

$$\frac{W\,l}{\left(4\,a'\dfrac{d''^2}{d} + 1.155\,t'\,d^2\right)} = \frac{4\,d\,E\,D}{l^2};\ \text{whence,}$$

$$W\,l^3 = 4\,d\,E\,D\left(4\,a'\,\frac{d''^2}{d} + 1.155\,t'\,d^2\right) = 4\,E\,D\,(4\,a'\,d''^2 + 1.155\,t'\,d^3).$$

From this equation, the following values of D and E are deduced:—

$$D = \frac{W\, l^3}{4\,E\,(4\,a'\,d''^2 + 1.155\,t'\,d^3)} \quad \text{.................} \quad (5)$$

$$E = \frac{W\, l^3}{4\,D\,(4\,a'\,d''^2 + 1.155\,t'\,d^3)} \quad \text{.................} \quad (6)$$

The values of E, by formula (6), for the rails tested by Mr. Price Williams, as detailed in table No. 232, page 663, are as follows:—

Iron Rails, double-headed.	E.		Steel Rails, double-headed.	E.
L	11,146	....	A	13,038
N	10,571	....	B	13,588
M	9,683	....	C	12,982
P	12,457	....	D	13,007
Averages ...	10,964	....	....	13,154

That is, the iron rails were extended, say, $^1/_{11,000}$ of their length per ton of tensile stress per square inch of section; and the steel rails were extended, say, $^1/_{13,000}$ of their length per ton per square inch. It has already been deduced from direct experiments on the elongation of bars (see pages 623, 624), that the extension of iron was from $^1/_{10,000}$ to $^1/_{13,000}$ part of the length, and that of steel was $^1/_{13,000}$ part of the length, per ton per square inch. Thus, the deductions from experiment on transverse resistance, are strongly corroborated by the results of experiment on direct tensile resistance.

Substituting these values of E, in round numbers, in formula (5), the following formulas for the deflection of iron and steel rails, like those tested by Mr. Price Williams, are derived:—

Deflection of Double-headed Rails, within the Elastic Limit, Loaded at the Middle.

$$\text{Iron............}\quad D = \frac{W\, l^3}{44{,}000\,(4\,a'\,d''^2 + 1.155\,t'\,d^3)} \quad \text{............} \quad (7)$$

$$\text{Steel..........}\quad D = \frac{W\, l^3}{52{,}000\,(4\,a'\,d''^2 + 1.155\,t'\,d^3)} \quad \text{............} \quad (8)$$

D = the deflection at the middle, in inches.
W = the load at the middle, in tons.
a' = the net sectional area of one flange, in inches (excluding the central portion pertaining to the web).
d = the total depth of the rail, in inches.
d'' = the vertical distance apart of the centres of the flanges, in inches.
t' = the thickness of the web, in inches.
l = the length of span between the supports, in inches.

Flange-Rails.—By a similar process, equating the values of s, given by formula (4), page 665, and by formula (2), page 528, formulas are deducible for the deflection of flange-rails within elastic limits:—

$$\frac{W\,l\,h_1}{6.92\;d_3\,A} = \frac{4\,d\,E\,D}{l^2};\text{ and } W\,l^3\,h_1 = 6.92 \times 4 \times d\,d_3\,E\,D\,A;$$

whence the following values of D and E:—

$$D = \frac{W\,l^3\,h_1}{27.68\;d\,d_3\,E\,A}, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\,9\,)$$

$$E = \frac{W\,l^3\,h_1}{27.68\;d\,d_3\,D\,A}. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\,10\,)$$

To find the value of E by the formula (10), for Mr. Fowler's steel rail, investigated at page 666, for which the value of the quantity A is 6.983:—

$$E = \frac{11.964 \text{ tons} \times 60 \text{ inches}^3 \times 1.99 \text{ inches}}{27.68 \times 4.5 \text{ inches} \times 3.374 \text{ inches} \times .2414 \text{ inch} \times 6.983} = 7264.$$

That is to say, the flange of Mr. Fowler's steel rail is extended $^1/_{7264}$ part of its length per ton of tensile stress per square inch. This fraction is considerably greater than the fraction that was found for the double-headed steel rails tested by Mr. Price Williams, averaging $^1/_{13,000}$ part. The greater extensibility, nearly twice as much, is in accordance with the relative tensile strengths of the steels of which the different rails were made—35 tons per inch for the flange-rail, and 45 tons for the double-headed rails.

Substituting in formula (9), the value of E, just found, the formula is reduced to the following form for the deflection of steel flange-rails like Mr. Fowler's:—

Deflection of Steel Flange-Rails, within the Elastic Limit, Loaded at the Middle.

$$D = \frac{W\,l^3\,h_1}{200{,}000\;d\,d_3\,A} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\,11\,)$$

To find, in the absence of data, the probable numerical constant for the deflection of iron flange-rails, let the constant in this formula be reduced in the ratio of 52,000 to 44,000, the correlative constants for steel and iron, in formulas (7) and (8); or to $200{,}000 \times \frac{44}{52} = 170{,}000$:—

Deflection of Iron Flange-Rails, within the Elastic Limit, Loaded at the Middle.

$$D = \frac{W\,l^3\,h_1}{170{,}000\;d\,d_3\,A} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\,12\,)$$

D = the deflection at the middle, in inches.

W = the load at the middle, in tons.

h_1 = the height of the neutral axis of the reduced section, above the base of the section, in inches.

d = the total height of the rail, in inches.

d_3 = the vertical distance apart of the centres of tension and compression, in inches.

l = the span, in inches.

A = the sum of the products obtained according to Rule 3, page 665.

STEEL SPRINGS.

The author, in 1854–55, investigated the elastic strength of laminated springs, in his work on *Railway Machinery*,[1] and he deduced the following formulas for their elasticity and working strength:—

$$E = \frac{1.66\, l^3}{b\, t^3\, n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$S = \frac{b\, t^2\, n}{11.3\, l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

E = the elasticity, or deflection, in sixteenths of an inch per ton of load.
S = the working strength, or load, in tons.
l = the span when loaded, in inches.
b = the breadth of plates in inches, supposed uniform.
t = the thickness of plates in sixteenths of an inch.
n = the number of plates.

Rules for the Elasticity of Laminated Springs.

Rule 1.—*To find the elasticity of a laminated spring.* Multiply the breadth in inches by the cube of the thickness of each plate in sixteenths of an inch, and by the number of plates; multiply the cube of the span in inches by 1.66. Divide the second product by the first. The quotient is the elasticity in sixteenths of an inch per ton of load.

Rule 2.—*To find the span due to a given elasticity, and number and size of plates.* Multiply the elasticity by the breadth in inches, and by the cube of the thickness in sixteenths, and by the number of plates; and divide by 1.66. Find the cube root of the quotient. The result is the span in inches.

Rule 3.—*To find the number of plates due to a given elasticity, span, and size of plate.* Multiply the cube of the span in inches by 1.66. Multiply the elasticity by the breadth of plate in inches, and by the cube of the thickness in inches. Divide the first product by the second. The quotient is the number of plates.

Note 1.—The span and the elasticity are those due to the spring *when weighted.*

Note 2.—When extra thick back and short plates are used, they must be replaced by an equivalent number of plates of the ruling thickness, prior to the application of Rules 1 and 2. This is found by multiplying the number of extra-thick plates by the cube of their thickness, and dividing by the cube of the ruling thickness. Conversely, the number of plates of the ruling thickness given by Rule 3, required to be removed and replaced by a given number of extra-thick plates, are found by the same calculation.

Note 3.—It is assumed that the plates are similarly and regularly formed, and that they are of uniform width, and but slightly tapered at the ends.

Rules for the Working Strength of Springs.

Rule 4.—*To find the working strength of a laminated spring.* Multiply the breadth of plates in inches by the square of the thickness in sixteenths, and

[1] *Railway Machinery*, 1855, page 242. Also, *Railway Locomotives*, 1860.

by the number of plates; multiply the working span in inches by 11.3. Divide the first product by the second. The quotient is the working strength in tons of load.

RULE 5.—*To find the working span due to a given working strength, and number and size of plates.* Multiply the breadth of plate in inches by the square of the thickness in sixteenths, and by the number of plates; multiply the working strength in tons by 11.3. Divide the first product by the second. The quotient is the working span in inches.

RULE 6.—*To find the number of plates due to a given working strength, span, and size of plate.* Multiply the working strength in tons by the span in inches, and by 11.3; multiply the breadth of plate in inches by the square of the thickness in sixteenths. Divide the first product by the second. The quotient is the number of plates.

RULE 7.—*To find the required original compass of the spring.* Multiply the elasticity in sixteenths per ton by the working strength in tons, and add the product to the desired working compass. The sum is the whole original compass, to which an allowance of from ⅛ to 3/8 inch should be added, for the permanent setting of the spring.

Note 1.—The span is that due to the form of the spring when weighted.

Note 2.—Extra thick back or short plates must be replaced by an equivalent number of plates of the ruling thickness, before applying the Rules 4 and 5. To find this, multiply the number of extra-thick plates by the square of their thickness, and divide by the square of the ruling thickness. Conversely, the number of plates of the ruling thickness given by Rule 6, required to be removed and replaced by a given number of extra-thick plates, are found by the same calculation.

Helical Springs.—Most of the data on the strength of helical springs are indefinite and contradictory. It may be assumed that the elasticity, or deflection per unit of load, is as the fourth power of the diameter or of the side of the bar, if round or square, of which the spring is constructed; as the cube of the mean diameter of the coil or helix, as the number of free coils of the springs, and as the load applied. In the "Report on Safety Valves,"[1] the following formula is propounded:—

$$E = \frac{d^3 \times w}{D^4 \times C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

E = Compression or Extension of one coil, in inches.

d = diameter from centre to centre of steel bar composing the spring, in inches.

w = the weight applied, in pounds.

D = the diameter, or the side of square, of the steel bar of which the spring is made, in 16ths of an inch.

C = a constant which, from experiments made, may be taken as 22 for round steel, and 30 for square steel.

The deflection for one coil is to be multiplied by the number of free coils, to obtain the total deflection for a given spring.

[1] *Transactions of the Institution of Engineers and Ship-builders in Scotland*, 1874–75, page 39.

It is also stated in the Report that the relation between the safe load, size of steel, and diameter of coil, has been deduced from the works of Professor Rankine; and that it may be taken for practical purposes as follows:—

$$D = \sqrt[3]{\frac{w \times d}{3}}, \text{ for round steel,} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$D = \sqrt[3]{\frac{w \times d}{4.29}}, \text{ for square steel,} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

ROPES—HEMP AND WIRE.

Hempen Ropes.

By the old ropemakers' rule the breaking strength in hundredweights was equal to four times the square of the girth of the rope in inches. This is equivalent to Gregory's rule, by which the breaking strength in tons was equal to one-fifth of the square of the girth in inches. The square of half the girth represented the weight in pounds per fathom. The following table of the strength of cordage, is reduced from Mr. Glynn's[1] data. The ropes recorded in the second part of the table are machine-made ropes. Made by the warm register, the rope is stronger and more durable than by the cold register, as it is more thoroughly penetrated by the tar. But it is less pliable, and cold-register rope is now generally used for cranes, and block and tackle.

Table No. 235.—Breaking Strength of Tarred Hemp Ropes.

(Reduced from Mr. Glynn's data.)

Size of Rope.		Made by the Old Method.		Made by the Register.	
Girth.	Diameter.	Common Staple Hemp.	Best Petersburg Hemp.	Cold Register.	Warm Register.
inches.	inches.	tons.	tons.	tons.	tons.
3	.95	2.22	2.70	3.30	3.85
3½	1.11	3.33	3.87	5.00	5.25
4	1.27	3.92	4.67	5.85	6.85
4½	1.43	4.60	5.55	7.29	8.68
5	1.59	5.95	7.08	9.15	10.71
5½	1.75	6.90	8.31	11.07	13.00
6	1.91	8.10	9.65	10.94	14.80
6½	2.07	9.16	10.54	15.46	18.10
7	2.24	10.24	12.26	18.00	21.00
7½	2.39	11.15	13.73	20.60	24.10
8	2.54	12.00	14.30	23.43	27.42

Specimens of white 2-inch rope, exhibited at Kew Gardens, bore the following breaking weights:—

[1] *On the Construction of Cranes and Machinery*, page 94.

2-inch Neapolitan,................ 2.75 tons, breaking weight.
" Konigsberg,................ 1.97 " "
" French,.................... 2.17 " "
" St. Petersburg,............ 2.17 " "
" Italian, ... 2.32 " "

Specimens of rope supplied by the National Association of Rope and Twine Spinners, were tested by Mr. Kirkaldy.

Rope.	Circumference.	Weight per Fathom.	Ultimate Strength.	
			Total.	Per Lb.-w'ght per Fathom.
	inches.	pounds.	tons.	tons.
Russian rope, 48 threads,........	5.26	5.74	4.95	.863
Machine yarn, 50 "	5.07	5.35	5.14	.961
Hand-spun yarn, 51 "	5.39	6.04	8.16	1.350

	Extension in 50 inches Length. Stress per Pound-weight per Fathom.					
	500 lbs.	1000 lbs.	1500 lbs.	2000 lbs.	2500 lbs.	3000 lbs.
	inches.	inches.	inches.	inches.	inches.	inches.
Russian rope,.........	3.38	5.29	6.59	—	—	—
Machine yarn,........	3.25	4.53	5.56	6.56	—	—
Hand-spun yarn,.....	3.26	4.46	5.29	5.91	6.35	6.63

The bearing capacity of a hemp rope is proportional to its thickness, the number of its strands, the slackness with which they are twisted, and the quality of the hemp. Karl von Ott states that ropes 0.866 inch in diameter, whose threads had been shortened by twisting $^1/_5$ th, ¼th, and ⅓d of their original length, broke respectively with loads of 6834 lbs., 5335 lbs., and 4519 lbs. He adds that the ultimate strength of ropes, according as they are wetted, or tarred, or dry, usually varies between 7000 lbs. and 11,400 lbs. per square inch, and that a working strength of one-sixth, say 1422 lbs., or 0.63 ton per square inch may be adopted.

Wire Ropes.

The strength of wire ropes of iron and steel, manufactured by Messrs. R. S. Newall & Co., is given in table No. 236, together with that of hemp rope, for comparison. From the table, the following data are derived:—

1. The Breaking Strength is—

About 1 ton per lb. weight per fathom for round hemp rope.
2 " " " " iron "
3 to 3½ " " " steel "

2. The Working Load is—

3 cwts. per lb. weight per fathom for round hemp rope.
6 " " " " iron "
10 " " " " steel "

3. The Working Load in Cwts. is—

5/6ths of the square of the circumference in inches, for round hemp rope.
About 5 times " " " " " iron "
9 times " " " " " steel "

Table No. 236.—Strength of Ropes—Hemp, Iron, Steel.

(Messrs. R. S. Newall & Co.)

1. Round Ropes—for Inclined Planes, Mines, Collieries, Ships' Standing Rigging, &c.

Hemp.		Iron.		Steel.		Tensile Strength.	
Circumference.	Weight per Fathom.	Circumference.	Weight per Fathom.	Circumference.	Weight per Fathom.	Working Load.	Ultimate Strength.
inches.	pounds.	inches.	pounds.	inches.	pounds.	cwts.	tons.
2¾	2	1	1	...	...	6	2
		1½	1½	1	1	9	3
3¾	4	1⅝	2	...	...	12	4
		1¾	2½	1½	1½	15	5
4½	5	1⅞	3	...	...	18	6
		2	3½	1⅝	2	21	7
5½	7	2⅛	4	1¾	2½	24	8
		2¼	4½			27	9
6	9	2⅜	5	1⅞	3	30	10
		2½	5½			33	11
6½	10	2⅝	6	2	3½	36	12
		2¾	6½	2⅛	4	39	13
7	12	2⅞	7	2¼	4½	42	14
		3	7½			45	15
7½	14	3⅛	8	2⅜	5	48	16
		3¼	8½			51	17
8	16	3⅜	9	2½	5½	54	18
		3½	10	2⅝	6	60	20
8½	18	3⅝	11	2¾	6½	66	22
		3¾	12			72	24
9½	22	3⅞	13	3¼	8	78	26
10	26	4	14			84	28
...	...	4¼	15	3⅜	9	90	30
11	30	4⅜	16			96	32
...	...	4½	18	3½	10	108	36
12	34	4⅝	20	3¾	12	120	40
2. Flat Ropes.—For Pits, Hoists, &c. &c.							
4 × 1⅛	20	2¼ × ½	11	...	...	44	20
5 × 1¼	24	2½ × „	13			52	23
5½ × 1⅜	26	2¾ × ⅝	15	...	...	60	27
5¾ × 1½	28	3 × „	16	2 × ½	10	64	28
6 × 1½	30	3¼ × „	18	2¼ × ½	11	72	32
7 × 1⅞	36	3½ × „	20	„ × „	12	80	36
8¼ × 2⅛	40	3¾ × $\frac{11}{16}$	22	2½ × ½	13	88	40
8½ × 2¼	45	4 ×	25	2¾ × ⅜	15	100	45
9 × 2½	50	4¼ × ¾	28	3 × „	16	112	50
9½ × 2⅜	55	4½ × „	32	3¼ × „	18	128	56
10 × 2½	60	4⅝ × „	34	3½ × „	20	136	60

Table No. 237.—Strength of Cable Fencing Strands and Solid Fencing Wire.

(Reduced from Messrs. Francis Morton & Co.'s "Standard Quality" Table.)

Size of Fencing Strand.		Solid Wire of Equal Diameter.		Length of One Ton.		Ultimate Strength per Square Inch.	
				Strand.	Wire.	Strand.	Wire (annealed).
		No.	inch.	yards.	yards.	tons.	tons.
	No. 6/o	o	.326	3,000	2,700	2.419	2.000
	No. 5/o	1	.300	3,800	3,200	1.828	1.683
	No. 4/o	2	.274	5,600	3,850	1.562	1.402
	No. 3/o	3	.250	6,000	4,650	1.340	1.169
	No. oo	4	.229	6,200	5,500	1.160	.988
	No. o	5	.209	7,800	6,600	.893	.817
	No. 1	6	.191	9,800	7,900	.714	.682
	No. 2	7	.174	11,000	9,550	.627	.566
	No. 2A	8	.159	15,000	11,400	.491	.473

Note.—The number, size, and strength of the Iron Wire quoted in this Table are the same as those of Ryland's Warrington Wires, table No. 82, page 247.

Table No. 238.—Tensile Strength of American Iron Wire Rope and Hemp Rope.

(Mr. J. A. Roebling.)

Trade Number.	Circumference of Wire Rope.	Circumference of Hemp Rope of equal Strength.	Ultimate Strength.	Trade Number.	Circumference of Wire Rope.	Circumference of Hemp Rope of equal Strength.	Ultimate Strength.
No.	inches.	inches.	tons. (English.)	No.	inches.	inches.	tons. (English.)
Fine Wire.				14	3.26	8¼	18.2
1	6.62	15½	67.3	15	2.98	7¼	14.5
2	6.20	14½	59	16	2.68	6¼	11.3
3	5.44	13	49	17	2.40	5½	8
4	4.90	12	39.6	18	2.12	5	6.9
5	4.50	10¾	32	19	1.9	4¾	5.3
6	3.91	9½	24.7	20	1.63	4	3.72
7	3.36	8	18.4	21	1.53	3.3	2.57
8	2.98	7	14.5	22	1.31	2.8	1.93
9	2.56	6	10.4	23	1.23	2.46	1.5
10	2.45	5	7.8	24	1.11	2.2	1.16
Coarse Wire.				25	.94	2.04	.94
11	4.45	10¾	33	26	.88	1.75	.74
12	4.00	10	27.3	27	.78	1.50	.51
13	3.63	9½	22.7				

French Wire Rope.—For mining purposes, each strand consists of a core of hemp and 12 wires; and the rope has 5 or 6 strands on a central hemp core. Flat ropes are formed by laying 3 or 4 ropes side by side, and binding or lacing them with annealed wire; but flat ropes are seldom employed.

Table No. 239.—French Iron Wire Ropes for Mining Service.

(Manufactured by MM. Harmegnies, Dumont, & Co., Anzin.)

Working depth, 400 metres, or 440 yards.

Flat Ropes.					Round Ropes.			
Number of Strands.	Width.	Thickness.	Weight per Yard.	Working Load.	Number.	Diameter.	Weight per Yard.	Working Load.
strands.	inches.	inch.	lbs.	tons.	No.	inches.	lbs.	tons.
8	5.1	.87	16	5	10	1.30	6.5	3
8	4.7	.79	13	4.5	11	1.10	5	2.5
6	3.9	.83	12	4	12	.98	3.8	2
8	4.3	.67	11	3.5	13	.83	3	1.5
6	3.5	.79	10	3	14	.71	2.6	1
6	3.2	.67	9	2.5	15	.63	2	.75
6	3.2	.63	8	2	16	.59	1.5	.5
6	2.8	.59	7	1.8	17	.51	1	.25
9	2.4	.55	6	1.5				

Note to Table.—1. Steel wire ropes may be a third less in weight than iron wire rope for the working load. 2. Hemp ropes should be a third heavier than iron wire rope for the same working load.

Steel Wire Ropes.—Ropes consisting of 26 steel wires, No. 14 W. G., or .085 inch in diameter, are made for steam ploughing purposes. The weight of the rope is about 2 lbs. per yard,—less than 1 ton per 1000 yards. Each wire, it is said, bears a tensile stress of from 2000 lbs. to 1 ton; and, at this rate, the rope should have a tensile resistance equal to 24 or 26 tons.

CHAINS.

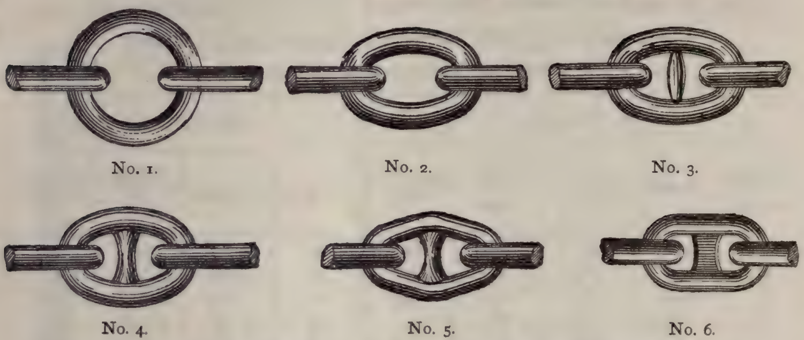
Chains are constructed either with open links, Figs. 274 and 275, or with stud-links, Figs. 276, 277, 278, and 279.

The standard proportions of the links of chains, in terms of the diameter of the bar iron from which they are made, are as follows:—

	Extreme Length.	Extreme Width.
Stud-link	6 diameters	3.6 diameters.
Close-link	5 ,,	3.5 ,,
Open-link	6 ,,	3.5 ,,
Middle-link	5.5 ,,	3.5 ,,
End-links	6.5 ,,	4.1 ,,

End-links are the links which terminate each 15-fathom length of chain; they are made of thicker iron, generally 1.2 diameters of the common links.

Ordinary Stud-link Chain-cable.—The admiralty test for the tensile strength of ordinary stud-link chain-cables, is at the rate of 630 lbs. per circular ⅛-inch section of one side of a link: equivalent to 22.92 tons per square inch of one side, or to 11.46 tons per square inch of both sides taken together,—just within the elastic limit.



Figs. 274–279.—Links of Chain-Cables. No. 1, Circular Link. No. 2, Oval Link. No. 3, Oval Stud-Link, with pointed stud. No. 4, Oval Stud-Link, with broad-headed stud. No. 5, Obtuse-angled Stud-Link. No. 6, Parallel-sided Stud-Link.

The weight of a link in similar cables, increases as the cube of any lineal dimension, say the thickness; and the weight per yard increases as the square of the thickness of chain. Whence the formula—

Weight per yard of Common Stud-link Chain-cable.

$$W = 26.9\,d^2;\ \text{or, in round numbers, } 27\,d^2 \qquad (1)$$

W = the weight per yard in pounds; d = the thickness of the chain, or the bar from which it is made, in inches.

The weight of a bar of iron a yard long is 10 lbs. per square inch of section, or 7.854 lbs. per circular inch; that is, a 1-inch round bar weighs 7.85 lbs. per yard, whilst a stud-chain cable of 1-inch iron, weighs 26.9 lbs. per yard, or 3.42 times the weight of a 1-inch bar. Generally, therefore, a stud chain-cable weighs 3.42 times as much as a bar of the same size and length.

The table No. 240 contains the dimensions, weights, and strengths of ordinary stud-link chain-cables. Column 4 shows the weight of 100 fathoms of cable in 8 lengths; including 4 swivels and 8 joining shackles. The sixth column gives the ultimate strength by actual tests made at Woolwich, in 1842–43, averaging, as shown in the last column, 16 tons per square inch, or two-thirds of the strength of the original bar, assumed at 24 tons per square inch.

The safe working-stress is 5.73 tons per square inch of both sides together, or half of the proof-stress.

Open-link Chains, Figs. 274 and 275.—The chain, Fig. 275, is sometimes called a close-link chain, to distinguish it from the circular-link chain, Fig. 274. The ultimate strength, generally, is the same as that of stud-link chains; but the elastic limit is less than that of the others, and the proof-stress for close-link chains is just two-thirds of that for stud-link chains, or

Table No. 240.—Ordinary Stud-Link Chain-Cable Weight and Strength.

Dimensions of Link.			Weight of 100 Fathoms.		Average Ultimate Strength.	Admiralty Proof-stress adopted by Lloyds'.		Ultimate Strength per square inch of Both Sides of Link.
Diameter of each Side.	Length of One Link.	Width of One Link.	Total.	Per Fathom (6 Feet).				
inches.	inches.	inches.	cwts.	lbs.	tons.	tons.	per cent.	tons.
7/16	2⅝	1.575	9.20	11.3	—	3½	—	—
½	3	1.8	12	13.4	—	4½	—	—
9/16	3⅜	2.025	15.2	17.2	—	5½	—	—
⅝	3¾	2.25	18.75	21	9.58	7	73	15.6
11/16	4⅛	2.475	22.7	25.4	—	8½	—	—
¾	4½	2.7	27	30.2	13.51	10⅛	75	15.3
⅞	5¼	3.15	36.75	41.2	20.38	13¾	67	16.9
1	6	3.6	48	53.8	24.25	18	74	15.4
1⅛	6¾	4.05	60.75	69	29.54	22¾	77	14.9
1¼	7½	4.5	75	84	—	28⅛	—	—
1⅜	8¼	4.95	90.75	101.6	—	34	—	—
1½	9	5.4	108	121	59.58	40½	68	16.9
1⅝	9¾	5.85	126.75	142	—	47½	—	—
1¾	10½	6.3	147	164.6	74.12	55⅛	74	15.4
1⅞	11¼	6.75	168.75	189	92.88	63¼	68	16.8
2	12	7.2	192	215	99.54	72	72	15.8
2⅛	12¾	7.65	216.75	242.8	—	81¼	—	—
2¼	13½	8.1	243	276.2	—	91⅛	—	—
2⅜	14¼	8.55	270.75	303.2	—	101½	—	—
2½	15	9.0	300	336	—	112½	—	—
2¾	16½	9.9	363	406.6	—	136⅛	—	—
Averages,							72	15.9

Note 1.—The *Safe Working-stress* is taken at half the Proof-stress.

2.—The *Proof-stress* and *Safe Working-stress* for close-link chains are respectively two-thirds of those of stud-link chains.

7.64 tons per square inch of section of both sides, or 410 lbs. per circular ⅛-inch of section of one side. The safe working-stress is half the proof-stress, or 3.82 tons per square inch of section.

The weight of close-link chain is about three times the weight of the bar from which it is made, for equal lengths.

Karl von Ott, comparing the weight, cost, and strength of the three materials, hemp, iron wire, and chain iron, concludes that the proportion between the cost of hemp rope, wire rope, and chain, is as 2 : 1 : 3; and that, therefore, for equal resistances, wire rope is only of half the cost of hemp rope, and a third of the cost of chains.

LEATHER BELTING.

According to the experiments of Messrs. Briggs and Towne, the tensile strength of single leather belts, .219 inch thick, was,

	Per inch wide.		Per square inch of Section.
Through the lace-holes,	210 lbs.	......	960 lbs.
Through the rivet-holes,	382 „	......	1740 „
Through the solid parts,	675 „	......	3080 „

Messrs. Norris & Co.'s beltings, as tested by Mr. Kirkaldy, gave the following results for ultimate tensile strength:—

Table No. 241.—TENSILE STRENGTH OF LEATHER BELTING.

SIZE.	English Belting.	Helvetia Belting.
DOUBLE.	lbs.	lbs.
12 inches,...........................	14,861	17,622
7 "	6,193	11,089
6 "	5,603	10,456
4 "	4,365	6,207
2 "	2,942	4,237
SINGLE.		
10 "	8,846	11,888
5 "	4,060	5,426
4 "	3,248	3,948
3½ "	3,007	3,377

Spill's machinery belting is manufactured from flax-yarn, saturated with a compound substance said to be incapable of decomposition. According to the annexed results of tests it is stronger than leather belts:[1]—

		Tensile Strength. per inch Wide.
No. 1,	5 inches wide, ...	1254 lbs.
No. 2,	5 "	1489 "
No. 3,	10 "	1563 "
Leather belt,	4 "	525 "

Untanned leather belts are said to be half as strong again as tanned leather belts. Mr. John Mason, of Bulkley, Barbadoes, uses belts cut from raw cowhide, simply dried in the sun. They last longer, he says, than leather belts, and are made at a fourth of the cost of the latter.[2]

India-rubber belts, made of American cotton canvas, cemented in layers by vulcanized india-rubber, and covered by a compound of rubber, have been proved to possess considerably greater frictional adhesion than leather belts.

STRENGTH OF BOLTS AND NUTS.

Mr. Brunel's Experiments.[3]—Mr. Brunel tested the tensile strength of screwed bolts and nuts of Shropshire iron, from ⅝ inch to 1¼ inches in

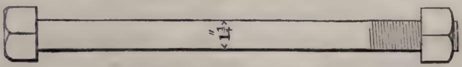


Fig. 280.—Screw Bolt and Nut.

diameter, applying the stress between the head and the nut, when 16 inches

[1] *Exhibited Machinery of* 1862, page 423.
[2] *Engineering*, June 19, 1874.
[3] The particulars of these experiments are derived from the Author's work on *Railway Locomotives*, 1860.

apart, and placed as in Fig. 280. The length of the screwed part was 3¼ inches. In most instances, the bolt snapped at the base of the screwed part.

Diameter inches.	Total Breaking Weight. tons.	Breaking Weight per sq. inch. tons.
⅝	10¼	32
¾	12	27
⅞	15¾	25
1	20	25
1⅛	21	21
1¼	29	23

To find to what extent the screwing of a bolt diminishes its tensile strength, Mr. Brunel tested four 1¼-inch bolts and nuts to the annexed form, Fig. 281, on which the screwed part was enlarged to 1½ inches in diameter. The bolts were broken in the shank, and the average breaking weight was equal to 25.2 tons per square inch, showing an addition of 2.2 tons per inch, as compared with the screwed shank, Fig. 280. Inversely, it may be inferred that the strength of 1¼-inch bolts was reduced 2.2 tons, or 8 per cent., by screwing.

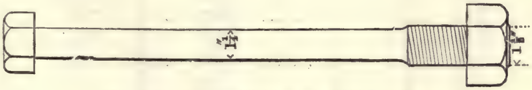


Fig. 281.—Screwed Enlarged Bolt and Nut.

The heads of the 1¼-inch bolts were 1¼ inches thick, and they stood fast during all the trials. The depth of the nuts of these bolts varied from 1¼ inch to ¾ inch.

Nuts 1 inch deep,	or $^8/_{10}$ths of the diameter,			stood well.
Do. ⅞	,,	or $^7/_{10}$ths ,,	,,	thread strained.
Do. ¾	,,	or $^6/_{10}$ths ,,	,,	thread stripped.

The thread, it appears, was stripped when the depth of the nut was only $^3/_5$ ths of the diameter. Nevertheless, in ordinary good practice, a depth of half the diameter has been found sufficient for both the head and the nut. But it may well be better to make them deeper, to allow for contingencies.

Working Stress for Screwed Bolts.—A working stress of 1½ tons per square inch has been assigned for screwed bolts. In France, it has been taken as high as 3¾ tons per square inch.

Whitworth's System of Standard Sizes of Bolts and Nuts.—The thickness of the bolt head is ⅞ths of the diameter, and that of the nut is equal to the diameter. The angle of the triangular thread is, in this system, 55°. The top and the bottom of the thread are rounded off, and the reduction so made of the exact height of the triangle is one-third; that is, one-sixth from the top, and one-sixth from the bottom. The actual height of the thread becomes rather more than $^3/_5$ ths, and less than $^2/_3$ ds, or about 63 per cent., of the pitch. See table No. 242, next page.

For screws with square threads, the number of threads per inch is one-half of the number for triangular threads.

2⅞	2.509	3½
3	2.634	3½..
3¼		3¼
3½	......	3¼..
3¾		3
4	......	3 ..
4¼		2⅞
4½	......	2⅞..
4¾		2¾
5	......	2¾..
5¼		2⅝
5½	......	2⅝..
5¾		2½
6	......	2½..

The American standard pitches are nearly identical with the Whitworth standards.

American Standard Sizes of Bolts and Nuts.

(United American Railway Master Car-builders' Association, in Convention at Richmond, Va., June 15, 1871.)

ROUGH BOLTS.—The breadth across the flats of the bolt-head and the nut = 1½ diameters + ⅛ inch.

The thickness of the head = ¾ diameter + 1/16 inch.

The thickness of the nut = 1 diameter.

FINISHED BOLTS.—The breadth across the flats of the bolt-head and the nut = 1½ diameters + 1/16 inch.

The thickness of the head and of the nut = 1 diameter − 1/16 inch.

Diameter.	Number of Threads per Inch.	Diameter.	Number of Threads per Inch.	Diameter.	Number of Threads per Inch.
inches.	threads.	inches.	threads.	inches.	threads.
¼	20	1⅜	6	3¾	3
5/16	18	1½	6	4	3
⅜	16	1⅝	5½	4¼	2⅞
7/16	14	1¾	5	4½	2¾
½	13	1⅞	5	4¾	2⅝
9/16	12	2	4½	5	2½
⅝	11	2¼	4½	5¼	2½
¾	10	2½	4	5½	2⅜
⅞	9	2¾	4	5¾	2⅜
1	8	3	3½	6	2¼
1⅛	7	3¼	3½		
1¼	7	3½	3¼		

Table No. 243.—WHITWORTH'S STANDARD PITCHES FOR SCREWED IRON PIPING.

Diameter of Piping.	Number of Threads per inch.	Diameter of Piping.	Number of Threads per inch.	Diameter of Piping.	Number of Threads per inch.
inches.	threads.	inches.	threads.	inches.	threads.
⅛	28	⅝	14	1½	11
¼	19	¾	14	1¾	11
⅜	19	1	11	2	11
½	14	1¼	11	above 2	8

M. Armengaud gives a table of the dimensions of bolts and nuts, based on the average practice in France. It is here translated into English measures, for threads of triangular and of square section. The thickness of the nut for triangular threads is equal to the diameter of the bolt, as in Whitworth's system. The depth of the square thread is nearly equal to half the pitch, or to the thickness of the thread.

Table No. 244.—Armengaud's French Standard Bolts and Nuts.

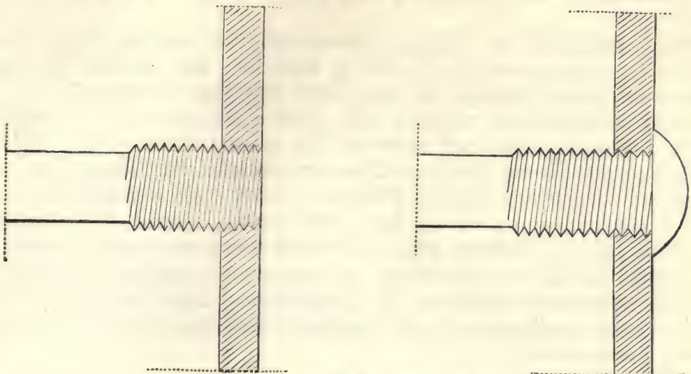
With Hexagonal Heads and Nuts.

1. Triangular Thread—(Equilateral Triangle).

Screw.				Head and Nut.			Working Tensile Stress.
Diameter of Bolt and Screw.		Diameter at Bottom of Thread.	Number of Threads per inch.	Thickness of Head.	Thickness of Nut.	Breadth across the Flats.	
millimetres.	inches.	inches.	threads.	inches.	inches.	inches.	lbs.
5	.20	.13	18.1	.24	.20	.55	44
7.5	.30	.22	16	.30	.30	.68	99
10	.39	.31	14.1	.38	.39	.88	178
12.5	.49	.39	12.7	.44	.49	1.04	277
15	.59	.48	11.5	.52	.59	1.20	400
17.5	.69	.58	10.6	.58	.69	1.40	545
20	.79	.66	9.8	.66	.79	1.50	713
22.5	.89	.76	9.1	.72	.89	1.68	902
							tons.
25	.98	.84	8.5	.80	.98	1.84	.50
30	1.18	1.02	7.5	.94	1.18	2.16	.73
35	1.38	1.20	6.7	1.08	1.38	2.48	.99
40	1.58	1.40	6.0	1.22	1.58	2.80	1.30
45	1.77	1.56	5.5	1.36	1.77	3.20	1.64
50	1.97	1.74	5.1	1.50	1.97	3.44	2.03
55	2.17	1.92	4.7	1.64	2.17	3.76	2.45
60	2.36	2.08	4.4	1.74	2.36	4.08	2.92
65	2.56	2.26	4.1	1.92	2.56	4.40	3.42
70	2.76	2.44	3.8	2.06	2.76	4.70	3.97
75	2.95	2.60	3.5	2.20	2.95	5.00	4.56
80	3.15	2.78	3.4	2.34	3.15	5.35	5.12
2. Square Thread.							
		Depth of Thread.				tons.	
20	.79	.072	6.57	—	1.82	.32	—
25	.98	.081	5.97	—	2.01	.51	—
30	1.18	.093	5.40	—	2.22	.73	—
35	1.38	.10	4.93	—	2.41	.99	—
40	1.57	.106	4.53	—	2.63	1.30	—
45	1.77	.114	4.20	—	2.85	1.64	—
50	1.97	.128	3.91	—	3.07	2.03	—
55	2.17	.13	3.65	—	3.30	2.45	—
60	2.36	.14	3.43	—	3.50	2.92	—
65	2.56	.15	3.23	—	3.70	3.42	—
70	2.76	.158	3.06	—	3.92	3.97	—
75	2.95	.166	2.92	—	4.13	4.56	—
80	3.15	.174	2.76	—	4.36	5.18	—
85	3.35	.183	2.63	—	4.58	5.85	—
90	3.54	.192	2.51	—	4.78	6.56	—
95	3.74	.200	2.41	—	5.00	7.30	—
100	3.94	.209	2.31	—	5.22	8.10	—
105	4.13	.220	2.22	—	5.43	8.93	—
110	4.33	.226	2.13	—	5.66	9.80	—
115	4.53	.230	2.06	—	5.87	10.71	—
120	4.72	2.24	2.00	—	6.08	11.66	—

SCREWED STAY-BOLTS AND STAYED SURFACES.

Screwed Stay-Bolts.—Sir William Fairbairn tested the strength of ¾-inch stay-bolts, with enlarged ends, screwed into ⅜-inch plates of copper and



Figs. 282, 283.—Flat Stayed Plates.

of iron, some of them being rivetted or headed in addition, as in the Figs. 282 and 283.

	Bolts.	Plates.	Breaking Weight.
1.	Copper	into copper, screwed and rivetted,............	7.2 tons.
2.	Iron	into copper, do. do.	10.7 „
3.	Iron	into copper, screwed only........................	8.1 „
4.	Iron	into iron, screwed and rivetted..................	12.5 „

Notes.—1*st Test.* The bolt broke through the shank.

2*d Test.* The rivet-head was broken off, and the bolt was drawn out of the plate, stripping the thread.

3*d Test.* The bolt stripped the thread of the plate.

4*th Test.* The bolt broke through the shank; screw and plate uninjured.

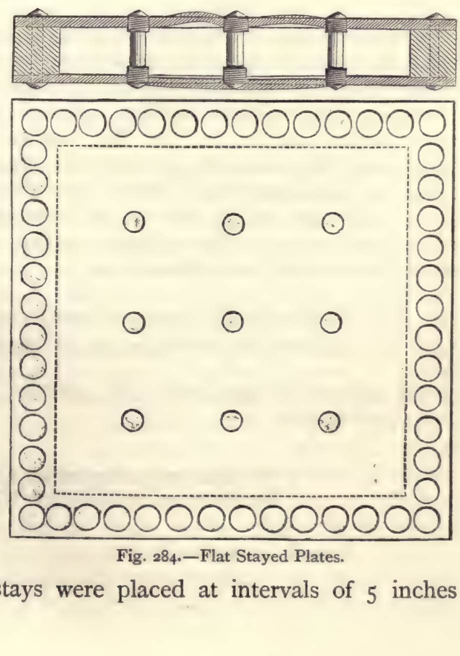


Fig. 284.—Flat Stayed Plates.

Flat Stayed Plates.—Sir William Fairbairn tested two flat boxes, Fig. 284, 22 inches square, having top and bottom plates of ½-inch copper and ⅜-inch iron respectively, inclosing a 2½-inch water-space; stayed with $^{13}/_{16}$-inch iron stays, having enlarged ends screwed and rivetted into the plates, to represent the conditions of the firebox of a locomotive. The stays were placed at intervals of 5 inches

in the first box, and 4 inches in the second. Under the pressure of water, the sides of the first box commenced to bulge or swell between the stays at a pressure of 455 lbs. per square inch, and the box was burst at 815 lbs., by the drawing of the head of a stay bolt through the copper plate, as shown.

In the second box, the bulging commenced at a pressure of 515 lbs. per square inch; it amounted to ⅓ inch at 1600 lbs; and at 1625 lbs. one of the stays was drawn through the iron plate, stripping the thread.

At the 5-inch intervals, rupture took place when the stress on each stay-bolt was 9 tons; at the 4-inch intervals, the ultimate stress was 11½ tons.

Flat Plates of Marine Boilers.—Two experimental flat boxes were tested at Plymouth Dockyard, by Mr. Phillips.[1] They were constructed respectively of 7/16-inch and of ½-inch iron plates, stayed at intervals of 15¾ inches by 15¼ inches, with 1⅜-inch screwed stay-bolts rivetted over at the ends, giving a superficies of 240 square inches for each bolt. Tested by hydrostatic pressure, the plates were bulged between the stay-bolts, and were finally pushed off, or drawn away from the bolts, under the following pressures:—

Plates.	Sectional Area of Stay-bolts.		Bursting Pressure per Square Inch of Surface.	Total Pressure for each Stay-bolt.
	In Body.	At Thread.		
inch.	square inch.	square inch.	lbs.	tons.
7/16	1.48	(say) 1.2	105	11.25
½	1.48	(say) 1.2	140	14.73

When nuts were applied to the ends of the stay-bolts through the 7/16-inch plate, they bore a pressure amounting to 165 lbs. per square inch, on the plate; when the box gave way at a rivetted joint.

Rules for Flat Stayed Surfaces.—Mr. Wm. Bury[2] propounds the following rules for the staying of the flat surfaces of marine boilers:—1st. The diameter of the screwed stays over the threads, should never exceed three times the thickness of the plates. 2d. The working steam-pressure allowed per square inch of section of the stay-bolts, at the threads, is 5000 lbs. 3d. The formula for the working pressure in pounds per square inch, with the above-named proportions, is,—

$$\frac{112\ (\text{thickness of plate in sixteenths inch})^2}{\text{area of stayed surface for each stay, in square inches}} = \text{lbs. per square inch},$$

which appears to agree with safe practice. Mr. Bury gives the following data, by this rule:—

[1] See *Engineering*, September 1, 1876, page 185.
[2] *Engineering*, September 15, 1876, page 236.

Table No. 245.—PROPORTIONS OF FLAT STAYED SURFACES OF BOILERS.

For a working pressure of 30 *lbs. per square inch.*

Diameter of Stay-bolts.	Sectional Area at Threads.	Area of Surface for each Stay-bolt.	Distance of Centres of Stay-bolts.
inch.	square inch.	square inches.	inches.
1 ⅛	0.8	133	11 ½
1 ¼	1.0	166	13
1 5/16	1.1	183	13½
1 ⅜	1.2	200	14⅛
1 ½	1.5	250	15¾

Mr. Bury recommends that nuts should be applied on the uptake ends of the bolts outside the plates, where they are above the water-line. He reckons on a bursting pressure six times the working pressure.

HOLLOW CYLINDERS:—TUBES, PIPES, BOILERS, &c.

RESISTANCE TO INTERNAL OR BURSTING PRESSURE. TRANSVERSE RESISTANCE.

The action of a centrifugal pressure within a cylinder is illustrated by Fig. 94, page 274. The resistance offered by the sides of the cylinder to internal pressure transversely, is not uniformly exerted throughout the thickness of the sides. On the contrary, the resistance varies, and is a maximum at the inner surface of the cylinder, and when the stress on the inner surface does not exceed the limit of elastic resistance, the tensional stress diminishes uniformly through the thickness of the sides, and is a minimum at the outer surface.

For cast-iron, in which the strain increases approximately in proportion to the stress, this simple ratio of decrease holds approximately up to the bursting strength, which is measured by the total resistance opposed to breakage when the internal surface is strained to the ultimate limit of its tensile strength. But in the stretching of wrought iron and steel, there is a break in the uniformity of the stretching, at the yielding point, as is shown very clearly by Fig. 222, page 624; for, beyond the yielding point, the extension proceeds in a greatly accelerated ratio with the stress.

Take, for instance, the cast-iron cylinder of a hydraulic press, 10 inches in diameter internally and 20 inches externally, shown in cross section in Fig. 285. Divide the thickness of it into an indefinite number of concentric rings of equal thicknesses, *a*, *b*, *c*, *d*, *e*; and suppose, only for the sake of argument, that the first or innermost ring is stretched by internal pressure to 11 inches in diameter inside. All the other rings will be stretched to larger diameters, in such proportions that, whilst the circumferential extension is the same for all the rings, the increase of diameter will be inversely as the original diameter of each ring, so that the outermost, or 20-inch ring, will be stretched only ½ inch in diameter, or half the diamet-

rical stretch of the innermost ring. The comparative stretches of the successive rings are shown by shadings on the right side of the figure; and the shadings at the same time show by their thicknesses the relative stresses on the successive rings.

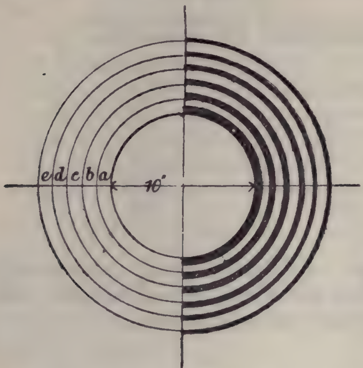


Fig. 285.—Diagram to show the Stretching of Hollow Cylinders by Internal Pressure.

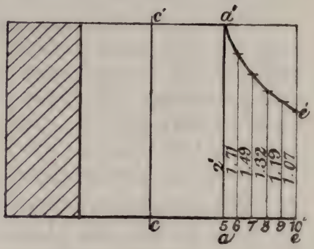


Fig. 286.—Diagram to show the Hyperbolic Ratio of the Stress throughout the Thickness, by Internal Pressure.

Since the stress is inversely as the radial distance from the centre, if its values be represented by ordinates to the radius treated as a base-line, $c\,a\,e$, in the longitudinal section, Fig. 286, they will, if connected at the ends, form a hyperbolic curve $a'\,e'$; and the area comprised between the curve and the base-line, is a measure of the total resistance of the section. Let

r = the inside radius $c\,a$,
r' = the outside radius $c\,e$,
s = the maximum tensile stress, in tons per square inch,
d = the inside diameter = $2\,r$,
d' = the outside diameter = $2\,r'$,
R = the ratio of the outside diameter to the inside diameter $= \dfrac{d'}{d} = \dfrac{r'}{r}$,
p = the internal pressure in tons per square inch.

Then, the rectangular area $c\,c'\,a'\,a$ is a measure of, or is equal to, $r \times s$, for a length of 1 inch parallel to the axis; and the area of resistance under the hyperbolic curve $a\,a'\,e'\,e$, is equal to, for both sides $(r \times s) \times \text{hyp log R} \times 2$. The internal pressure to be resisted, for 1 inch of length, is equal to $p\,d$, the product of the inside diameter and the hydrostatic pressure per square inch; and it is equal to the resistance; that is, $p\,d = (r \times s \times \text{hyp log R} \times 2)$. Or, $p\,d = 2\,r\,s \times \text{hyp log R} = d\,s \times \text{hyp log R}$; and

$$p = s \times \text{hyp log R} \quad (1)$$

$$s = \frac{p}{\text{hyp log R}} \quad (2)$$

$$\text{hyp log R} = \frac{p}{s} \quad (3)$$

These formulas express the relations of the internal pressure, and the maximum tensile stress on the metal at the inner surface, within the limits of elastic strength. They are given as rules, below.

Bursting Strength.—For cast-iron cylinders, the foregoing formulas may also be employed in calculations for the bursting strength, and the corresponding ultimate breaking strength.

To calculate the bursting strength of wrought iron and of steel cylinders, let the base-line $c\,a\,e$, Fig. 286, represent, as before, the inside radius $c\,a$, and the outside radius $c\,e$. Draw the verticals $c\,c'$ and $a\,a'$, to measure the ultimate tensile strength of the metal per square inch; conceive the verticals to be bisected at points which may be indicated as c'' and a'', and draw $c''\,a''\,e''$ parallel to the base. The rectangle $a\,a''\,e''\,e$ would represent the resistance of the section due to the elastic strength of the material, which is uniform throughout the thickness and is taken as half the ultimate tensile strength. Draw intermediate vertical ordinates through the radial intervals of the thickness, between a and c; and set off the lengths of the upper segments, above the middle level $a''\,e''$, to represent the values of the uniformly varying tensions in excess of the elastic limit, forming a hyperbolic curve, say, $a'\,e'$. The resistance of the section thus treated, consists of two parts:—the uniform resistance $a\,a''\,e''\,e$, equal to $(r'-r) \times \frac{1}{2}\,s$; and the varying resistance $a''\,a'\,e'\,e''$, equal to $(r \times \frac{s}{2}) \times \text{hyp log R}$. Twice the sum of these resistances is equal to the internal pressure per inch of length of the cylinder; whence,[1]

$$p = s\,\frac{(\text{R} + \text{hyp log R} - 1)}{2} \quad\ldots\ldots\ldots\ldots (4)$$

$$s = \frac{2\,p}{(\text{R} + \text{hyp log R} - 1)} \quad\ldots\ldots\ldots\ldots (5)$$

$$(\text{R} + \text{hyp log R}) = \frac{2\,p}{s} + 1 \quad\ldots\ldots\ldots\ldots (6)$$

Rules for the Strength of Hollow Cylinders, within the Limits of Elastic Strength.

Rule 1. *To find the Internal Pressure for a given maximum tensile stress on the material.* Multiply the hyperbolic logarithm of the ratio of the external to the internal diameter, by the maximum tensile stress in tons per square inch of the metal. The product is the internal pressure in tons per square inch.

Rule 2. *To find the maximum Tensile Stress on the sides for a given internal pressure.* Divide the pressure in tons per square inch by the hyperbolic logarithm of the ratio of the external to the internal diameter. The quotient is the maximum tensile stress on the metal in tons per square inch.

[1] The formula (4) is thus deduced:—

$$p\,d = (\,(r'-r) \times \frac{s}{2} \times 2\,) + (\,2\,(\,r \times \frac{s}{2}) \times \text{hyp log R}); \text{ or,}$$

$$p\,d = (\frac{d'-d}{2} \times s) + (\frac{d}{2} \times s \times \text{hyp log R}).$$

Dividing both sides by d,

$$p = s\,(\frac{d'-d}{2\,d} + \frac{\text{hyp log R}}{2}) = s\,(\frac{d'}{2\,d} - \tfrac{1}{2} + \frac{\text{hyp log R}}{2});$$

and, substituting R for $\frac{d'}{d}$, the formula for the pressure becomes,

$$p = s\,\frac{(\text{R} + \text{hyp log R} - 1)}{2} \quad\ldots\ldots\ldots\ldots (4)$$

RULE 3. *To find the Ratio of the Outside Diameter to the Inside Diameter, for a given maximum tensile stress on the sides, and a given internal pressure.* Divide the pressure by the stress, both in tons per square inch. The quotient is the hyperbolic logarithm of the ratio of the diameters, for which the ratio may be found in a table of hyperbolic logarithms.

RULES FOR THE BURSTING STRENGTH OF HOLLOW CYLINDERS.

Cast Iron.

The rules and formulas (1), (2), and (3), may be employed for calculating the bursting strength, and the corresponding ultimate tensile strength, of cast-iron hollow cylinders.

Wrought Iron and Steel.

RULE 4. *To find the Bursting Pressure for a given ultimate tensile strength of the material.* To the ratio of the outside to the inside diameter, add the hyperbolic logarithm of this ratio, and from the sum deduct 1. Multiply half the remainder by the ultimate tensile strength in tons per square inch. The product is the bursting pressure in tons per square inch.

RULE 5. *To find the Ultimate Tensile Strength of the material for a given bursting pressure.* To the ratio of the outside to the inside diameter, add the hyperbolic logarithm of this ratio, and from the sum deduct 1. Divide twice the bursting pressure in tons per square inch by the remainder just found. The quotient is the ultimate tensile strength in tons per square inch.

RULE 6. *To find the Ratio of the Outside to the Inside Diameter, for a given bursting pressure and ultimate tensile strength.* Divide twice the bursting pressure by the tensile strength, and add 1 to the quotient. The sum is equal to the ratio plus the hyperbolic logarithm of the ratio. The value of the ratio is found by trial and error, in a table of hyperbolic logarithms.

Notes to Rules 1 *to* 6.—1. The hyperbolic logarithm of a number is equal to the product of its common logarithm by 2.3026. 2. The pressure and the tensile stress may be expressed in pounds or in hundredweights, instead of tons.

1*st Example.*—Let the inside diameter of the cast-iron cylinder of a hydraulic press be 10 inches, the outside diameter 30 inches, and the ultimate strength of the metal 7 tons per square inch; to find the bursting pressure. The ratio of the diameters is $(\frac{30}{10}=)$ 3, of which the hyperbolic logarithm is 1.0986 (see table No. 2, page 61). By rule 1, the bursting pressure is $(1.0986 \times 7 =)$ 7.69 tons per square inch.

Average Stress on the Metal.—As the total transverse resistance per inch of length of cylinder is equal to $p\,d$, which is the product of the inside diameter by the bursting pressure per square inch, the average stress on the metal is equal to $\frac{p\,d}{d'-d}$; that is to say, it is equal, in tons per square inch, to the product of the inside diameter by the bursting pressure in tons per square inch, divided by the difference of the inside and outside diameters.

In the foregoing example, the average stress, which bursts the cylinder, is equal to $\left(\frac{10 \times 7.69}{30-10}=\right)$ 3.845 tons per square inch of section—little more than half the direct tensile resistance of the metal.

2d Example.—A steam-boiler, 7 feet in diameter inside, of $^7/_{16}$-inch wrought-iron plates, was burst at a longitudinal double-rivetted joint by a pressure of 310 lbs. per square inch. The outside and inside diameters, d and d', were 84.875 inches and 84 inches respectively, the ratio of which is 1.0104. By formula (5) the ultimate tensile strength was

$$\frac{310 \times 2}{1.0104 + \text{hyp log } 1.0104 - 1} = \frac{620}{1.0104 + .010345 - 1} = \frac{620}{.020745} = 29{,}886 \text{ lbs.},$$

or 13.34 tons per square inch of the section of the solid plate.

3d Example.—A cast-iron pipe, 10 inches in diameter inside, is ¾ inch in thickness. What is the bursting strength when the ultimate tensile strength of the material is equal to 7 tons per square inch? The ratio of the outside to the inside diameter is as 11.5 to 10, or as 1.15 to 1; and, by formula (1),

$$7 \times \text{hyp log } 1.15 = 7 \times .1398 = .9786 \text{ ton},$$

or 2192 lbs. per square inch, is the bursting pressure.

Approximate Rules for Transverse Resistance to Bursting Pressure.

When the diameter is very considerable, compared to the thickness, the transverse resistance to bursting pressure may be taken approximately as directly proportional to the thickness of the metal, and inversely proportional to the diameter. The total pressure on a 1-inch length of section of both sides together, is equal to the product of the diameter by the pressure per square inch.

Let d = the diameter, in inches; t = the thickness of metal at each side, in inches; s = the ultimate tensile strength of the metal, in tons per square inch; and p = the pressure in pounds per square inch. Then dp is the total pressure on a 1-inch length of both sides together; $2t$ is the sectional area of both sides; and $2ts \times 2240 = dp$, or,

$$p = \frac{4480\,ts}{d};\ t = \frac{dp}{4480\,s};\ \text{and } s = \frac{dp}{4480\,t}. \ \ldots\ldots (7), (8), (9)$$

Rule 7.—*The bursting pressure* in pounds per square inch of surface is equal to 4480 times the product of the thickness by the ultimate tensile strength per square inch, divided by the diameter.

Rule 8.—*The thickness of metal* required at each side is equal to the product of the diameter and the pressure per square inch, divided by the ultimate tensile strength in tons per square inch, and by 4480.

Rule 9.—*The ultimate tensile strength* in tons per square inch of section of metal is equal to the product of the diameter by the bursting pressure in pounds per square inch; divided by the thickness of metal and by 4480.

Note.—When the material is made of jointed plates, the tensile strength of the whole plate is to be multiplied by the coefficient of strength of the joint, to give the reduced strength to be employed as the value of s in the calculation.

Longitudinal Resistance to Bursting Pressure.

When the ends of a cylinder are closed, and make one piece with the cylindrical portion, the total longitudinal resistance to internal pressure is

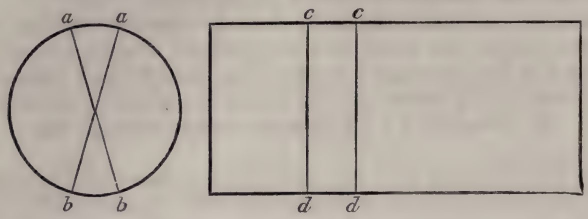


Fig. 287.—Diagram for the Resistance of a Flat-headed Cylinder to Bursting Pressure.

directly proportional to the thickness of the metal, and to the diameter; whilst the total bursting force, acting on the ends, is proportional to the

Table No. 246.—Experimental Resistance of Solid-Drawn Tubes to Bursting Pressure and Collapsing Pressure.

(Deduced from Messrs. Russell's data.)

Wrought-Iron Tubes.

External Diameter.	Thickness.		Internal Diameter.	Bursting Pressure.		Collapsing Pressure.		Difference of Bursting and Collapsing Pressures.
				Per square inch of Surface.	Per square inch of Section of Metal.	Per square inch of Surface.	Per square inch of Section of Metal.	
inches.	B.W.G.	inch.	inches.	lbs.	tons.	lbs.	tons.	tons.
3¼	10	.134	2.982	4800	23.84	3300	17.86	5.98
3⅛	10	.134	2.857	4500	21.42	3150	16.40	5.02
3	11	.120	2.760	4500	23.10	3500	19.53	3.57
2¾	11	.120	2.510	5200	24.28	3500	17.89	6.39
2½	11	.120	2.260	5000	21.02	3600	16.74	4.28
2¼	11	.120	2.010	5900	22.06	4500	18.82	3.24
2	12	.109	1.782	5900	21.53	4900	20.07	1.46
1¾	12	.109	1.532	5600	17.57	4000	14.33	3.24
Averages, omitting the last tube				—	22.40	—	18.20	—
Homogeneous Metal Tubes.								
3	13	.095	2.810	3600	23.77	3150	22.20	—
2¼	13	.095	2.060	7600	36.78	4600	24.32	—
2	15	.072	1.856	4000	23.02	3500	21.70	—
1⅝	17	.058	1.409	4600	24.94	4000	25.02	—
Bessemer Steel Tubes.								
1¾	13	.095	1.56	7800	28.92	4600	18.91	—
Hydraulic Tubes.								
				proved to				
2½	—	5/16	1⅞	11,000	14.73	—	—	—
1⅝	—	5/16	1	6,000	4.29	—	—	—
1⅛	—	¼	⅝	4,000	2.23	—	—	—
⅞	—	5/16	¼	12,000	2.15	—	—	—

square of the diameter. It results that the longitudinal resistance per square inch to bursting force is inversely proportional to the diameter. Let the circle and the rectangle, Fig. 287, be a cross section and a longitudinal section of a cylindrical boiler; the area of the circle is a measure of the longitudinal pressure of the steam on the ends of the boiler. Set off the interval *a a* on the circular section, and the interval *c c* on the longitudinal section; and draw the diameters *a b*, *a b*, and the parallels *c d*, *c d*. The areas of pressure to be resisted are respectively the two triangular spaces *a b*, and the rectangle *c d*; and, since the former have only half the surface of the latter, it follows that the longitudinal stress per square inch on the shell is only half the transverse stress, and that the amount of the longitudinal resistance is proportionally twice as much as that of the transverse resistance.

On the same showing, a hollow sphere resists twice the pressure per square inch, that a tube of equal diameter and equal thickness can do.

Wrought-iron Tubes.—Messrs. J. Russell & Sons tested the resistance of solid-drawn wrought-iron tubes to bursting pressure, and to collapsing pressure, on the results of which table No. 246 is based.

The bursting pressure of the wrought-iron tubes in tons per square inch of section of metal, appears to be practically constant; and it may be taken for practical purposes that the ultimate strength is measured by the tensile strength of the material.

Resistance of a Lancashire Boiler to Bursting Pressure.—A boiler 7 feet in diameter, made of $^7/_{16}$-inch plates, was tested by Mr. L. E. Fletcher, and bore a pressure of 310 lbs. per square inch, when it failed at one of the longitudinal seams, which were double-rivetted. Applying rule 9, page 691, the ultimate tensile strength was equal to $\left(\frac{84 \text{ inches} \times 310 \text{ lbs.}}{.4375 \times 4480} = \right)$ 13.29 tons per square inch of section of the entire plate. This instance formed the subject of the 2d example, page 638. In this instance, the tensile strength, as calculated by the exact rule 5, page 690, is 13.34 tons per square inch. This is .05 ton, or about $^2/_5$ ths of 1 per cent. more than is given by the approximate rule.

Take the correctly calculated strength, 13.34 tons, with the net section of plate between the rivets, which was two-thirds of the section of the continuous plate. Then $13.34 \times \frac{3}{2} = 20.01$ tons per square inch, the tensile strength of the plate between the rivet-holes.

Resistance of a Cylindrical Marine Boiler and a Superheater to Bursting Pressure.[1]—A cylindrical boiler, 11 feet 3 inches in diameter inside, of ¾-inch plates, double-rivetted, was burst by a hydraulic pressure of 230 lbs. per square inch, equivalent, by rule 9, to $\frac{135'' \times 230 \text{ lbs.}}{.75'' \times 4480} = 9.241$ tons per square inch of section of the solid plate. The rivet-holes were 1 $^1/_{16}$ inch in diameter, at 2¾ inches pitch, leaving 61.36 per cent. of solid metal between; and the ultimate tensile strength of metal left between the rivet-holes was $9.241 \times \frac{100}{61.36} = 15.06$ tons per square inch.

A superheater, 99.915 inches in diameter inside, of $^9/_{16}$-inch plates,

[1] The data are derived from *Engineering*, July 21, 1876, page 47.

double-rivetted, with $^{13}/_{16}$-inch rivet-holes at $2\frac{1}{2}$ inches pitch, was burst at the same time by a hydraulic pressure of 245 lbs. per square inch, equivalent to $\frac{99.915 \times 245}{.562'' \times 4480} = 9.655$ tons per square inch of the solid plate. The metal left between the rivet-holes was 67.6 per cent. of the entire section, and the resistance of that metal was $9.655 \times \frac{100}{67.6} = 14.28$ tons per square inch of its net section.

Now the tensile strength of the plates of the boiler and the superheater, tested by Mr. Kirkaldy, averaged 20.5 tons per square inch for the boiler, and 20.2 tons for the superheater. These instances have already been noticed at page 638, in the discussion of rivet-joints, and they forcibly demonstrate the essential weakness of rivetted lap-joints in very thick plates. The net tensile resistance of the plates between the holes was reduced a fourth.

Cast-Iron Pipe.—For a 10-inch pipe, $\frac{3}{4}$ inch thick, having an ultimate tensile strength of 7 tons per square inch, the bursting pressure is, by formula (7), page 691,

$$\frac{4480 \times .75 \times 7}{10} = 2352 \text{ lbs. per square inch.}$$

By a previous calculation, page 691, with the exact formula (1), the bursting pressure was found to be 2192 lbs. per square inch, showing that the ordinary approximate formula (7) gives 160 lbs., or $7\frac{1}{4}$ per cent., more than the correct formula.

Resistance of Hollow Cylinders to External Collapsing Pressure.

Solid-drawn Tubes.—By the action of a centripetal force on the outside of a hollow cylinder, compressive stress is produced, tending to collapse the cylinder. In table No. 246, the resistance of wrought-iron tubes to collapse is given. It is less than the resistance to bursting, and the difference between the bursting and the collapsing pressures increases with the diameter, as shown in the last column. When plotted and arranged into a curve, or, as in this case, a straight line, the value of the difference, in terms of the diameter, is $2\frac{2}{3}(d-1)$, which probably holds for diameters up to 6 inches. When the diameter d is only one inch, $d-1=0$, and the difference vanishes. The average bursting pressure being 22.40 tons per square inch of section of metal, the collapsing pressure is $22.40 - 2\frac{2}{3}(d-1)$; or,—

Collapsing Pressure per square inch of Longitudinal Section of Metal for Solid-drawn Iron Tubes up to 6 inches in diameter.

$$p' = 25 - 2\tfrac{2}{3}\,d, \quad \ldots\ldots\ldots\ldots \quad (4)$$

in which d is the external diameter in inches, and p' is the collapsing pressure in tons per square inch of longitudinal section. The thickness is taken as from $^1/_{15}$th to $^1/_{25}$th of the diameter.

The corresponding pressure per square inch on the external surface of the tube, is equal to the total collapsing pressure for 1 inch in length of the tube, divided by the diameter; or it is $\frac{p' \times 2\,t \times 2240}{d}$; and, reducing,

Collapsing Superficial Pressure per square inch (1st formula).

$$p = \frac{4480\, p'\, t}{d}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

p being the superficial pressure in pounds per square inch, and t and d the thickness and the diameter in inches.

The collapsing pressure may be expressed in terms of the diameter, by substituting in the above formula (5) the value of p' in (4), when it becomes, by reduction, $p = t \times (\frac{112000}{d} - 11947)$, or, in round numbers,—

Collapsing Superficial Pressure per square inch (2d formula).

$$p = t \times (\frac{112000}{d} - 12000). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

The table No. 247 shows the bursting and collapsing pressures of solid wrought-iron tubes of the usual diameters and thicknesses, calculated by means of the preceding formulas:—

Table No. 247.—Solid-drawn Iron Tubes—Calculated Bursting and Collapsing Pressures.

External Diameter.	Thickness.		Internal Diameter.	Bursting Pressure.		Collapsing Pressure.	
				Per square inch of Internal Surface.	Per square inch of Section of Metal.	Per square inch of External Surface.	Per square inch of Section of Metal.
inches.	B.W.G.	inch.	inches.	lbs.	tons.	lbs.	tons.
1¼	14	.083	1.084	7700	22.4	6500	21.7
1⅜	14	.083	1.209	6900	22.4	5800	21.3
1½	14	.083	1.334	6200	22.4	5200	21.0
1⅝	14	.083	1.459	5700	22.4	4700	20.7
1¾	14	.083	1.584	5300	22.4	4300	20.3
1⅞	14	.083	1.709	4900	22.4	4000	20.0
2	14	.083	1.834	4500	22.4	3700	19.7
2⅛	13	.095	1.935	4900	22.4	3800	19.3
2¼	13	.095	2.060	4600	22.4	3600	19.0
2½	12	.109	2.282	4800	22.4	3600	18.3
2¾	12	.109	2.532	4300	22.4	3100	17.7
3	11	.120	2.760	4400	22.4	3000	17.0
3¼	11	.120	3.010	4000	22.4	2700	16.3
3½	10	.134	3.232	4200	22.4	2700	15.7
3¾	10	.134	3.482	3900	22.4	2400	15.0
4	10	.134	3.732	3600	22.4	2100	14.3
4¼	10	.134	3.982	3400	22.4	1900	13.7
4½	10	.134	4.232	3200	22.4	1700	13.0
4¾	10	.134	4.482	3000	22.4	1600	12.3
5	10	.134	4.732	2800	22.4	1400	11.7
5¼	9	.148	4.954	3000	22.4	1400	11.0
5½	9	.148	5.204	2800	22.4	1200	10.3
5¾	9	.148	5.454	2700	22.4	1100	9.7
6	9	.148	5.704	2600	22.4	1000	9.0

Large Furnace-Tubes.—The furnace-tubes of Lancashire and Cornish boilers vary in diameter from 18 inches to 4 feet, and they are usually composed of rings of plates rivetted together. The resistance to collapse under external steam pressure is derived partly from the longitudinal tension to which they are subject; and partly from their direct resistance to compression and collapse. It is supposed that longitudinal tensional resistance is brought into action to an important extent, for supporting the tube. This is a mistaken supposition. The records of collapses of furnace-tubes show that the length of the tube was the least influential factor; and of small value unless for very short lengths of from 3 feet to 6 feet or 9 feet, defined by stiffening rings or seams, if not by the actual length between the end plates. Practically, all the resistance to collapse of unfortified lengths of plain furnace-tubes is supplied by the compressive resistance and the stiffness of the tube.

From the monthly reports of Mr. Lavington E. Fletcher, 21 cases of collapsed iron tubes have been extracted, comprising flues of from 32 to 48 inches in diameter, ⅜ inch and 7/16 inch in thickness, and from 18 to 40 feet in length, under collapsing pressures of from 40 lbs. to 70 lbs. per square inch. In some instances, no doubt, the tubes had been sensibly worn; but in most instances, they had been in good order.

By plotting the results of these collapsed tubes, and tracing a mean curve through the plots, the following formula is derived:—

Collapsing Pressure of Plain Iron Furnace-tubes of Cornish and Lancashire Steam Boilers.

$$p = \frac{200000\,t^2}{d^{1.75}} \quad \text{(3)}$$

p = collapsing pressure, in pounds per square inch.
t = thickness of the plates of the furnace-tube, in parts of an inch.
d = internal diameter of the furnace-tube, in inches.

This formula is directly applicable to furnace-tubes of any length greater than 9 feet.

From the results of hydraulic tests made at the Leeds Forge, it appears that a plain flue-tube, 3 feet 1 inch inside diameter, of ⅜-inch plate, 7 feet long, bore a pressure of 175 lbs. per square inch before giving way by collapse; and that a like tube, corrugated, on Fox's system, bore a pressure of 450 lbs. per square inch before giving way.

Lead Pipes.

Mr. Jardine found that a 1½-inch lead pipe, .20 inch thick, sustained a pressure of 1000 feet of water, or 29½ atmospheres, without any alteration of form. Under 1200 feet of water, or 35 atmospheres, it began to enlarge; and it burst under 1400 feet, or 40 atmospheres, having swollen to a diameter of 1¾ inches. A 2-inch pipe, .20 inch thick, sustained a pressure of 800 feet of water, or 23½ atmospheres, with scarcely any enlargement; but it burst under 1000 feet, or 29 atmospheres. From these results it appears, by the aid of rule 2, page 689, that the elastic strength of lead is equal to 15 cwts. per square inch of sectional area and that the ultimate strength is equal to 1 ton per square inch.

[1] *Monthly Reports to the Manchester Steam-Users' Association*, 1862–69.

FRAMED WORK:—CRANES, GIRDERS, ROOFS, &c.

When the Weight or Force is parallel to one of the Members of the Frame.

For the purpose of resisting the stress of heavy loads, the triangle consti-

Fig. 288.

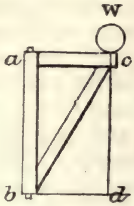


Fig. 289.

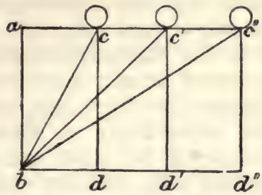


Fig. 290.

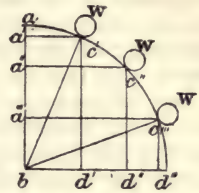


Fig 291.

Illustrations of Stress in Framed Work.

tutes the fundamental feature of framed work, as distinguished from solid work or web-work. When a load W is applied direct to a vertical pillar *ab*, Fig. 288, the resistance is in the line of the stress, and no framework is employed. But, if the load be applied at *c*, Fig. 289, horizontally apart from *a*, the triangular frame *a b c* is constructed to carry it. Complete the parallelogram *a d*, and it is seen that, if the vertical stress of W be measured by *ab* or *cd*, the horizontal tensile stress in *a c*, and the diagonal compressive stress in *c b*, are measured by the lengths of these members respectively. It is obvious here, as in other cases, that when a counteracting resistance is opposed obliquely to a weight or other force, the resisting stress is necessarily greater than the force; and that the diagonal stress increases with the overhang, as in Fig. 290, where, under the weights *c*, *c'*, and *c"* successively further from the origin *a*, the diagonal compressive resistances *c b*, *c' b*, and *c" b* are successively increased. The horizontal tensional resistances, measured by *c a*, *c' a*, and *c" a*, are likewise

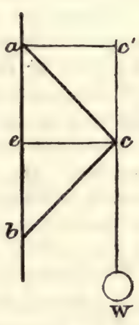


Fig. 292.

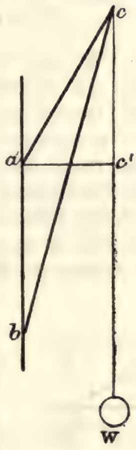


Fig. 293.

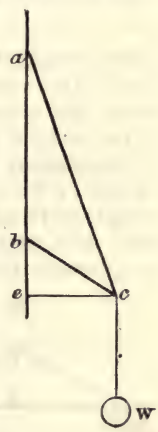


Fig. 294.

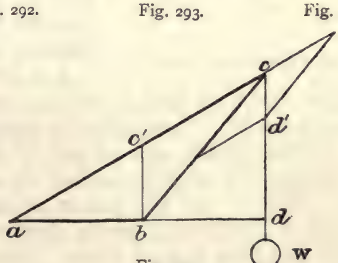


Fig. 295.

Illustrations of Stress in Oblique Framed Work.

successively increased. These, in fact, being perpendiculars to *ab*, are the leverages of the weights tending to pull over the upper end *a* of the upright *ab*; and simultaneously to push out the lower end *b*. The verticals *cd*, *c'd'*, and *c''d'*, equal and parallel to *ab*, are successively the measures of the same weight at the several positions.

If a diagonal of constant length, equal, say, to *ab*, be moved into various positions, *bc'*, *bc''*, and *bc'''*, on the lower end *b*, with a given weight W supported at the end, as in Fig. 291; the weight is to the thrust in the diagonal, proportionally, as the vertical *c'd'*, *c''d''*, or *c'''d'''*, is to the diagonal; showing that the thrust in the diagonal increases as the elevation is diminished; and that the horizontal tension in *a'c'*, *a''c''*, and *a'''c'''*, also increases.

In oblique-angled frames, like Figs. 292, 293, and 294, the stresses in the three members are respectively as their lengths. The horizontal pulling and thrusting stresses at the upper and lower points *a* and *b* respectively, are measured by the perpendicular *c' a* or *ce*, in Figs. 292 to 294; and they are the same as if the upper member had been horizontal, as at *ac'*.

When the Weight or Force is not parallel to any Leading Member of the Frame.

The weight W, Fig. 295, is supported by the slanting triangular frame *abc*. The vertical *cd* represents the weight. By the parallelogram of forces, the stresses, tensile and compressive, in *ac* and *bc*, proportionally to the weight, are ascertained. Draw the vertical *bc'*, then the triangle *bc'c* represents the three forces in equilibrium:—*bc'* for the weight, and *bc* and *c'c* for the thrust and the pull in the respective members. This triangle of forces is the same as if the members *cc'* and *cb* had, in fact, been fixed to a vertical wall or member *bc'*, as in the bracket, Fig. 293; and it is apparent that wherever the members *ac* and *bc* be extended to or attached, the stresses for a given weight remain unaltered. The compressive stress in the horizontal member *ab*, is expressed by the horizontal *bd*.

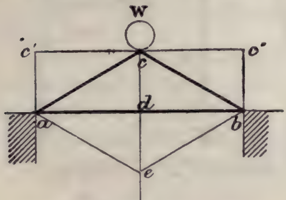


Fig. 296.—Elementary Truss.

The construction of the parallelogram may be dispensed with by simply drawing the vertical *bc'* forming up the triangle of forces *bc'c*. The horizontal *bd*, measures the thrust in the member *ab*.

Let *abc*, Fig. 296, be a triangular frame, with equal limbs *ca* and *cb* resting on supports at *a* and *b*, and loaded at the apex by W. Complete the parallelogram *ce*; then *ce* is the weight, *cd* is half the weight, and *ca* and *cb* are the compressive stresses in the sides. Again, the stresses *ac* and *bc* are resolved into the vertical pressures *ac'* and *bc''* on the points of support, each equal to *cd*, half the load; and the horizontal tensile stresses *ad* and *bd*, in the lower member, equal and opposite to each other. One of these, or the half of *ab*, is the measure of the tension in this member.

Otherwise, by equality of moments:—$\frac{1}{2}\,W \times \frac{1}{2}\,l = d \times$ tension in *ab*; in which d $(= \frac{1}{2}\,W)$ is the depth *cd*, and l is the span *ab*. Thence, $\frac{1}{2}\,l =$ *ad*, is the tension in *ab*.

Further, reducing the equation of moments,

$$\text{tension in } ab = \frac{W\,l}{4\,d} \quad \text{..................} \quad (A)$$

Or, the tension in the horizontal member *ab* is equal to the product of the weight by the span, divided by 4 times the rise.

The horizontal thrust at the apex is equal to the tension in *ab*.

The length of the inclined members *ac* and *cb*, in terms of the span and the rise, $= \sqrt{(\frac{1}{2}\text{ span})^2 + \text{rise}^2}$; from which the stress in these members is given.

Framed Girders—The Warren-Girder, Loaded at the Middle.

Let two equilateral triangles or bays, like Fig. 296, be framed together as in Fig. 297, forming in all three triangles; and loaded at the middle. Complete the parallelogram *de*, and the weight measured by *de* is resolved into the tensile stresses dc' and dc''. Each of these is resolved into two compressive stresses:—$c'a$ and $c''b$ to the points of support, and $c'c''$ and $c''c'$ equal and opposed to each other. The thrusts at *a* and *b* are resolved into the vertical components ac''' and bc_4, each equal to *cd*, half the weight, resisted and carried by the supports at *a* and *b*; and the horizontal components ad' and bd'', equal and opposite to each other, and each of them exhibiting the tensile stress in the lower member *ab*.

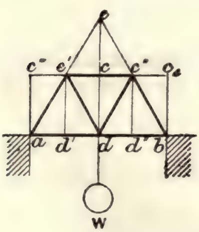


Fig. 297.

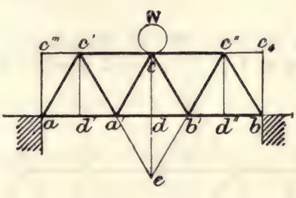


Fig. 298.

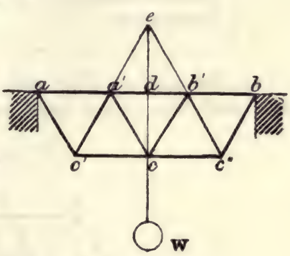


Fig. 299.

Framed Girders—the Warren-Girder.

The compressive stress in the upper member $c'c''$ is double the tensile stress in the lower member *a b*.

Suppose a girder of five equilateral triangles, Fig. 298, having three bays below and two above, loaded at the central apex *c*. Complete the parallelogram *ce*; the weight *ce* is resolved into the thrusts ca' and cb', which are resolved into opposing tensions $a'b'$ and $b'a'$, and tensions $a'c'$ and $b'c''$. These tensions are resolved into opposing thrusts $c'c$ and $c''c$, and thrusts $c'a$ and $c''b$ which terminate at the points of support *a* and *b*. These final thrusts are resolved into the vertical components ac''' and bc_4, each equal to half the weight, and the horizontal tensions ad' and bd''. These horizontal tensions, which are half the tension exerted in the middle bay, are transmitted to the middle bay $a'b'$, where they balance each other. The middle bay is thus subjected to two tensile stresses:—the stress due to the thrust of the weight on the middle diagonals ca' and cb', measured by $a'b'$, the length of a bay; and the transmitted stress excited at the supports, measured by ad' or bd'', the length of half a bay. The total tension on the

middle bay is therefore measured by 1½ times the length of a bay, and that on each side bay by half a length, or a third of the tension in the middle bay.

The compressive stress in each of the upper bays cc', cc'', is measured by the length of a bay, and is twice the stress in the end lower bay.

The successive stresses in the lower and upper bays consecutively, it is seen, increase uniformly from each end towards the middle, thus:—

Tablet *a*.

Bays in compression,........	—	$c'c$	—	cc''	—
Bays in tension,...............	aa'	—	$a'b'$	—	$b'b$
Stresses as	1	2	3	2	1

At the same time the function of the diagonals, or braces, is to transmit the incidence of the weight to the supports, by compression and tension alternately.

Invert this girder, as in Fig. 299, and suspend the weight from the inverted apex c. The stresses in the several members are of the same intensity, but reversed, thus:—

Tablet *b*.

Bays in compression,........	aa'	—	$a'b'$	—	$b'b$
Bays in tension,...............	—	$c'c$	—	cc''	—
Stresses as	1	2	3	2	1

Let the girder, Fig. 298, be doubled in length, to comprise six lower bays, and five upper bays, as in Fig. 300; and loaded at the middle. The

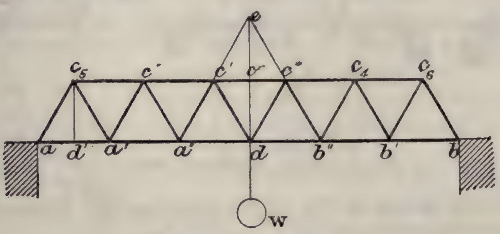


Fig. 300.—Warren-Girder.

horizontal stresses in the flanges are accumulated from each end towards the middle, where they are a maximum, as in tablet c.

Tablet *c*.

Bays in compression, ..	—	c_5c'''	—	$c'''c'$	—	$c'c''$	—	$c''c_4$	—	c_4c_6	—
Bays in tension,.........	aa'	—	$a'a''$	—	$a''d$	—	db''	—	$b''b'$	—	$b'b$
Stresses as..............	1	2	3	4	5	6	5	4	3	2	1

Valuation of the horizontal stress in terms of the load.—The unit-stress 1, in the tablet c, is measured by ad', Fig. 300, the horizontal component of the oblique thrust c_5a, of which c_5d' is the vertical component, or half the weight. The value of the unit-stress relatively to that of c_5d', or half

the weight, may be measured by means of a scale of parts. Or, trigonometrically, let the angle at a be signified by α, then,—

$$c_5 d' : a d' : : \text{sine } \alpha : \text{cosine } \alpha : : \tfrac{1}{2} \text{W} : \text{unit-stress; and}$$

$$\text{unit-stress at } a = \tfrac{1}{2} \text{W} \frac{\text{cosine } \alpha}{\text{sine } \alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In the Warren-girder, the angle α is 60°; and

$$\text{unit-stress at } a = \tfrac{1}{2} \text{W} \frac{.500}{.866} = \tfrac{1}{2} \text{W} \times .577; \text{ or}$$

$$\text{unit-stress at } a = .2885 \text{ W} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

showing that the horizontal unit-stress in each of the end bays is equal to the weight at the centre multiplied by .2885.

The stresses in the other bays, above and below, are in simple proportion to their numerical order from the support at each end towards the centre:—

$$\text{stress on any bay} = \text{unit-stress} \times \text{N}, \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which N is the order-number of the bay. The stress on the central bay is also expressed by the equation,—

$$\text{stress on the central bay} = \text{unit-stress} \times \frac{n+1}{2}, \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which n is the total number of bays. Also,

$$\text{stress on the middle pair of bays} = \text{unit-stress} \times \frac{n-1}{2} \ldots\ldots\ldots (5)$$

In the example, Fig. 300, the stress on the central bay $c' c''$, by formula (3) or (4), is,—

$$(.2885 \text{ W} \times 6), \text{ or } (.2885 \text{ W} \times \frac{11+1}{2}) = 1.731 \text{ W}; \ldots\ldots\ldots\ldots (a)$$

and the stress in the central pair of bays is,—

$$(.2885 \text{ W} \times 5), \text{ or } (.2885 \text{ W} \times \frac{11-1}{2}) = 1.443 \text{ W}. \ldots\ldots\ldots\ldots (b)$$

It appears that the stress at the middle of the longer boom is greater than the stress at the middle of the shorter boom by one unit-stress.

Valuation by moments.—The tension in the central bay is given by the expression (A), page 699, namely $\frac{\text{W}\, l}{4\, d}$, in which W is the weight, l the span, = 6 bays, and d the depth = .866 (sine α) proportionally, the length of a bay being 1. The tension is, then, $\frac{\text{W} \times 6}{4 \times .866} = 1.732$ W, as already found (a).

Stress in the braces.—The stress in the braces is to half the weight, as the length of a brace is to the depth of the girder, or as radius to sine α, therefore,—

$$\text{Stress in each brace} = \tfrac{1}{2} \text{W} \times \frac{1}{\text{sine } \alpha} = \frac{\text{W}}{2 \text{ sine } \alpha} \quad \ldots\ldots\ldots (6)$$

In the Warren-girder, sine α = .866, and

$$\text{the stress in the brace (Warren-girder) is } \frac{\text{W}}{2 \times .866} = .577 \text{ W}, \ldots (7)$$

which is twice the unit-stress in the flange.

The Warren-Girder Loaded at an Intermediate Point other than the Centre.

In a Warren-girder, Fig. 301, loaded at d, as in an ordinary loaded beam, the weight on the supports at a and b are respectively

$$\frac{n}{l}\,W \text{ and } \frac{m}{l}\,W, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

in which l is the total number of bays in the longest flange, and m and n the number of bays to the left and to the right of the weight.

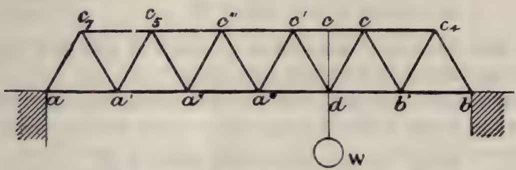


Fig. 301.—Warren-Girder, loaded at any intermediate point.

Stress in the Braces.—The stresses in the braces $d\,c'$ and $d\,c''$, which immediately support the weight, are as n and m, or inversely as the lengths of the two segments; and, adapting formula (6),

$$\text{Stress in the braces of the longer side} = \frac{n}{l} \times \frac{W}{\text{sine } \alpha} \quad \ldots\ldots\ldots \quad (9)$$

$$\text{Do.} \qquad \text{do.} \qquad \text{shorter do.} = \frac{m}{l} \times \frac{W}{\text{sine } \alpha} \quad \ldots\ldots\ldots \quad (10)$$

Sine $\alpha = .866$, and in this example the stresses are,

$$\text{In the longer side} = \frac{2}{6} \times \frac{W}{.866} = .385\ W, \quad \ldots\ldots\ldots\ldots \quad (c)$$

$$\text{In the shorter side} = \frac{4}{6} \times \frac{W}{.866} = .770\ W, \quad \ldots\ldots\ldots\ldots \quad (d)$$

transmitted to the supports a and b, and there resolved into vertical and horizontal components.

Second Process for the stress in the braces—Unit-coefficient of diagonal stress.—The sum of the stresses (c) and (d) is 1.155 W, which bears to the weight W the ratio of the length of a brace to the depth of the girder; since 1. : .866 : : 1.155 : 1. Divide the coefficient 1.155 by the number of diagonals, $2\,l$ or 12, and the quotient .09625 is a unit-coefficient per diagonal. Multiply this unit-coefficient by $2\,m$ and $2\,n$, or the number of diagonals in the longer and the shorter sides:

$$\begin{array}{lr} .09625 \times 4 \text{ diagonals} = & .385 \\ .09625 \times 8 \quad \text{do.} \quad = & .770 \\ \hline & 1.155 \end{array}$$

The products are the coefficients (c) and (d). This process for arriving at the stresses in the diagonals is the simplest where a number of calculations are to be made for one girder.

Horizontal Stress in the Booms.—The unit-stress at each end of the girder, adapting formula (1), is as follows:—

$$\text{Unit-stress at support } a = \frac{n'}{l} \text{W} \frac{\text{cosine } \alpha}{\text{sine } \alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$\text{Do.} \qquad \text{support } b = \frac{m'}{l} \text{W} \frac{\text{cosine } \alpha}{\text{sine } \alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

In Fig. 301 the number of bays m' and n' are respectively four and two bays, to the left and to the right; together, six bays $= l$. Then,

$$\text{Unit-stress at support } a = \frac{2}{6} \text{W} \times .577 = .192 \text{ W};$$

$$\text{Do.} \qquad \text{support } b = \frac{4}{6} \text{W} \times .577 = .385 \text{ W};$$

and the unit-stress at b is equal to twice the unit-stress at a.

The stress in the intermediate bays, between each support and the weight, is as before (formula 3),

For the long end $a\,d$, (unit-stress at a) $\times$ N; .... (14)
For the short end $b\,d$, (do.   at b) $\times$ N; .... (15)

in which N is the order-number of the bay, on either side of the weight, reckoned from the point of support at the same side.

The successive stresses thus calculated are given in the following tablet f, in which the unit-stress at a is taken as 1, and that at b is proportionally as 2.

Tablet f (Fig. 301).

In compression	—	$c_7\, c_5$	—	$c_5\, c'''$	—	$c'''\, c'$	—	$c'\, c''$	—	$c''\, c_4$	—
In tension	$a\, a'$	—	$a'\, a''$	—	$a''\, a'''$	—	$a'''\, d$	—	$d\, b'$	—	$b'\, b$
The horizontal stresses are as.................	1	2	3	4	5	6	7	8	6	4	2

The maximum stress is in the bay $c'\, c''$, over the weight, in compression.

The tensile stress in the two bays $a'''\, d$ and $d\, b'$, contiguous to the weight, are as 7 and 6 respectively, and they do not balance each other. But, as a matter of fact, there *is* a balance of stress, and it is completed by the difference of the horizontal components of the stresses in the two braces $d\,c'$, $d\,c''$, from which the weight is directly suspended, being respectively equal to the unit-stresses for the long and short ends. The difference of these is $2 - 1 = 1$, or one unit-stress in the direction $d\,b$; and $(6 + 1 =)7$ is the total stress in the bay $d\,b'$, which balances the opposite stress, also 7, in the bay $d\,a'''$.[1]

The Warren-Girder Uniformly Loaded.

A uniform load on a Warren-girder is, in fact, a load equally divided and applied to the apices of the web, as, for example, in Fig. 302, in which the

[1] With this explanation, it may be said with propriety that the sum of the increments of stress on the one side of the weight is equal to the sum of the increments of stress on the other side. But, abstractly, it is an erroneous assumption.

shorter flange, which is uppermost, comprises six apices, on which the weights W′, W″, &c., are placed.

Stress in the Braces.—The stress in the braces caused by each weight may be calculated separately in the manner already explained. The unit-

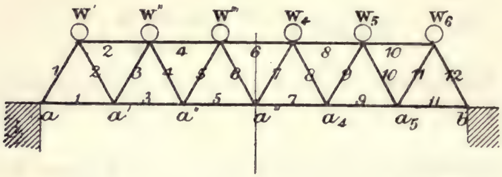


Fig. 302.—Warren-Girder, uniformly loaded—Longer Flange Undermost.

coefficient of stress for one diagonal in 12, as in Fig. 302, is $1.155 \div 12 = .09625$. The stresses caused by the weight W′, which is 1 diagonal from a, and 11 diagonals from b, are,

In the brace 1 .... $.09625 \times 11$ diagonals $= 1.058\ W_1$;
In the braces 2 to 12...... $.09625 \times 1$ do. $= .096\ W_1$.

Calculating, in the same way, the stresses caused by the other weights, the constituents of stress on each diagonal are obtained, the coefficients of which are given in the following table, No. 249, in which compressive stress is distinguished as +, and tensile stress as −. The resulting coefficient of stress in each brace is given in the second last column:—

Table No. 249.—Coefficients of Stress in the Braces of a Warren-Girder Uniformly Loaded, with the Longer Flange Undermost. Fig. 302.

Ratio of Segments of Girder.	W′ 1 to 11.	W″ 3 to 9.	W‴ 5 to 7.	W4 7 to 5.	W5 9 to 3.	W6 11 to 1.	Resultant Stress in each Diagonal.	Units of Resultant Stress.
braces.							in parts of W′.	
1	+1.058	+.866	+.674	+.481	+.289	+ .096	+3.464	6
2	+ .096	−.866	−.674	−.481	−.289	− .096	−2.310	4
3	− .096	+.866	+.674	+.481	+.289	+ .096	+2.310	4
4	+ .096	+.289	−.674	−.481	−.289	− .096	−1.155	2
5	− .096	−.289	+.674	+.481	+.289	+ .096	+1.155	2
6	+ .096	+.289	+.481	−.481	−.289	− .096	±0.000	0
7	− .096	−.289	−.481	+.481	+.289	+ .096	±0.000	0
8	+ .096	+.289	+.481	+.674	−.289	− .096	+1.155	2
9	− .096	−.289	−.481	−.674	+.289	+ .096	−1.155	2
10	+ .096	+.289	+.481	+.674	+.866	− .096	+2.310	4
11	− .096	−.289	−.481	−.674	−.866	+ .096	−2.310	4
12	+ .096	+.289	+.481	+.674	+.866	+1.058	+3.464	6

When the longer flange is uppermost, as in Fig. 303, and the weights are applied to the upper apices, one weight is supposed to be divided into

halves, of which one half is placed directly over each support, leaving 5 weights supported by the girder. The stresses in the braces, of which

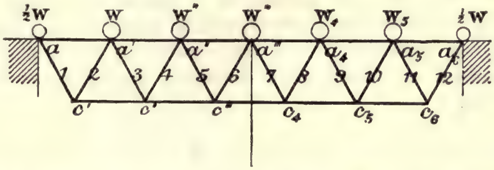


Fig. 303.—Warren-Girder, uniformly loaded—Longer Flange Uppermost.

there are 12, as in Fig. 302, being calculated in terms of the unit-coefficient .09625, the constituent and resulting stresses are given in table No. 250:—

Table No. 250.—Coefficients of Stress in the Braces of a Warren-Girder Uniformly Loaded, with the Longer Flange Uppermost. Fig. 303.

Ratio.	W' 2 to 10.	W'' 4 to 8.	W''' 6 to 6.	W_4 8 to 4.	W_5 10 to 2.	Resultant Stress in each Brace.	Units of Resultant Stress.
braces.						in parts of W'.	
1	−.963	−.770	−.577	−.385	−.193	−2.888	5
2	+.963	+.770	+.577	+.385	+.193	+2.888	5
3	+.193	−.770	−.577	−.385	−.193	−1.732	3
4	−.193	+.770	+.577	+.385	+.193	+1.732	3
5	+.193	+.385	−.577	−.385	−.193	− .577	1
6	−.193	−.385	+.577	+.385	+.193	+ .577	1
7	+.193	+.385	+.577	−.385	−.193	+ .577	1
8	−.193	−.385	−.577	+.385	+.193	− .577	1
9	+.193	+.385	+.577	+.770	−.193	+1.732	3
10	−.193	−.385	−.577	−.770	+.193	−1.732	3
11	+.193	+.385	+.577	+.770	+.963	+2.888	5
12	−.193	−.385	−.577	−.770	−.963	−2.888	5

The unit of resultant stress in the braces in these tables, Nos. 249 and 250, is taken as .577 W', being the stress caused in a brace by a half-weight (formula (7), page 701); and the respective values of the stress are expressed in units of that value in the last column of the tables.

In the girder, Fig. 302, having the longer flange undermost, the stress on the middle pair of braces is = 0, and on the successive pairs towards the supports each way, the stress increases in arithmetical progression, thus:—

On braces............... 1 ... 2 & 3 ... 4 & 5 ... 6 & 7 ... 8 & 9 ... 10 & 11 ... 12
The units of stress are 6 ... 4 ... 2 ... 0 ... 2 ... 4 ... 6

In the girder, Fig. 303, having the longer flange uppermost, the stress in the braces increases also in arithmetical progression, but by a different distribution, being as 1 in the two middle pairs, thus:—

On braces............... 1 & 2 ... 3 & 4 ... 5 & 6 ... 7 & 8 ... 9 & 10 ... 11 & 12
The units of stress are 5 ... 3 ... 1 ... 1 ... 3 ... 5

The maximum stress in the braces exists at the extremities, and amounts to 6 units in the first girder, and 5 units in the second; these are the stresses due to half the load on each girder, or 6 half-weights and 5 half-weights respectively. The stress in any intermediate brace is that due to the number of half-weights between it and the centre of the girder. Putting n'' equal to the number of half-weights between the brace and the centre of the Warren-girder,

$$\text{Stress in a given brace} = .577 \times 2\,n''\,W'$$
$$= 1.155\,n''\,W' \quad \ldots\ldots\ldots\ldots \quad (16)$$

In its general form, for any angle of brace, made with the flange, the formula is a modification of formula (6), page 701:—

$$\text{Stress in a given brace} = \frac{n''\,W}{\text{sine }\alpha} \quad \ldots\ldots\ldots\ldots \quad (17)$$

The braces which meet at an unloaded apex are equally stressed:—one by compression, the other by tension.

Stress in the Flanges.—The flanges receive increments of stress at each apex, advancing from the supports to the centre, where the total stress is a maximum. The increment of stress at any apex is equal to the horizontal component of the resultant of the two resultant diagonal stresses at the apex.

Radius : cosine α : : resultant diagonal stress : horizontal component,

and therefore,

$$\text{Horizontal component} = \text{diagonal stress} \times \text{cosine }\alpha; \ldots\ldots \quad (18)$$

that is to say, each unit of resultant diagonal stress, or .577 W′, causes a unit of horizontal stress, or .2885 W′ (formula (2), page 701).

The process of deducing the horizontal stresses from the diagonal stresses, and summing them up, is shown in the following analyses:—

WARREN-GIRDER UNIFORMLY LOADED—ANALYSIS OF STRESS IN FLANGES.

Longer Flange Undermost, Fig. 302—*Half of Girder.*

1. No. of braces, and No. of bays,	1	2	3	4	5	6
2. Units of resultant stress in braces,	+6	−4	+4	−2	+2	0
3, 4. Resultant stress of braces at apices, ...	—	6+4	4+4	4+2	2+2	2
or	—	10	8	6	4	2
5. Horizontal components of these, or increments of horizontal stress in bays, units,..	—	10	8	6	4	2
6. Accumulated increments of stress in bays, units,..........	—	10	18	24	28	30
7. Total horizontal stress in bays, units,...........................	6	16	24	30	34	36

Longer Flange Uppermost, Fig. 303—*Half of Girder.*

1. No. of braces, and No. of bays,	1	2	3	4	5	6
2. Units of resultant stress in braces,	−5	+5	−3	+3	−1	+1
3, 4. Resultant stress of braces at apices,	—	5+5	5+3	3+3	3+1	1+1
or	—	10	8	6	4	2
5. Horizontal components of these, or increments of horizontal stress in bays, units,..	—	10	8	6	4	2
6. Accumulated increments of stress in bays, units,..........	—	10	18	24	28	30
7. Total horizontal stress in bays, units,........................	5	15	23	29	33	35

The resultant stress at the central apex, between braces 6 and 7, is, in both cases, equal to 0; and therefore there is no increment of horizontal stress at the centre.

The increment of horizontal stress in the central bay, No. 6, is, in both cases, equal to 2 units, and the increments increase by 2 units, from bay to bay, up to bay No. 2, where the increment amounts to 10 units.

So that, inversely, the increments of horizontal stress in the flange diminish in arithmetical progression as they approach the centre.

Let n = the number of braces, or the total number of bays, in half the girder;

N = the order-number of a given bay, counted from the end of the girder, above and below;

W′ = the weight on one apex, for which the unit of horizontal stress, transmitted through one of a pair of diagonals, is .2885 W′;

The equations, for the horizontal stress in the given bay, based upon the foregoing analysis, are as follows:

1st. *When the longer flange is undermost:—*

Horizontal stress in a given bay $= .2885\,W'(n + (N-1)(2n - N))$...... (19)

2d. *When the longer flange is uppermost:—*

Horizontal stress in a given bay $= .2885\,W'((n-1) + (N-1)(2n - N))$ (20)

These formulas are very easy of application. The reasoning by which they have been constructed by the author is given in the foot-note.[1]

[1] The increments of horizontal stress, at the several bays, line 5, in the "Analysis of stress," are in arithmetical progression, having the common difference, 2, originating at the centre. The order-number of the terms of the progression, counting from the centre, is expressed by $(n-(N-1))$; and $(n-(N-1))\times 2$, is the value of the increment in units. For the first increment, for example, in bay No. 2, N = 2, and $n = 6$; and the value of the increment is $(6-(2-1))\times 2 = 10$, as given in the analysis. If N′, N″, N‴, &c., represent for the moment the successive order-numbers of the bays following No. 2 bay, the values of the successive accumulated increments of stress, line 6, are as follows:—

In No. 2 bay, $(n-(N-1))\times 2$
No. 3 ,, $(n-(N-1))+(n-(N'-1))\times 2$
No. 4 ,, $(n-(N-1))+(n-(N'-1))+(n-(N''-1))\times 2$,

and so on. The value for each bay, putting N for the order-number of the bay, and condensing the expression, is—

Rolling Load on the Warren-Girder.

Concentrated Rolling Load on the Warren-Girder.—The stress caused in successive diagonals by a passing weight is alternately tensile and compressive; and the stress is a maximum when the load is on the apex.

Distributed Rolling Load on the Warren-Girder.—Suppose that the rolling load is practically of uniform distribution, as a railway train, the stresses in the diagonals and flanges may be tabulated and analyzed, as exemplified at pages 704 and 705. If the train extend over the whole of the girder, the case becomes one of a girder uniformly loaded. The stresses in partially-covered girders may be analyzed in like manner, and the changes in direction and intensity of stress determined.

But it is essential, at the same time, that the stresses caused by the permanent weight of the bridge should be determined; since the actual ultimate stress in any member is the resultant of the action of the whole of the load, both permanent and passing. The maximum stress in the flanges takes place when the passing load covers the whole of the girder.

Parallel Lattice-Girder.

Latticing is the combination of two or more systems of triangulation in the web of a girder, in which the diagonals cross each other. The number of apices is proportionally multiplied, and the length of the bays is proportionally shortened. The effect is that the weight is distributed over a greater number of points in the flange, the graduations of stress in the flange are reduced, whilst also the stress in the diagonals is proportionally reduced. There is a special advantage in lattice-work, in affording the means of stiffening the braces by simple connections at the intersections.

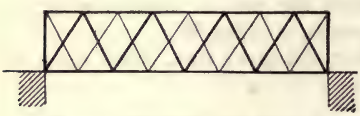


Fig. 304.—Parallel Lattice-Girder.

If the diagonal stresses be calculated, in the first instance, as for a single triangular system, let them be divided by the number of systems in the lattice; the quotient is the aliquot part of the stress, as distributed to each diagonal. The fundamental triangulation is shown by thick lines, Fig. 304; and in this instance, where only one additional system is interpolated, the stress in the fundamental diagonals is reduced to a half of what they would sustain if they stood alone.

The Parallel Strut-Girder.

In the parallel strut-girder, Fig. 305, having vertical and diagonal bracing, supporting a single weight, W, on the upper flange at the centre, the vertical

$$(n \times \overline{2 \times (N-1)}) - (N \times \overline{2 \times (N-1)}) + \frac{1+(N-1)}{2} \times 2 \times (N-1) = (N-1) \times (2(n-N)+N)$$

To this is to be prefixed the initial horizontal stress at bay No. 1, which is n units, or 6 units, for Fig. 302, and $(n-1)$ units, or 5 units, for Fig. 303. Thence the formulas (19) and (20):—

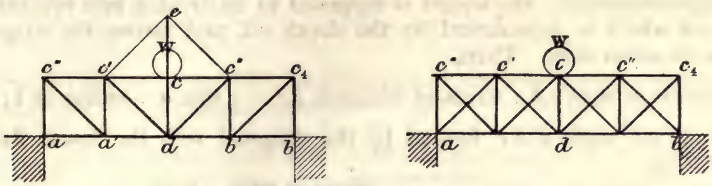
Horizontal stress in a given bay $= .2885\ W_1 (n + (N-1)(2n-N))$, ... (19)

when the longer flange is undermost; and

Horizontal stress in a given bay $= .2885\ W_1 (n-1) + (N-1)(2n-N))$, ... (20)

when the longer flange is uppermost.

brace or strut cd receives and supports the whole of the weight; and the compressive stress is divided and transmitted by tension through the diagonals dc' and dc'', according to the parallelogram of forces, of which



Figs. 305 and 306.—Parallel Strut-Girders.

the diagonal de, equal to twice dc, the depth of the girder, represents the weight. The depth dc represents half the weight, and the tensional stress in each of the diagonals dc' and dc'' is represented in magnitude and direction by the diagonals.

The tensions of these diagonals balance each other horizontally at their intersection at the lower flange at d, and thus they do not throw any horizontal stress on the lower flange.

Strut-Girder with a Concentrated Moving Weight.

When the load moves over the girder, each strut requires to be braced by a pair of diagonals intersecting at the foot of the strut, in the same way as the strut cd, under the fixed weight in Figs. 305 and 306, is braced by the diagonals dc' and dc''. The result is a system of cross-bracing, or counter-bracing, by crossed ties, as in Fig. 306. The extra braces at the outer struts ac', bc'' (Fig. 305), are not necessary, but they are introduced to complete the design. The maximum stress is imposed on each strut when the weight passes over its summit.

If the weight move on the lower flange, the maximum stress on a given strut is imposed when the weight passes the lower end of the next strut on the side of the more distant support; and the maximum stress on any strut, by the lower flange, never exceeds half the weight. (Fig. 305.)

The tension in the diagonal dc' is resolved into compressive stress in the upper bay $c'c$ and the strut $c'a'$. The compressive stress in the strut $c'a'$ is resolved into tensile stress in the lower bay $a'd$ and the outer diagonal $a'c'''$; and lastly, the stress in the outer diagonal $a'c'''$ is resolved into compressive stress in the outer bay $c'''c'$ and in the strut $c'''a$, of which the former is transmitted to the middle bay $c'c$.

A similar action takes place in the other half of the girder, and the horizontal stresses in one half balance those in the other.

The vertical stress in the lateral struts is obviously, by transmission, equal to ½ W. That is, the stress in the lateral struts is only half the stress on the central strut, which supports the whole of the weight. The compressive stress in the outermost struts, or half of the weight, is received and resisted by the supports at a and b. The outer bays of the lower flange, aa' and $b'b$, are not subjected to any transmitted stress.

The horizontal stresses in the bays of the upper and lower flanges are, then, in the following ratios:—

No. of bay	1,	2,	3,	4
Compressive stress in upper flange, as	1,	2,	2,	1
Tensile stress in lower flange, as	0,	1,	1,	0

Trigonometrically, the weight is supposed to be divided into two halves, each of which is represented by the depth cd, and causes the diagonal stress on either side. Then,

Stress in strut cd : stress in diagonal dc' : : sine α : radius or 1;

α being the angle $a'dc'$ formed by the diagonal with the lower flange. Therefore,

$$\text{Stress in each diagonal} = \frac{\text{stress in strut}}{\text{sine } \alpha} = \frac{\frac{1}{2}\text{W}}{\text{sine } \alpha}; \text{ or}$$

$$\text{Stress in each diagonal} = \frac{\text{W}}{2 \text{ sine } \alpha} \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

When the distance apart of the struts is equal to the depth of the girder, the angle $\alpha = 45°$, and sine $\alpha = .707$; then,

$$\text{Stress in every diagonal} = \frac{\text{W}}{2 \times .707} = .707 \text{ W}.$$

In the upper flange, the stress caused by each diagonal being as the half weight dc to the length of a bay cc'', or as sine α to cosine α; then

$$\text{Stress caused in each upper bay} = \tfrac{1}{2}\text{W} \frac{\text{cosine } \alpha}{\text{sine } \alpha} \ldots\ldots \quad (22)$$

When the angle $\alpha = 45°$, cosine α = sine α; and

Stress caused in each upper bay = ½ W, and
Stress accumulated in middle upper bay = W.

In the lower flange, the middle lower bays are in tension = ½ W, due to the thrust of the struts at a' and b'.

If the extreme bays of the lower flange be removed, and the girder be supported direct at the ends of the upper flange, as in Fig. 307, the stresses

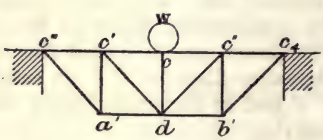


Fig. 307.—Parallel Strut-Girder.—Lower end bays removed.

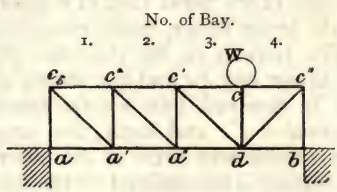


Fig. 308.—Parallel Strut-Girder, loaded at an intermediate point.

in the structure remain unaltered, since there is no horizontal stress in the extreme bays of the parallel girder, Fig. 305. It was seen that the function of the end struts was only to support the girder.

Strut-Girder Loaded at an Intermediate Point, off the Centre.

The strut-girder, Fig. 308, four bays in length, is loaded at cd, one bay from the support b, and three bays from the support a. Let l = the total

number of bays, and m' and n' = the numbers of bays to the left and to the right of the weight. The loads on the supports at a and b, as well as the compressive stress on the struts to the right and left, are respectively—

$$\frac{n'}{l}\text{W, and }\frac{m'}{l}\text{W}, \qquad (23)$$

being in the inverse ratio of the distances of the weight from the supports; or they are, in the present example—

Stress in the struts of the longer side................ ¼ W,
Do. do. shorter side................ ¾ W.

The stresses in the diagonals are in the same proportion, thus:—

$$\text{Stress in the diagonals of the longer side} = \frac{n'}{l}\,\frac{\text{W}}{\text{sine }\alpha} \qquad (24)$$

$$\text{Do. do. shorter side} = \frac{m'}{l}\,\frac{\text{W}}{\text{sine }\alpha} \qquad (25)$$

If the angle $\alpha = 45°$, then sine $\alpha = .707$, and the stresses are,

In the diagonals of the longer side......... $\frac{1}{4} \times \frac{\text{W}}{\cdot 707} = \cdot 3535$ W,

Do. do. shorter side......... $\frac{3}{4} \times \frac{\text{W}}{\cdot 707} = 1.0605$ W.

The unit or increment of horizontal stress in the bays of the upper and lower flanges is as follows:—

$$\text{Unit of stress in the bays of the longer side, } \frac{n'}{l}\text{W}\,\frac{\text{cosine }\alpha}{\text{sine }\alpha} \qquad (26)$$

$$\text{Do. do. shorter side, } \frac{m'}{l}\text{W}\,\frac{\text{cosine }\alpha}{\text{sine }\alpha} \qquad (27)$$

When the angle $\alpha = 45°$, $\frac{\text{cosine }\alpha}{\text{sine }\alpha} = 1$, and the unit of stress is,

In the bays of the longer side............................ ¼ W,
Do. do. shorter side .... ¾ W,

the accumulated stress in the several bays is, by formula (3), page 701,

$$\text{For the longer side} \ldots\ldots \frac{n'}{l}\text{W}\,\frac{\text{cosine }\alpha}{\text{sine }\alpha} \times \text{N} \qquad (28)$$

$$\text{For the shorter side} \ldots\ldots \frac{m'}{l}\text{W}\,\frac{\text{cosine }\alpha}{\text{sine }\alpha} \times \text{N} \qquad (29)$$

in which N is the order-number of the bay, in the upper flange, on either side of the weight, reckoned from the point of support. For the lower flange, N is the order-number less 1, seeing that, as before explained, there is no transmitted horizontal stress in the lower bays situated next the points of support.

The resultant stresses in the several bays are, in the example, Fig. 308, relatively, as follows:—

No. of bay	1,	2,	3,	4
Compressive stress in upper flange as	1,	2,	3,	3
Tensile stress in lower flange as	0,	1,	2,	0

Here it is apparent that, whilst the resultant stresses in the bays 3 and 4 of the upper flange balance each other, there is no tensile stress in No. 4 bay of the lower flange to balance the stress in No. 3. But the balance is supplied by the difference of the horizontal components of the diagonal stresses which meet below the weight at *d*.

Strut-Girder Uniformly Loaded.

The general conditions of stress in the strut-girder uniformly loaded, as in Fig. 309, are similar to those in the Warren-girder, as elucidated, page 704.

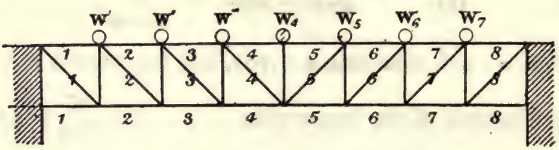


Fig. 309.—Strut-Girder uniformly loaded.

Stress in the Struts.—The stress is calculated by an adaptation of formula (17), page 706, in which sine α becomes $= 1$, seeing that the angle of the strut with the flange is a right angle. The formula becomes,

$$\text{Stress in a given strut} = n'' \text{W}, \quad \ldots\ldots\ldots\ldots \quad (30)$$

in which n'' = the number of weights between the strut and the centre of the girder.

Stress in the Diagonals.—This stress is found by formula (17)—

$$\text{Stress in a given diagonal} = \frac{n'' \text{W}}{\text{sine } \alpha} \quad \ldots\ldots\ldots\ldots \quad (31)$$

For illustration, the diagonals 1 and 8 carry the seven weights W′ to W_7 suspended between them; and each sustains half the number, or 3½ W, and the stress in each is $3\frac{1}{2} \frac{\text{W}}{\text{sine } \alpha}$. Similarly, the diagonals 2 and 7 carry the five weights W″ to W_6 between them, each sustaining 2½ weights, or 2½ W′; and the stress in each is $2\frac{1}{2} \frac{\text{W}}{\text{sine } \alpha}$. The diagonals 3 and 6 carry the weights W‴ W_4 W_5 between them, each supporting 1½ W′, and the stress is $1\frac{1}{2} \frac{\text{W}}{\text{sine } \alpha}$. Lastly, the diagonals 4 and 5 carry the weight W_4 between them, each supporting ½ $W_{,,}$, and the stress is $\frac{1}{2} \frac{\text{W}}{\text{sine } \alpha}$. No diagonal stress is transmitted across the centre of the girder: in this respect the strut-girder differs from the Warren-girder.

Stress in the Flanges.—The stress in the flanges increases by diminishing increments at each apex towards the centre, where it is a maximum. The increments consist of the horizontal components of the diagonal stresses developed at each apex. The accumulated horizontal stress in each bay is expressed by the following formulas, which have been constructed in a manner similar to that by which formulas (19) and (20) were constructed:—

Let n = the number of bays in the length of half the girder;
N = the order-number of a given bay, in the upper or the lower flange,
α = the angle between the diagonal and the flange,
W′ = the weight on one strut, for which the unit of horizontal stress in the flange, transmitted through the next diagonal, is $W' \frac{\text{cosine } \alpha}{\text{sine } \alpha}$.

For the horizontal stress in a given bay:—

1st. *In the Upper Flange:—*

$$\text{Stress in a given bay} = W' \frac{\text{cosine } \alpha}{\text{sine } \alpha} N \left(n - \frac{N}{2}\right) \quad (32)$$

2d. *In the Lower Flange:—*

$$\text{Stress in a given bay} = W' \frac{\text{cosine } \alpha}{\text{sine } \alpha} (N - 1) \times \left(n - \frac{N-1}{2}\right) \quad (33)$$

When the distance apart of the struts is equal to the depth of the girder, $W' \frac{\text{cosine } \alpha}{\text{sine } \alpha} = W'$.

The gradation of stress in the flanges may be given for Fig. 309, containing 8 bays.

No. of diagonal and bays,	1	2	3	4	
Increments of stress in diagonals, (unit = $\frac{W'}{\text{sine } \alpha}$)	3½	2½	1½	½	units.
Increments of stress in bays of upper flange (unit = $W' \frac{\text{cosine } \alpha}{\text{sine } \alpha}$)	3½	2½	1½	½	units.
Accumulated stress in do. do.,	3½	6	7½	8	units.
Increments of stress in bays of lower flange,	0	3½	2½	1½	units.
Accumulated stress in do. do.,	0	3½	6	7½	units.

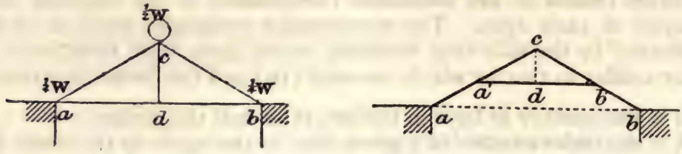
Strut-Girder Traversed by a Load Uniformly Distributed.

The struts require to be counter-braced, and the stresses are calculated as in the immediately preceding case.

Roofs.

1. The weight of and load upon a roof are taken as uniformly distributed over the surface of the roof; and the total weight on each pair of rafters, couple, or truss, is equal to the sum of the weight of the truss itself, and of so much of the roof as is carried between two trusses.

In the triangular roof-truss, *a b c*, Fig. 310, the total weight, W, may be considered as localized at the supports *a* and *b*, and the ridge *c*: a fourth



Figs. 310 and 311.—Triangular Roof-Trusses.

each at *a* and *b*, and a half at *c*. The tension in *a b* is that due to the weight, ½ W, at *c*, which is, by equation (A), page 699, ½ W $l \div 4\,d$, in which l is the span, and d the depth; or

$$\frac{W\,l}{8\,d} \qquad (34)$$

Or the tension in the tie-member *a b*, under a uniformly distributed weight, is equal to the product of the weight by the span, divided by 8 times the rise.

The horizontal thrust at the ridge *c*, is equal to the tension in the horizontal tie.

The rafters *c a* and *c b* are subject to two stresses:—1st. Compressive thrust, as pillars, by the weight; the thrust is cumulative, beginning as nothing at the apex, and ending at the maximum for the whole weight at the abutments *a* and *b*, where it is equal to $(\frac{1}{2}\,W \times \frac{c\,a}{c\,d})$. 2d. Transverse stress from the weight, ½ W, uniformly distributed, reduced in the ratio of the slant height *a c* to the half-span *a d;* the moment of which is equal to, $\frac{W\,l}{4 \times (a\,c)}$.

2. When the horizontal tie is applied at any higher level, *a′ b′*, Fig. 311, the tension in it is inversely as the depth *c d*, according to the expression (34). In addition to the stresses in the rafters, already noticed in the previous case, there are compressive and transverse stresses excited by the pull of the tie *a′ b′*.

3. In the A-truss roof, Fig. 312, the stresses are mixed. Let the span be 40 feet, the rise 10 feet, and the depth *c d* 8 feet. The rafters *a c* and *b c*

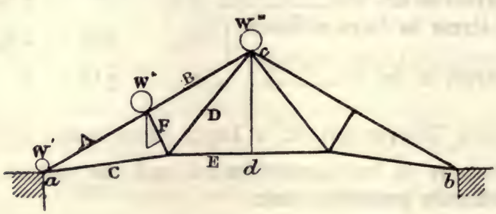


Fig. 312.—A-Truss Roof.

are 22.5 feet long, the struts F are 3.33 feet long, and the tension bars C and D 11.75 feet long. The weight on the couple is 8 tons, uniformly distributed, of which 4 tons is supported on each rafter, say 1 ton at *a* over the abutment, 2 tons at F, and 1 ton at the ridge *c*.

The pressure, 2 tons on F, being vertical, is resolved, as indicated by diagram, into 1.8 tons stress on F, and .875 tons on A. The stress on F is resolved into 3.18 tons in C and in D, tie-rods.

The tension in E is by formula (34), $\frac{8 \text{ tons} \times 40 \text{ feet}}{8 \times 8 \text{ feet}} = 5$ tons; and it is resolved into $4\frac{3}{4}$ tons in C, and .875 tons in F. This tension in F is resolved into 1.54 tons in C and in D.

The total stresses in the three tension-rods C, D, and E are, then, as follows:—

			Totals.
In C,	stress through F by direct weight of roof,..	3.18 tons.	
	stress by tensile force of E,....................	4.75 "	
	stress from F by do. do.,.......................	1.54 "	9.47 tons.
In D,	stress through F by direct weight of roof,..	3.18 "	
	stress from F by tensile force of E,.........	1.54 "	4.72 "
In E,	stress,..		5.00 "

So much by way of analysis. But Mr. Stoney shows a method of deducing the stresses, by starting from the stress on the abutment and working thence towards the centre. Referring to Fig. 312, and adopting the same data as above, the reaction of the left abutment is 4 tons, of which 1 ton is directly balanced by the weight W′ concentrated there, leaving 3 tons to be resolved in the directions of A and C, into 10.35 tons and 9.47 tons respectively. The pressure of W″, 2 tons, is resolved into 1.8 tons on F and .875 tons on A; and (10.35 − .875 =) 9.475 tons is the thrust in B. At *a*, the stresses in C and F, which are known, are resolved by the intermediate substitution of their resultant into the stress 4.72 tons in D, and 5 tons in E.

4. In Fig. 313, the middles of the rafters are strutted by struts *c′ d* and *c″ d*, meeting at *d* in the horizontal tie-bar, and tied to the ridge by the vertical rod *c d*. The weight of the roof is localized at *a*, *c′*, *c*, *c″*, and *b*, in the proportions $\frac{1}{8}$ W, $\frac{1}{4}$ W, $\frac{1}{4}$ W, $\frac{1}{4}$ W, and $\frac{1}{8}$ W. In the truss *a c′ d*, the weight on *c′* is equally sustained by the limbs *c′ a* and *c′ d*, $\frac{1}{8}$ W being borne by the abutment, and $\frac{1}{8}$ W being transmitted through the tie-rod *c d* to the ridge *c*. As $\frac{1}{8}$ W is also transmitted from *c″*, in the right hand rafter, the total weight at the ridge is ($\frac{1}{4}$ + $\frac{1}{8}$ + $\frac{1}{8}$) W = $\frac{1}{2}$ W. This is just what the ridge would have borne, without the intervention of the struts; and the function of the struts is chiefly to assist the rafters in resisting transverse stress. The pull in the vertical tie-rod *c d* is ($\frac{1}{8}$ + $\frac{1}{8}$ =) $\frac{1}{4}$ W.

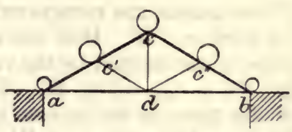


Fig. 313.—A-Truss Roof.

To find the stresses in the rafters and the horizontal member *a b*, Dr. Rankine distinguishes the main truss *a c b* and the secondary trusses *a c′ d* and *d c″ b*. The tension in *a b* is the sum of the tensions due to the first and second trusses; the thrust in *a c′*, likewise, is the sum of the thrusts, and that in *c′ c* is the thrust of the first truss only. Suppose the span l = 20

feet, and the rise $d = 7\frac{1}{2}$ feet; the pull in $a\,d$ by the first truss is by formula (A), $\frac{\frac{1}{2} W l}{4 d} = \frac{1}{3}$ W; and by the second truss, $\frac{\frac{1}{4} W \times \frac{1}{2} l}{4 \times \frac{1}{2} d} = {}^1/_6$ W. The sum of these pulls is $(\frac{1}{3} + {}^1/_6 =)\ \frac{1}{2}$ W, the resultant tension in $a\,b$ or $a\,d$.

The thrust in $a\,c$ by the first truss, is to the relative tension in $a\,d$, as $a\,c$ to $a\,d$. The length of $a\,c = \sqrt{(a\,d)^2 + (c\,d)^2} = 12.5$ feet, and the thrust is, $\frac{1}{3} W \times \frac{12.5}{10} = .417$ W. In the second truss, the thrust in $a\,c'$ is the same fraction of the tension in $a\,d$, due to the truss; or it is ${}^1/_6 W \times \frac{12.5}{10} =$ .242 W. The sum of the first and second thrusts, or .658 W, is the resultant thrust in $a\,c'$.

The value of the thrust in $a\,c$, by the first truss, may be found in terms of the relative tensions in $a\,d$ and $c\,d$; for it is equal to

$\sqrt{\text{tension in } (a\,b)^2 + \text{tension in } (c\,d)^2} = \sqrt{(\frac{1}{3})^2 + (\frac{1}{8})^2} = .417$; as has already been found.

5. In Fig. 314, each rafter is divided into three equal parts, which are supported by two struts, $c'\,d$ and $c'''\,d'$ for the left-hand rafter, and $c_4\,d''$ and $c''\,d$ for the right-hand rafter; suspended by vertical rods $c'\,d'$, $c\,d$, and $c''\,d''$; united by the main tie-rod $a\,b$. The total weight W, uniformly distributed, is localized at a, c''', c', c, c'', c_4, b in the proportions, ${}^1/_{12}$ W, ${}^1/_6$ W, ${}^1/_6$ W, ${}^1/_6$ W, ${}^1/_6$ W, ${}^1/_6$ W, ${}^1/_{12}$ W.

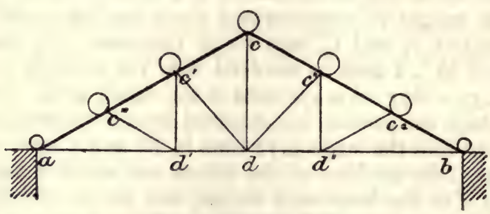


Fig. 314.—Trussed Roof.

Three trusses are recognized here: the first truss $a\,c\,b$, the second $a\,c'\,d$, and the third $a\,c'''\,d'$. Half the stress at c''', the summit of the isosceles or third truss, is transmitted by the vertical rod $c'\,d'$ to c', where the stress is increased to $({}^1/_6 + {}^1/_{12} =)\ \frac{1}{4}$ W. This load is transmitted to a and d' at the base of the truss, in the inverse ratio of the segments $a\,d'$ and $d'\,d$; that is, two thirds, or $(\frac{1}{4} \times \frac{2}{3} =)\ {}^1/_6$ W, is transmitted through the strut $c'd$ and rod $c\,d$, to c. An equal quantity, ${}^1/_6$ W, is transmitted from the right-hand rafter, and the sum ${}^1/_3$ W added to ${}^1/_6$ W, makes $\frac{1}{2}$ W, the resultant load at the ridge.

Suppose the span $l = 60$ feet and the rise $d = 15$ feet, the pull in $a\,b$ or $a\,d$ due to the first truss, is by formula (A), $\frac{\frac{1}{2} W l}{4 d} = \frac{1}{2}$ W; by the second truss, the pull is ${}^8/_9$ ths of what it would be if the truss were isosceles, or it is $\frac{\frac{1}{4} W \times \frac{1}{2} l}{4 \times \frac{2}{3} d} \times {}^8/_9 = \frac{W l}{24 d} = {}^1/_6$ W; by the third truss, it is $\frac{{}^1/_6 W \times {}^1/_3 l}{4 \times {}^1/_3 d} = \frac{W l}{24 d} = {}^1/_6$ W. The sum of the three pulls, or $(\frac{1}{2} + {}^1/_6 + {}^1/_6 =)\ {}^{20}/_{24}$ W,

is the resultant stress in *a d′* and *d″ b;* and the sum of the first and second pulls is the resultant stress in *d′ d″*, or $\frac{2}{3}$ W.

The pull in the main tie-rod *a b*, due to the second strut, was said to be $^8/_9$ ths of what it would have been for an isosceles truss. In general terms, the fraction of what it would be for an isosceles truss of the same height and length is the ratio of the product of the segments into which the tie-rod or base of the truss is divided by the vertical rod from its apex, to the square of half the base. In this instance the base is divided into $\frac{2}{3}$ and $^1/_3$, and $^2/_3 \times {}^1/_3 = {}^2/_9$; also $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ or $^2/_8$; and the ratio of $^2/_9$ to $^2/_8$ is 8 to 9, or $^8/_9$ ths.[1]

The thrusts in the rafters may be found by the method already applied, in the previous case; and the same general process is applicable to roofs of more extensive construction. Professor Rankine gives general equations, for the stresses in roofs of the strut-and-rod class, Fig. 314; and Mr. Stoney shows how the stresses may be found in employing the parallelogram of forces, from the pressure on an abutment.[2]

By the application of the principle of the parallelogram of forces, the stresses in crescent and other forms of girders and roofs may be determined.

[1] See page 508, at top.
[2] *Civil Engineering*, page 472. *The Theory of Strains*, page 159.

WORK, OR LABOUR.

UNITS OF WORK OR LABOUR.

The fundamental units of work—the foot-pound and the kilogrammetre—have been defined at page 312; and their relations with those of horse-power have been stated at page 158.

Horse-power.—Horse-power is a measure of the rate at which work is performed. One horse-power is the expression of 33,000 foot-pounds of work done per minute, or 550 foot-pounds of work done per second. It is nearly identical with the French horse-power (*cheval-vapeur*, or *cheval*), which is equal to 75 kilogrammetres of work done per second. As a kilogrammetre is equal to 7.233 foot-pounds, the "*cheval*" is equal to ($75 \times 7.233 =$) 542.5 foot-pounds of work per second, which is 1.37 per cent. less than the English measure of a horse-power.

Mechanical equivalent of heat.—The values of the mechanical equivalent of heat in English and in French measures are defined at page 332, and their relations are stated at page 159. An English unit of heat is the quantity which is required to raise the temperature of water at or near 39.1° F., the temperature of its maximum density, through 1° F.; and its mechanical value or equivalent is equal to 772 foot-pounds. One horse-power is therefore equivalent to ($33{,}000 \div 772 =$) 42¾ heat-units per minute.

A French unit of heat is equal to that which is required to raise the temperature of 1 kilogramme of water through 1° C.; and its mechanical equivalent is 424 kilogrammetres = 3063.5 foot-pounds.

Labour of Men.

Mr. Smeaton concluded that the power of an ordinary labourer at ordinary work was equivalent generally to work done at the rate of 3762 foot-pounds per minute. But, according to a particular estimate made by him for pumping up water 4 feet high, by good English labourers, their power was equivalent to 3904 foot-pounds of work per minute; and this he estimated as twice that of ordinary persons "promiscuously picked up."

Mr. John Walker found that the force exerted by an ordinary labourer in raising weights for driving piles, average daily work, was 12 lbs. In working daily at a winch or a crane-handle, the average force was 14 lbs. moving at the rate of 220 feet per minute, equivalent to ($14 \times 220 =$) 3080 foot-pounds per minute.

Mr. Glynn says that a man may exert a force of 25 lbs. at the handle of a crane for short periods; but that, for continuous work, a force of 15 lbs. is all that should be assumed, moving through 220 feet per minute. The power of a man would thence be ($15 \times 220 =$) 3300 foot-pounds per minute.

Mr. G. B. Bruce states that, in average work at a pile-driver, a labourer exerts a force of 16 lbs., plus the resistance of the gearing, at a velocity of 270 feet per minute, for 10 hours a day, making one blow every four minutes. The power is $(16 \times 270 =)$ 4320 foot-pounds per minute.

Mr. Joshua Field, in 1826, tested the performances of men at a crane of rough construction, in ordinary use. The barrel was 11¾ inches in diameter to the centre of the chain; and the driving-gearing consisted of two pinions and two wheels, geared successively, with an 18-inch handle. The ratio of the power to the weight was 1 to 105. The loads were thus so proportioned as to be reduced successively to from 10 to 35 lbs. at the handle; frictional resistance being additional. The load was raised through a height of 16½ feet in each experiment. The results were as follows:—

Table No. 251.—Power of Men at a Crane.

No. of Experiment.	Statical Resistance of the Load at the Handle.	Load Raised.	Time in Raising.	Equivalent Power in Foot-pounds per Minute.	Remarks.
	lbs.	lbs.	minutes.	ft.-lbs.	
1	10	1050	1.5	11,550	Easily done by a stout Englishman.
2	15	1575	2.25	11,505	Tolerably easily by the same man.
3	20	2100	2.0	17,325	Not easily by a sturdy Irishman.
4	25	2625	2.5	17,329	With difficulty by a stout Englishman.
5	30	3150	2.5	20,790	Do. by a London man.
6	35	3675	2.2	27,562	With the utmost difficulty by a tall Irishman.
7	"	"	2.5	24,255	Do. do. by a London man.
8	"	"	2.83	21,427	With extreme labour by a tall Irishman.
9	"	"	3.0	20,212	With very great exertion by a sturdy Irishman.
10	"	"	4.05	15,134	With the utmost exertion by a Welshman.
11	"	"	—	—	Given up at this time by an Irishman.

Mr. Field states that No. 4 gave a near approximation to the maximum power of a man for 2½ minutes. In all the succeeding trials, the men were so much exhausted as to be unable to let down the load.

It would appear from this table that the maximum net pressure at the handle for constant working would not exceed 15 lbs., exclusive of frictional resistance. The loads were only from ½ to 1½ tons,—much below the capacity of the crane; and the frictional resistance was disproportionally great for the work of one man. The author has found that when cranes were worked up to their capacity, the men could without difficulty exert a net pressure of 30 lbs. at the handle, exclusive of frictional resistance, for a short time. In one instance, he observed that a very strong man raised 23 cwt. by a 30 cwt. crane, when he exerted a net force of 100 lbs. at the handle. An ordinary man at the same crane, raised 14 cwt. with difficulty, with a net force of 56 lbs. at the handle; and the same man raised without difficulty 10 cwt., with a force of 40 lbs. at the handle.

A man can exert on the handle of a screw-jack, of say 11 inches radius, a net force of 20 lbs., without difficulty.

From the foregoing data, it appears that the average net daily work of an ordinary labourer at a pump, a winch, or a crane, may be taken at

3300 foot-pounds per minute; and, allowing one-third more for the frictional resistance of the machine or apparatus, the total work done would be at the rate of 4400 foot-pounds per minute.

It may be added that, taken generally, well-fed English labourers can turn a crank by hand, at the rate of from 25 to 30 turns per minute, for a continuance, exerting a pressure of 20 lbs. at the handle; or they can apply a pressure of 28 or 30 lbs. for a short time, or from 50 to 56 lbs. at an emergency.

M. Cornet's estimate of the work of a labourer in France, turning a crank, amounts to 6 kilogrammetres per second, equivalent to 2604 foot-pounds per minute, for 8 hours a day above ground, or 6 hours a day in a mine. This, it is presumed, is the net work.

Labour of Horses.

According to Messrs. Boulton & Watt's estimate of the power of a dray-horse, it could do work equivalent to 33,000 foot-pounds per minute, for 8 hours a day.

Tredgold estimated the work of a horse at 27,000 foot-pounds per minute, for 8 hours a day.

Simms tested the labour of horses in raising water:—

23,412	foot-pounds	per minute, for	8	hours	a day.
24,360	"	"	6	"	"
27,056	"	"	4½	"	"
32,943	"	"	3	"	"

He preferred the performances for 6 hours and 3 hours a day, as they were unobjectionable to the health and durance of the horses.

Rennie found that a horse weighing 11 cwts. could draw a canal boat at a speed of 2½ miles per hour, with a pull of 108 lbs., over a distance of 20 miles per day. This performance is equivalent to a work of 23,760 foot-pounds per minute. He estimated that the average work of horses, strong and weak, is at the rate of 22,000 foot-pounds per minute.

Mr. Beardmore found that a horse eight years old, weighing 10¾ cwts., performed 39,320 foot-pounds of work per minute, for 8 hours a day.

It is inferred from the foregoing data, that the maximum work done by an average horse, per day of 8 hours, is at the rate of 25,000 foot-pounds per minute. At the same time, it appears from the results of trials at Bedford, to be noticed subsequently, that the average work of a horse is 20,000 foot-pounds per minute. See page 963.

Good horses can draw a load of 1 ton at the rate of 2½ miles per hour, during from 10 to 12 hours.

Mr. A. Wilson found that, in India, a pair of well-fed bullocks raised 82 bags of water 22 feet high in 1 hour, for a morning's work of 4½ hours. Each bag contained 4¼ cubic feet of water, and the work was equivalent to 8000 foot-pounds per minute.

Work of Animals in Carrying Loads.[1]

Men—Carrying by Hand.—Labourers wheeling millstone in barrows, on the quays of Paris, a distance of 22 to 27 yards, making 25 to 30 trips per

[1] Data derived from *Les Moyens de Transport*, by M. Alfred Evrard, 1872, vol. i.

hour; the daily work performed is equivalent to the carriage of from 330 to 400 lbs. 1 mile.

The following are other cases of daily labour, showing the useful weight carried 1 mile:—

In Belgium, working in couples, one man carries 560 lbs. 1 mile.

At Port Royal, loading pig-iron, one man carries 160 lbs. 1 mile.

At Paris, loading sugar-loafs, 86 lbs.

Men—Carrying on the Back.—Carrying tiles or bricks, net load 106 lbs.; day's work 600 lbs. carried 1 mile.

Carrying coal in mines, net load 90 to 100 lbs.; day's work 344 lbs. 1 mile. Another case, net load 100 to 130 lbs.; day's work 340 lbs. 1 mile.

Loading coke into waggons, net load 100 lbs.; day's work 270 lbs. 1 mile.

Discharging coke on the ground, net load 100 lbs.; day's work 330 lbs. 1 mile.

Discharging coal on the ground, Port Royal; net load 106 lbs.; day's work 370 lbs. 1 mile.

Discharging coal on the ground, Paris; net load 110 lbs.; day's work 560 to 960 lbs. 1 mile.

Charging small coal into boats, Rive-de-Gier; 190 lbs. net load; day's work 1230 lbs. 1 mile.

On the back of a Horse.—The load carried by a horse on its back varies generally from 220 to 390 lbs., about 27½ per cent. of the weight of the animal.

Carrying a man of 176 lbs. weight, at about 3½ miles per hour; day's work 4400 lbs. 1 mile.

Carrying a load of 260 lbs. for 10 hours at 2½ miles per hour, 6540 lbs. 1 mile.

Trotting with a man of 176 lbs. weight, at 5 miles per hour, for 7 hours; day's work, 6100 lbs. 1 mile.

On the back of a Mule.—At Buenos Ayres; net load, 170 to 220 lbs.; day's work 6400 lbs. 1 mile.

In Spain, net load 400 lbs. at 2.9 miles per hour; day's work 5300 lbs. 1 mile.

In France, net load 330 lbs. at 2 miles per hour; day's work 5000 lbs. 1 mile.

On the back of an Ass.—Load 176 lbs., carried 19 miles; day's work 3300 lbs. 1 mile.

The asses in Syria can carry from 450 to 550 lbs. of grain.

On the back of a Camel.—Load 550 lbs. carried 30 miles per day for 4 days, resting on the 5th day. For 4 days, day's work 16,500 lbs. 1 mile. For 5 days, 13,000 lbs. 1 mile.

The ordinary load for a dromedary is 770 lbs.

On the back of a Lama.—Load 110 lbs.; day's work 2000 to 3000 lbs. 1 mile.

FRICTION OF SOLID BODIES.

The friction between surfaces pressed together, whether flat or round, of which one is moved on the other, is said to be in the direct ratio of the pressure, and to be independent of the velocity, and of the area of the surfaces pressed together, up to what may be called the elastic limit, or the

Table No. 252.—Friction of Journals in their Bearings.

Diameters from 2 to 4 inches. Speeds varied as 1 to 4. Pressures up to 2 tons nearly.

(Reduced from M. Morin's data.)

Description of Surfaces in Contact.	Lubricant.	Coefficient of Friction. Ordinary Lubrication.	Coefficient of Friction. Continuous Lubrication.
Journals. Bearings.		pressure=1	pressure=1.
Cast iron on cast iron	Lard, olive oil, or tallow	.07 to .08	.03 to .054
	The same lubricants, and wetted	.08	—
	Asphalte	.054	—
	Surfaces unctuous	.14	—
	Unctuous and wetted	.14	—
Cast iron on gun metal	Lard, olive oil, or tallow	.07 to .08	.03 to .054
	Surfaces unctuous	.16	—
	Unctuous and wetted	.16	—
	Slightly unctuous	.19	—
Cast iron on lignum vitae	Wood slightly unctuous	.18	—
	Oil, or lard	—	.09
	Unctuous	.10	—
	Unctuous, with mixture of lard and plumbago	.14	—
Wrought iron on cast iron	Olive oil, tallow, or lard	.07 to .08	.03 to .054
Wrought iron on gun metal	Olive oil, tallow, or lard	.07 to .08	.03 to .054
	Grease	.09	—
	Unctuous and wetted	.19	—
	Slightly unctuous	.25	—
Wrought iron on lignum vitae	Oil, or lard	.11	—
	Unctuous	.19	—
Gun metal on gun metal	Oil	.10	—
	Lard	.09	—
Lignum vitae on cast iron	Lard	.12	—
	Unctuous	.15	—
Lignum vitae on lignum vitae	Lard	—	.07

limit beyond which abrasion takes place. Friction is of two kinds:—sliding or rubbing, and rolling; and the frictional resistance to the commencement of motion, after two bodies have rested sometime in contact, is greater than the friction between bodies of which one is already in motion upon the other.

M. Morin's experiments afford the principal available data for use. Though the constancy of friction holds good for velocities not exceeding 15 or 16 feet per second; yet, for greater velocities, the resistance of friction appears, from the experiments of M. Poirée, in 1851, to be diminished in some proportion as the velocity is increased.

Table No. 253.—FRICTION:—RESULTS OF EXPERIMENTS.

(Reduced from M. Morin's data.)

Description of Surfaces in Contact.	Disposition of Fibres.	State of the Surfaces.	Coefficient of Friction.
			pressure=1.
Oak on oak...........................	parallel	dry	.48
Do.	„	soaped	.16
Do.	perpendicular	dry	.34
Do.	on end, on side	„	.19
Different woods on oak..........	parallel	„	.36 to .40
Wrought iron on oak.............	„	„	.62
Do. do.	„	wetted	.26
Do. do.	„	soaped	.21
Cast iron on oak..................	„	dry	.49
Do. do.	„	wetted	.22
Do. do.	„	soaped	.19
Brass on oak..........................	„	dry	.62
Wrought iron on elm.............	„	„	.25
Cast iron on elm...................	„	„	.20
Leather on oak......................	„	„	.30 to .35
Do. do.	„	wetted	.29
Leather belt on oak (flat).........	„	dry	.27
Do. on oak pulley.......	perpendicular	„	.47 (after resting in contact.)
Leather on cast iron & on bronze	—	„	.56
Do. do. do.	—	wetted	.36
Do. do. do.	—	unctuous and wet	.23
Do. do. do.	—	oiled	.15
Wrought iron on cast iron.......	—	slightly unctuous	.18
Do. on bronze	—	„	.18
Cast iron on cast iron............	—	„	.15
Do. on bronze...............	—	„	.15
Bronze on bronze.................	—	dry	.20
Do. on cast iron...............	—	„	.22
Do. on wrought iron	—	slightly unctuous	.16
Oak, elm, yoke elm, cast iron, wrought iron, steel, and bronze, sliding on each other, or on themselves......	—	lubricated in the ordinary way, with lard, tallow, oil, &c.	.07 to .08
		continuously lubricated	.05
		slightly unct'ous	.15

It appears from the table No. 252 that the frictional resistance of metal journals revolving in metal bearings is uniform for all metals, with one exception; and that the resistance, with continuous lubrication, is only 56 per cent. of the resistance with ordinary lubrication:—

Journal. Bearing.	Lubrication.	Coefficient.
Cast iron in cast iron...... Cast iron in gun metal....	ordinary	.07 to .08, or $^1/_{14}$ to $^1/_{12.5}$ mean .075, or $^1/_{13.3}$
Wrought iron in cast iron Wrought iron in gun metal	continuous	.03 to .054, or $^1/_{33}$ to $^1/_{18.5}$ mean .042, or $^1/_{24}$

Gun metal in gun metal; ordinary, .10, or $^1/_{10}$th.

Additional data derived from the friction of mill-shafting, are given under Shafting.

Friction on Rails.

M. Poirée's experiments on the Paris and Lyons railway were made with a waggon, presumably having four wheels, of which the brake was screwed up, so that the wheels were skidded. The resistance to traction, or the friction on the rails, at various velocities, was as follows:—

Table No. 254.—Sledging Friction of a Waggon on Rails.

M. Poirée's Coefficients.

Empty Waggon, 3.40 tons. Velocity of Waggon.		State of the Rails.				
		Dry.	Very Dry.	Damp.	Dry and Rusty.	Dry. Springs gagged.
feet per second.	miles per hour.	weight=1.	weight=1.	weight=1.	weight=1.	weight=1.
13 to 20	9 to 14	.208	—	—	.201	—
20 to 26	14 to 18	.179	.246	—	.182	.200
26 to 33	18 to 22	.167	—	.110	.175	—
33 to 46	22 to 30	—	.222	—	.162	.172
46 to 60	30 to 40	.144	.202	—	—	.154
60 to 72	40 to 50	—	.187	.083	.136	.132

Comparative Coefficients of Friction for Different Weights; Rails Dry.

Speed, 30 feet per second; or 20 miles per hour.					
Empty waggon, 3.40 tons	.175	—	—	—	—
Loaded waggon, 6.45 „	.169	—	—	—	—

At speeds under 20 miles per hour, it appears from the table that, when the rails are dry, the coefficient of friction, or the adhesion, is one-fifth of the weight, and that on very dry rails it is one-fourth. As the speed is increased, the adhesion is reduced.

These data are corroborative of the results of the author's experiments on the ultimate tractive force of locomotives on dry rails, from which he obtained a coefficient of friction equal to one-fifth of the weight, at speeds of about 10 miles per hour.

They are corroborated by the experience of train-brakes on the District Railway, London. The gripping force of the brake is greatest just before the train is brought to a state of rest.

It is seen from the table that, on damp rails, the coefficient of friction, at 20 miles per hour, was reduced to one-ninth.

In the second part of the table, it is seen that the coefficient of friction increases in a ratio rather less than that of the weight.

Work Absorbed by Friction.

The product of the total pressure between the rubbing surfaces by the coefficient of friction, is the total frictional resistance; and the product of this resistance by the space through which it acts, is the work done, or absorbed, by friction. Let—

W = the load or pressure, in pounds,
f = the coefficient of friction between the two surfaces,
s = the space passed through by one surface on the other, in feet,
t = the time in minutes,
v = the velocity at the surface, in feet per minute,
S = the speed of revolution, or number of turns per minute,
U = the work absorbed, in foot-pounds,
H = the horse-power absorbed,
d = the diameter of the axle-journal, or of the pivot, in inches,
r = the radius of the axle-journal, or of the pivot, in inches,
α = half the angle at the apex of a conical journal or a conical pivot,
l = the axial length of the rubbing surface of a conical pivot, in inches.

The space described by a cylindrical journal for one turn, is equal to $3.14\,d$, in inches, or to $.26\,d$, in feet; and by the flat end of a cylindrical pivot, the space described is equal to two-thirds of this quantity, in the ratio of the mean diameter to the extreme diameter, or $.175\,d$, in feet. For a conical pivot, the mean diameter is, as for a flat pivot, two-thirds of the extreme diameter of the rubbing surface of the pivot, and the space described for one turn is expressed by $.175\,d$, in feet. But the pressure on the surface of the conical pivot, compared with the pressure on a flat pivot, is greater in the ratio of the slant length of the pivot to the extreme radius of the rubbing surface, or as radius to sine α; or as $\sqrt{r^2+l^2}$ to r.

The pressure on the surface of a conical journal, is to that on a cylindrical journal of the same length and extreme diameter, as radius to cosine α.

Work Absorbed by Friction.

1. On a flat surface.............................. $U = fW \times s$ .... (1)
2. On a cylindrical journal, for one turn..... $U = fW \times .26\,d$..... (2)
3. On the square end of a cylindrical pivot, for one turn.............................. $U = fW \times .175\,d$... (3)
4. On a conical journal, for one turn...... $U = \dfrac{fW \times .26\,d}{\cos\alpha}$..... (4)
5. On a conical pivot, for one turn........ $U = \dfrac{fW \times .175\,d}{\text{sine}\,\alpha}$... (5)

The second formula, (2), is applicable to the cases of friction-couplings or friction-brakes.

The horse-power absorbed by continuous frictional action on a flat surface, or surface of other form, is equal to the product of the resistance by the velocity, divided by 33,000. On a journal or a pivot, it is equal to the work absorbed in one turn, multiplied by the speed, and divided by 33,000.

Horse-Power Absorbed by Friction.

1. On a flat surface......... $\left\{ H = \dfrac{f W \times v}{33{,}000} \right.$ .... (6)

2. On a cylindrical journal $\left\{ H = \dfrac{f W S \times .26\, d}{33{,}000} = \dfrac{f W S d}{127{,}000} \right.$ .... (7)

3. On a cylindrical pivot.. $\left\{ H = \dfrac{f W S \times .175\, d}{33{,}000} = \dfrac{f W S d}{189{,}000} \right.$ .... (8)

4. On a conical journal.... $\left\{ H = \dfrac{f W S \times .26\, d}{33{,}000 \times \cos \alpha} = \dfrac{f W S d}{127{,}000 \times \cos \alpha} \right.$ ...... (9)

5. On a conical pivot...... $\left\{ H = \dfrac{f W S \times .175\, d}{33{,}000 \times \sin \alpha} = \dfrac{f W S d}{189{,}000 \times \sin \alpha} \right.$ ...... (10)

MILL-GEARING.

TOOTHED GEAR.

In a pair of toothed wheels, or pinions, geared together, the relative angular velocities or speeds, or number of turns per minute, are inversely as their diameters, or as their radii; and in a train of wheels and pinions, like that in Fig. 315, suppose the axle A makes 27 turns per minute, and that the diameters of the wheel and pinion through which it drives the axle C, are as 7 and 3, the axle C makes $(27 \times \frac{7}{3} =)$ 63 turns per minute. Again, the axle C drives the axle D by a wheel and a pinion having diameters also as 7 to 3, and the speed of D is $(63 \times \frac{7}{3} =)$ 147 turns per minute. Finally, the axle B is driven by D by gearing as 7 to 3, and D makes $(147 \times \frac{7}{3} =)$ 343 turns per minute. The successive accelerations of speed are, then, as follows:—

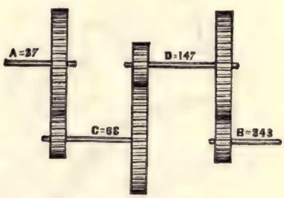


Fig. 315.—Toothed Gear.

Axle A driving axle C	...	as 3 to 7
„ C „ D	...	„ 3 to 7
„ D „ B	...	„ 3 to 7
Total acceleration	... as	27 to 343

Here, the total ratio of acceleration is found by multiplying together the first terms of the ratios, and the last terms of the ratios; equivalent to the ratio of 1 to 12.7, since $343 \div 27 = 12.7$.

It is seen that, in this example, the acceleration of speed takes place by equal additions, in the ratio of 3 to 7. To find what this common ratio must be, when the initial and final speeds alone are given, take the cube root of the compound ratio of total acceleration, that is,

$$\sqrt[3]{\frac{343}{27}} = \frac{7}{3}; \text{ or, } \sqrt[3]{\frac{12.7}{1}} = \frac{2.333}{1};$$

giving the common ratio 3 to 7, as already employed, or 1 to 2⅓, which is the same.

The third root is taken, because there are three accelerations of speed. If there were only two accelerations, the square root would be taken; if

four accelerations, the fourth root. In general, to find the common ratio, take that root of the total ratio, the index of which is equal to the number of accelerations. The same rule applies in cases of reduction of speed.

When the speeds are increased by ratios which are different from each other, the ratios are nevertheless to be multiplied together, as already exemplified, to find the resultant ratio.

In making this multiplication, in the above example, the ratio 3 to 7 might have been replaced by the equivalent ratio 1 to $2\frac{1}{3}$. This ratio multiplied twice by itself, produces the total ratio 1 to 12.7; and $27 \times 12.7 = 343$, the final speed in turns per minute.

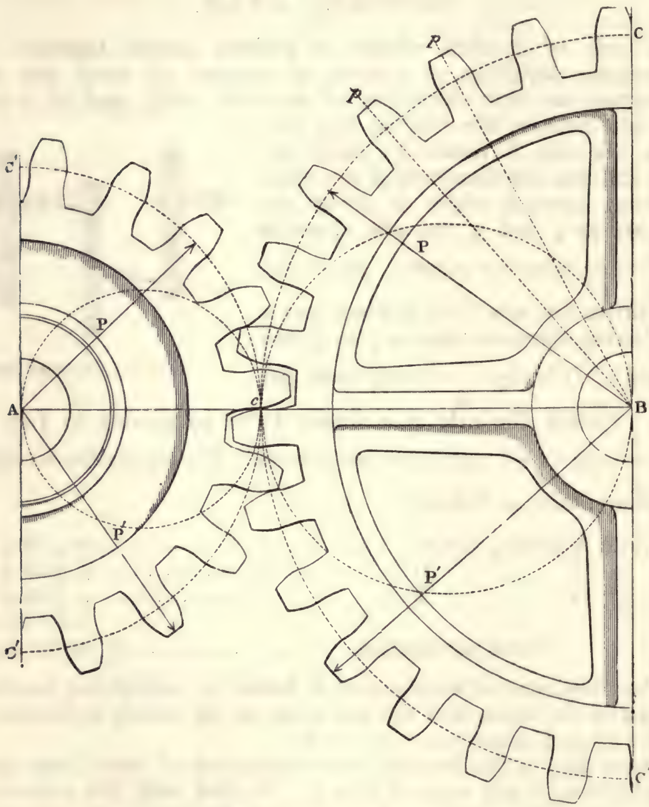


Fig. 316.—Toothed Gear.

The numbers of teeth in the several wheels and pinions of each pair, are necessarily proportional to their diameters; and they may be employed instead of the diameters, to express the ratios; and be multiplied together to find the resultant ratio.

Pitch of the Teeth of Wheels.

The *pitch* of the teeth of wheels is the distances apart from centre to centre of the teeth, measured on the pitch-circle. The *pitch-circle*, or *pitch-*

line, is the circle passing through the body of the teeth, which expresses the virtual or normal circumference of the wheel; it would be the actual circumference if the teeth were indefinitely small, when the wheel and pinion might work together by frictional contact. In the annexed Fig. 316, showing the halves of a wheel and a pinion in gear, A B is the line of centres, and C C and C C are the pitch circles touching at *c*; the divisions A *c* and B *c*, of the line of centres, being the pitch-radii of the wheels. The arc of the pitch-circle, between *p* and *p*, is the pitch of the teeth, and it comprises a tooth and a space.

The numbers of teeth in the wheel and the pinion are in the same ratio as the diameters of their pitch-circles. Let N = the diameter of any wheel at the pitch-line, P = the pitch, n = the number of teeth; then,

$$n = \frac{3.1416}{P} \times D; \quad P = 3.1416 \times \frac{D}{n}; \quad D = n \times \frac{P}{3.1416} \ldots\ldots (1), (2), (3).$$

In ordinary practice, the pitches most commonly used range from 1 inch to 4 inches, advancing by eighths to 2 inches, and thence by fourths of an inch. Below 1 inch, the pitches decrease by eighths down to ¼ inch. Sir William Fairbairn employed the following pitches:—

Spur Fly-Wheels.—5, 4½, 4, 3½, 3¼, 3, 2½, 2, 1½ inches.

Spur and Bevil Wheels for Millwork.—5, 4½, 4, 3½, 3¼, 3, 2¾, 2½, 2¼, 2⅛, 2, 1¾, 1⅝, 1⅜, 1¼, 1⅛, 1, ⅞ inches.

For toolwork the pitches usually range from 1 inch to ¼ inch.

To save calculation in the application of the formulas (1) & (3), the values of $\frac{3.1416}{P}$, and $\frac{P}{3.1416}$, for particular pitches, are given in the table No. 255.

Table No. 255.—Toothed Wheels—Multipliers for the Number of Teeth and the Diameter.

(Rules 1 and 3).

Pitch.	Multiplier for the Number of Teeth.	Multiplier for the Diameter.	Pitch.	Multiplier for the Number of Teeth.	Multiplier for the Diameter.
inches.	$\frac{3.1416}{\text{pitch.}}$	$\frac{\text{pitch}}{3.1416.}$	inches.	$\frac{3.1416}{\text{pitch.}}$	$\frac{\text{pitch}}{3.1416.}$
6	.5236	1.9095	1⅝	1.9264	.5141
5	.6283	1.5915	1½	2.0944	.4774
4½	.6981	1.4720	1⅜	2.2848	.4377
4	.7854	1.2732	1¼	2.5132	.3979
3½	.8976	1.1141	1⅛	2.7926	.3568
3¼	.9666	1.0345	1	3.1416	.3183
3	1.0472	.9547	⅞	3.5904	.2785
2¾	1.1333	.8754	¾	4.1888	.2387
2½	1.2566	.7958	⅝	5.0266	.1989
2¼	1.3963	.7135	½	6.2832	.1592
2	1.5708	.6366	⅜	8.3776	.1194
1⅞	1.6755	.5937	¼	12.5664	.0796
1¾	1.7952	.5570			

Table No. 256.—Diameter of Toothed Wheels.

(Given the pitch and the number of teeth.)

Number of Teeth.	Pitch in Inches.									
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼
	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.
1	.318	.398	.477	.557	.637	.714	.796	.875	.955	1.04
2	.636	.796	.955	1.14	1.27	1.43	1.59	1.75	1.91	2.07
3	.955	1.193	1.43	1.67	1.91	2.14	2.39	2.63	2.86	3.10
4	1.27	1.591	1.91	2.23	2.55	2.85	3.18	3.50	3.82	4.14
5	1.59	1.989	2.39	2.79	3.18	3.57	3.98	4.38	4.77	5.17
6	1.91	2.387	2.86	3.34	3.82	4.28	4.78	5.25	5.73	6.21
7	2.23	2.785	3.34	3.90	4.45	4.99	5.57	6.13	6.68	7.24
8	2.55	3.182	3.82	4.46	5.09	5.71	6.37	7.00	7.64	8.28
9	2.86	3.580	4.30	5.01	5.73	6.42	7.16	7.88	8.59	9.31
10	3.18	3.98	4.77	5.57	6.37	7.14	7.96	8.75	9.55	10.35
20	6.36	7.96	9.55	11.14	12.73	14.27	15.92	17.51	19.10	20.69
30	9.55	11.93	14.32	16.71	19.10	21.41	23.87	26.26	28.64	31.04
40	12.73	15.91	19.10	22.28	25.46	28.54	31.83	35.02	38.19	41.38
50	15.91	19.89	23.87	27.85	31.83	35.67	39.79	43.77	47.74	51.73
60	19.09	23.87	28.64	33.42	38.20	42.81	47.75	52.52	57.29	62.07
70	22.27	27.85	33.42	38.99	44.56	49.94	55.71	61.28	66.84	72.42
80	25.46	31.82	38.19	44.56	50.93	57.08	63.66	70.03	76.38	82.76
90	28.64	35.80	42.97	50.13	57.29	64.21	71.62	78.78	85.93	93.11
100	31.83	39.78	47.74	55.70	63.66	71.35	79.58	87.54	95.48	103.45
110	35.00	43.76	52.51	61.27	70.03	78.48	87.54	96.29	105.03	113.80
120	38.18	47.74	57.28	66.84	76.39	85.62	95.50	105.05	114.58	124.14
130	41.36	51.72	62.06	72.41	82.76	92.75	103.45	113.80	124.12	134.50
140	44.54	55.70	66.84	77.98	89.12	99.89	111.41	122.56	133.67	144.83
150	47.73	59.67	71.61	83.55	95.49	107.03	119.37	131.31	143.22	155.18
160	50.91	63.65	76.38	89.12	101.86	114.16	127.33	140.06	152.77	165.52
170	54.10	67.63	81.16	94.69	108.22	121.29	135.29	148.82	162.32	175.87
180	57.28	71.60	85.93	100.26	114.59	128.43	143.24	157.57	171.86	186.21
190	60.46	75.58	90.71	105.83	120.95	135.57	151.20	166.33	181.41	196.56
200	63.64	79.56	95.48	111.40	127.32	142.70	159.16	175.08	190.96	206.90
210	66.81	83.55	100.26	116.97	133.68	149.82	167.13	163.84	200.52	217.26
220	70.00	87.52	105.02	122.54	140.06	156.96	175.08	192.58	210.06	227.60
230	73.19	91.49	109.80	128.11	146.42	164.11	183.03	201.34	219.60	237.94
240	76.36	95.48	114.56	133.68	152.78	171.24	191.00	210.10	229.16	248.28
250	79.55	99.45	119.35	139.25	159.15	178.37	198.95	218.85	238.70	258.63
260	82.72	103.44	124.12	144.82	165.52	185.50	206.90	227.60	248.24	269.00
270	85.92	107.40	128.91	150.39	173.87	192.63	214.86	236.34	257.79	279.33
280	89.08	111.40	133.68	155.96	178.24	199.78	222.82	245.12	267.34	289.66
290	92.28	115.36	138.45	161.53	184.61	206.91	230.78	253.86	276.89	300.01
300	95.49	119.34	143.22	167.10	190.98	214.05	238.74	262.62	286.44	310.35

Appendix to Table.—To find the diameters for pitches under 1 inch, namely,

¼ | 5/16 | ⅜ | 7/16 | ½ | 9/16 | ⅝ | 11/16 | ¾ | 13/16 | ⅞ inch,

refer to the columns in the table, respectively, for

1 | 1¼ | 1½ | 1¾ | 1 | 2¼ | 1¼ | 2¾ | 1½ | 3¼ | 1¾ inches;

and divide the quantities in these columns by the corresponding divisors, following:—

4 | 4 | 4 | 4 | 2 | 4 | 2 | 4 | 2 | 4 | 2

To find the diameter for pitches above 3¼ inches, namely,

3½ | 4 | 4½ | 5 | 5½ | 6 inches,

refer to the columns in the table, respectively, for

1¾ | 2 | 2¼ | 2½ | 2¾ | 3 inches;

and multiply the quantities in these columns by 2.

To find the diameter for any number of teeth between 10 and 300, not specified in the table. Find the diameters for the tens and hundreds in the number, and for the units; the sum of these diameters is the required diameter. For a wheel of 1-inch pitch, with 135 teeth, for example:—the diameter for 130 teeth is 41.36 inches, and for 5 teeth it is 1.59 inches; and 41.36 + 1.59=42.95 inches, the required diameter for 135 teeth.

RULE 1. *To find the Number of Teeth in a wheel of a given diameter and pitch.*—Multiply the diameter in inches, by the multiplier in the second column of the table, opposite the given pitch. The product is the number of teeth.

RULE 2. *To find the Diameter of a Wheel having a given number and pitch of teeth.*—Multiply the number of teeth by the multiplier opposite the given pitch, in the third column of the table. The product is the diameter in inches.

Note to Rule 1.—When the answer contains a fraction, the diameter requires to be slightly altered, so as to bring out a whole number of teeth. For this purpose, take the nearest whole number of teeth to the answer given by the rule, and find the modified diameter for that number, by Rule 2.

FORM OF THE TEETH OF WHEELS.

That the teeth of wheels and pinions may work together smoothly, steadily, without rubbing friction, and with uniform motion, they should be shaped to epicycloidal forms on the faces, and hypocycloidal forms on the flanks,—being the portions of the sides of the teeth respectively above and below the pitch-line. These forms are explained and exemplified at pages 19 and 20.

With respect to the formation of the flanks of the teeth of the wheel, when the generating circle has half the diameter of the pitch circle, the hypocycloid described by it is a straight diametrical line, and therefore the flanks are straight and radial to the centre, as in the Fig. 318, next page.

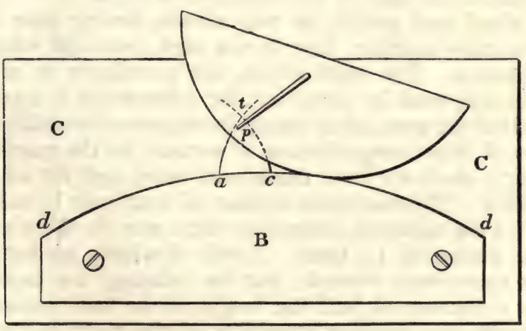


Fig. 317.—Formation of Teeth of Wheels.

Similarly, with a generating circle having half the diameter of the pitch-circle of the pinion, the flanks of the teeth of the pinion are also straight and radial. If now the same generating circles be employed to form the faces of the teeth of the other wheel or pinion respectively, the wheel and pinion will work truly together. For example, let the template B, Fig. 317, cut to the pitch-circle of the wheel, be screwed to a hardwood board C. Set off the thickness *a c* of a tooth, on the pitch-circle *d d*, and apply the segment D, of which the radius is equal to half the radius of the pinion. With a tracing point *p*, inserted obliquely at the edge of D, roll the segment D on the template B, so as to describe the epicycloidal curve *a t;* and, in the same way, describe the counterpart *c t.* The two faces of the tooth are

thus formed. To draw the flanks of the tooth, trace the arc *d d* on the board C C, and remove B; screw the board to a slip of deal *h*, Fig. 318, and find the centre of the pitch-circle, on the slip *h*; draw the radii *a b* and *c b*, to form the flanks, which are slightly rounded at the base *b*, to join the arc *f f*, drawn through the roots of the teeth. The end of the tooth is defined by the arc *g g*.

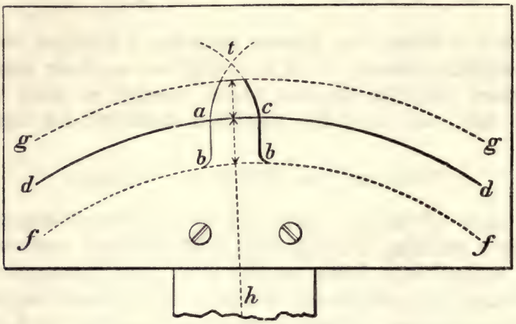


Fig. 318.—Formation of Teeth of Wheels.

Performing the same operation, conversely, for the teeth of the pinion, the wheel and the pinion so constructed work truly together. In the wheel and pinion, Fig. 316, are shown the generating circles, having diameters equal to the radii of the respective wheels.[1]

Whilst a wheel and pinion, or two wheels, having their teeth so constructed, work truly together, they do not work truly with wheels or pinions of other diameters. For bevil-wheels, this peculiarity is of no moment, since they can only work by pairs; but, for spur-wheels, it is convenient that any two wheels of the same pitch should be capable of working truly together. This property of interchangeableness is secured by the employment of the same generating circle for both flanks and faces, and for all diameters, as well as for racks. The minimum number of teeth may be taken as 12, and the diameter of the common generating circle may be taken as equal to the radius of the pinion of 12 teeth. Teeth of wheels so formed have the flanks rather excessively tapered; but, by reducing the taper, or thinning the tooth, for a distance of half the height of the flank measured from the root of the tooth, the working durability of the tooth is much increased, and steadiness of action is promoted even when the tooth has become considerably worn. If the full form of the flank of the tooth be retained, a shoulder is gradually worn into it, by which a tendency is induced to force the wheel and pinion out of gear; but in cutting away the flanks near the base, whilst the working portion of the flank-surface remains, shoulders can only be formed after very excessive wear.[2]

Involute Teeth.—The teeth of two wheels will work truly together, when their acting surfaces are involutes. The involute curve may be described

[1] There is an error in the form of the flanks of the teeth as shown in Fig. 316; they should have been straight and radial.

[2] Mr. Robert Wilson, of Patricroft, employs this method of forming the teeth of spur-wheels. The author is indebted to him for the above particulars about it.

mechanically:—Let A, Fig. 319, be the centre of a wheel, and *m n a* a thread lapped round its circumference. The curve *a b h*, described by a pin at the end of the thread when unwound from the circle, is an involute.

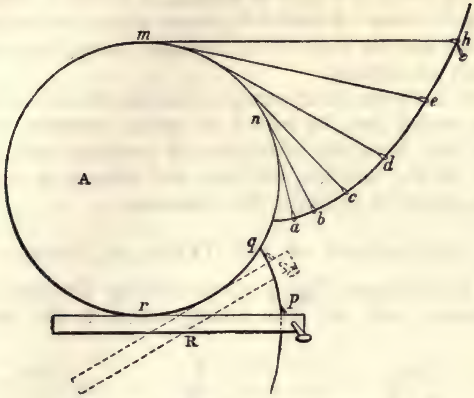


Fig. 319.—Formation of Teeth of Wheels.

The curve is also formed by causing a straight ruler R to roll on the circle, Fig. 319, with a pin at the end, tracing the involute *q p*.

That two wheels with involute teeth should work truly, the circles from

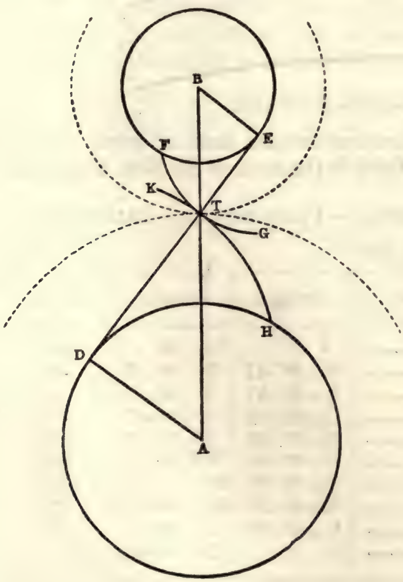


Fig. 320.

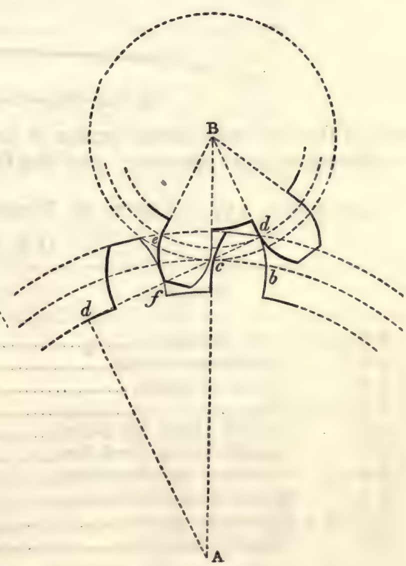


Fig. 321.

Formation of Teeth of Wheels.

which the involute forms for each wheel are generated, must be concentric with the wheels, with diameters in the same ratio as those of the wheels. Let A T, B T, Fig. 320, be the pitch-radii of two wheels to work together;

through T draw any straight line D E, and with the perpendiculars A D and B E, describe the circles D H and E F. The involutes K T H and G T F give the forms of the teeth.

To describe the teeth of a pair of wheels, of which A *c* and B *c*, Fig. 321, are the pitch radii, draw *c d* and *c d* perpendicular to the radials B *d* and A *d*; these radials are the radii of the involute circles from which the acting faces of the teeth are formed.

Involute teeth have the disadvantage of being, when in contact, too much inclined to the radial line, by which an undue pressure is excited on the bearings. But they have the advantage of working truly, even at varying distances apart of the centres, and any two wheels of a pitch will work together in sets, however different the diameters.

Proportions of the Teeth of Wheels.

Referring to the annexed Fig. 322, the leading dimensions are indicated by literal references, and are thus distinguished in the table No. 257, in

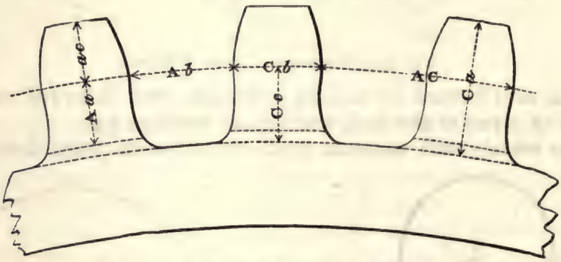


Fig. 322.—Proportions of Teeth of Wheels.

which the first and second scales of proportions are, both of them, used by engineers in good practice;[1] and the third is the scale of Sir Wm. Fairbairn.[2]

Table No. 257.—Teeth of Wheels:—Proportional Dimensions.
(Fig. 322.)

Elements.		1st Scale.	2d Scale.	3d Scale.
A *b* + C *b*	Pitch of teeth,........................	1	15 or 1	1.00
C *b*	Thickness of teeth,..................	5/11 or .45	7 or .47	.45
A *b*	Width of space, ...	6/11 or .55	8 or .53	.55
A *b* − C *b*	Play,..................................	1/11 or .09	1 or .07	.10
a c	Length above pitch-line,..........	3/10 or .30	5½ or .37	.35
C *c*	Length below pitch-line,..........	4/10 or .40	6½ or .43	.40
A *a* + *a c*	Working length of tooth,.........	6/10 or .60	11 or .73	.70
C *a*	Whole length of tooth,.............	7/10 or .70	12 or .80	.75
C *c* − A *a*	Clearance at root,..................	1/10 or .10	1 or .07	.05
C *b*	Thickness of rim,....................	—	7 or .47	—

Note to table.—The proportion of clearance at the root of the tooth is usually varied from, say, 1/10th of the pitch for the smaller wheels, to 1/20th for the larger wheels.

[1] *Engineer and Machinist's Assistant*, page 89. [2] *Mills and Millwork*, Part 2, page 33.

Strength of the Teeth of Wheels.

The tooth of a wheel in action, is a beam fixed at one end and loaded at the other; and, as the available strength of a wheel-tooth is that of its weakest exposure, the strength should be calculated for the contingency that the whole of the force may be applied to the tooth at one corner, D, Fig. 323. The tooth, it is conceivable, may be broken across at any of the lines B *b*, B *c*, or B *d*; but, under uniform conditions, the actual line of fracture would be B *b*, which is the base of a right-angle triangle having the equal sides D B and D *b*, and of which the height D *e* is equal to half the base B *b*. Now, the height of the tooth D B, is the slope of the right-angled triangle D B *e*, and is 1.414 times D *e*; and the values of the elements for the application of the formula, according to the 3d scale of proportions in table No. 257, are:—

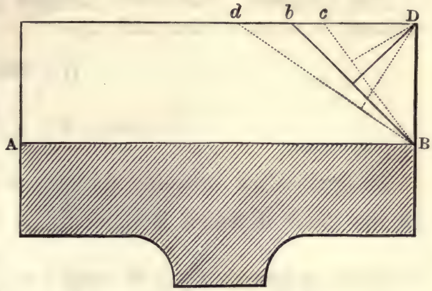


Fig. 323.—Strength of Teeth of Wheels.

P = the pitch = 1.
l = the length of the "beam" = D e = .75 P ÷ 1.414 = .53 P.
b = the breadth of the beam = B b = .53 P × 2 = 1.06 P.
d = the depth of the beam, or the thickness of the tooth = .45 P.
s = the tensile strength of the material in tons per square inch.
W = the breaking weight at the corner of the tooth, in pounds.

Adapting the general formula (4), page 507, the coefficient .289*s* becomes (.289 × 2240 *s* =) 647, and, in terms of the above symbols,—

$$W = \frac{(647\ s)\ b\ d^2}{l} \qquad (4)$$

To express the breaking strength of the tooth in terms of the pitch, substitute the equivalent values of *b*, *d*, and *l*, in formula (4):—

$$W = \frac{647\ s \times 1.06\ P \times (.45\ P)^2}{.53\ P} = 647\ s \times .405\ \frac{P^3}{P}; \text{ or}$$

$$W = (262\ s)\ P^2 \qquad (5)$$

Values of the Numerical Coefficient in Formula (5).

Ultimate Tensile Strength per Square Inch. tons.		Coefficient (262 *s*).	
7.	Cast iron,	262 × 7 = 1834,	say 1800
8.	Do.,	262 × 8 = 2096,	„ 2100
9.	Do.,	262 × 9 = 2358,	„ 2400
10.	Do.,	262 × 10 = 2620,	„ 2600
11.	Do.,	262 × 11 = 2882,	„ 2900
12.	Do.,	262 × 12 = 3144,	„ 3100
12.	Gun-metal,	262 × 12 = 3144,	„ 3100
20.	Wrought iron,	262 × 20 = 5240,	„ 5200
30.	Steel,	262 × 30 = 7860,	„ 8000

For wheels of ordinary cast iron, when the tensile strength is not given, assume 7 tons per square inch as the tensile strength, and adopt the coefficient 1800. Then,

The Ultimate Transverse Strength of the Teeth of ordinary Cast-Iron Wheels, in terms of the *pitch*, is,—

$$W = 1800\ P^2 \dots\dots\dots\dots \quad (6)$$

$$\text{Inversely, } P = \sqrt{\frac{W}{1800}} \dots\dots\dots\dots \quad (7)$$

To express the breaking strength of the tooth in terms of the thickness of the tooth, d, which is equal to .45 P. The pitch $P = \frac{d}{\cdot 45}$, and by substitution in formula (5), $W = 262\ s \times (\frac{d}{\cdot 45})^2$; or,

$$W = (1294\ s)\ d^2 \dots\dots\dots\dots \quad (8)$$

VALUES OF THE NUMERICAL COEFFICIENT IN FORMULA (8).

Ultimate Tensile Strength per Square Inch. tons.		Coefficient (1294 s).
7.	Cast iron,	1294 × 7 = 9058, say 9000
8.	Do.,	1294 × 8 = 10,352, „ 10,000
9.	Do.,	1294 × 9 = 11,646, „ 12,000
10.	Do.,	1294 × 10 = 12,940, „ 13,000
11.	Do.,	1294 × 11 = 14,234, „ 14,000
12.	Do.,	1294 × 12 = 15,528, „ 16,000
12.	Gun-metal,	1300 × 12 = 15,528, „ 16,000
20.	Wrought iron,	1294 × 20 = 25,880, „ 26,000
30.	Steel,	1294 × 30 = 38,820, „ 39,000

Again assuming a tensile strength of 7 tons per square inch for ordinary castings,—

The Ultimate Transverse Strength of the Teeth of ordinary Cast-Iron Wheels in terms of the *thickness*, is,

$$W = 9000\ d^2 \dots\dots\dots\dots \quad (9)$$

$$\text{Inversely, } = d \sqrt{\frac{W}{9000}} \dots\dots\dots\dots \quad (10)$$

The excess of transverse strength of cast iron, in thicknesses of from 1 to 3 inches, above that which is calculated from the tensile strength, as detailed at page 555, affords a margin for weak forms of teeth, and for the loss of strength by trimming or by wear, and especially by the removal of the skin. The excess, it is true, diminishes as the thickness increases; but, on the contrary, the diminution of strength is less by the wear of the thicker teeth than by that of the thinner teeth. Thus, there is a natural adjustment of the supply of strength in excess to the requirement.

WORKING STRENGTH OF WHEEL-TEETH.

It is usual to act on a factor of safety of 10, for wheel-teeth: Formulas (6), (7), (9), and (10), thus adapted, become,—

For the Working Strength of Wheel-teeth of ordinary Cast Iron:—

In terms of the pitch,.........$W = 180\ P^2$ .... (11)

Do., ... $P = \sqrt{\frac{W}{180}}$ .... (12)

In terms of the thickness,....$W = 900\ d^2$ .... (13)

Do., ...$d = \sqrt{\frac{W}{900}} = 30\sqrt{W}$ .... (14)

For wheels of cast iron of greater strength, or of gun-metal, wrought iron, or steel, the coefficients for a factor of safety of 10, are one-tenth of those which are given at pages 735 and 736, and they may be substituted in the above formulas, thus,—

For Working Strength.	In formulas (11) and (12).	In formulas (13) and (14).
Coefficient for gun-metal,	310	1600
Do. wrought iron,	520	2600
Do. steel,	800	3900

Sir William Fairbairn states that, for wooden teeth, a thickness 1½ times that of cast-iron teeth is sufficient; $^3/_5$ ths of the pitch goes to the thickness of the wooden cogs, and $^2/_5$ ths to that of the iron teeth of the wheel geared with the wooden teeth. There is no clearance, as the teeth are accurately trimmed.

Breadth of the Teeth of Wheels.

When the breadth of a tooth is just twice its whole length, its ultimate resistance to transverse stress is approximately equal to its resistance to diagonal stress applied at one corner. A greater breadth than twice the length, therefore, is not reckoned to add to the transverse resistance of the tooth; but it is necessary for durability.

Breadth in relation to Working Stress.—Tredgold fixed the maximum average working stress at the pitch-line at 400 pounds per inch of breadth of teeth. Sir William Fairbairn adopted this datum.

Horse-power Transmitted by Toothed Wheels.

A horse-power is work done at the rate of 33,000 pounds through 1 foot, per minute; or 550 foot-pounds per second. Let,—

H = the horse-power transmitted.
W = the stress in pounds, at the pitch-line.
v = the velocity at the pitch-line, in feet per second.
S = the speed in turns per minute.
D = the diameter in feet.

$$H = \frac{W\,v}{550}; \quad W = \frac{550\ H}{v}; \quad v = \frac{550\ H}{W}; \text{ .... } (15), (16), (17).$$

That is to say;—1st. *The horse-power transmitted* is equal to the product of the stress by the velocity at the pitch-line, divided by 550. 2d. *The stress at the pitch-line* is equal to 550 times the horse-power divided by the velocity at the pitch-line. 3d. *The velocity at the pitch-line* is equal to 550 times the horse-power, divided by the stress.

The speed of the wheel, in turns per minute, is equal to $\frac{v \times 60}{3.1416\,D}$; or

$$S = \frac{19.1\,v}{D}; \qquad v = \frac{D\,S}{19.1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18), (19)$$

That is to say;—1st. *The speed of the wheel* is equal to 19.1 times the velocity at the pitch-line, divided by the diameter. 2d. *The velocity at the pitch-line* is equal to the product of the diameter by the speed, divided by 19.1.

To find the horse-power in terms of the stress, the diameter and the speed; or W, D, and S. By formula (15), $H = \frac{W\,v}{550}$; and, substituting the value of v in (19), $H = \frac{W\,D\,S}{550 \times 19.1}$, or

$$H = \frac{W\,D\,S}{10{,}500} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

$$S = \frac{10{,}500\,H}{W\,D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

That is to say, *the horse-power transmitted* is equal to the product of the stress at the pitch-line, by the diameter and by the speed; divided by 10,500.

For the horse-power in terms of the pitch. Substitute in formula (20), the value of W in terms of the pitch, that is, by formula (11), for cast iron, $180\,P^2$; then $H = \frac{P^2\,D\,S \times 180}{10{,}500}$; or

$$\text{(for cast iron)} \qquad H = \frac{P^2\,D\,S}{58.3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

That is to say, *the horse-power that may be transmitted by a cast-iron wheel* is equal to the product of the square of the pitch by the diameter and by the speed; divided by 58.3.

The horse-power per foot of diameter and per turn per minute, is

$$\text{(for cast iron)} \qquad H = \frac{P^2}{58.3}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

being equal to the square of the pitch divided by 58.3.

The formulas (22) and (23) are available for the calculation of the horse-power of wheels made of other metals, by using the proper constants, as follows:—

For Horse-power.	In Formulas (22) and (23).
Coefficient for gun-metal	$10{,}500 \div 310 = 33.9$
Do. wrought-iron	$10{,}500 \div 520 = 20.2$
Do. steel	$10{,}500 \div 800 = 13.1$

Table No. 258 gives particulars of dimensions, stress, and horse-power of toothed spur-wheels of ordinary cast iron, for various pitches. The thickness, column 2, is given as 45 per cent of the pitch; the working

stress at the pitch-line, column 3, is calculated from the pitch by formula (11); the horse-power transmitted at 1 foot per second, column 6, is calculated from the stress by formula (15), and the horse-power per foot of diameter and per turn per minute, in the last column, is calculated by formula (23). This table affords data for performing all the usual calculations for the horse-power of toothed wheels.[1]

Table No. 258.—STRENGTH AND HORSE-POWER OF SPUR-WHEELS,

Made of ordinary cast iron.

Pitch of Teeth.	Thickness of Teeth.	Working Stress at Pitch-line.	Breadth of Teeth.		Horse-Power Transmitted.	
			Least Breadth.	Usual Breadth.	At 1 Foot per Second at the Pitch-line.	Per Foot of Diameter and per Turn per Minute.
inches.	inches.	pounds.	inches. / thickness × 2.	inches.	H. P.	H. P.
1	.45	180	.90	2½	.327	.0172
1¼	.56	281	1.12	2½ and 3	.511	.0268
1½	.67	405	1.34	3¼ and 3½	.736	.0386
1¾	.79	551	1.58	4¼	1.000	.0525
2	.90	720	1.80	6	1.310	.0686
			for 400 lbs. per inch.			
2	.90	720	1.80	6	1.310	.0686
2¼	1.01	911	2.28	6	1.656	.0868
2½	1.12	1125	2.81	6	2.045	.1007
2¾	1.24	1361	3.40	7	2.474	.1297
3	1.35	1620	4.05	9	2.945	.1544
3¼	1.46	1901	4.75	9	3.456	.1811
3½	1.57	2205	5.51	10	4.010	.2100
4	1.80	2880	7.20	14	5.236	.2744
4½	2.02	3645	9.11	14	6.627	.3472
5	2.25	4500	11.25	15 to 16	8.182	.4028
6	2.73	6480	16.20	16 to 18	11.782	.6176

Note.—For mitre and bevil wheels, the mean diameter, breadth, and thickness of teeth are to be used in calculation.

WEIGHT OF TOOTHED WHEELS.

Spur-wheels.—The weight of spur-wheels of usual proportions, per inch of breadth, increases in a ratio greater than the diameter, but less than the square of the diameter. As ascertained by plotting the particulars of a great number of wheels, the relation of the diameter and the weight per inch wide, is represented by the general expression $a\,d + b\,d^2$, in which d is the diameter, and a and b are constants for each pitch.

Within the ordinary practical limits of breadth for each pitch, the weight varies as the breadth of the wheel, and may be taken at a constant per inch of breadth.

[1] The author is aware that Tredgold, Fairbairn, and others, give higher values for the strength and power of the teeth of wheels than he gives in the text.

Table No. 259.—Weight of Cast-Iron Spur-Wheels,

Per Inch of Breadth.

Pitch.	Diameters in Feet.								Usual Breadth.
	.50	.75	1	1.5	2	2.5	3	4	
inches.	cwt.	cwt.	cwt.	cwt.	cwt.	cwt.	cwt.	cwt.	inches.
6	—	—	—	—	1.27	1.66	2.07	2.97	16 to 18
5	—	—	—	—	1.08	1.40	1.75	2.52	15 to 16
4½	—	—	—	.707	.984	1.28	1.60	2.30	14
4	—	—	—	.638	.888	1.16	1.44	2.07	14
3½	—	—	—	.569	.792	1.03	1.29	1.85	10
3¼	—	—	—	.535	.744	.968	1.21	1.74	9
3	—	—	.319	.500	.696	.905	1.13	1.62	9
2¾	—	.218	.297	.466	.648	.844	1.05	1.51	7
2½	—	.202	.275	.431	.600	.781	.98	1.40	6
2¼	—	.185	.253	.397	.552	.719	.90	1.29	6
2	.110	.169	.231	.362	.504	.656	.82	1.18	6
1¾	.100	.153	.209	.328	.456	.594	.74	1.06	4¼
1½	.089	.137	.187	.293	.408	.531	.66	.95	3¼ not exc'ding 5 ft. 3½ above 5 feet.
1¼	.077	.121	.165	.259	.360	.469	.59	.84	2½ not exc'ding 4 ft. 3 above 4 feet.
1	.068	.105	.143	.224	.312	.406	.51	.73	2½
	lb.	lb.	lb.	lb.	lb.	lb.	lb.	lb.	
⅞	6	9	13	22	32	43	55	84	2
¾	6	9	13	22	32	43	55	84	2
½	5	8	12	20	31	44	—	—	1¼
⅜	6	10	15	27	42	—	—	—	1⅛

Pitch.	Diameters in Feet.							
	5	6	7	8	9	10	11	12
inches.	cwt.	cwt.	cwt.	cwt.	cwt.	cwt.	cwt.	cwt.
6	3.98	5.09	6.30	7.63	9.06	10.60	12.24	14.00
5	3.38	4.32	5.36	6.48	7.70	9.00	10.40	11.88
4½	3.08	3.94	4.88	5.90	7.01	8.20	9.47	10.82
4	2.78	3.55	4.40	5.33	6.33	7.40	8.55	9.77
3½	2.48	3.17	3.93	4.75	5.64	6.60	7.62	8.71
3¼	2.33	2.98	3.69	4.46	5.30	6.20	7.16	8.18
3	2.18	2.78	3.45	4.18	4.96	5.80	6.70	7.66
2¾	2.03	2.59	3.21	3.89	4.62	5.40	—	—
2½	1.88	2.40	2.98	3.60	4.28	5.00	—	—
2¼	1.73	2.21	2.74	3.31	3.93	4.60	—	—
2	1.58	2.02	2.50	3.02	3.59	—	—	—
1¾	1.43	1.82	2.26	2.74	—	—	—	—
1½	1.28	1.63	2.02	—	—	—	—	—
1¼	1.13	1.44	—	—	—	—	—	—
1	.98	—	—	—	—	—	—	—

For pitches of 1 inch and upwards, the weight per inch of breadth, for a given diameter, increases directly as the pitch.

The following formulas are deduced from the actual weights of spur-wheels of from ⅜ inch to 4 inches pitch, and from 6 inches to 12 feet in diameter.

Weight of cast-iron spur-wheels of from 1 inch to 6 inches pitch, per inch of breadth:—

$$W = (.05 + .08\,p)\,d \times (1 + .10\,d) \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

Weight of cast-iron spur-wheels of pitches less than 1 inch:—

For ⅞ inch pitch, $W = .0935\,d + .0235\,d^2$ .... (25)
For ¾ " $W = .0935\,d + .0235\,d^2$ .... (26)
For ½ " $W = .069\,d + .0345\,d^2$ .... (27)
For ⅜ " $W = .080\,d + .0530\,d^2$ .... (28)

W = the weight of the wheel in hundredweights per inch of breadth.
d = the diameter in feet.
p = the pitch in inches.

The first formula would probably be suitable for finding the weights of wheels up to 20 feet in diameter.

The results given by the above formulas are average results.

Mortise-wheels.—The weight of mortise-wheel castings is the same as that of spur-wheels, having the same leading dimensions.

Bevil-wheels and Mitre-wheels.—The weight is less than that of spur-wheels of the same leading dimensions, varying from two-thirds or three-fourths of the weight of spur-wheels, for the larger diameters, to about seven-eighths for the smaller diameters.

The table No. 259 of the weight of spur-wheels is calculated by means of formulas (24) to (28).

FRICTIONAL WHEEL-GEARING.

When one smooth cast-iron wheel is employed to drive another by direct contact at the circumferences, the adhesion or driving force is produced by interpressure between the wheels, and may be taken as one-sixth of the pressure.

Robertson's grooved-surface frictional-gearing consists of wheels or pulleys geared together by frictional contact, in which the driving surfaces are grooved or serrated annularly, the ridges of one surface entering the grooves of the other. A lateral wedging action is obtained, which augments the adhesion of the surfaces, as compared with flat friction-surfaces, in the ratio of 9 to 1. That is, the grooved wheels require a force of 3 lbs. acting at their circumference to make them slip, for every 2 lbs. applied on the axis; whereas, two flat surface-wheels would require ($2 \times 9 =$) 18 lbs. of pressure on the axis, to enable them to resist a force of 3 lbs. acting at the circumference.

Compared with leather belts, frictional gearing, worked under a pressure equal to the tension of the belts, has been proved to have greater adhesive force:—in one experiment, about 30 per cent. more.

The grooves are made of V shape, for which 50° is the most suitable angle. The pitch of the grooves is varied according to the velocity and the power to be transmitted:—from ⅛ inch to ¾ inch; the ordinary pitch is ⅜ inch. The general laws of friction are found to apply to the action of frictional gearing.[1]

BELT-PULLEYS AND BELTS.

The acceleration or reduction of the angular velocity, or speed, of shafts driven by means of belts and drum-pulleys, is, like that of shafts driven by toothed gearing, in the inverse ratio of the diameters of the pulleys; and, when speed is brought up by means of successive shafts and small and large pulleys, the ratio of the initial to the final speed is the product of

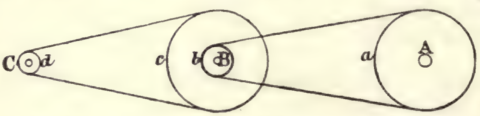


Fig. 324.—Belt-pulleys and Belts:—Multiplication of Speed.

the ratios of the successive accelerations of speed; and these may be expressed in terms of the diameters of the successive pairs of pulleys. For example, the driving pulley *a*, 36 inches in diameter, on the shaft A, makes 60 revolutions per minute, and drives by a belt the 12-inch pulley *b* on the shaft B; which carries the 36-inch pulley *c*, which drives the 8-inch pulley *d* on the third shaft C. The speeds are calculated thus:—

	Turns per Minute.
Shaft A..	60
Shaft B $60 \times \frac{36}{12}$, or $60 \times 3 =$	180
Shaft C $60 \times \frac{36}{12} \times \frac{36}{8}$, or $60 \times 3 \times 4.5 =$	810

In these calculations, it is assumed that there is no slip of the belt on the pulley; and M. Morin supports this assumption. Mr. R. H. Buel[2] deduced from direct observation that the reduction of speed by the slip or "creep," of five belts employed to communicate motion from an engine-shaft to a distant shaft, through intermediate shafts and pulleys, varied from just one-half to one-quarter per cent.; whilst M. Krest[3] deduced a slip for one belt only, amounting to 2 per cent. The size of the belts observed by Mr. Buel were greatly in excess of those actually required for the transmission of the power.

Tensile Strength of Driving Belts.

Several particulars of the strength of belts are given at pages 679, 680. It has there been noticed that Messrs. Briggs and Towne[4] tested the strength of

[1] See a paper by Mr. James Robertson on "Grooved-Surface Frictional-Gearing," in the *Proceedings of the Institution of Mechanical Engineers*, 1856.
[2] *Journal of the Franklin Institute*, vol. lxviii., 1874, page 256.
[3] *Annales des Mines*, 1862. [4] *Journal of the Franklin Institute*, January, 1868.

3-inch leather belts, $^7/_{32}$ inch or .219 inch thick. They found that the weakest parts of an ordinary belt are the ends through which the lacing holes are punched, and that the belt is usually weaker than the lacing itself. The rivetted splices are the next weakest points. The strengths of new and partially used belts were found to be almost identical.

	Ultimate Tensile Strength.		
	Total.	Per inch of Width.	Per square inch of Section.
At lacings	629 lbs.	210 lbs.	958 lbs.
At splicings	1146 „	382 „	1744 „
At solid part	2025 „	675 „	3086 „

Messrs. Briggs and Towne adopt 200 lbs. per inch wide, as the ultimate strength of laced belts .22 inch thick; and they take a third of this, or 66⅔ lbs. per inch wide, as the working strength. This is just one-sixth of the working stress, 400 lbs. per inch of width, adopted for toothed wheels. It is equivalent to 304 lbs. per square inch of section.

M. Morin adopts $^1/_5$ kilogramme, and M. Claudel ¼ kilogramme, per square millimetre; equivalent to 284 lbs., and 355 lbs., per square inch of section, for the working strength of leather belts.

Dr. Hartig found, from the results of experiments made by him in a woollen mill, that the tension on the driving belts varied from 30 lbs. to 532 lbs. per square inch of section, and that it averaged 273 lbs. per square inch.[1]

An average working strength of 300 lbs. per square inch of section of leather belts may be accepted for purposes of calculation.

Calculation for Horse-Power Transmitted by Leather Belts.

The formulas for the power of toothed wheels, (15) to (21), pages 737 and 738, may be adapted for the calculation of the power of belts. Let

H = the horse-power.
W = the working stress transmitted per inch wide, in pounds.
b = the breadth of the belt, in inches.
b' = do. do. in feet.
Wb = the total working stress transmitted, in pounds.
v = the velocity of the belt, in feet per second.
v' = do. do. in feet per minute.
D = the diameter of the pulley, in feet.
S = the speed of the pulley, in turns per minute.

In Terms of Horse-Power, Working Stress, and Velocity of Belt.

$$H = \frac{W b \times v}{550}; \quad W b = \frac{550\,H}{v}; \quad v = \frac{550\,H}{W b} \quad \ldots\ldots\ (1)\ (2)\ (3)$$

[1] "Essais Dynamométrique" (la Laine Cardée), by Dr. Ernest Hartig, translated by M. E. Simon. *Annales du Conservatoire des Arts et Métiers*, vol. viii., page 611.

In Terms of the Velocity of the Belt, and the Diameter and the Speed of the Pulley.

$$S = \frac{19.1\ v}{D};\ \text{and}\ v = \frac{D\,S}{19.1} \quad \dots\dots\dots\dots\dots\dots \quad (4), (5)$$

$$H = \frac{W\,b \times D\,S}{10{,}500} \quad \dots\dots\dots\dots\dots\dots \quad (6)$$

$$S = \frac{10{,}500\ H}{W\,b \times D} \quad \dots\dots\dots\dots\dots\dots \quad (7)$$

The performances of belts may be compared by calculating the number of square feet of belt-surface passed over either pulley per minute per horse-power:—involving the elements of working stress and velocity. It is found by multiplying the velocity in feet per minute by the breadth of the belt in feet, and dividing the product by the horse-power transmitted:—

Belt-surface described per Minute per Horse-Power Transmitted.

$$\text{Belt-area in square feet} = \frac{v' \times b'}{H} = \frac{5\ v \times b}{H}; \quad \dots\dots\dots \quad (8)$$

In the second expression of value, the velocity is expressed in feet per second, and the breadth is in inches.

Adhesion and Power of Leather Belts.

The normal pressure per square inch of a belt on the surface of a pulley is equal to the quotient of the tension on a belt 1 inch wide by the radius of the pulley in inches. Otherwise, the tension on a belt 1 inch wide, is equal to the product of the normal pressure on the pulley per square inch by the radius in inches. This is the same equation as is used to find, reversely, the action of steam within a boiler:—the transverse stress on each side of the shell of the boiler, per inch of length, is equal to the product of the internal pressure per square inch by the radius in inches.

M. Morin's Experiments.

M. Morin[1] ascertained that, when one pulley is driven by another, by means of a leather belt, the sum of the tensions in the two sides of the belt, is the same when in motion and at rest. The tension on the pulling side, when in motion, is therefore increased by as much as the tension on the return side is reduced. When the belt just slides on the pulley by tension, the relation of the sliding and the slack tensions is expressed by the formula—

$$\text{Log}\ T = \log t + 0.434\, c\, \frac{C}{R} \quad \dots\dots\dots\dots\dots\dots \quad (9)$$

T = the greater tension.
t = the slack tension.
c = the coefficient of friction between the band and the pulley.
C = the length of the arc of the circumference embraced by the belt.
R = the radius of the pulley.

[1] *Aide-Memoire de Mecanique Pratique*, 1864; page 289.

RULE.—*To find the tension just sufficient to cause a leather belt to slide over a pulley with a given slack tension.* Divide the length of the arc embraced by the belt, by the radius of the pulley, and multiply the quotient by the coefficient of friction, and by 0.434; add the product to the logarithm of the slack tension. The sum is the logarithm of the sliding tension.[1]

The coefficients of friction deduced by M. Morin are as follows:—

For leather belts in ordinary condition on wooden pulleys 0.47
For new belts on wooden pulleys.. 0.50
For belts in ordinary condition on cast-iron pulleys, either turned or rough 0.28
For wet belts on cast-iron pulleys 0.38
For hemp ropes on wooden pulleys 0.50

To facilitate calculation, M. Morin gives the following tablet of the ratios of the sliding tension to the slack tension, for various proportions of the arc of the circumference embraced by the band, and for the several coefficients of friction:—

Tablet A.

Ratio of the Arc Embraced, to the Circumference.	Ratio of Sliding to Slack Tension. K.					
	New Belts on Wooden Pulleys.	Belts in Ordinary Condition.		Wet Belts on Cast-iron Pulleys.	Cords on Wooden Pulleys or Winches.	
		Wooden Pulleys.	Cast-iron Pulleys.		Rough.	Polished.
.20	1.87	1.80	1.42	1.61	1.87	1.51
.30	2.57	2.43	1.69	2.05	2.57	1.86
.40	3.51	3.26	2.02	2.60	3.51	2.29
.50	4.81	4.38	2.41	3.30	4.81	2.82
.60	6.59	5.88	2.87	4.19	6.58	3.47
.70	9.00	7.90	3.43	5.32	9.01	4.27
.80	12.34	10.62	4.09	6.75	12.34	5.25
.90	16.90	14.27	4.87	8.57	16.90	6.46
1.00	23.14	19.16	5.81	10.89	23.90	7.95
1.50	—	—	—	—	111.31	22.42
2.00	—	—	—	—	535.47	63.23
2.50	—	—	—	—	2575.80	178.52

Example.—The slack tension of a belt in ordinary condition, on a wooden pulley, is 100 lbs.; and the belt embraces half the circumference. Opposite .50 in column 1, find the multiplier 4.38 in column 3. Then, $100 \times 4.38 = 438$ lbs., the sliding tension.

It is found by experience that the resistance of pulleys to sliding of belts is independent of the diameter.

When a rope is wound once round a wooden barrel, it is seen by the table that the resistance is 24 times the pull at the slack end, on a rough barrel; and that, when wound 2½ times round the barrel, the resistance is 2575 times the pull.

Calculation of the Power of Belts, by M. Morin's Data.—The pull available for driving is equal to the difference of the sliding and slack tensions, or

[1] Messrs. Briggs and Towne show clearly how this formula is arrived at; *Journal of the Franklin Institute*, January, 1868, page 18.

$T - t = Kt - t = t\,(K - 1)$, in which K is the ratio of T to t, as in tablet A; and,

$$t = \frac{T - t}{K - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (a)$$

The value of the driving pull, or the difference of tensions $(T - t)$, is equal to $\frac{550\,H}{v}$, in which H is the horse-power, and v the velocity of the belt in feet per second. Substituting this value for $(T - t)$ in equation (a),

$$t = \frac{550\,H}{(K - 1)\,v} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$H = \frac{t\,v\,(K - 1)}{550} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

That is to say, the *minimum slack tension* is equal to 550 times the horse-power, divided by the velocity, and by the ratio of the sliding to the slack tension minus 1.

And, *the available horse-power* is equal to the product of the slack tension by the velocity of the belt, and by the ratio of the tensions minus 1; divided by 550.

M. Morin recommends that the value of the slack tension, formula (10), should be increased by $^1/_{10}$th, to cover the friction of the journals.

M. Claudel's Data for Belts.

M. Claudel gives the following empirical formula, in common use, for finding the breadth of a leather belt enveloping half the circumference of a pulley. Altering the measures:—

$$b = c\,\frac{H}{v}; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

in which b = the breadth in inches, H = the horse-power, v = the speed of the belt in feet per second; and c a constant, 26 for upright shafts, and 20 for horizontal shafts.

This formula gives values for the breadth, averaging about double what would be given by the table No. 261, when the belt laps half round the pulley. The belt then works to half its power; and M. Claudel instances the common experience that a belt 3¼ inches broad, moving at a velocity of 9 feet per second, can very well transmit 1 horse-power, with ordinary tension, and without overstraining, working on turned and smooth pulleys of equal diameter. This example, if adopted as a basis, would give a coefficient of 29 in formula (12). The working tension is only about 20 lbs. per inch wide.

At the same time, the values given by the empirical formula (12) are little more than those deducible from the data of M. Morin.

Mr. Evan Leigh's Rules for Belting.[1]

Mr. Leigh is of opinion that a main driving-belt, to be rightly applied, should pass through 3000 or 4000 lineal feet per minute; and should be of sufficient width to drive all the machinery and shafting to be driven, quite

[1] *The Science of Modern Cotton Spinning*, 1873, page 37.

easily, running in a slack condition. The belt should work from the periphery of the fly-wheels of quick-running engines.

RULE 1.—*To find the horse-power of a main driving double belt, working slackly and easily.* Multiply the number of square inches covered by the belt, on the surface of the driven pulley, by half the speed of the belt in feet per minute; and divide the product by 33,000. The quotient is the horse-power.

RULE 2.—*To find the proper width of a main driving double belt for a given horse-power.* Multiply the horse-power by 33,000, and divide the product by the length in inches of periphery of driven pulley covered by the belt, and by half the speed of the belt in feet per minute. The quotient is the width in inches.

For existing establishments, where it is desired not to disturb actual arrangements, the following rule, for single belts, approaches nearer to ordinary practice:—

RULE 3.—*To find the width of a single belt for any given horse-power* [actual practice]. Multiply the horse-power by 33,000, and divide the product by the length in inches of periphery of the smaller pulley covered by the belt, and by the speed of the belt in feet per minute. The quotient is the width in inches.

By this rule, which is based on ordinary practice, single belts are calculated to do twice as much duty as double belts of the same width by Rule 2; and, comparatively, the stronger double belts, calculated by Rules 1 and 2, have exceedingly easy work. Hence their great durability. Applying Rule 1 to the second example of wide double belts quoted from Mr. Cooper, below:—there are two driven pulleys, 7 feet in diameter, lapped by the belt for $^2/_5$ ths of their circumference, equal to 8.8 feet, or 105.6 inches on each; the width of each belt is 14½ inches, and $105.6 \times 14.5 \times 2 = 3062$ square inches covered. The velocity of the belts is 3498 feet per minute; and $3062 \times 3498 \div 2 \div 33{,}000 = 162.3$ horse-power. The horse-power actually transmitted by the two belts together is 250, or fully a half more than is allowed by Mr. Leigh's rule; yet those belts are said to have lasted upwards of 22 years.

Examples of Very Wide Belts.

Mr. J. H. Cooper[1] details some examples of the performance of belts in taking off the power of steam engines from the fly-wheel to one, two, or three line-shafts—

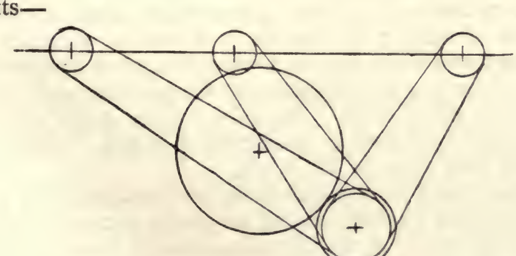


Fig. 325.—Main Driving Belts, with intermediate gearing.

1. Horizontal condensing Corliss engine, with intermediate gearing, at Conestoga Mills, No. 2, Lancaster, Pa. Three belts. Fig. 325.

[1] *Journal of the Franklin Institute,* vol. lxviii., 1874, page 256.

2. Engine of the same class; belts driven by fly-wheel. Two belts. Fig. 326.
3. Horizontal Corliss engine; belt driven by fly-wheel. One belt.

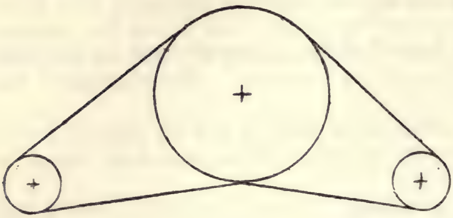


Fig. 326.

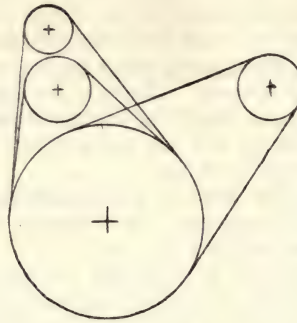


Fig. 327.

Main Driving Belts, off the fly-wheel.

4. Horizontal Corliss engine at Manayunk; belts driven by fly-wheel. Three belts. Fig. 327.
5. Corliss engine, at Great Bend, Indiana. One belt.

The chief particulars and results are reduced in table No. 260.

No. 1 belts have been at work upwards of twenty years; No. 2, upwards of twenty-two years; No 3 has been eight or nine years at work. Nos. 1 to 4, double-thickness belts, transmit a tension of from 50 to 86 lbs. per inch wide, or from 25 to 43 lbs. per inch wide for one thickness. The single-thickness belt, No. 5, averages 126 lbs. per inch wide; and Mr. Cooper mentions a 6-inch belt employed to drive a wheel-forcing press on a 24-inch pulley, transmitting a stress of 104 lbs. per inch wide.

Messrs. Briggs and Towne's Experiments.

Messrs. Briggs and Towne's experiments on the friction or adhesion of leather belts, were made with 3-inch belts, and the arc of contact equal to 180°, on ordinary cast-iron pulleys. The average results of 168 experiments, under tensions of from 7 to 110 lbs. per inch of width of belt, gave a sliding tension 6.294 times the slack tension, and a friction co-efficient of 0.5833. To cover the contingencies of temperature and moisture of the atmosphere, they adopt only $^6/_{10}$ths of 6.294, or 3.776 as the maximum practical value, giving a friction co-efficient of 0.423. These values are by one-half greater than those of M. Morin.

Messrs. Briggs and Towne calculate the maximum working stress per inch of width, transmitted by belts having a working strength of 66 $^2/_3$ lbs. per inch of width, by means of the formula deduced from Rankine's formula (*Applied Mechanics*), and based upon the foregoing data:—

$$W = 66\tfrac{2}{3}\left(1 - 10^{-0.003206\,\alpha}\right) \quad \ldots\ldots\ldots\ldots \quad (12)$$

W = the working stress transmitted per inch of width, in pounds.
α = the arc of contact, in degrees.

Table No. 260.—FIRST-MOTION DRIVING BELTS IN THE UNITED STATES.

(Mr. Cooper.)

	Cylinder. — Diameter and Stroke.	Revolutions per Minute.	Fly-wheel. — Diameter.	Intermediate Gearing.	Driven Pulleys. Number and Diameter.	Driven Pulleys. Speed per Minute.
	ins. ft.	turns.	feet.	ft. ins.		turns.
1	30 × 6	52½	spur, 22; 25 tons	spur,...... 9 7½; 3 pulleys, 9 6; speed,.... 120	3 pulleys, 5 ft.	228
2	28 × 5	50½	pulley, 22; 17 tons		2 ,, 7 ,,	159
3	16 × 4	65	14		1 ,, 5 ,,	182
4	30 × 5	52	24		1 ,, 6 ,,	208
					2 ,, 8 ,,	156
5	18 × 4	65	12		1 ,, 3 ft. 6 in.	223

Table continued.

	Thickness and Width of Belt.	Velocity of Belt. Feet per Minute.	Velocity of Belt. Feet per Second.	Horse-power Transmitted.	Tension on Belt. Total.	Tension on Belt. Per inch Wide.	Belt Surface per Minute per H.P.
		feet.	feet.	H. P.	lbs.	lbs.	sq. feet.
1	double, 23½ ins.	3582	59.7	125	1152	49	54.9
	,, 29 ,,	3582	59.7	175	1612	56	49.5
2	,, 14½ ,,	3498	58.3	125	1180	81	33.6
	,, 14½ ,,	3498	58.3	125	1180	81	33.6
3	,, 12 ,,	2859	47.7	90	1038	86.5	31.8
4	,, 17 ,,						
	,, 21½ ,,						
	,, 26½ ,,						
	total, 65 ,,	3920	65.3	457	3849	59	46.5
5	single, 22 ,,	2453	40.9	190 to 222	2555	116	23.6
					2985	136	20.2

Concluding Table of the Driving Power of Leather Belts.

On the whole, it may be concluded that Messrs. Briggs and Towne's data afford a satisfactory basis for the application of general practical rules. Table No. 261 gives particulars of the practical driving-power of leather belts .22 inch thick, per inch wide, based on their data for arcs of contact, of from 90° to 270°, assuming 66⅔ lbs. per inch wide, as the maximum working strength. The maximum transmitted working stress, calculated by them by means of formula (12), is given in column 2; the 3d and 4th columns were calculated by means of formulas (1) and (6); and the horse-power in column 4, multiplied by 33,000, gives column 5. The sum of the tensions, in column 6, is calculated by adding to the transmitted stress in column 2, twice the difference between it and 66⅔. Thus, for the first case, $66^2/_3 - 32.33 = 34.33$, which is the slack tension, and $32.33 +$

$(34.33 \times 2) = 101.00$, in column 6. The last column gives the resultant stress, by the parallelogram of forces, caused by the tensions of the belt, on the bearings of the shaft.

Messrs. Briggs and Towne give many instances in practice, in corroboration of their deductions.

Table No. 261.—Driving-Power of Leather Belts.

Maximum Working Strength 66⅔ lbs. per inch wide, single thickness, .22 inch.

(Based on Messrs. Briggs and Towne's data.)

Arcs of Contact.	Maximum Working Stress transmitted, per inch wide.	Power transmitted, per inch wide. At 1 foot per second, Velocity of Belt.	Per foot of Diameter of Pulley, and per Turn per Minute.		Sum of the Tensions on both sides of a Belt, per inch wide.	Resultant Pressure on the Journals, per inch width of Belt.
degrees.	lbs.	horse-power.	horse-power.	foot-lbs.	lbs.	lbs.
90°	32.33	.059	.00308	102	101.00	71.42
100	34.80	.063	.00331	109	98.53	75.47
110	37.07	.067	.00353	116	96.26	78.85
120	39.18	.071	.00373	123	94.15	81.53
135	42.06	.076	.00400	132	91.27	84.32
150	44.64	.081	.00425	140	88.69	85.67
180	49.01	.089	.00467	154	84.32	84.32
210	52.52	.095	.00500	165	80.81	78.05
240	55.33	.100	.00527	174	78.00	67.59
270	57.58	.105	.00548	181	75.75	53.56

Note.—The thickness of belt is .22 inch for a maximum working strength of 66⅔ lbs. per inch wide. For any other thickness the data in the table are to be altered in the ratio of .22 to the thickness.

India-Rubber Belting.

Driving Belts[1] manufactured from American cotton canvas, cemented in layers by vulcanized india-rubber, and coated with the same material, have been tested for strength and adhesion. It is stated that a strip 1 inch wide bears a tensile stress of 200 lbs., and that the india-rubber belt possesses about three times the surplus or effective adhesion of leather belts.

Weight of Belt-Pulleys.

The weight of pulleys of the same diameter, varies within much wider limits than that of spur-wheels, not merely because the breadth varies very much, but also that there is greater variation per inch of breadth. The following formulas for the weight of drum-pulleys are, therefore, the expression of average weights for pulleys of medium proportions. For pulleys designedly strong and heavy, up to 30 inches in diameter, the weight per inch of breadth may be as much as 25 per cent. more than the average, or, for particularly light pulleys, as much lighter. As the diameter increases, the percentage of variation diminishes; and for 6-feet or 7-feet pulleys, it may never exceed 10 per cent. either way.

[1] Manufactured by the North British Rubber Company.

The author is indebted to Mr. R. Heber Radford for the examples of the actual weights of pulleys as used in the Sheffield, Manchester, and Bradford districts, given in tables No. 262, 263, and 264. The averaged weights per inch wide of the Sheffield and the Manchester finished pulleys, in terms of the diameter ranging from 1 foot to 4 feet, are found, by plotting, to be expressed by the same formula. For the pulleys of the Bradford district, the formula is slightly different from that; but the chief interest of the Bradford examples, consists in the data they afford of the reduction of the weight of the rough castings, by the operations of turning, boring, and slotting. From these data, the following formulas have been deduced, showing that the increase of weight per inch wide, is simply as the increase of diameter:—

Table No. 262.—Weight of Finished Cast-Iron Pulleys, Sheffield District.

(Examples contributed by Mr. R. Heber Radford.)

Diameter.		Width.	Weight, turned, bored, and slotted.		Diameter.		Width.	Weight, turned, bored, and slotted.	
			Total.	Per inch wide.				Total.	Per inch wide.
feet.	ins.	inches.	lbs.	lbs.	feet.	ins.	inches.	lbs.	lbs.
1	0	6	28	4.7	2	6	10	224	22.4
1	3	6	63	10.5	2	6	12	232	19.3
1	3	10	86	8.6	2	8	9	110	12.2
1	3	12	102	8.5	2	11½	6	140	23.3
1	5	7	66	9.4	3	0	8	120	15.0
1	6	9	75	8.3	3	0	12	108	9.0
1	9	9	80	9.0	3	0	14	136	9.7
1	9	10	84	8.4	3	5	8	160	20.0
2	0	6½	114	17.5	3	6	6	160	26.7
2	0	10	120	12.0	3	7	12	175	14.6
2	0	10	158	15.8	4	0	8	212	26.5
2	0	16	170	10.6	4	0	8	225	28.1
2	5	6	110	18.3	4	1½	12	338	28.2

Table No. 263.—Weight of Finished Cast-Iron Pulleys, Manchester District.

(Examples contributed by Mr. R. Heber Radford.)

Diameter.		Width.	Diameter of Hole.	Weight—turned, bored, and slotted.		Diameter.		Width.	Diameter of Hole.	Weight—turned, bored, and slotted.	
				Total	Per inch wide.					Total.	Per inch wide.
feet.	ins.	inches.	inches.	lbs.	lbs.	feet.	ins.	inches.	inches.	lbs.	lbs.
1	4	3	2½	28	9.3	2	6	6	2½	105	17.5
2	0	6	2½	74	12.3	2	6	8	2	116	14.5
2	0	6	2	76	12.7	3	0	6	2½	129	21.5
2	0	8	2½	84	10.5	3	0	6	2	134	22.3
2	2	6	2	87	14.5	3	6	6	2½	137	22.8
2	3	6	2½	93	15.5	3	6	6	2	144	24.0

Table No. 264.—Weight of Rough Castings, and Finished Cast-Iron Pulleys, Bradford District.

(Examples contributed by Mr. R. Heber Radford.)

Diameter.		Width.	Diameter of Hole.	Weight, Rough Castings.		Weight, turned, bored, & slotted,		Reduction of Weight.
				Total.	Per inch wide.	Total.	Per inch wide.	
feet.	inches.	inches.	inches.	lbs.	lbs.	lbs.	lbs.	per cent.
	10	3	1⅝	16	5.3	13	4.3	19
1	0	4¼	2	21	5.0	18	4.0	14
1	2	4	2	31	7.75	27	6.75	13
1	4	4¾	2¼	44	9.26	38	8.0	13.6
1	6	5½	2⅛	63	11.4	53	9.6	16
1	8	9	2¾	104	11.6	92	10.2	11.5
1	10	10	2¾	132	13.2	118	11.8	10.6

Weight of Pulleys per inch wide, in the Lancashire and Yorkshire Districts, from 1 *foot to* 4 *feet in Diameter.*

Rough Castings,..................... $W = 7.625d - 1.5$ .... (13)
Turned and Finished Pulleys,..... $W = 7d - 1.75$ .... (14)

W = the weight of the pulley in pounds per inch wide.
d = the diameter in feet.

Note.—These formulas are probably applicable for pulleys of from 10 inches to 10 feet in diameter.

From the weights of a very large number of rough castings of pulleys, ranging from 1 foot to 7 feet in diameter, as used in the London district, supplied by Mr. Charles Mackintosh, the following formulas have been deduced. For rough castings above 2 feet in diameter, the weight increases simply as the diameter increases. For diameters less than 2 feet, the weight increases with the square of the diameter. The same proportional reduction for the finished weight may be applied to the London pulleys as was done to the Lancashire pulleys:—

Weight of Pulleys per inch wide, in the London District, from 1 *foot to* 7 *feet in diameter.*

Rough castings { not exceeding 2 feet in diameter,........ $W = 3d^2 + 3$...... (15)
Rough castings { 2 feet in diameter and upwards,.... $W = 12\frac{1}{4}d - 9.5$...... (16)

Turned and finished pulleys { not exceeding 2 feet in diameter, $W = 3d^2 - .625d + 2.75$ (17)
Turned and finished pulleys { 2 feet in diameter and upwards, $W = 11.625d - 9.25$..... (18)

The weights of pulleys, rough as cast, and turned and finished, have been calculated by means of the foregoing formulas, for diameters increasing from 10 inches to 8 feet; given in table No. 265. It is apparent that the London pulleys are much heavier than the country pulleys. The reduction of the weight of the rough castings by turning and finishing, varies from 13 per cent. for 12-inch pulleys, to 10½ per cent. for 2-feet pulleys, and 9 per cent. for 8-feet pulleys, for the country pulleys; and from 15 to 7 per cent. for the London pulleys.

Table No. 265.—Belt Pulleys—Calculated Weights.

Diameter.	Lancashire and Yorkshire. Weight per inch wide.		London. Weight per inch wide.		Diameter.	Lancashire and Yorkshire. Weight per inch wide.		London. Weight per inch wide.	
	Rough Castings.	Turned and Finished.	Rough Castings.	Turned and Finished.		Rough Castings.	Turned and Finished.	Rough Castings.	Turned and Finished.
inches.	lbs.	lbs.	lbs.	lbs.	feet.	lbs.	lbs.	lbs.	lbs.
10	4.8	4.08	5.1	4.33	2.75	19.5	17.50	24.2	22.2
11	5.5	4.67	5.5	4.68	3	21.4	19.25	27.2	25.1
12	6.1	5.25	6.0	5.13	3.25	23.3	21.00	30.2	27.9
13	6.7	5.83	6.6	5.67	3.5	25.2	22.75	33.2	30.1
14	7.5	6.42	7.1	6.12	4	29.0	26.25	39.5	36.8
15	8.0	7.00	7.7	6.67	4.5	32.8	29.75	45.5	42.4
16	8.7	7.58	8.3	7.22	5	36.6	33.25	51.5	48.1
18	9.9	8.75	9.7	8.51	5.5	40.4	36.75	57.5	53.8
20	11.2	9.91	11.3	10.00	6	44.2	40.25	64.0	60.0
21	11.6	10.50	12.2	10.9	6.5	48.1	43.75	70.0	65.7
feet.									
2	13.7	12.25	15.0	13.50	7	51.9	47.25	76.3	71.7
2.25	15.7	14.00	18.1	16.4	7.5	55.7	50.75	82.5	77.3
2.5	17.6	15.75	21.1	19.3	8	59.5	54.25	90.5	84.3

ROPE-GEARING.[1]

Round hemp-ropes working in grooved wheels are occasionally employed instead of belts or toothed wheels for transmitting power from the engine. The fly-wheel is made considerably wider than a spur fly-wheel would be, but rather less than a belt-wheel would be, and V grooves are turned out of the circumference, the sides of which are at an angle of 40°, and the number and size of which are regulated by the quantity of power to be taken off. The ropes are usually 5¼ and 6½ inches in circumference for larger powers, and 4¼ inches for smaller powers. To prevent wear and tear of rope, the circumference or the diameter of a pulley should be at least 30 times that of the rope, and the shafts should be at a distance apart of from 20 to 60 feet.

The number of ropes required for the transmission of a given power is determined from the circumferential velocity of the fly-wheel, which is generally between 3000 and 6000 feet per minute. Mr. Durie instances the rope-gearing of Messrs. Nicoll's factory at Dundee, which was erected in 1870. The power of the engine varies from 400 to 425 indicator horse-power. The fly-wheel, 22 feet in diameter, makes 43 turns per minute, with a surface velocity of 2967 feet per minute; it is 4 feet 10 inches wide, and has 18 grooves, each of which is occupied by a 6½-inch rope, transmitting the power of say 23 indicator horse-power. Five ropes are employed to transmit the power to the ground floor, over a 7½-feet pulley;

[1] See a paper read by Mr. James Durie, at the Institution of Mechanical Engineers, published in *Engineering*, November 3, 1876, page 394.

four ropes drive a 5½-feet pulley on the first floor; and six ropes drive a 5½-feet pulley on the second floor. Lastly, on the other side of the engine-shaft, the power is transmitted by three ropes to a weaving-shed, on a 7½-feet pulley. For 23 horse-power, the stress on each rope is $(\frac{33000 \times 23}{2967} =)$ 256 lbs., less the resistance of the engine. When the ropes become too slack, they are cut and re-spliced, and the work of a rope under such treatment is temporarily performed by the other ropes driving the same pulley.

In another example 40 indicator horse-power is disposed of, for each 6½-inch rope, at a velocity of 3784 feet per minute; and the equivalent stress is $(\frac{33000 \times 40}{3784} =)$ 349 lbs. for each rope.

Taking the ultimate strength of a 6½-inch rope at 10 tons, or 22,400 lbs., it would appear that the working stress is only about 1½ per cent. of the ultimate strength; giving a factor of safety, 67.

It is believed that an economy of power is effected by the substitution of rope-gearing for toothed-gearing. A 6½-inch rope is equivalent, according to Mr. Durie, to a leather belt 4 inches wide, for the transmission of work, at say 3000 feet per minute. From some comparative experiments made by Mr. W. A. Pearce, Dundee, it appears that a 6-inch rope in a grooved pulley possessed four times the adhesive resistance to slipping exhibited by a half-worn ungreased 4-inch single belt.

The ropes used for gearing are made of carefully selected hemp:—the fibres very long, well twisted and laid, yet soft and elastic. The splice should be uniform, of the same diameter as the rope, and 9 or 10 feet long.

Transmission of Power by Rope to Great Distances.

Wire-Ropes.—M. Hirn, in 1850, made many trials with endless bands of steely iron passing over pulleys for the conveyance of power to great distances; but he finally adopted iron wire-ropes, unannealed, working over grooved pulleys of large diameter. M. Umber,[1] in 1859, described the apparatus. The pulleys may be of hard wood; they are formed with a groove slightly rounded, about 2 inches deep and 1½ inches wide, lined at the bottom with leather or gutta-percha. They should be at least 1 metre, or 3.28 feet, in diameter, and should be driven at the greatest practicable speed. A diameter equal to 200 times that of the cable, is the most suitable proportion. The distance apart of the driving and the driven pulleys should be not less than from 130 to 160 feet, and the pulleys may be placed at any greater distance apart. The greater the distance apart, the steadier the movement. The velocity of the rope is about 50 feet per second, or 3000 feet per minute. At this rate, a force of 11 lbs. would be equivalent to 1 horse-power.

The most common sizes of wire-rope employed are as follows:—

Diameter.	Weight per Metre.	Weight per Yard.
4 millimetres, or .16 inch.	.10 kilogramme.	.20 pound.
6 „ or .24 „	.17 „	.34 „
9 „ or .35 „	.31 „	.62 „
12 „ or .47 „	.45 „	.90 „

At Colmar, a force of 47 horse-power is transmitted a distance of 250

[1] *Annales des Ponts et Chaussées*, 1859.

yards by a ½-inch wire-rope, over two pulleys of 3 metres, or about 10 feet in diameter, making 95 turns per minute. The rope is supported at the middle of the span by pulleys of 1 metre in diameter. The frictional resistance is less than 3 per cent. The ropes receive a coat of a mixture of oil and tar twice per month, and they wear well.

The ropes, in all cases, consist of 36 wires, in six strands of 6 wires each, on a core of hemp. Each strand likewise is formed on a hempen core. The hempen cores are favourable for flexibility.

From another account, it appears that 100 horse-power can be transmitted 120 yards without any intermediate support, by an endless wire-rope of .40 inch in diameter, over pulleys from 13 to 14 feet in diameter, making 100 turns per minute, equivalent to a velocity of rope of upwards of 4000 feet per minute. For longer distances, the rope is supported at intervals of 160 yards by 7-feet pulleys. The calculated loss of power in transmitting 120 horse-power is 17½ per cent., or 21 horse-power. The sources of loss are:—1st. The resistance of the air to the arms of the wheels. 2d. The resistance of rigidity of the rope in passing over the wheels. 3d. Axle-friction:—fixed loss 2½ per cent. for the large pulleys, and 1 per cent. for every 1000 yards.

In an excellent illustrated account of M. Hirn's rope-transmitter, by Mr. H. M. Morrison,[1] he states that soft willow wood succeeds best as lining for the large pulleys. The pulleys were constructed successively of copper, hardwood, and polished cast iron, and were also faced with leather, horn, india-rubber, lignum-vitæ, and boxwood; but all these materials failed, as the facings were soon worn out, and when the groove was of metal or hardwood, and did not itself wear, it destroyed the rope. The tension in the upper rope, he says, is just double that in the lower rope. The best method of changing the direction of transmission of the power, at any point in its course, has been found by experience to be by the use of bevil-wheels. Directing pulleys are not so good for the purpose. For high speeds, the pulleys should be of best cast steel, as iron pulleys may fly to pieces by centrifugal force. Mr. Morrison states that the fine makes of ropes are constructed of 6 strands of 12 wires each—72 wires in all; and that in America, the wires are still finer and closer, and as many as 135 in number.

Cotton Ropes.—Mr. Ramsbottom, in 1863, applied cotton ropes or cords, for driving the traversing cranes at Crewe workshops.[2] The cords are made of soft white cotton, ⅝ inch in diameter when new, and weighing 1½ oz. per foot; they soon become reduced to $^9/_{16}$ inch thick by stretching, and they last about eight months. They are, when new, rubbed over with a little tallow and wax. The total lengths of each of the two cords, in three different shops, are respectively 800, 320, and 560 feet. The pulleys over which they are passed are not less than 18 inches in diameter, or 32 diameters of the cord; and in the first of the above shops, alone, the cord makes from 12 to 20 bends according to the machinery in action. The groove of the driving-pulleys is V-shaped, at an angle of 30°, and the cord is gripped between the inclined sides. The cord is supported at intervals of 12 or 14 feet by flat slippers of chilled cast iron.

The velocity of the cord is 5000 feet per minute; and as some of the pulleys make 1000 turns per minute, they require to be perfectly self-balanced,

[1] *Proceedings of the Institution of Mechanical Engineers*, 1874.

[2] See his paper on the subject, in the *Proceedings of the Institution of Mechanical Engineers*, 1864.

in order to run with steadiness and ease. In the overhead traversers, the total leverage is slightly over 3000 to 1; and in lifting a load of 9 tons, the actual pull on the rope is 17 lbs. A tightening stress on the cord of 109 lbs., applied by means of a weighted pulley, is found to keep it steady, and to give the required degree of hold on the main driving-pulley.

In the wheel-shop, two dozen pairs of blocks and ropes had previously been employed, requiring a large number of labourers to work them; whilst now the two traversing cranes, with two men to work them, do the whole work of the shop, and it is done much more quickly than before.

SHAFTING.

Shafting is subject to two kinds of stress:—transverse and torsional. The dimensions of shafts are settled by conditions of stiffness, or resistance to deflection, under the action of either kind of stress.

Transverse Deflection of Shafts.

The deflections of cast-iron, wrought-iron, and steel bars or shafts, loaded at the middle, are given by formulas (8) and (10), page 564; (5) and (6), page 590; and (3) and (4), page 619. The deflection under the same weight uniformly distributed, is ⅝ths of that under the weight when placed at the middle. Altering some of the measures, let

D = the deflection, in inches.
W = the weight, in pounds.
l = the length or distance between centres of bearings, in feet.
d = the diameter of the round shaft, in inches.
b = the side of the square shaft, in inches.

The modified formulas, adapted for uniformly distributed weight, are given in two series; first, for shafts simply supported at the ends; second, for shafts fixed at both ends, as are continuous shafts. In settling the divisors for the second series, it is assumed that the deflections are one-half of those of the first series. The deflections, actually, are not so great as one-half; and margin is thus left for deflection arising from the mode of application of the torsional force, and for the excess of deflection at the loose end of the line of shafting.

Transverse Deflection of Shafts, under Uniformly Distributed Weight.

Supported at the ends. Fixed at the ends.

Cast-iron shafts:—Round, $D = \dfrac{W\,l^3}{39{,}400\,d^4}$; $D = \dfrac{W\,l^3}{79{,}000\,d^4}$ .... (1)

Square, $D = \dfrac{W\,l^3}{58{,}000\,b^4}$; $D = \dfrac{W\,l^3}{116{,}000\,b^4}$ ... (2)

Wrought-iron shafts:—Round, $D = \dfrac{W\,l^3}{66{,}400\,d^4}$; $D = \dfrac{W\,l^3}{133{,}000\,d^4}$ ... (3)

Square, $D = \dfrac{W\,l^3}{97{,}500\,b^4}$; $D = \dfrac{W\,l^3}{195{,}000\,b^4}$ ... (4)

Steel shafts:—Round, $D = \dfrac{W\,l^3}{78{,}800}$; $D = \dfrac{W\,l^3}{158{,}000}$ ...... (5)

Square, $D = \dfrac{W\,l^3}{116{,}000}$; $D = \dfrac{W\,l^3}{232{,}000}$ ...... (6)

The working limit of deflection is taken as $^1/_{100}$ inch per foot of length, or of the distance of bearings; and the limiting value of D in inches is $\frac{l}{100}$. By substitution of this value for D; and by reduction and inversion:—

The Diameter and the Side for the Limiting Transverse Deflection.

Supported at the ends. Fixed at the ends.

Cast-iron shafts:—Round, $d^4 = \frac{W\,l^2}{394}$; $d^4 = \frac{W\,l^2}{790}$.......... (7)

Square, $b^4 = \frac{W\,l^2}{580}$; $b^4 = \frac{W\,l^2}{1160}$.......... (8)

Wrought-iron shafts:—Round, $d^4 = \frac{W\,l^2}{664}$; $d^4 = \frac{W\,l^2}{1330}$......... (9)

Square, $b^4 = \frac{W\,l^2}{975}$; $b^4 = \frac{W\,l^2}{1950}$......... (10)

Steel shafts:—Round, $d^4 = \frac{W\,l^2}{788}$; $d^4 = \frac{W\,l^2}{1576}$......... (11)

Square, $d^4 = \frac{W\,l^2}{1160}$; $d^4 = \frac{W\,l^2}{2320}$......... (12)

The Distributed Weight for the Limiting Transverse Deflection.

Supported at both ends. Fixed at the ends.

Cast-iron shafts:—Round, $W = \frac{394\,d^4}{l^2}$; $W = \frac{790\,d^4}{l^2}$....... (13)

Square, $W = \frac{580\,b^4}{l^2}$; $W = \frac{1160\,b^4}{l^2}$...... (14)

Wrought-iron shafts:—Round, $W = \frac{664\,d^4}{l^2}$; $W = \frac{1330\,d^4}{l^2}$...... (15)

Square, $W = \frac{975\,b^4}{l^2}$; $W = \frac{1950\,b^4}{l^2}$...... (16)

Steel shafts:—Round, $W = \frac{788\,d^4}{l^2}$; $W = \frac{1576\,d^4}{l^2}$...... (17)

Square, $W = \frac{1160\,d^4}{l^2}$; $W = \frac{2320\,d^4}{l^2}$...... (18)

[*Overhung shafts.*—When the weight is overhung on a length l from the bearing, the deflection is approximately 16 or 20 times the deflection as found by the above formulas, for the same weight, on the same length, l, when supported at both ends.]

Let the total distributed weight, including the weight of the shaft with wheels and pulleys, and the resultant stresses of driving bands, be taken at 1¾ times the weight of the shaft. The weight, in pounds per foot, of a wrought-iron round shaft, is 3.33 lbs. per square inch of section; and with wheels, &c., it is (3.33 × 1¾ =) 5.83 lbs. per square inch. In terms of the diameter d, for a length in feet l, the gross weight W for wrought-iron is $.7854\,d^2\,l \times 5.83$; for cast iron, in the same way, it is $.7854\,d^2\,l \times 5.47$; and for steel, it is $.7854\,d^2\,l \times 5.95$. Whence:—

Gross Distributed Weight of Round Shafts with Mounting, for the Limiting Span.

(Shaft fixed at both ends.)

Cast-iron round shafts, ...$W = 4.30\ d^2\ l$.................. (19)
Wrought-iron round shafts, ...$W = 4.58\ d^2\ l$.................. (20)
Steel round shafts, ...$W = 4.67\ d^2\ l$.................. (21)

Substituting these values of W, in formulas (13), (15), and (17), for the shaft fixed at the ends, and reducing:—

Length of Span for the Limiting Transverse Deflection under the Gross Distributed Weight.

(Shaft fixed at both ends.)

Cast-iron round shafts,......$l^3 = 184\ d^2$; and $l = \sqrt[3]{184\ d^2}$... (22)

Wrought-iron round shafts, $l^3 = 290\ d^2$; and $l = \sqrt[3]{290\ d^2}$... (23)

Steel round shafts,...........$l^3 = 337\ d^2$; and $l = \sqrt[3]{337\ d^2}$... (24)

When the shaft is employed to transmit power without giving off any, the distributed weight is only that of the shaft itself. The formulas (22), (23), and (24) may be adapted for the less weight by increasing the coefficients in the ratio of 1 to 1¾.

Length of Span for the Limiting Transverse Deflection under the Net Weight of the Shaft only.

(Shaft fixed at both ends.)

Cast-iron round shafts,......$l^3 = 322\ d^2$; and $l = \sqrt[3]{322\ d^2}$... (25)

Wrought-iron round shafts, $l^3 = 508\ d^2$; and $l = \sqrt[3]{508\ d^2}$... (26)

Steel round shafts,...........$l^3 = 591\ d^2$; and $l = \sqrt[3]{591\ d^2}$... (27)

The length of span under the gross distributed weight, is to that under the net weight of the shaft, as 1 to $\sqrt[3]{1.75}$, or as 1 to 1.205, or 1 to 1 $^1/_5$.

THE ULTIMATE TORSIONAL STRENGTH OF ROUND SHAFTS.

Modifying formulas (14), page 566; (9), page 591; and (5), page 620; to express the relations of the moment W′ R, in statical foot-pounds, being the diameter in inches.

Torsional Strength of Round Shafts.

For cast iron,..................$W' R = 373\ d^3$; $d^3 = \dfrac{W' R}{373}$ ... (28)

For wrought iron, ...$W' R = 933\ d^3$; $d^3 = \dfrac{W' R}{933}$ ... (29)

For steel, tensile strength 30 tons per sq. in....... $W' R = 1120\ d^3$; $d^3 = \dfrac{W' R}{1120}$ ... (30)

Torsional Deflection of Round Shafts.

The deflections of round bars or shafts of cast iron, wrought iron, and steel, under torsional stress, within elastic limits, are given by formulas (18), page 566; (14), page 592; and (8), page 621. Altering some of the measures, let

D' = the angular deflection in parts of a revolution.
W' = the twisting force, in pounds.
R = the radius of the force, in feet.
$W'R$ = the moment of the force, in statical foot-pounds.
l' = the length of the shaft, in feet.
d = the diameter of the shaft, in inches.

The modified formulas are as follows; the coefficients are given in the nearest round numbers.

Torsional Deflection of Round Shafts.

Cast-iron shafts,............ $D' = \dfrac{W'Rl}{11,100\, d^4}$ .... (31)

Wrought-iron shafts,......... $D' = \dfrac{W'Rl}{16,600\, d^4}$ .... (32)

Steel shafts,................. $D' = \dfrac{W'Rl}{34,300\, d^4}$ .... (33)

A torsional deflection of 1°, in a length equal to 20 diameters of the shaft, is a good working limit of deflection; that is, $^1/_{360}$ th part of a turn, or .00278 turn, for 20 diameters. Now, for cast-iron, $W'R = \dfrac{11,100\, d^4 D'}{l}$; wrought iron, $W'R = \dfrac{16,600\, d^4 D'}{l}$; steel, $W'R = \dfrac{34,300\, d^4 D}{l}$; and, substituting .00278 for D', and $\dfrac{20\,d}{12}$ for l, in these equations, and reducing:—

The Working Moment of the Force, and the Diameter, for the Limiting Torsional Deflection.

For cast iron,....... $W'R = 18.5\, d^3$; $d^3 = \dfrac{W'R}{18.5}$ .... (34)

For wrought iron, $W'R = 27.7\, d^3$; $d^3 = \dfrac{W'R}{27.7}$ .... (35)

For steel,........... $W'R = 57.2\, d^3$; $d^3 = \dfrac{W'R}{57.2}$ .... (36)

By these convenient transformations, the diameter is reduced to the third power; and, since the ultimate torsional strength is also in the ratio of the third power, the same margin of strength, or factor of safety, is provided by the formulas for all diameters. Comparing the coefficients in the foregoing formulas, for the ultimate and the working moments, the factors of safety are found to be,—

For cast-iron round shafts, ... $\frac{373}{18.5} = 20$, factor of safety.

For wrought-iron round shafts, $\frac{933}{27.7} = 34$, factor of safety.

For steel round shafts, ... $\frac{1120}{57.2} = 19.5$, factor of safety.

The formulas (34), (35), and (36) are reduced to rules as follows:—

RULE 1. *To find the maximum Torsional Stress that may be transmitted by a shaft, within good working limits.*—Multiply the cube of the diameter in inches, by 18.5 for cast iron; by 27.7 for wrought iron; or by 57.2 for steel. The product is the torsional stress in statical foot-pounds.

RULE 2. *To find the Diameter of a shaft capable of transmitting a given torsional stress, within good working limits.*—Divide the torsional stress in statical foot-pounds, by 18.5 for cast iron; by 27.7 for wrought iron; or by 57.2 for steel. The cube-root of the quotient is the diameter in inches.

Note.—The torsional stress is expressed by the product of the actual torsional force in pounds, by the radial distance in feet at which it is applied.

POWER THAT MAY BE TRANSMITTED BY ROUND SHAFTS, WITHIN GOOD WORKING LIMITS.

The working moments of the force, in statical foot-pounds, formulas (34), (35), and (36), are equivalent to as many pounds acting at a radius of 1 foot; and for one turn of the shaft, the work done is equivalent to the product of the moment by (2 feet × 3.1416 =) 6.28:—

The Work for One Turn of a Round Shaft.

$$\text{Cast iron, ... } U = 116\,d^3 \qquad (37)$$
$$\text{Wrought iron, } U = 174\,d^3 \qquad (38)$$
$$\text{Steel, ... } U = 359\,d^3 \qquad (39)$$

The horse-power developed is equal to the work done in one turn multiplied by the speed, or number of turns per minute, divided by 33,000:—

Horse-power of a Round Shaft.

$$\text{Cast iron, ... } H = \frac{116\,d^3\,S}{33{,}000} = \frac{d^3\,S}{285}. \qquad (40)$$

$$\text{Wrought iron, } H = \frac{174\,d^3\,S}{33{,}000} = \frac{d^3\,S}{190}. \qquad (41)$$

$$\text{Steel, ... } H = \frac{359\,d^3\,S}{33{,}000} = \frac{d^3\,S}{92}. \qquad (42)$$

in which S = the speed in turns per minute, and H = the horse-power.

RULE 3. *To find the maximum Horse-power of a shaft, within good working limits.*—Multiply the cube of the diameter in inches, by the speed in turns per minute; and divide by 285 for cast iron, by 190 for wrought iron, or by 92 for steel. The quotient is the horse-power.

The following additional rules are obtained by inversion of the formulas (40), (41), and (42):—

RULE 4. *To find the Diameter of a shaft capable, within good working limits, of transmitting a given horse-power.*—Multiply the horse-power by 285 for cast iron, by 190 for wrought iron, or by 92 for steel; and divide by the speed in turns per minute. The cube-root of the quotient is the diameter in inches.

RULE 5. *To find the Speed required for transmitting a given horse-power, within good working limits.*—Multiply the horse-power by 285 for cast iron, by 190 for wrought iron, or by 92 for steel; and divide the product by the cube of the diameter in inches. The quotient is the speed in turns per minute.

The table No. 266 shows the net weight of round wrought-iron shafting per lineal foot, extracted from table No. 76, page 240; and the gross weight per lineal foot, comprising weight of pulleys, stress of belts, &c., taken at 1¾ times the net weight of the shafting.

Table No. 266.—WEIGHT OF ROUND WROUGHT-IRON SHAFTING.

Gross weight = 1¾ times net weight of shaft.

Diameter of Shaft.	Weight per lineal foot.		Diameter of Shaft.	Weight per lineal foot.		Diameter of Shaft.	Weight per lineal foot.	
	Net.	Gross.		Net.	Gross.		Net.	Gross.
inches.	lbs.	lbs.	inches.	lbs.	lbs.	inches.	lbs.	lbs.
1	2.62	4.58	3¾	33.5	58.8	9	212	371
1¼	4.09	7.15	4	41.9	73.5	9½	236	413
1½	5.89	10.3	4¼	47.3	82.8	10	262	458
1⅝	6.91	12.1	4½	53.0	92.8	11	317	555
1¾	8.02	14.0	4¾	59.1	104	12	377	660
1⅞	9.20	16.1	5	65.5	115	13	398	697
2	10.5	18.3	5¼	72.2	126	14	462	808
2⅛	11.8	20.6	5½	79.2	139	15	530	928
2¼	13.3	23.3	5¾	86.6	152	16	670	1176
2⅜	14.8	25.9	6	94.2	165	17	759	1330
2½	16.4	28.7	6½	111	196	18	848	1484
2¾	19.8	34.6	7	128	224	19	945	1652
3	23.6	41.3	7½	147	257	20	1040	1834
3¼	27.7	48.4	8	168	294			
3½	32.1	56.2	8½	189	331			

The table No. 267 gives the torsional strength and horse-power of round shafts of wrought iron, within the good working limits already defined, from 1 inch to 20 inches in diameter. The 2d column, ultimate torsional resistance reduced to statical foot-tons, is calculated by means of formula (29), page 758; the 3d column, working torsional stress, is calculated with formula (35), page 759; the 4th column, work done for one turn, with formula (38), page 760; the 5th column, horse-power at the rate of one turn per minute, with formula (41), page 760; the 6th column, speed required for one horse-power, contains the reciprocals of the values in column 5; they may be calculated by rule 5, above; the 7th and 8th columns, distance of bearings and distributed weight, are calculated with formulas (23) and (20), page 758; and the 9th column, distance of bearings under net weight, with formula (26). Multipliers for shafts of cast iron and of steel, are subjoined to the table.

Table No. 267.—STRENGTH OF ROUND WROUGHT-IRON SHAFTING.

Diameter of Shaft.	Torsional Action.					Transverse Action.		
	Ultimate Resistance.	Working Stress.	Work for One Turn per Minute.	Horse-Power at the rate of One Turn per Minute.	Speed in Turns per Minute for One Horse-Power.	Under the Gross Distributed Weight. Distance of Bearings for the Limiting Defl'tion.	Under the Gross Distributed Weight. Gross Weight for the Span.	Under the net Weight of Shaft. Distance of Bearings for the Limiting Deflection.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
inches.	statical ft.-tons.	statical ft.-pounds.	ft.-pounds.	H. P.	turns.	feet.	lbs.	feet.
1	.42	27.7	174	.00526	190	6.6	30	7.9
1¼	.82	54.1	340	.01028	97.3	7.7	55	9.2
1½	1.42	93.5	587	.01779	56.2	8.6	89	10.3
1⅝	1.80	118.9	746	.02259	44.3	9.2	112	11.0
1¾	2.25	148.4	932	.02820	35.4	9.6	134	11.5
1⅞	2.77	182.6	1147	.03469	28.8	10.1	163	12.1
2	3.36	221.6	1391	.04211	23.7	10.5	193	12.7
2⅛	4.00	265.8	1669	.05062	19.8	11.0	227	13.2
2¼	4.80	315.5	1981	.05995	16.7	11.4	264	13.7
2⅜	5.62	371.1	2330	.07051	14.2	11.8	305	14.2
2½	6.56	432.8	2718	.08224	12.2	12.5	359	15.0
2¾	8.73	576.1	3618	.1094	9.14	13.0	450	15.6
3	11.3	747.9	4697	.1421	7.04	13.7	566	16.5
3¼	14.4	951.0	5972	.1807	5.54	14.5	701	17.4
3½	18.0	1188	7458	.2257	4.43	15.2	854	18.3
3¾	22.1	1461	9173	.2775	3.60	16.0	1029	19.2
4	26.9	1773	11,130	.3368	2.97	16.7	1225	20.1
4¼	32.2	2127	13,360	.4040	2.48	17.4	1439	20.9
4½	38.2	2524	15,850	.4796	2.09	18.1	1679	21.7
4¾	45.0	2969	18,650	.5642	1.77	18.8	1943	22.6
5	52.5	3463	21,740	.6579	1.52	19.4	2220	23.3
5¼	60.7	4008	25,170	.7616	1.31	20.0	2525	24.0
5½	69.8	4609	28,950	.8758	1.14	20.6	2854	24.7
5¾	79.8	5266	33,070	1.000	1.00	21.2	3210	25.4
6	90.6	5983	37,570	1.137	.880	21.8	3600	26.2
6½	117	7606	47,770	1.445	.692	22.9	4421	27.5
7	144	9501	59,670	1.805	.554	24.2	5426	29.0
7½	177	11,680	73,390	2.220	.450	25.3	6518	30.4
8	215	14,180	89,070	2.694	.371	26.5	7774	31.8
8½	258	17,010	106,800	3.232	.309	27.6	9133	33.1
9	306	20,190	126,800	3.837	.261	28.7	10,650	34.4
9½	360	23,750	149,200	4.512	.222	29.8	12,320	35.7
10	420	27,700	174,000	5.260	.190	30.8	14,100	36.9
11	559	36,870	231,500	7.005	.143	32.8	18,180	39.4
12	725	47,860	300,600	9.095	.110	34.7	22,880	41.7
13	922	60,860	382,200	11.83	.0865	36.6	28,330	44.0
14	1152	76,010	477,300	14.44	.0693	38.5	34,560	46.2
15	1417	93,490	587,100	17.76	.0563	40.3	41,530	48.4
16	1720	113,500	712,500	21.56	.0464	42.1	49,330	50.5
17	2062	136,100	854,800	25.86	.0387	43.8	57,970	52.6
18	2447	161,500	1,015,000	30.69	.0326	45.5	67,490	54.6
19	2880	190,000	1,193,000	36.10	.0277	47.2	78,040	56.6
20	3360	221,600	1,391,000	42.11	.0237	48.8	80,660	58.5

Note.—To find the corresponding values for shafts of cast iron and of steel, multiply the tabular values by the following multipliers:—

Cast Iron,	2/5	2/3	2/3	2/3	1.5	.86	.81	.86
Steel,	1.2	2.06	2.06	2.06	.48	1.05	1.07	1.05

Frictional Resistance of Shafting.

The frictional resistance of horizontal shafting running on cylindrical journals, is calculable by means of formulas (2) and (7), pages 725, 726; where the coefficient of friction, f, is determined by experiment. M. Morin's data, page 722, show that the coefficient is .075 with ordinary oiling, and .042 with continuous oiling.

The table No. 268, next page, is based on the results of extensive observations on the resistance of mill-shafting in America, by Mr. S. Webber,[1] of Manchester, N.H. Take the average of his frictional coefficients with those of M. Morin:—

	Ordinary Oiling.	Continuous Oiling.
M. Morin's coefficients,...	.075	.042
Mr. Webber's coefficients,	.066	.044
Means,............	.070 or, $^1/_{14}$th	.043 or, $^1/_{23}$d

Substituting these values of the coefficient f in formulas (2) and (7), pages 725, 726:—

Work Absorbed by Friction for One Turn of a Horizontal Shaft.

Ordinary oiling,............... $U = .0182\ W d$ .... (43)

Continuous oiling, ... $U = .0112\ W d$ .... (44)

Horse-power Absorbed by Friction of a Horizontal Shaft.

Ordinary oiling,......$H = \frac{.0182\ W d S}{33{,}000} = \frac{W d S}{1{,}800{,}000}$ .... (45)

Continuous oiling,...$H = \frac{.0112\ W d S}{33{,}000} = \frac{W d S}{2{,}950{,}000}$ .... (46)

U = work absorbed, in foot-pounds.
W = total weight of shafting and pulleys, plus the resultant stress of belts, in pounds.
H = horse-power absorbed.
d = diameter of journals, in inches.
S = the number of turns per minute.

The resistance of upright shafting is probably about three-fourths of that of horizontal shafting:—in the ratio of the resistance of a cylindrical pivot to that of a journal.

Ordinary Data for the Resistance of Shafting.—Mr. R. H. Tweddell gives the following results of observations:—

Indicator horse-power,	driving the shafting of a tool-shop alone,		6.65
Do.	do.	resistance of steam-engine alone,...........	3.51
Do.	do.	net power absorbed by shafting alone,.....	3.14

The shafting was 300 feet long, and so consumed 1 horse-power per 100 feet of length in turning it. The speed is not stated. This resistance was equal to from 15 to 17 per cent. of the total indicator horse-power, for ordinary full work.

[1] *Journal of the Franklin Institute*, vol. lxviii., 1874, p. 261.

Table No. 268.—Frictional Resistance of Shafting—United States, 1871–72.

(Based on Mr. S. Webber's Data.)

Continuous Oiling.

Diameters.	Length.	Weight of Shafting.	Weight of Pulleys.	Weight of Total.	Average Length per Bearing.	Speed, in Turns per Minute.	Horse-power. Total.	Horse-power. Per 100 Feet of Length.	Coefficient of Friction.	Place and Conditions.
inches.	feet.	lbs.	lbs.	lbs.	feet.	turns.	H.P.	H.P.		
2⅛	8.5	104	577	678	—	216	.089	1.05	.034	Amoskeag Mills. Counter-shaft. Dreyfus oilers. Sperm and kerosene oils mixed.
2⅛	34	404	1974	2378	5	216	.357	1.05	.041	Amoskeag Mills. Four counter-shafts connected with belts. Same oils mixed.
2⅛	114	1366	1859	3225	7.6	216	.590	.52	.050	Amoskeag Mills. Single line. Same oiling.
2⅛	228	2732	3617	6349	7.6	216	1.181	.52	.050	Amoskeag Mills. Two lines like above, connected with belts. Oiling as above.
2⅛	342	4098	5331	9429	7.6	216	1.858	.54	.055	Amoskeag Mills. Three lines like above, connected with belts. Oiling as above,
2⅝ 2⅛	16 178 } 194	2427	2988	5415	7.2	216	.687	.35	.034	Amoskeag Mills. Single line.
4⅛ 2⅞ 2⅝ 2⅜ 2⅛	10.3 80 32 48 32 } 202.3	3910	5393	9303	7.8	210	1.587	.78	.033	Amoskeag Mills. Single line. Lubrication the same.
										Ordinary Oiling.
2⅝ 2⅜ 2⅛	10 48 32 } 90	1289	1456	2745	7.5	150	.499	.56	.064	Amoskeag Mills. Single line. Oiled in ordinary way, daily. Tallow in boxes as safeguard in case of heating.
Similar line	90	1289	1006	2296	7.5	150	.394	.44	.061	Do. do. do.
2⅝ 2⅜ 2⅛	34 32 32 } 98	1484	1736	3220	7.6	150	.537	.55	.059	Do. do. do.

Table No. 268 (*continued*).

Diameters.	Length.	Weight of Shafting.	Weight of Pulleys.	Weight of Total.	Average Length per Bearings	Speed, in Turns per Minute.	Horse-power. Total.	Horse-power. Per 100 Feet of Length.	Coefficient of Friction.	Place and Conditions.
inches.	feet.	lbs.	lbs.	lbs.	feet.	turns.	H.P.	H.P.		
2⅞ 2⅛	10.3 176 } 186.3	2336	2999	5335	7.8	211	1.442	.78	.076	Amoskeag Mills. Single line. Had been oiled in the morning. Tested at 11 a.m.
Similar line	186.3	2336	2999	5335	7.8	211	1.234	.66	.065	Do. do. Tested just after oiling.
2¼	200	2700	—	—	—	155	.571	.29	—	Whittenton Mills. Single line.
2⅛	24.8	295	350	645	8	210	.260	1.04	.114	Langdon Mills. Single line. Sprung in centre by pull of belt.
2⅛	42	428	—	—	10	120	.267	.64	—	Haydensville.
4 2⅛	9 231 } 240	3151	2354	5805	8	211	1.558	.65	.071	Salmon Falls, N.H. Single line. Tallow in boxes. Tested at noon. Had been oiled in the morning.
Same shaft	......	3151	2354	5805	8	211	1.120	.47	.052	Salmon Falls, N.H. Single line. Tallow removed, and sponge saturated in oil substituted. Tested at noon.
3 2⅜ 2⅛ 1⅞	9.8 32 32 96 } 169.8	1987	2000	3987	5.7	185	.685	.40	.057	Rockport Mass. Oiled daily in the usual manner.
3 2½ 2¼ 2 1¾	10.7 24 127.1 64 64.2 } 290	3554	4268	7882	8	245	1.870	.65	.059	Masconom't Mills, Newbury Port. Oiled daily in the usual manner.
1⅝	100	—	—	—	9	170	.335	.34	—	Paterson, N.J. Dreyfus oilers.
2	200	—	—	—	—	200	.549	.27	—	Granite Mills, Fall River. Dreyfus oilers.
General Averages.										
Continuous Oiling.........		—	—	36,757	—	—	—	—	.044	
Ordinary Oiling............		—	—	35,173	—	—	—	—	.066	

Mr. Westmacott states that 1200 feet of shafting, having an average diameter, 2¾ inches, and that had been running for years, absorbed 1 indicator horse-power per 100 feet of length to drive it alone—all belts off—at 120 turns per minute.

Mr. B. Walker states that the resistance of the shafting at the flax mills of Messrs. Marshall, Leeds, with belts, absorbed less than 10 per cent. of the total indicator power of the engine.[1]

M. E. Cornut found, by careful experiments, that, in the flax mills at Hamégicourt, when all the machines were at work, the total power required to drive the mill was 150 indicator horse-power; and the power required to drive the engine and shafting alone, was about 30 indicator horse-power, or 20 per cent. of the total power.[2]

Mr. R. Davison, about 1842, tested the power absorbed by shafting at Truman, Hanbury, and Buxton's brewery. There were 190 feet of horizontal shafting, and 80 feet of upright shafting; total length, 270 feet, on thirty-four bearings having 3300 square inches of area, together with eleven pairs of spur and bevil wheels, from 2 to 9 feet in diameter. They absorbed 7.65 indicator horse-power. The shafting had probably an average diameter of 4¾ or 5 inches; and the resistance was at the rate of 2.73 horse-power per 100 feet. The speeds were not given.[3]

Mr. Webber, table No. 268, shows that, taking great lengths only, from 0.33 to 0.78 horse-power per 100 feet is absorbed, with constant oiling; and that from 0.40 horse-power to nearly 1½ horse-power per 100 feet,—averaging about 1 horse-power per 100 feet,—is absorbed with ordinary oiling.

Journals of Shafts.

The journals or bearings of shafts should be proportioned with reference to the pressure or load to be sustained by the journal and its pedestal. The simplest measure of the bearing capacity of a journal is the product of its length by its diameter, in square inches; and the axial area thus obtained may be multiplied by a proper unit of pressure per square inch, to give the bearing capacity. Sir Wm. Fairbairn and Mr. Box give instances of the weights on bearings of shafts,[4] from which the following deductions are made, showing the pressure per square inch of axial section of journal:—

		in. in.		lbs.
Fly-wheel shafts;—journal,		18 × 14;	pressure per square inch,	178
„	„	11 × 9½	„	225
„	„	10¼ × 8¼	„	222
		Average,..................................		208

	lbs. lbs.		Pressure.
Link-bearings,......	456 to 690;	average per square inch,.....	573 lbs.
Crank-pins,.........	687 to 1152;	„ „	874

Mr. Box says, that the pressure on bearings, in most cases, should not exceed 500 lbs. per square inch, measured on the circumference of the journal, equivalent to 750 lbs. per square inch of the axial section.

[1] *Proceedings of the Institution of Mechanical Engineers*, 1874.
[2] *Essais Dynamometriques*, 1873.
[3] *Proceedings of the Institution of Civil Engineers*, 1843.
[4] *Mills and Millwork*, part ii. page 73; *Mill-Gearing*, page 52.

Dr. Rankine[1] gives the following as ordinary values of the intensity of pressure between a pair of greased surfaces:—

	Per square inch.			Per square inch.		
	lbs.		lbs.		lbs.		lbs.	
For journals,	450	to	150 circumferentially, or	675	to	225 axially
For flat pivots,...					2240	

whilst Sir Wm. Fairbairn limited the pressure on pivots to 240 lbs. per square inch.

The length of the journals of shafts is ordinarily 1½ times the diameter. With this proportion, the gross weight on the journals of shaftings, as tabulated in table No. 267, varies from 20 lbs. per square inch axially, for 1-inch shafting, to about 40 lbs. for 2-inch shafting, and 134 lbs. per square inch for a 20-inch shaft.

Journals of Railway Axles.—The journals of the axles of railway carriages and waggons are usually made with a length equal to more than twice the diameter. A common size is 3½ inches diameter by 8 inches or 9 inches long. With a brass bearing having a width of 2½ inches measured on the chord, the horizontal area of bearing-surface is, for a length of 8 inches ($8 \times 2\frac{1}{2} =$) 20 square inches; and a load of 6000 lbs. on each journal is equivalent to a pressure of 300 lbs. per square inch of horizontal area of bearing surface: a satisfactory proportion.

Again, the proportion of the load to the horizontal area of the journal itself, say ($8 \times 3\frac{1}{2} =$) 28 square inches, or, for a smaller diameter, say, ($8 \times 3\frac{1}{4} =$) 26 square inches, averages, say, 224 lbs., or 2 cwts. per square inch of horizontal section, or 10 square inches per ton of load. This is an ordinary working proportion, both for carrying and for locomotive stock.[2]

[1] *Steam-Engine and other Prime Movers*, page 16.

[2] See, on this subject, papers "On the Construction of Railway Waggons," by Mr. W. R. Browne, and on "Railway Rolling Stock Capacity," by Mr. W. A. Adams, in the *Proceedings of the Institution of Civil Engineers*, vol. xlvi., 1875–76, pages 81, 100.

EVAPORATIVE PERFORMANCE OF STEAM-BOILERS.

NORMAL STANDARDS.

The *evaporative efficiency*, or the *efficiency*, of a steam-boiler is measured by the proportional quantity of the whole heat of combustion of a given fuel, absorbed into the boiler and applied to the conversion of water into steam. *Efficiency* is also expressed by the weight of water evaporated by one pound of the fuel, in the sense of the French word *rendement*—yield; and, for the purpose of directly comparing performances effected at various pressures and temperatures, it is customary to reduce them to a normal standard of efficiency, expressed by the equivalent weight of water which would be converted into steam if it were supplied to the boiler at 212° F. and evaporated at 212°, and of course under one atmosphere of pressure:—briefly, evaporated from and at 212° F.

The standard temperature of the water as supplied to the boiler, is sometimes taken at 62° F., the average natural temperature of cold water; or at 100° F., which is about the temperature of the condensing water of steam-engines.

The uniform standard, of water evaporated from and at 212°, is adopted in the following discussions.

Evaporative rapidity, or *evaporative power*, is expressed by the quantity of water evaporated per hour by a steam-boiler. It may be the total quantity of water, or it may be the quantity of water per square foot of grate-area, or per square foot or per square yard of heating surface.

Evaporative performance comprises both the elements, efficiency and rapidity; though it is also used to express simply the evaporative efficiency of the boiler, or of the fuel.

Let w = the weight of water evaporated per pound of a fuel, from water supplied at the temperature t, into steam of the total heat H, measured from 32° F. Let w', t', and H$'$, be any other corresponding values for the same expenditure of heat. Then, the total heat expended in evaporating 1 lb. of water is $H + 32 - t$, or $H' + 32 - t'$, and

$$w' = w \frac{H + 32 - t}{H' + 32 - t'} \qquad (1)$$

Let H$'$ be the total heat of steam generated at 212° F., or 1146°, and $t' = 212°$ F.; and, by substitution in formula (1), and reduction,

$$w' = w \times \frac{H + 32 - t}{966}, \qquad (2)$$

in which w' is the equivalent weight of water as evaporated from and at 212° F.

RULE.—*To find the equivalent weight of water evaporated from and at 212° F., when a given weight of water is supplied at a given temperature, and evaporated at a given pressure.* Find, in table No. 128, page 387, the total heat of the steam generated at the given absolute pressure; add 32° to it, and from the sum subtract the temperature of the feed-water; and divide the remainder by 966. Multiply the given weight of water by the quotient. The product is the equivalent weight of water evaporated from and at 212° F.

When the water is to be taken as evaporated at 212°, but supplied

at $t' = 100°$ F., use the divisor 1078, in formula (2)
at $t' = 62°$ F., „ 1116, „

HEATING POWER OF FUELS.

The heating powers of fuels, treated in detail, in pages 409 to 458, are here collected for ready reference, in table No. 269.

Table No. 269.—HEATING POWER OF FUELS.

No.	FUEL.	Heating Power of a Pound of Fuel.	
		Units of Heat.	Water Evaporated per Pound of Fuel, from and at 212° F.
		units.	lbs.
1	Warlich's Fuel	16,495	17.07
	(units. lbs.)		
	Coal:—Ebbw Vale, 1848 ... 16,221 16.79		
	Powell's Duffryn, 1848 ... 15,715 16.25		
	Llangennech, 1848–71 ... 14,765 15.28		
2	Average (best Welsh) 15,567 16.11	15,567	16.11
3	Haswell Wallsend (Newcastle)	15,502	16.04
4	British coals, average	14,133	14.62
5	Coke	13,550	14.02
6	Lignite, perfect	11,678	12.10
7	Asphalte	16,655	17.24
8	Wood, perfectly dry	7,792	8.07
9	Do. 25 per cent. moisture	5,565	5.80
10	Wood-charcoal, dry	12,696	13.13
11	Peat, perfectly dry	9,951	10.30
12	Do. 25 per cent. moisture	7,156	7.41
13	Peat-charcoal, 85 per cent. carbon, dry	12,325	12.76
14	Tan, perfectly dry, 15 per cent. of ash	6,100	6.31
15	Do. 30 per cent. moisture	4,284	4.44
16	Straw, 15¼ per cent. moisture	5,231	5.44
17	Petroleum	20,240	20.33
18	Petroleum oils	27,531	28.50
19	Coal-gas (mean of Ross and Harcourt)	34,292	35.50

EVAPORATIVE PERFORMANCE OF STATIONARY AND MARINE STEAM-BOILERS, WITH COAL.

The chemical history of the combustion of coal in furnaces has been briefly outlined, page 426. Regarded mechanically, there are three modes of supplying coal to ordinary furnaces by hand-firing, namely,—first, spreading-firing, in which the charge of coal is scattered evenly over the whole surface of the grate; second, alternate firing, in which the coal is laid evenly along half the width of the grate at a time, each side alternately; third, coking-firing, in which the coal is thrown on to the dead-plate in front of the bars and left there for a time, in order that the mass may become coked, after which the mass is pushed towards the bridge, and another charge is thrown on to the front of the fire in its place.

The proportion of surplus air, the presence of which is required for the combustion of coal, in ordinary furnaces, in excess of the quantity which is chemically consumed, is diminished as the rate of combustion is increased; and the diminution of the excess is one of the reasons why the temperature in the furnace rises as the rapidity of combustion is increased. The following are the results of observations on the proportion of surplus air admitted into the furnace, in parts of the air that was chemically consumed:—

	Rate of Combustion. Coal per Square Foot of Grate per Hour.		Surplus Air.
R. Hunt, Cornish Boilers,...	2 to 4 lbs.	......	100 per cent.
Professor Johnson, America,	7 "	......	100 "
Delabèche and Playfair,......	10 to 16 "	......	25 to 50 "
J. A. Longridge, Newcastle trials,......................	20 lbs. and upwards.	......	9 "

The evidence is not sufficient to settle the question; and it is doubtful whether Mr. Longridge's deduction is applicable to boilers in general. It is very probable that surplus air only ceases to be present when the rates of combustion are very much higher than 20 lbs. per square foot.

With very slow and uniform rates of combustion, all, or nearly all the air required, may be drawn through the grate. If the combustion be rapid, a considerable proportion of the air must be introduced directly above the fuel, to consume the gases. It was seen, page 428, that, allowing a total of 140 cubic feet of air, chemically consumed for the combustion of one pound of coal of average composition, 36 per cent. is consumed by the volatilized portions, and 64 per cent. by the fixed portion, or coke. Mr. Longridge mentions an instance in which, with ordinary stoking and a closed doorway, dense smoke was given off, the quantity of air that passed through the furnace, exclusively through the grate, only amounted to 100 cubic feet per pound of coal. This quantity was little more than equal to what was sufficient to burn the fixed portion of the coal. The smoke was prevented when an additional supply of air was admitted from the doorway, above the fuel.

Experiments on the Evaporative Power of Coals, conducted by Messrs. Delabèche and Playfair, 1847–50.

Referring to the averaged results of performance of these coals, page 414, it is well understood that they were not burned under the conditions most

favourable for each variety of coal. Yet they throw light on the conditions of composition, which appear to control the heat-producing power of coals, and probably of wood and other fuels too. Neither the variations of the quantity of constituent hydrogen, nor those of the carbon, are commensurate with the wide range of performance; but it is evident that the evaporative performance decreases regularly as the oxygen increases, thus,—

	Oxygen.		Water Evaporated per pound of Coal from and at 212°.
Patent fuels,	2.79 per cent.	......	9.20 lbs.
Welsh coals,	4.15 „	......	9.05 „
Newcastle coals,	5.69 „	......	8.37 „
Lancashire coals,	9.53 „	......	7.94 „
Scotch coals,	9.69 „	......	7.70 „
Derbyshire and Yorkshire coals,	10.28 „	......	7.58 „

Taking averages, it is seen that the evaporative efficiency of coal varies directly with the quantity of constituent carbon, and inversely with the quantity of constituent oxygen; and that it varies, not so much because there is more or less carbon, as, chiefly, because there is less or more oxygen. The percentages of constituent hydrogen, nitrogen, sulphur, and ash, taking averages, are nearly constant, though there are individual exceptions, and their united effect, as a whole, appears to be nearly constant also.

Evaporative Performance of Lancashire Stationary Boilers, at Wigan, 1866–68.[1]

The coal selected for trial was Hindley Yard coal, from Trafford Pit, which ranks with the best coals of the district. Three stationary boilers were selected; 1st, an ordinary double-flue Lancashire boiler, 7 feet in diameter, and 28 feet long; the flue-tubes were 2 feet 7½ inches in diameter inside, of ⅜-inch plate. 2d, Another Lancashire boiler of the same dimensions, in which the tubes were of 5/16-inch steel plate. 3d, A Galloway or water-tube boiler, 26 feet long, and 6 feet 6 inches in diameter; with two furnace-tubes 2 feet 7⅜ inches in diameter, opening into an oval flue 5 feet wide by 2 feet 6½ inches high, containing 24 vertical conical water-tubes. The first and second boilers were new and specially constructed for the trials; the third boiler was a second-hand one. These three boilers were set side by side, on side walls and with two dampers. The flame passed through the flue-tubes, back under the boiler, then along the sides to the chimney. The chimney was 105 feet high, above the floor; octagonal, 6 feet 10 inches wide at the base, and 5 feet wide at the top, where the sectional area was 21 square feet.

Total grate-area in each boiler:—	6 feet long;	31.5 square feet.	
„ „	4 „	21.0 „	

[1] The Author is indebted for the particulars of these trials to "The South Lancashire and Cheshire Coal Association's" *Report on the Boiler and Smoke-Prevention Trials, conducted at Wigan*, 1869. The experiments were conducted, and the report, excellently reasoned, was written by Mr. Lavington E. Fletcher.

	Lancashire.	Galloway.
Heating surface, in flue-tubes,	464.34 square feet.	431.12 square feet.
In external flues	303.08 „	288.24 „
Total surface,	767.42 „	719.36 „
Ratio of grate-area, 6 feet long, to heating surface,	1 to 24.4 „	1 to 22.8 „
Ratio of grate-area, 4 feet long, to heating surface,	1 to 36.5 „	1 to 34.3 „
Circuit, or length of heating surface, traversed by the draught from the centre of the grate,	80 feet.	74 feet.
Total distance from centre of grate to base of chimney,	117 „	101 „
Height of chimney above level of floor,	100 feet.	
Height of chimney above level of grates,	96 feet 9 inches.	

A Green's fuel-economizer was placed in the main flue; it had 12 rows of 4½-inch cast-iron pipes, 8 feet 9 inches long, placed vertically—84 tubes in all—having a collective heating surface of 850 square feet, exclusive of the connecting pipes at top and bottom. The feed-water was passed through the economizer on its way to the boiler, and absorbed a portion of the waste heat.

The fire-grates were tried at 2 lengths, 6 feet and 4 feet. The shorter grate gave the more economical result, but it generated steam less rapidly. Three modes of firing were tried;—spreading, coking, and alternate firing. With round coal, on the whole, the greatest duty was obtained by coking firing, with the least smoke. With slack, alternate-side firing had the advantage.

Fires of different thicknesses were tried: 6 inches, 9 inches, and 12 inches. It was found that 9 inches was better than 6 inches, and 12 inches better than 9 inches. Air admitted at the bridge gave a slightly better result than by the door; and the admission of air in small quantity on the coking system, prevented smoke. The doors were double, slotted on the outside, and pierced with holes on the inner side. The maximum area of opening was 31½ square inches for each door, being at the rate of 2 square inches per square foot for the 6-feet grates, and 3 square inches for the 4-feet grates. The amount of opening was regulated by a slide.

The standard fire adopted for trial was 12 inches thick, of round coal, treated on the coking system, with a little air admitted above the grate, for a minute or so after charging.

The water was evaporated under atmospheric pressure.

The quantity of refuse from the Hindley Yard coal, averaged in the trials with the marine boiler, to be afterwards described, 2.8 per cent. of clinker, 2.8 per cent. of ash, and .8 per cent. of soot; in all, 6.4 per cent. Making allowance for the difference of soot, the total refuse may be taken, in the trials of the stationary boilers, at 6 per cent.

General Deductions.—The advantage of the 4-feet grate over the 6-feet grate, was manifested by comparative trials with round coal 12 inches thick,

and slack 9 inches thick. With the 4-feet grate, the evaporative efficiency, taking averages, was 9 per cent. greater than with the 6-feet grate; though the rapidity of evaporation was 15 per cent. less, at the same time that 19½ per cent. more coal was burned per square foot per hour.

When equal quantities of coal were burned per hour, the fires being 12 inches thick, 8 per cent. more efficiency and 12 per cent. greater rapidity of evaporation were obtained from the shorter grate. Thus:—

	Coking Firing.	
Length of grate,	6 feet.	4 feet.
State of damper,	two-thirds closed.	fully open.
Coal per hour,	4.0 cwts.	4.14 cwts.
Coal per square foot of grate per hour,	14 lbs.	23 lbs.
Water at 100° evaporated per hour,	65 cubic feet.	72.6 cubic feet.
Water at 212° per pound of coal,	10.10 lbs.	10.91 lbs.
Smoke per hour:—		
Very light,	4.3 minutes.	4.1 minutes.
Brown,	0.4 ,,	0.3 ,,
Black,	0.0 ,,	0.0 ,,

To compare the performances with coking and spreading firing, having 12-inch fires for round coal, and 9-inch fires with slack:—Whilst, with round coal, the rapidity of evaporation was the same with both modes of firing, the efficiency was from 3 to 4 per cent. greater with coking. With slack, on the contrary, the spreading fire evaporated a fourth more water per hour than the coking fire, though with 4¼ per cent. less efficiency.

With thicknesses of coking fire, 6 inches, 9 inches, and 12 inches, for round coal; and 6 inches and 9 inches for slack; the results were in all respects decidedly in favour of the thicker fires rather than the thinner fires. Comparing the thinnest and the thickest fires, from 5½ to 20 per cent. more water was evaporated per hour by the thickest fires, and from 11 to 18 per cent. more per pound of fuel.

The effect of the admission of air above the grate, continuously or intermittently, for the prevention of smoke, as compared with that of its non-admission, was ascertained with round coal, and with slack. The averaged results showed that by admitting the air above, the evaporative efficiency was increased 7 per cent.; but that the rapidity of evaporation was diminished 3½ per cent.

Comparing the admission of air above the fuel at the door, and at the bridge through a perforated cast-iron plate; it was found that the admission at the bridge made a better performance, by about 2½ per cent., than at the door.

To try the effect of increasing the supply of air above the fuel, the door-frame was perforated to give an additional square inch of air-way per foot of grate, making up 3 square inches; an allowance of 1 square inch was also provided at the bridge. Round coal was burned on the coking system, 12 inches thick, on 6-feet grates, with a constant admission of air above the fuel. When the supply by the door was increased from 2 inches to 3 inches per square foot of grate, the evaporative efficiency fell oft 8½ per cent., and the rapidity 3 per cent. When an extra inch was supplied at the bridge, making up 4 square inches per foot of grate, the evaporative efficiency only fell off 0.65 per cent., and the rapidity 1¼ per

cent. The effect of this evidence is, that the bridge is the better place for the admission of air, and that if the air be admitted by the bridge alone, the area of supply may be beneficially raised to 4 square inches per square foot of grate.

Comparing the effect of the admission of air in a body, undivided, with that of its admission in streams, on a 6-feet grate, with coking fires 12 inches thick of round coal; there was 6¼ per cent. of loss of efficiency by the admission in a body, though the smoke was equally well prevented.

Mr. Fletcher concludes that the greatest rapidity of evaporation was obtained, when the passages for the admission of air above the fuel were constantly closed; that the next degree of rapidity was obtained when they were open only for a short time after charging, and the lowest when they were kept open continuously. He also concludes that, whilst, in realizing the highest power of a free-burning and gaseous coal, smoke is prevented; yet, in realizing the highest power of the boiler, smoke is made.

In burning slack, smoke was prevented as successfully as in burning round coal, though its evaporative efficiency was from 1 to 1½ lbs. of water per pound of fuel, less than with round coal.

To work out the problem of firing slack without smoke, and without loss of rapidity of evaporation; trials were made at the boilers of 16 mills, when the slack was fired on the alternate-side system. No alterations were made in the furnaces in preparation for these trials; in many instances, the fire-doors had no air-passages through them. The grates were from 3 feet 7 inches to 7 feet long; they averaged 6 feet in length.

Number of boilers fired,..................................	65 boilers.
Slack burned per boiler per week of 60 hours,......	17.35 tons.
Slack per square foot of grate per hour,..............	19.25 lbs.
Smoke per hour:—	
Very light,..	11.5 minutes.
Brown,...	2.3 ,,
Black,...	0.3 ,,
	14.1 ,,

In 12 instances, no black smoke whatever was made. It is said that the steam was as well kept up, and the speed of the engines as well maintained, as before the trials were made.

Comparative Performance of the Stationary Boilers at Wigan.

There were made altogether about two hundred and ninety trials with the three boilers, of which sixty may be regarded as comparative trials of the boilers. The results of these sixty trials are embodied in the table No. 270, page 776. The second part gives the best results that had been obtained from each boiler, supplied with round coal, on the coking system; and with air admitted through the doors for a few minutes after charging. Suffice it, meantime, to remark that the performance of the double-flue boilers amounted practically to the same as that of the water-tube boiler. Thus,

the means of the double-flue boilers compare as follows with the results of the conical water-tube boiler:—

AVERAGES OF SIXTY TRIALS, WITHOUT ECONOMIZER—

	Water at 100° consumed per hour.		Water at 212° per lb. of coal.
Double-flue boilers,..................	79.65 cubic feet	......	10.31 lbs.
Conical water-tube boiler,..........	78.95 „	......	10.34 „

BEST RESULTS OBTAINED:—WITHOUT ECONOMIZER—

Double-flue boilers,..................	81.92 „	......	10.86 „
Conical water-tube boiler,..........	79.17 „	......	10.58 „

WITH ECONOMIZER—

Double-flue boilers,..................	90.72 „	......	11.56 „
Conical water-tube boiler,..........	86.31 „	......	11.82 „

In doing the same work, it is to be noted that the water-tube boiler was 2 feet shorter, and 6 inches less in diameter, than the double-flue; and that it had 48 square feet, or 6 per cent. less area of heating surface.

A trial was made with the object of testing the comparative merits of the plain double-flue and the water-tube flue, by shutting off the draught from the external flues, and leading it direct from the internal flues to the chimney, with the following results (grates 6 feet long, coking firing, 12 inches thick):—

WITHOUT ECONOMIZER—

	Water at 100° consumed per hour.		Water at 212° per lb. of coal.
Iron double-flues,.....................	82.97 cubic feet	......	8.23 lbs.
Water-tube flue,.......................	80.00 „	......	8.50 „

WITH ECONOMIZER—

Iron double-flues,.....................	98.85 „	......	10.08 „
Water-tube flue,.......................	89.08 „	......	10.16 „

Showing that the double flues, having 33 square feet, or nearly 8 per cent. more heating surface than the water-tube flue, evaporated more water per hour, but with rather less efficiency than the water-tube flue.

The evaporative power of the boilers was rather increased than diminished by the closing of the external flues, though there was a sacrifice of evaporative efficiency.

Water-tubes.—Four water-tubes were inserted in each flue of the iron-flue boiler, 5½ inches in diameter inside, and 2 feet 7½ inches long, making an addition of 30 square feet, or 6½ per cent., to the flue-heating surface, or 4 per cent. of the total heating surface. The result of the insertion showed equal rapidity of evaporation, and a gain of 3 per cent. in efficiency; as follows:—

	Water at 100° consumed per hour.		Water at 212° F. per pound of coal.
Without water-tubes, ...	91.15 cubic feet.	......	10.43 lbs.
With water-tubes,...............	91.12 „	......	10.77 „

Table No. 270.—COMPARATIVE RESULTS OF PERFORMANCE OF HINDLEY YARD (LANCASHIRE) COAL, WITH THREE STATIONARY BOILERS, AT WIGAN. 1867–68.

(Compiled from Tabular Statements in the Report of Mr. L. E. Fletcher.)

FUEL—HINDLEY YARD COAL.

BOILER, AND MODE OF FIRING.	Area of Fire-grate.	Coal Consumed per Week of 60 Hours.	Coal Consumed per Hour.	Coal per Square Foot of Grate per Hour.	Water Consumed from 100° per Hour.	Water per Square Foot of Grate per Hour.	Water Evaporated from 212° per Pound of Coal.	Smoke per Hour.
	sq. feet.	tons.	cwts.	lbs.	cub. feet.	cub. feet.	lbs.	minutes.
ROUND COAL, COKING FIRING, 12 ins. thick, WITHOUT ECONOMIZER.	I. AVERAGES OF SIXTY TRIALS—							
Double-flue boiler, with iron tubes	31.5	15.69	5.23	18.6	86.73	2.75	10.32	very light 2.9, brown 1.2, black 0.7
Double-flue boiler, with steel tubes	,,	16.15	5.38	19.1	87.74	2.79	10.17	,, 1.5 ,, 0.2 ,, 0.0
Conical water-tube boiler	,,	15.43	5.15	18.3	84.21	2.67	10.19	,, 2.1 ,, 1.2 ,, 0.5
Double-flue boiler, with iron tubes	21	12.13	4.04	21.5	70.62	3.36	10.88	,, 0.3 ,, 0.0 ,, 0.0
Double-flue boiler, with steel tubes	,,	12.75	4.25	22.7	73.40	3.50	10.77	,, 4.5 ,, 0.0 ,, 0.0
Conical water-tube boiler	,,	12.23	4.08	21.8	70.52	3.36	10.77	,, 4.7 ,, 0.1 ,, 0.0
SPREADING FIRING, 12 inches thick.								
Double-flue boiler, with iron tubes	31.5	16.92	5.64	20.1	86.77	2.75	9.56	,, 5.3 ,, 4.9 ,, 3.3
Double-flue boiler, with steel tubes (no trials)	,,	—	—	—	—	—	—	— — —
Conical water-tube boiler	,,	16.84	5.61	20.0	87.98	2.80	9.75	,, 4.4 ,, 3.6 ,, 2.8
Double-flue boiler, with iron tubes	21	12.11	4.04	21.6	71.44	3.40	10.62	,, 13.7 ,, 3.4 ,, 2.5
Double-flue boiler, with steel tubes	,,	12.93	4.31	23.0	73.72	3.51	10.60	,, 14.5 ,, 3.3 ,, 1.9
Conical water-tube boiler	,,	12.79	4.26	22.7	73.10	3.48	10.66	,, 14.1 ,, 4.7 ,, 1.8
AVERAGES.								
Double-flue boiler, with iron tubes	—	14.21	4.74	20.5	78.89	3.06	10.34	,, 5.6 ,, 2.4 ,, 1.6
Double-flue boiler, with steel tubes	—	14.69	4.90	21.6	80.41	3.27	10.28	,, 6.4 ,, 2.1 ,, 1.3
Conical water-tube boiler	—	14.31	4.77	20.7	78.95	3.08	10.34	,, 6.3 ,, 2.4 ,, 1.3
Average of the three boilers	—	14.40	4.80	20.9	79.40	3.14	10.32	,, 6.1 ,, 2.3 ,, 1.4

Table No. 270 (*continued*).

Second Part:—BEST RESULTS OBTAINED FROM THE THREE STATIONARY BOILERS.

BOILER, AND MODE OF FIRING.	Area of Fire-grate.	Coal Consumed per Week of 60 Hours.	Coal Consumed per Hour.	Coal per Square Foot of Grate per Hour.	Water Consumed from 100° per Hour.	Water per Square Foot of Grate per Hour.	Water Evaporated from 212° per Pound of Coal.	Smoke per Hour.
	sq. feet.	tons.	cwts.	lbs.	cub. feet.	cub. feet.	lbs.	minutes.
ROUND COAL, COKING FIRING, 12 ins. thick, WITHOUT ECONOMIZER.								
Double-flue boiler, with iron tubes	31.5	16.47	5.49	19.52	92.07	2.92	10.43	very light 2.7, brown 0.9, black 0.0
Double-flue boiler, with steel tubes	,,	14.90	4.97	17.67	84.65	2.69	10.62	,, 0.5 ,, 0.0 ,, 0.0
Conical water-tube boiler	,,	16.00	5.33	18.95	85.22	2.71	9.95	,, 0.0 ,, 0.0 ,, 0.0
Double-flue boiler, with iron tubes	21	12.14	4.05	21.60	73.12	3.48	11.22	,, 0.9 ,, 0.0 ,, 0.0
Double-flue boiler, with steel tubes	,,	12.91	4.30	22.93	77.84	3.71	11.22	,, 4.2 ,, 0.0 ,, 0.0
Conical water-tube boiler	,,	12.14	4.05	21.60	73.12	3.48	11.22	,, 6.3 ,, 0.0 ,, 0.0
Means of the Two Sizes of Grate.								
Double-flue boiler, with iron tubes	—	14.31	4.77	20.39	82.59	3.20	10.82	,, 1.8 ,, 0.4 ,, 0.0
Double-flue boiler, with steel tubes	—	13.91	4.64	19.79	81.24	3.20	10.91	,, 2.3 ,, 0.0 ,, 0.0
Conical water-tube boiler	—	14.07	4.69	20.05	79.17	3.10	10.58	,, 1.3 ,, 0.0 ,, 0.0
Average of the three boilers	—	14.10	4.70	20.05	81.00	3.09	10.77	,, 1.8 ,, 0.1 ,, 0.0
WITH ECONOMIZER.								
Double-flue boiler, with iron tubes	31.5	17.57	5.86	20.84	107.78	3.42	11.46	,, 3.2 ,, 1.3 ,, 0.0
Double-flue boiler, with steel tubes	,,	14.27	4.76	16.92	88.02	2.79	11.51	,, 1.6 ,, 0.0 ,, 0.0
Conical water-tube boiler	,,	15.53	5.18	18.42	97.98	3.11	11.78	,, 6.5 ,, 0.5 ,, 0.0
Double-flue boiler, with iron tubes (no trials)	21	—	—	—	—	—	—	— — —
Double-flue boiler, with steel tubes	,,	13.39	4.46	23.79	83.54	3.98	11.65	,, 0.6 ,, 0.0 ,, 0.0
Conical water-tube boiler	,,	11.75	3.92	20.91	74.73	3.56	11.86	,, 0.7 ,, 0.6 ,, 0.0
Means of the Two Sizes of Grate.								
Double-flue boiler, with iron tubes	—	15.48	5.16	22.06	95.66	3.65	11.56	,, 1.9 ,, 0.7 ,, 0.0
Double-flue boiler, with steel tubes	—	13.83	4.61	19.92	85.78	3.39	11.57	,, 1.1 ,, 0.0 ,, 0.0
Conical water-tube boiler	—	13.64	4.55	19.45	86.31	3.33	11.82	,, 3.6 ,, 0.5 ,, 0.0
Average of the three boilers	—	14.32	4.77	20.39	89.25	3.40	11.65	,, 2.2 ,, 0.4 ,, 0.0

Green's Fuel-Economizer.—From the average results of various comparative trials, burning round coal and slack, and with coking firing, on 6-feet and 4-feet grates, it appeared that, burning equal quantities of coal per hour, the rapidity of evaporation was increased 9.3 per cent., and the efficiency 10 per cent., by the addition of the economizer.

Temperature of the Products of Combustion, and of the Feed-water, when the water is passed through the Economizer. Average results:—

	With 6-feet Grate. Before.	After.	With 4-feet Grate. Before.	After.
Temperature of gases in the flues before and after traversing the economizer,	649°	340°	501°	312°
Temperature of the feed-water,..........	47	157	41	137

Whence it follows that, to raise the temperature of the feed-water through 100° F., the gases were cooled down through an average of 250° F.

Temperature of the Products of Combustion, without the Economizer.—The variations to which the temperature of the escaping gases is subject, are illustrated in the annexed statement, showing the temperature with different thicknesses of fire, burning round coal with coking firing, without the economizer.

Round Coal.	With 6-feet Grate.			With 4-feet Grate.		
Thickness of fires,.....................inches	12	9	6	12	9	6
Coal per foot of grate per hour,...pounds	19	20	20	23	24	24
Water at 100° evaporated per hour, cu. feet	85.7	85.5	81.2	72.8	70.7	61.7
Water at 212° per pound of coal, pounds	10.12	9.79	9.16	10.90	9.95	9.21
Temperature in chimney-flue,..............	630°	556°	539°	505°	451°	445°
Smoke per hour—						
Very light, ...minutes	2.0	0.0	0.5	2.8	0.4	0.0
Brown,............................minutes	0.4	0.0	0.1	0.1	0.0	0.0
Black,............................minutes	0.0	0.0	0.0	0.0	0.0	0.0

It is shown that the temperature in the chimney flue is lower with the 4-feet grate, than with the 6-feet grate; it averages 107° lower, and correspondingly, the evaporative efficiency averages higher. But, with the same grate, both the evaporative efficiency and the temperature become less with the thinner fire, due, no doubt, as Mr. Fletcher points out, to the passage of a greater surplus of air through the thinner fire.

Volume of Air Supply and Products of Combustion.—The volume of air entering the ash-pit and passing through the grate, when the doors were closed, was found, by means of Biram's anemometer, to be, for grates 4 feet long, with fires 9 inches thick, from 245 to 250 cubic feet per pound of coal burned; the average velocity of entrance into the ash-pit, which was 2 feet square, having been observed to be 9.3 feet per second. As the composition of the coals has not been given, it may only be assumed roughly, that the coal chemically consumed 140 cubic feet of air for the combustion of one pound; and, if the above-noted quantities of air supplied be exact, it would follow that a surplus of air amounting to from 75 to 80 per cent. was present. This is questionable, and it is probable, in the scarcity of data, that the observations for velocity were made at the centre of the draught-way, where the velocity was a maximum, and that no correction was made for the inferior velocities at other parts of the section.

From an analysis of the products of combustion in the chimney, it appeared that there was no appreciable quantity of carbonic oxide present.

Trials under Steam of more than one Atmosphere of Pressure.—As the experiments at Wigan were made under one atmosphere of pressure, a few trials were made under an effective pressure of 40 lbs. per square inch, with the following comparative results:—

	At atmospheric pressure.	At 40 lbs. per square inch.
Water at 100° evaporated per hour, cubic feet,	83.6	80.4
Water at 212° per pound of coal, pounds,	10.76	9.53

showing a reduction of 1¼ pounds of water in evaporative efficiency, at the higher pressure, which is more or less accounted for, first, by the greater total heat of steam at the higher pressure, requiring more fuel-heat for its formation; secondly, by the higher temperature of the water in the boiler at the higher pressure, which would to some extent check the absorption of the last portions of heat from the gases before they escaped into the chimney-flue. Still, the difference is excessive.

Trials with D. K. Clark's Steam-induction Apparatus for the Prevention of Smoke.—In Clark's smoke-preventer, the air was admitted through the door, regulated in quantity by a flap-valve, and deflected upwards upon an air-plate placed across the furnace above the dead-plate, and against the furnace-front. Steam from an auxiliary boiler was conducted by a pipe above the air-plate, and was discharged in four jets over the fire, towards the bridge. In passing over and beyond the edge of the air-plate, the steam induced the air which passed forward from the door under the air-plate, and carried it onward above the fire—thus forcibly mingling it with the combustible gases, and at the same time increasing the draught.

The trials were made in three ways—1st, with the jets and the air-valves constantly open; 2d, with the jets and the air-valves open for a minute or so only, after each charge; 3d, with the jets constantly open, while the air-valves were closed. It was found that, when the jets were constantly open, the quantity of steam consumed from the auxiliary boiler to supply them amounted to one-thirtieth of the quantity of water evaporated.

The following are the comparative results of performance on 6-feet grates, with the steam-inductor, and with the ordinary fire-door and the split bridge. The jets and air-valves of the steam-inductor were open for a minute or so only after each charge; and, taking the interval between the charges at fifteen minutes, it is evident that the quantity of steam consumed by the nozzles was insignificant:—

Round Coal, 6-feet Grate; Firing, 12 inches thick.	Without Economizer. Coking.	Without Economizer. Spreading.	With Economizer. Coking.
Coal per sq. foot of grate per hour, steam inductor, pounds,	18.77	23.86	18.20
Water at 100° per hour, steam-inductor, ...cubic feet,	87.70	101.80	91.77
Water at 100° per hour, ordinary door,......... "	89.81	86.77	99.88
Water at 212° per pound coal, steam inductor, pounds,	10.38	9.41	11.15
Water at 212° per pound coal, ordinary door, "	10.32	9.76	—
Smoke per hour, ordinary door—			
Very light, ...minutes,	3.1	5.3	—
Brown, ... "	0.8	4.9	—
Black, ... "	0.0	3.3	—

SLACK, 6 feet Grate; Firing, 9 inches thick.

		Without Economizer. Coking.	Without Economizer. Spreading.	With Economizer. Coking.
Coal per sq. foot of grate per hour, steam-inductor, pounds,		18.7	—	20.7
Water at 100° per hour,	steam-inductor, ...cubic feet,	78.52	—	101.09
	ordinary door, ... „	67.55	—	71.58
Water at 212° per pound coal,	steam-inductor, pounds,	9.17	—	10.65
	split bridge, ... „	8.88	—	9.23
Smoke per hour, ordinary door—				
Very light, ...minutes,		0.2	—	0.0
Brown, ... „		0.0	—	0.0
Black, ... „		0.0	—	0.0

It is shown that, with round coal and coking firing, there was no advantage by the steam-inductor, except in reducing the smoke, whilst the evaporation was rather less rapid than with the ordinary door; but that, with spreading firing, the evaporation was more rapid by 17 per cent. With slack, the evaporation was decidedly superior, both in rapidity and efficiency, with the steam-inductor.

Trials with Self-feeding Fire-grates (*Vicars' System*).—The fire-bars are impressed with a slow reciprocating movement, the effect of which is to cause the fuel to travel gradually and steadily from the front to the back.

The comparative performances of Vicars' grate and the ordinary grate, are shown by the subjoined results:—

ROUND COAL, 4-feet Grate.

Water at 100° per hour,	Vicars' grate, ...cubic feet,	83.62
	coking firing, ... „	75.92
Water at 212° per pound coal,	Vicars' grate, pounds,	10.11
	coking firing, „	10.91

SLACK, length of Grate, 4 feet.			6 feet.	With Economizer.
Water at 100° per hour,	Vicars' grate, ...cubic feet,	64.95	77.94	78.97
	coking firing, ... „	60.72	67.56	71.58
	spreading „ ... „	77.72	—	—
Water at 212° per pound coal,	Vicars' grate, pounds,	9.82	9.52	10.56
	coking firing, „	9.58	8.88	9.23
	spreading „ „	8.94	—	—

It is seen that, with slack, Vicars' grate had the advantage both in rapidity and efficiency of evaporation over hand-firing coking, and that it also evaporated more rapidly with round coal, but less efficiently; though, if the rapidity had been the same, the efficiency would probably also have been the same. Compared with spreading firing, Vicars' grate was superior in evaporative efficiency as well as in the prevention of smoke, though it did not evaporate so rapidly.

In burning large quantities of coal continuously on Vicars' grate, the rapidity of evaporation fell off in the longer trials, and to some extent also the efficiency. The 6-feet grates were very little behind the 4-feet grates in efficiency.

Comparative Performance in Calm and Windy Weather.—A high wind invariably increased the performance. The average results under all conditions showed that 10 per cent. more coal and 12 per cent. more water were consumed, and that the evaporative efficiency was increased 4.4 per cent.

Comparative Performance when the Natural Draught was increased by the aid of an Auxiliary Furnace.—An auxiliary furnace was put in action at the bottom of the chimney, so as to increase the draught. The effect, taking the mean of a number of trials, was to raise the rapidity of evaporation from 72.96 to 84.09 cubic feet of water at 100°, per hour, whilst the water evaporated per pound of fuel was raised from 10.77 to 10.81 pounds. The mean efficiency, thus slightly raised, was in fact an average of two opposite effects; for, with round coal, the efficiency was reduced, whilst with slack it was increased, by the additional draught.

Mr. Fletcher's Conclusions.—Mr. Fletcher draws the following conclusions from the experiments on stationary boilers at Wigan:—1st. That the coals of the South Lancashire and Cheshire district, though of a bituminous and free-burning character, can be economically burned in the ordinary class of mill-boiler, without smoke. 2d. That the double-flue Lancashire boiler, whether with steel or iron flues, and the Galloway, or water-tube boiler, are practically equal in performance; and that both of them develop, when suitably set and fired, high economic results. 3d. That external brickwork flues, though adding but little to the yield of steam, save fuel. 4th. That the addition of a feed-water heater or economizer is a decided advantage, not only in increasing the yield of steam, but also in diminishing the annual cost of boiler repairs and coal.

EVAPORATIVE PERFORMANCE OF SOUTH LANCASHIRE AND CHESHIRE COALS, IN A MARINE BOILER, AT WIGAN. 1866–68.[1]

The marine-boiler was a copy of the test-boiler at Keyham Dockyard. The shell was rectangular, 5 feet wide, for two furnaces, 7 feet 8 inches long, and 8 feet 10 inches high. The furnaces were 1 foot 8⅝ inches wide, 2 feet 8⅜ inches high at the front, rising to 3 feet high at the back, and 6 feet deep from front to back or tube plate. There were 124 flue-tubes, 2¼ inches in diameter inside, and 5 feet long, placed at a pitch of 3⅛ inches from centre to centre. The chimney was 18 inches in diameter, and 52 feet 8 inches high above the boiler, or 59 feet 8 inches above the level of the grates. The proportions of furnaces which were finally adopted, after many preliminary trials, were as follows:—Dead-plate, 10 inches long, 16 inches below the crown of the furnace; grates, 3 feet long, inclined ¾ inch to a foot; bars, ½ inch thick, air-spaces ½ inch; bridge built up to a level 9 inches below the crown, and 9¼ inches above the grate. The fire-doors were fitted with a sliding grid for the admission of air into a perforated box inside the door. In the first instance, there were 730 perforations, giving an area of 33 square inches, or 3.2 inches per square foot of grate. They were afterwards reduced to 342 in number, 16½ square inches in area, or 1.6 inch per foot of grate.

During the preliminary experiments, it was found of advantage to reduce the length of grate from 4 feet to 3 feet, to adopt a blind dead-plate in preference to a perforated one, and to slightly lower the grate. Fires of 6 inches, 9 inches, 12 inches, and 14 inches in thickness were tried; the greater the thickness the better was the performance. The firing was tried on the spreading and on the coking systems.

[1] The author is indebted for the particulars of these trials to Mr. Lavington E. Fletcher's Report. See note, page 771.

Coking-firing was adopted as the standard method, with fires of 14 inches and 12 inches thickness. The furnaces were charged alternately, and the entrance for air through the door was allowed to remain open for a few minutes after each charge was delivered, for the prevention of smoke. For each trial, 1000 lbs. of round coal was consumed, lasting 3 hours 27 minutes as an average; average rate of consumption, 290 lbs. per hour, or 28 lbs. per square foot of grate per hour. The feed-water was supplied at ordinary temperatures. The steam was generated under one atmosphere of pressure, and escaped direct into the air.

Total grate-area,		10.3 square feet.
Total heating surface:—		
Plate, above the grate,	95 square feet.	
Tubes, outside surface,	413 „	508 „

Ratio of grate-area to heating surface, say 1 to 50.

For some trials, an inverted bridge was added at the back of the furnace, 9 inches clear of the first bridge. By thwarting the current, it was instrumental in preventing smoke, and in slightly increasing the evaporative efficiency,—by 1.7 per cent.; though at a loss of 7½ per cent. of evaporative rapidity.

The 14-inch fire excelled the 9-inch fire, burning coal at the rate of 27 lbs. per square foot, by 7½ per cent. for rapidity and efficiency of evaporation.

Comparing the coking and the spreading systems, there was 6½ per cent. gain by the coking system in efficiency, with a loss of 10 per cent. in rapidity. When air was shut off at the doorway, the smoke-making was accompanied by 4 per cent. loss of efficiency, with a small advance of 1½ per cent. of rapidity.

When the trials were prolonged, to burn 1500 lbs. of coal, as against the standard of 1000 lbs., the rate of consumption of fuel was 1 lb. more per square foot per hour, and of water 2¼ per cent. less; the average efficiency was reduced 5 per cent.

Table No. 271 shows the general results arrived at by Messrs. Richardson and Fletcher, compiled from the Report of Mr. Fletcher. The vacuum in the chimney [at the base, probably] was observed to vary from ⅜ inch to fully $7/16$ inch of water; and in the flame-box from ¼ inch to fully $5/16$ inch. The fires were maintained at 14 inches thick, and the coal was stoked, on the coking plan, in charges of from 29 to 38 lbs., at intervals of from 11 to 17 minutes. The perforations in the fire-doors were opened intermittently, and the doors were opened a little, occasionally, after firing. Each trial lasted for from 3 to 4 hours. The quantity of ash varied from 1½ to 7 per cent., and of clinker from 0.6 to 3 per cent.

From the table, it appears that the quantity of water evaporated varied from 44.12 to 51.63 cubic feet per hour, at the rate of from 10.37 to 12.54 lbs., at 212°, per pound of coal, averaging 11.54 lbs.; and that the coal was burned at the rate of from 25½ to 31⅓ lbs. per square foot of grate per hour. The duration of the smoke, which was very light, varied from 0.2 to 6 minutes per hour; the mean duration was 2.4 minutes in the hour.

A mixture of Hindley Yard coal and Welsh coal-dust, in the proportion

of 2 to 1, was tried, and the results are given in the table, showing an evaporation of 11.83 lbs. of water per pound of the fuel, and at the rate of 41.38 cubic feet per hour.

Table No. 271.—SOUTH LANCASHIRE AND CHESHIRE COALS—RESULTS OF TRIALS IN A MARINE BOILER AT WIGAN. 1866–68.

(Compiled from the Report of Mr. Lavington E. Fletcher to the Association for the Prevention of Steam-Boiler Explosions.)

Total area of fire-grates, 10.3 square feet.

COAL.	Coal Consumed per Hour.	Coal per Square Foot of Grate per Hour.	Water Consumed from 100° per Hour.	Water per Square Foot of Grate per Hour.	Water Evaporated from 212° per Pound of Coal.	Smoke per Hour.
	cwt.	lbs.	cub. feet.	cub. feet.	lbs.	minutes. very light
FIRST SERIES OF TRIALS.						
Hindley Yard	2.32	25.24	46.17	4.48	12.39	0.2
Worsley Top Four Feet	2.88	31.36	48.50	3.04	10.37	4.0
Upper Crumbouke	2.64	28.74	48.13	4.67	11.31	5.3
Lower Crumbouke	2.43	26.41	48.60	4.72	12.45	1.8
Upper Three Yards	2.48	27.00	46.26	4.49	11.60	3.3
Six Feet Rams	2.44	26.50	44.35	4.31	11.34	2.0
Great Seven Feet	2.73	29.71	51.34	4.98	11.71	5.9
Blackrod Yard	2.31	25.14	45.37	4.40	12.18	2.4
Pemberton Four Feet	2.84	30.87	51.63	5.01	11.31	2.9
Haigh Yard	2.35	25.53	47.38	4.60	12.54	0.5
Furnace Mine	2.66	28.93	44.49	4.32	10.40	1.4
Bickerstaffe Four Feet	2.54	27.67	45.28	4.40	11.08	0.0
Rushy Park and Little Delf, mixed	2.80	30.29	50.67	4.92	11.29	4.3
Ince mixed	2.63	28.64	46.52	4.51	10.99	1.6
Arley Mine	2.26	24.46	44.12	4.28	12.18	0.4
Average results of 15 samples of coal	2.55	27.63	47.25	4.59	11.54	2.4
Mixture of 2 Hindley Yard coal and 1 Welsh coal-dust	2.21	24.00	41.38	4.02	11.83	0.0
1	2	3	4	5	6	7

NOTE.—The quantities in column 6 have been recalculated.—D. K. C.

The effect of reducing the flue-surface was tried by plugging up one-half of the number of flue-tubes, in alternate diagonal rows, so that the tube-surface was reduced by 206.5 square feet. The comparative results obtained with 12-inch fires were as follows:—

		Flue-tubes all open.	Half the Tubes plugged up.
Coal per square foot of grate per hour	lbs.,	25	24
Water at 100° evaporated per hour	cubic feet,	45.78	43.01
Water per pound of coal, as supplied at 212°,	lbs.,	12.41	12.23
Smoke per hour—very light	minutes,	2.8	8.0

Showing that with half the tubes, the performance was nearly as good as with them all open.

Messrs. Nicoll & Lynn, for the Board of Admiralty, made two independent series of trials of the South Lancashire and Cheshire coals; in the second of which the draught was increased by a steam-jet from a neighbouring boiler. The average results of these trials are placed beside those of Mr. Fletcher's trials in the annexed table, No. 272.

Table No. 272.—South Lancashire and Cheshire Coals—Summary Results of Performance in the Wigan Marine Boiler,

In three series of trials, by Messrs. Richardson & Fletcher, and by Messrs. Nicoll & Lynn.

Particulars.	Trials by Messrs. Richardson and Fletcher.	Trials of Messrs. Nicoll and Lynn.	
		Without Jet.	With Jet.
Area of fire-grate, ... square feet	10.3	10.3	10.3
Coal per hour,.................................. cwt.	2.55	2.53	3.70
Coal per sq. foot of grate per hour,........ lbs.	27.63	27.50	41.25
Water from 100° per hour,.......... cubic feet	47.25	48.30	69.13
Water per sq. foot of grate per hour, „	4.59	4.69	6.71
Water from 212°, per pound of coal, ... lbs.	11.54	11.92	11.36
Duration of smoke, in the hour, very light,...................... minutes	2.4	1.1	0.0

Trials of Newcastle and Welsh Coals in the Wigan Marine Boiler.

A mixture of Davidson's Hartley and Hasting's Hartley (Newcastle coals), and a mixture of Powell's Duffryn, Nixon's Navigation, and Davis's Abercwomboy (Welsh), were tried in the Wigan boiler—being the same as some coals that had been tried at Keyham Dockyard in 1863. The general results of the trials are, for comparison, placed together with those of the South Lancashire and Cheshire coals, thus:—

Table No. 273.—Newcastle and other Coals:—Comparative Results of Evaporative Performance.

Coals.	Coal per Square Foot of Grate per Hour.	Water at 100° per Hour.	Water at 112° per lb. of Coal.
	lbs.	cubic feet.	lbs.
Newcastle,..	28.83	51.33	11.95
Welsh,..	26.20	48.60	12.44
South Lancashire and Cheshire :—			
Average,..	27.63	47.25	11.54
Highest evaporative efficiency (Haigh Yard), ...	25.53	47.38	12.54
Lowest evaporative efficiency (Worsley Top Four Feet),	31.36	48.50	10.37

Showing that the average of the South Lancashire and Cheshire coals is inferior in rapidity and in efficiency of evaporation to both of the other coals, and that though the best of the South Lancashire coals has a greater evaporative efficiency than the others, the rapidity of evaporation was less.

This comparison is corroborative of the deductions made from Delabèche and Playfair's analysis and trials of coals from the several districts (see page 413).

Evaporative Performance of Newcastle Coals in a Marine Boiler, at Newcastle-on-Tyne, 1857.[1]

These experiments were made to test the evaporative power of the steam-coal of the Hartley district of Northumberland. The experimental boiler was of the marine type, 10 feet 3 inches long, 7 feet 6 inches wide, and 10 feet high; with 2 internal furnaces, 3 feet by 3 feet 3 inches high, and 135 flue-tubes above the furnaces, in 9 rows of 15 each, 3 inches in diameter inside, 5½ feet long. The dead-plates were 16 inches long, and 21 inches below the crown of the furnace. As the result of many preliminary trials, two standard lengths of fire-grates were fixed upon—4 feet 9 inches, and 3 feet 2½ inches, with a fall of ½ inch to a foot; and the fire-bars were cast ½ inch thick, with air-spaces from ⅝ to ¾ inch wide. The fire-doors were made with slits ½ inch wide and 14 inches long, for the admission of air. The chimney was 2 feet 6 inches in diameter. A water-heater was applied at the base of the chimney, in the thoroughfare; it contained 76 vertical tubes, 4 inches in diameter, surrounded by the feed-water.

Total area of fire-grates, 4 feet 9 inches long, 28½ square feet.
Do. do. 3 „ 2½ „ „ 19¼ „
Heating surface of boiler (outside), 749 square feet.
Do. water heater,.....320 „
Ratio of larger grate-area to heating surface of boiler, 1 to 26.28
Do. smaller do. do. 1 to 38.91

Two systems of firing were adopted, as "standards of practice:"—First, ordinary or spreading firing, in which the fuel was charged over the grate, and the whole of the supply of air was admitted through the grate. Second, coking-firing, in which the fuel was charged, 1 cwt. at a time, upon the dead-plate, and subsequently pushed on to the grate, making room for the next charge; and air was admitted by the doorway as well as by the grate. Four systems of furnace were tried, of which Mr. C. W. Williams' was adjudged by the experimentalists to have rendered the best performance. According to this system, air was admitted above the fire at the front of the furnace, by means of cast-iron casings, having apertures on the outside, with slides, and perforated through the inner face, next the fire, with numerous ⅝-inch and ½-inch holes, having a total area of 80 square inches, or 5.33 square inches per square foot of grate. Alternate firing was adopted by Mr. Williams. The general results of the experiments are given in table No. 274.

[1] The author has derived the particulars of those trials from the *Report of Messrs. Longridge, Armstrong, & Richardson to the Steam Collieries Association of Newcastle-on-Tyne.* 1857.

Table No. 274.—Newcastle Coals (of the Hartley District of Northumberland)—Results of Evaporative Performance in an Experimental Marine Boiler at Newcastle-on-Tyne. 1857.

(Compiled from the Report of Messrs. Longridge, Armstrong, and Richardson to the Steam Collieries Association of Newcastle-on-Tyne.)

Numerical Order.	Plan of Furnace.	Area of Fire-grate.	Coal Consumed per Hour.	Coal per Sq. Foot of Grate per Hour.	Water Consumed from 60° per Hour.	Water per Square Foot of Grate per Hour.	Water Evaporated from and at 212° per Pound of Coal.	Remarks on the Prevention of Smoke, &c.
		sq. feet.	cwt.	lbs.	cubic feet.	cubic feet.	lbs.	
1	Standard grate, ordinary management,	28.5	5.38	21.15	74.80	2.62	8.94	Air admitted entirely through the grate. Much smoke, often very dense.
2	Do. best management, ...	,,	4.88	19.00	79.12	2.93	11.13	Air admitted through both the grate and the door. No smoke.
3	Do. ordinary management,	19.25	3.61	21.00	56.01	2.91	10.00	Air through the grate alone; used 100 cu. feet per pound of coal. Temperature in uptake, 448°. Much smoke.
4	Do. best management, ...	,,	3.00	17.25	57.78	2.995	12.53	Air through the grate and the door; used 70 cubic feet through the grate and 88 cubic feet through the doors, per pound of coal. Temperature in uptake, 480°. No smoke.
5	C. W. Williams' plan, ...	22	3.33	17.27	61.59	2.80	11.70	Prevention of smoke, practically perfect.
6	Do. do.	,,	5.30	26.98	88.96	4.04	10.80	Do. do.
7	Do. do.	18	4.40	27.36	76.92	4.31	11.37	Do. do.
8	Do. do.	15.5	5.18	37.40	85.30	5.51	10.63	Prevention of smoke, practically perfect; temperature at base of chimney above 600°.
	1	2	3	4	5	6	7	8

Notes to Table.—1. When the temperature was 600° in the uptake of the boiler, it was reduced by from 40° to 50° after having passed through the water-heater.

2. In another case, working with Williams' apparatus, when no air was admitted through the door, and with much smoke, the temperature in the uptake was 600°. With one aperture in the door opened, it was raised to 625°; with two apertures, 633°; with three, 638°; with five it fell to 620°.

3. The quantities in column 7 have been recalculated.—D. K. C.

The experimentalists reported that Mr. Williams' plan gave the best results, and they concluded: "1st. That by an easy method of firing, combined with a due admission of air in front of the furnace, and a proper arrangement of fire-grate, the emission of smoke may be effectually prevented in ordinary marine multitubular boilers whilst using the steam-coals of the Hartley district of Northumberland. 2d. That the prevention of smoke *increases* the economic value of the fuel and the evaporative power of the boiler. 3d. That the coals from the Hartley district have an evaporative power fully equal to that of the best Welsh steam-coals, and that, practically, as regards steam navigation, they are decidedly superior."

These gentlemen made a trial of Aberaman Welsh coal, and they found that its practical evaporative power, when it was hand-picked, and the small coal rejected, was at the rate of 12.35 lbs. of water per pound of coal, evaporated from 212°; this may be compared with the best result from Hartleys coal, large and small together, in table No. 274, which was 12.53 lbs. water from 212° per pound of coal, or with another result of experiment, with Hartley coal, not given in the table, showing 12.91 lbs. water per pound of coal. As a check on these results, they ascertained the total heat of combustion of the two coals here compared, by means of an apparatus constructed by Mr. Wright, of Westminster, so contrived that a portion of coal is burned under water, and the products of combustion actually passed through the water, which absorbs the whole heat of combustion. The following are the comparative values:—

	Water practically Evaporated per Pound of Coal.	Total Heat of Combustion in Evaporative Efficiency.
Welsh coal, hand-picked,	12.35 lbs.	14.30 lbs.
Hartley coal, large and small,	12.91 „	14.63 „

The experimentalists also point out the "elasticity of action" of the Hartley coals: they burned them at rates varying from 9 to 37½ lbs. per square foot of grate per hour without difficulty, and without smoke. The Welsh coal, burned at the rate of 34½ lbs. per foot per hour, melted, it is said, the fire-bars after an hour and a half's work.

Trials of Newcastle and Welsh Coals in the Marine Boiler at Newcastle, for the Board of Admiralty. By Messrs. Miller & Taplin. 1858.

Messrs. Miller & Taplin, representing the Board of Admiralty, conducted, in 1858, a series of trials at Newcastle, with the same marine boiler as was employed by Messrs. Longridge, Armstrong, & Richardson, the object of which was to investigate the comparative evaporative power and other properties of Hartley coal and Welsh steam-coal, and the merits of Mr. Williams' plan of smoke-prevention.

The fire-bars were 1¼ inch in thickness, and had ⅝-inch air-spaces. The feed-water was passed through the heater, except when otherwise stated. Mr. Williams' apparatus was constantly in action when Hartley coal was burned without smoke; and it was closed when this coal was tried for smoke making, also when Welsh coal was burned.

Table No. 275.—NEWCASTLE AND WELSH COALS:—RESULTS OF EVAPORATIVE PERFORMANCE IN THE SAME MARINE BOILER AS FOR TABLE NO. 274, WITH C. W. WILLIAMS' APPARATUS FOR THE PREVENTION OF SMOKE. 1858.

(Compiled from the Report of Messrs. Miller & Taplin to the Board of Admiralty.)

Numerical Order.	COAL.	Area of Fire-grate.	Coal Consumed per Hour.	Coal per Square Foot of Grate per Hour.	Water Consumed from 60° per Hour.	Water per Square Foot of Grate per Hour.	Water Evaporated from 212° per Pound of Coal.	Remarks on the Prevention of Smoke, &c.
	NEWCASTLE.	sq. feet.	cwt.	lbs.	cub. feet.	cub. feet.	lbs.	
9	West Hartley, direct from collieries...	42	6.0	16.00	89.84	2.14	9.65	Long grates, air-passages fully open. No smoke.
10	Do. do. ...	,,	6.6	17.60	93.77	2.23	9.14	Do. do. do.
11	Do. do. ...	,,	6.8	18.13	94.46	2.25	8.96	Do. do. do.
12	Do. do. ...	33	6.0	20.36	86.24	2.61	9.25	Do. do. do.
13	Do. do. ...	22	4.34	22.08	76.77	3.49	11.41	Fire heavily charged at intervals to test the apparatus. No smoke. Maximum rate of consumption, 55½ lbs. per square foot.
14	Do. do. ...	,,	4.53	23.04	74.74	3.39	10.62	No smoke.
15	Do. do. ...	,,	5.1	25.97	88.43	4.02	11.17	No smoke.
16	Do. do. ...	,,	5.12	26.05	81.98	3.73	10.33	No smoke.
17	Do. do. ...	,,	5.6	28.51	92.00	4.18	10.58	No smoke.
18	Do. do. ...	,,	5.8	29.53	86.63	3.94	9.63	Air-passages above the fuel closed. Dense black smoke.
19	Do. do. (small)	,,	3.4	17.31	55.20	2.51	10.47	No smoke. Trial of small broken coal.
20	Do. do. ...	18	3.0	18.67	51.97	2.89	11.17	Trial with slow combustion. Damper in chimney closed to an area of 250 sq. ins. Grate raised 5 ins. Bars ⅝ in. thick, spaces ½ in.
21	Do. do. ...	,,	4.0	24.89	68.09	3.78	10.96	No smoke.
22	Buddle's West Hartley, direct from the colliery	22	5.4	27.49	83.33	3.79	9.95	These trials were made to compare the efficiency of coal brought direct from the collieries with that of the same coal after having been carried about and transshipped. But there was a doubt whether the coal from the dockyard was Buddle's or another quality.
23	Buddle's West Hartley, from Woolwich Dockyard...	,,	4.6	23.42	71.93	3.27	10.08	
24	Do. do.	,,	4.6	23.42	77.40	3.52	10.85	
25	Do. do.	,,	4.7	24.00	73.73	3.35	10.07	
26	Lambton's Wallsend House Coal, direct from colliery	,,	3.2	16.29	59.64	2.71	12.01	Test for bituminous or highly smoky coal. No smoke.
	1	2	3	4	5	6	7	8

Table No. 275 (*continued*).

Numerical Order.	Coal.	Area of Fire-grate.	Coal Consumed per Hour.	Coal per Square Foot of Grate per Hour.	Water Consumed from 60° per Hour.	Water per Square Foot of Grate per Hour.	Water Evaporated from 212° per Pound of Coal.	Remarks on the Prevention of Smoke, &c.
	NEWCASTLE COAL, when the feed-water was passed directly into the boiler, without the heater.	sq. feet.	cwt.	lbs.	cub. feet.	cub. feet.	lbs.	
27	West Hartley	22	5.29	26.92	84.31	3.83	10.27	No smoke.
28	Do.	,,	5.30	26.98	82.21	3.73	9.98	Do.
29	Do.	,,	7.0	35.64	102.76	4.67	9.46	Do. Forced draught by steam jet in chimney.
30	Do.	,,	6.2	31.56	78.78	3.58	8.19	Air-passages above fire, closed. Dense black smoke.
	Air-passages above the grate closed, with Welsh coal.							
	SOUTH WELSH COAL, with the heater in action.							
31	Blaengwarn Merthyr	22	3.6	18.33	69.54	3.16	12.44	No smoke, except very light smoke when firing.
32	Powell's Dufferin	,,	4.06	20.65	79.34	3.60	12.58	Do. do. do.
33	Welsh coal	,,	4.1	20.87	72.74	3.31	11.44	Do. do. do.
34	Sent from Woolwich Dockyard	,,	4.3	21.89	77.26	3.51	11.57	Do. do. do.
35	Welsh coal	,,	4.4	22.40	88.35	4.02	12.95	Do. do. do.
36	Powell's Dufferin (small)	,,	1.8	9.16	30.43	1.38	10.87	No smoke. Trial of the small coal to which the pieces are reduced by exposure to weather, or by being kept in store for some time.
37	Powell's Dufferin	18	3.87	24.12	66.72	3.71	11.10	No smoke, except very light when firing.
	WELSH COAL, when the feed-water was passed directly into the boiler, without the heater.							
38	Blaengwarn Merthyr, sent from Woolwich Dockyard	22	4.55	23.18	74.35	3.38	10.11	No smoke, except light brown when firing.
	1	2	3	4	5	6	7	8

NOTE.—The quantities in column 7 have been recalculated.—D. K. C.

During the trials of Hartley coal, the fires were maintained at from 12 to 14 inches in thickness on the grates, the coal was stoked on the coking system, the fresh charges of coal having been delivered at the front, on each side of each grate alternately; and the incandescent fuel pushed forward towards the bridge before charging.

During the trials of Welsh coal, the fires were maintained at from 8 to 10 inches in thickness; and, in charging, the fresh coal was thrown where it was required, all over the fire; the burning fuel never being touched by any firing tool.

The cinders that fell through the grates were constantly raked together and thrown upon the fires.

The results of the trials made by Messrs. Miller & Taplin have been analyzed and compiled into the table No. 275, in which the results of the performances of the West Hartley coals are grouped, to which is added the results obtained from Lambton's Wallsend house coal, as a bituminous or highly smoky coal. The results of the trials of the South Welsh coal are likewise grouped in the table. Separate trials of each coal were made, in which the feed-water was delivered direct into the boiler, the heater having been for this purpose disconnected.

Messrs. Miller & Taplin concluded from the results of their experiments: 1st, that when the smoke from Hartleys coal is consumed, the evaporative value of this coal is nearly equal to that of Welsh coal, whilst its rapidity of combustion and evaporation of water is greater; 2d, that the Hartleys coal is less liable to be broken up by movement than Welsh coal; and that it is less disintegrated by long exposure to the atmosphere than Welsh coal; 3d, that Hartleys coal may be burned without making smoke, by the use of Mr. C. W. Williams' apparatus.

Trials of Welsh and Newcastle Coals in a Marine Boiler at Keyham Factory. 1863.

Trials of Welsh and Newcastle coals, singly and in combination, were conducted with the coal-testing boiler at Keyham Factory, by Mr. T. W. Miller. The boiler is the pattern from which the Wigan boiler, described at page 781, was made, and of which it was a copy, except that the flue-tubes are 2 inches. The dead-plate was 10 inches below the crown of the furnace, and 6 inches in length. The grate was made of two different lengths, 4 feet and 3 feet, and inclined 2 inches per foot. The bridge was 8 inches below the crown. Two different doors to each furnace were employed during the trials: one, a common door, with a few small perforations for air; the other was made double, and air entered from the bottom, and passed through numerous ½-inch holes into the furnace. The air-way in the second door amounted to 60 square inches, equal to 8.6 inches per square foot of the longer grates, and 11.4 inches for the shorter grates. The charges of coal were from 16 to 19 lbs.

Total grate-area, ...	4 feet long, ...		13.75	square feet.
„	3 „		10.3	„
Total heating surface, plate, ...		72.5		„
„ „	tubes (outside),	324.5	397.0	„

Ratios of grate-areas to heating surface: larger grates,... 1 to 29
„ „ „ smaller „ ... 1 to 38.5

Mr. Miller reported that "the combinations, in equal proportions, of Welsh and Newcastle coals, while they produced, on the average, nearly equal economical results, measured by the quantity of water evaporated by 1 lb. of fuel, they produced on the average greater rapidity in evaporation, and that they on the average produced the least amount of smoke." He also found that the small Welsh coal could be burned beneficially in mixture with Newcastle coal. The general results of his experiments are given in table No. 276.

Evaporative Performance of American Coals in a Stationary Boiler. 1843.

The American coals, of which the composition was given page 418, were tried for their evaporative performance by Professor Johnson, with a flat-ended cylindrical boiler, 3½ feet in diameter and 30 feet long, with two thorough internal flues, 10 inches in diameter. The grate was placed below the boiler at one end, and was 5 feet long by 3 feet wide; it was 9 inches below the boiler at the front, and 10 inches at the back. The bars were ¾ inch thick, with ½-inch air-spaces. The grate could be shortened 8 inches, by inserting a perforated air-plate at the bridge; and 11¼ inches at the front, by inserting a coking-plate. The air for combustion was heated in a chamber under the ash-pit, before passing through the grate.

The gases passed under the boiler to the back, returned through the inside flues, and made another circuit of the boiler by a wheel-draught through side flues. The chimney was 18 inches square, and 61 feet high above the grate.

Area of grate:—		
Full length,	16.25	square feet.
With air-plate,	14.07	„
With air-plate and dead-plate,	11.375	„
Heating surface:—		
Lower flue,	130	„
Two flue-tubes,	157	„
Two side-flues,	90.5	„
Total,	377.5	„

Ratio of grate-area to heating surface:—	Full length,	1 to 23.2
	With air-plate,	1 to 26.8
	With air-plate and dead-plate,	1 to 33.2

The water was evaporated into steam of from 6 lbs. to 7 lbs. per square inch above the atmosphere. The coal was delivered in charges of from 100 lbs. to 110 lbs. The condensed results of performance are given in table No. 277. The average results were that in burning 7 lbs. per square foot of grate per hour, 9¼ lbs. of water was evaporated from 212° F. per pound of coal.

The inferior evaporation for the bituminous caking coals is accounted for by its imperfect combustion, evidenced by the smoke which escaped in considerable quantity.

Table No. 276.—NEWCASTLE AND WELSH COALS:—RESULTS OF EVAPORATIVE PERFORMANCE IN THE COAL-TESTING MARINE BOILER AT KEYHAM STEAM FACTORY. 1863.

(Adapted and Condensed from the Report of Mr. T. W. Miller to the Board of Admiralty.)

COAL.	Area of Fire-grate.	Coal Consumed per Hour.	Coal per Sq. Foot of Grate per Hour.	Water Consumed from 100° per Hour.	Water per Sq. Foot of Grate per Hour.	Water Evaporated from 212° per Pound of Coal.	Remarks as to Prevention of Smoke, &c.
	sq. feet.	cwt.	lbs.	cub. feet.	cu. feet.	lbs.	
1st Series.—COALS IN STORE, recently delivered.							
With common doors—							
Waynes Merthyr, Resolven, Merthyr Dare, Gellia Cadoxton, } Welsh,	14	1.93	15.44	32.4	2.31	10.42	Mean of 4 trials. Smoke marks 70. Bars, ½-in. air-spaces.
Hartley Main, Newcastle,......	,,	2.32	18.56	34.5	2.46	9.22	,, 2 ,, ,, 278 ,, ,,
Half Welsh, half Hartley,....	,,	1.92	15.40	30.4	2.17	9.81	,, 3 ,, ,, 47 ,, ,,
Two Welsh, one Hartley,......	,,	1.76	14.08	28.7	2.05	10.12	,, 3 ,, ,, 28 ,, ,,
One Welsh, two Hartley,......	,,	1.96	15.70	30.7	2.20	9.72	,, 3 ,, ,, 71 ,, ,,
With perforated door—							
Hartley Main,....................	,,	2.06	16.50	30.2	2.16	9.10	1 trial. ,, 31 ,, ,,
2d Series.—FRESH COALS, specially ordered for trial.							
With common doors—							
Powell's Dufferin, Nixon's Navigation, Davis's Merthyr, } Welsh,	14	2.09	16.68	37.1	2.65	11.05	Mean of 2 trials. Smoke marks 30. Bars, ½-in. air-spaces.
Davidson's Hartley, Hasting's Hartley, } Newcastle,	,,	2.29	18.29	34.5	2.46	9.39	,, 3 ,, ,, 202 ,, ,,
Half Welsh, half Hartleys, ...	,,	2.03	16.24	34.4	2.46	10.56	,, 3 ,, ,, 23 ,, ,,
Two Welsh, one Hartleys, ...	,,	2.43	16.36	35.0	2.50	10.61	,, 3 ,, ,, 18 ,, ,,
One Welsh, two Hartleys, ...	,,	2.06	16.44	33.5	2.39	9.43	,, 3 ,, ,, 99 ,, ,,
Welsh,............................	,,	2.19	17.48	39.3	2.80	11.16	1 trial. ,, 14. Bars, ¾-in. ,,
1	2	3	4	5	6	7	8

Table No. 276 (*continued*).

COAL.	Area of Fire-grate.	Coal Consumed per Hour.	Coal per Sq. Foot of Grate per Hour.	Water Consumed from 100° per Hour.	Water per Sq. Foot of Grate per Hour.	Water Evaporated from 212° per Pound of Coal.	Remarks as to Prevention of Smoke, &c.
	sq. feet.	cwt.	lbs.	cub. feet.	cu. feet.	lbs.	
3d Series.							
With perforated doors—							
Welsh coal,	14	1.87	14.95	32.7	2.34	10.86	Mean of 2 trials. Smoke marks 1½. Bars, ½-in. air-spaces.
Hartleys,	,,	2.13	17.04	32.8	2.34	9.61	,, 3 ,, ,, 34 ,, ,,
Half Welsh, half Hartleys,	,,	2.18	17.44	37.1	2.65	10.54	,, 2 ,, ,, 14½ ,, ,,
Two Welsh, one Hartleys,	,,	2.08	16.64	35.7	2.55	10.64	,, 2 ,, ,, 10½ ,, ,,
One Welsh, two Hartleys,	,,	2.18	17.42	36.5	2.61	10.39	,, 2 ,, ,, 8½ ,, ,,
Davidson's Hartley,	,,	2.86	22.88	42.9	3.06	9.31	1 trial. ,, 65. Bars, ⅝-in. ,,
Half Hartley, half Welsh,	,,	2.30	18.40	31.0	2.22	10.80	1 trial. ,, 9. Bars, ⅞-in. ,,
4th Series.							
WITH SMALLER GRATE-AREA.							
With common doors—							
Welsh Coal,	10.5	2.11	22.46	38.3	3.65	11.31	Smoke marks 12. Bars, ½-inch air-spaces.
Half Welsh small, half Davidson's Hartley,	,,	2.02	21.60	36.0	3.43	11.06	,, 10 ,, ,,
Half Welsh beans, half Hasting's Hartley,	,,	2.14	22.85	36.7	3.50	10.65	,, 11 ,, ,,
5th Series.							
With perforated doors—							
Hartleys,	10.5	2.29	24.40	42.0	4.00	11.42	Smoke marks 25. Bars, ½-inch air-spaces.
Half Welsh, half Davidson's Hartley,	,,	2.10	22.34	39.3	3.74	11.65	,, 13 ,, ,,
1	2	3	4	5	6	7	8

NOTE.—1. The quantities in column 7 have been recalculated.—D. K. C.

2. The areas of grate assumed in the original calculations of this table were 14 and 10.5 square feet respectively. The areas are, accurately, only 13.75 and 10.3 square feet.—D. K. C.

Table No. 277.—AMERICAN COALS:—RESULTS OF THEIR EVAPORATIVE PERFORMANCE WITH A CYLINDRICAL STATIONARY BOILER AT THE NAVY YARD, WASHINGTON. 1843.

(Reduced from the Report of Professor W. R. Johnson.)

COAL.	Area of Fire-grate.	Coal Consumed per Hour.	Coal per Square Foot of Grate per Hour.	Water Evaporated from Ordinary Temperature per Hour.	Water per Square Foot of Grate per hour.	Water Evaporated from 212° F. per lb. of Coal.
	sq. feet.	lbs.	lbs.	cub. feet.	cub. feet.	lbs.
Anthracites (7 samples),...	14.30	94.94	6.64	12.37	0.87	9.63
Free-burning bituminous coals (11 samples),.....	14.14	99.16	7.01	13.73	0.97	9.68
Bituminous caking coals (Virginian, 10 samples),	14.15	105.02	7.42	12.16	0.86	8.48
Averages,	14.20	99.71	7.02	12.75	0.90	9.26

CONDITIONS OF SMOKE.

Anthracites, ...	No smoke.
Free-burning bituminous coals,.........	Little smoke, and mostly when charging.
Bituminous caking coals, ...	Smoke considerable, in one instance constant.

The air-plate was tried both open and closed for each coal, with various effect, good and bad. The average results for the open air-plate proved a gain in efficiency and a loss in rapidity, thus:—

COALS.	Gain of Efficiency.	Loss of Rapidity.
	per cent.	per cent.
Anthracites,................................	0.43	14.9
Free-burning coals, ...	2.13	2.68
Caking coals, ...	1.96	1.48
Foreign and western coals,.............	3.38	5.37

Showing that the open air-plate was most beneficial for efficiency and least injurious for rapidity with the smoke-making coals.

Surplus air in the products of combustion.—By analysis, it was found that twice the quantity of atmospheric air that was chemically necessary, passed through the furnace.

Temperature of the air and smoke.—The average temperatures were as follows:—

External air,................................	73° F.	
Air on arriving at the grate,............	250°.	Heated 177°.
Gases on arriving at the chimney,.....	292°.	Excess above steam 65°.
Draught-gauge, ...	.307 inch of water.	

Influence of soot in the flues.—It was observed that whilst, in the performance of the anthracites, day after day, the temperature at the chimney and the evaporative efficiency were practically constant, with the smoky coals the temperature rose and the efficiency fell off. In three instances of caking coal, the temperature rose 75° from an average of 298° F., on the

first day, to 373° F. on the last day; and the efficiency fell off 1 lb., from 8.66 to 7.68 lbs. These effects are due, of course, to the accumulation of soot on the surface of the boiler, and the impediment thus caused to the passage of heat.

Level of the grate.—The grate was tried at 7 inches and 12 inches below the crown, the standard level having been 9 inches. The trials showed that the 7-inch level was 5½ per cent. better than the 9-inch, and the 9-inch level 8 per cent. better than the 12-inch.

Effect of cutting off the two side-flues.—Reducing thus the heating surface by 90.5 square feet, the comparative performance with and without the side-flues was as follows:—

	Water per lb. of Coal.	Water per hour.
ANTHRACITE:—		
Without side-flues,	9.96 lbs.	13.33 cubic feet.
With do.	10.11 ,,	14.03 ,,
CAKING COAL:—		
Without side-flues,	7.80 ,,	11.72 ,,
With do.	8.52 ,,	11.30 ,,

Showing that, whilst the average rapidity was not affected, the efficiency was diminished by the closing of the side-flues.

EVAPORATIVE PERFORMANCE OF AN EXPERIMENTAL MARINE BOILER, NAVY YARD, NEW YORK, U.S.[1]

Mr. Isherwood made trials of an experimental multitubular marine boiler under cover, on land. The boiler was covered with felt stitched on canvas. Steam of 20 lbs. effective pressure per square inch was generated and blown off. The shell was 7 feet 7 inches deep, 3 feet 1½ inch wide, and 6 feet 5 inches high, with a single fire-flue 26 inches in diameter, and 24 flue-tubes above the fire-flue, 3 inches in diameter outside, 5 feet 10½ inches long. With a 4-inch dead-plate, the grate was 5 feet long, inclined, being 12 inches below the crown at the front, and 15½ inches at the bridge.

Area of fire-grate,			10.8 square feet.
Heating surface:—	furnace,	19.7 square feet.	
	smokebox,	25.75 ,,	
	tubes,	100.78 ,,	
	uptake,	4.02 ,,	150.30 ,,

Ratio of grate-area to heating surface, ...1 to 14.

The fires were 5 inches in thickness, using Pennsylvanian anthracite of medium quality. The refuse averaged about 20 per cent. of the fuel.

In the following selection of the results of the performance, the equivalent quantities of water as evaporated from and at 212° F., are substituted for the original quantities:—

[1] *Experimental Researches in Steam Engineering*, vol. ii. 1865.

Table No. 278.—Evaporative Performance of Experimental Marine Boiler, at the Navy Yard, New York, U.S.

	Area of Grate.	Heating Surface.	Ratio of Heating Surface to Grate.	Coal per Square Foot of Grate.	Water per Pound of Coal from and at 212°.
	1st Series of Trials:—Varying Rate of Combustion.				
	square feet.	square feet.	ratio.	lbs.	lbs.
A	10.8	150.3	14	5.57	9.27
B	10.8	150.3	14	10.99	8.95
C	10.8	150.3	14	16.57	7.94
D	10.8	150.3	14	22.10	7.80
E	10.8	150.3	14	27.76	7.40
	3d Series of Trials:—Varying Area of Grate.				
I	8.64	149.0	17.24	15	8.58
J	6.48	148.0	22.84	15	8.28
K	4.32	147.0	34.03	15	8.93
	4th Series of Trials:—Constant Total Quantity of Fuel Consumed per Hour, with Varying Grates.				
L	8.64	149.0	17.24	20.73	8.03
M	6.48	148.0	22.84	27.42	7.43
N	4.32	147.0	34.03	27.58	7.24
	8th Series of Trials:—Heating Surface of Tubes cut off.				
V	10.8	45.5	4.21	16.57	5.91
W	10.8	45.5	4.21	16.58	6.10
X	10.8	45.5	4.21	11.77	6.64

Experiments on the Comparative Evaporative Performance of Stationary Boilers in France. 1874.

A commission was appointed by the *Société Industrielle de Mulhouse* to test, under identical conditions, the comparative performance of a French or elephant boiler, a double-flue Lancashire boiler, and a so-called "Fairbairn" boiler.[1]

Lancashire boiler.—6.56 feet in diameter, 25.75 feet long; flues 27.5 inches in diameter, with internal fire-place 28.5 inches in diameter. Shell-plates .64 inch, flue-plates ½ inch thick. Grate inclined; mean level below crown, 16 inches. Fire-bars .6 inch thick, air-spaces ¼ inch.

"*Fairbairn*" *boiler.*—Two cylinders 4.1 feet in diameter, 25.75 feet long; central fire-tube 27.5 inches in diameter, enlarged at the end to form an internal fire-place 28.5 inches in diameter. The two cylinders were united to a third above them, 3.75 feet in diameter, 23 feet long, by three neckings or pipes, 14 inches in diameter, from each lower cylinder. Plates

[1] *Bulletin de la Société Industrielle de Mulhouse*, June, 1875. See also *Proceedings of the Institution of Civil Engineers*, vol. xliii. p. 377, where an abstract of the Report is published.

½ inch. Grate inclined; mean level below crown, 16 inches. Fire-bars .6 inch, air-spaces ¼ inch.

French boiler.—Body 3.74 feet in diameter, 29.5 feet long. Three heaters, 1.64 feet by 32.8 feet long, united to the body by three neckings to each. Plates of body ½ inch; of heaters .4 inch thick. Grate horizontal; level below middle heater, 18 inches, and below side heaters, 16 inches.

	"Fairbairn."	Lancashire.	French.
Length of boiler, ... feet	23 & 25.75	25.75	29.5 & 32.8
Total heating surface, ... square feet	1,017	612	607
Length of grates, ... feet	4.53	4.53	4.21
Combined width of grates, ... "	4.53	4.53	4.76
Total grate-area, ... square feet	20.5	20.5	20.1
Ratio of heating surface to grate-area, ...	1 to 49.5	1 to 29.8	1 to 30.3
Total capacity, ... cubic feet	642.5	637.5	531.1
Water-space, ... "	544.7	412.5	408.1
Steam-space, ... "	97.8	225.0	123.0
Heating surface per cubic foot of water, ... square feet	1.87	1.48	1.49
Total weight, with accessories, ... tons	19.6	16.6	14.5
Weight per square foot of heating surface, ... lbs.	42.4	59.7	52.5

The gases in the Fairbairn boiler passed from the flues by the sides of the lower cylinders, and returned by the sides of the upper cylinder, towards the chimney. In the Lancashire boiler, they passed from the inside flues on each side to the front, and thence under the boiler to the chimney. In the French boiler, the current was not divided, but after heating the three heaters it wound round the boiler. The flues delivered into the same chimney. The temperature in the flues, just at the chimney, about 4 inches above the bottom, was taken every five minutes. The steam was maintained at from 4.6 to 5 atmospheres. The feed-water was supplied at from 79° to 84° F. The regular daily work lasted from 6 a.m. to 6 p.m., with 1¼ hour interval; working time, 10¾ hours. The coal consumed in getting up steam was included in the consumption. Two days before the trial, each boiler was emptied and was thoroughly cleaned inside and outside. Each trial lasted several days consecutively. The coals consumed were Ronchamp and Saarbrucken, the general composition of which is indicated by the following analysis:[1]—

Gaseous Elements only, or "Pure Fuel."

	Carbon.	Hydrogen.	Oxygen and Nitrogen.	Actual Heat of Combustion of One Pound of Pure Fuel.
	per cent.	per cent.	per cent.	English units.
Ronchamp, ...	88.59	4.69	6.72	16,416
Saarbrucken, ...	81.10	4.75	14.15	15,320

[1] "Calorimetric Trials and Analysis of Coals and Lignites," by A. Scheurer-Kestner and C. Meunier-Dollfus. See *Proceedings of the Institution of Civil Engineers*, vol. xliii. p. 396.

Table No. 279—French and English Boilers.—Results of Evaporative Performance.

(Reduced from the Report of the Mulhouse Commission.)

Fuel.—Ronchamp and Saarbrucken Coals.

Boiler, and Fuel.	Coal Consumed per Hour.			Water Evaporated ℔ Hour from and at 212° F.		Water per lb. of Entire Coal.	Temperature of Gases.	Air drawn in per lb. of Coal.
	Total.	Per Sq. Foot of Grate.	Ash.	Total.	Per Sq. Foot of Grate.			
	cwt.	lbs.	pr. c't.	cub. ft.	cub. ft.	lbs.	Fahr.	cu. ft.
Ronchamp Coal. Heavy Firing.								
"Fairbairn,"	3.39	18.53	13.8	56.06	2.73	9.21	421°	226
Lancashire,	3.50	19.15	14.1	53.45	2.61	8.50	572	183
French,	3.69	20.57	14.1	54.73	2.72	8.26	562	194
Ronchamp Coal. Light Firing.								
"Fairbairn,"	1.96	10.70	13.5	31.14	1.52	8.86	337°	261
Lancashire,	1.91	10.41	14.6	30.52	1.49	8.92	406	194
French,	2.04	11.36	13.6	31.38	1.56	8.58	425	193
Saarbrucken Coal.								
"Fairbairn,"	3.04	16.59	10.6	43.20	2.11	7.93	402°	195
Lancashire,	3.02	16.50	9.7	40.69	1.99	7.51	554	180
French,	3.11	17.32	9.4	41.89	2.08	7.51	544	179
General Averages of the Foregoing Performances.								
"Fairbairn,"	2.80	15.27	12.6	43.74	2.12	8.67	387°	227
Lancashire,	2.81	15.35	12.8	41.55	2.03	8.33	511	186
French,	2.95	16.42	12.4	42.67	2.12	8.12	510	189
Averages of 3 days' Performance, when equal rates of evaporation were effected.								
Lancashire,	3.57	19.50	—	54.10	2.64	8.44	587°	165
French,	3.57	19.87	—	54.32	2.70	8.49	572	197

EVAPORATIVE PERFORMANCE OF LOCOMOTIVE BOILERS.

The author collected, from various trustworthy sources, the results of the performance of locomotive boilers, of the earliest as well as the most recent designs, and has reduced them and placed them together with the results of his own observations, in table No. 280.[1]

Boilers of nearly every size and variety that have been used in England, are represented in the table; the areas of grate vary from 6 to 24 square feet, the heating surfaces from 40 to 2000 square feet, and the ratios of surface to grate, or the surface-ratios, from 40 to 1 to 100 to 1. The fuel used was coke, except in a few specified instances of boilers designed for burning coal, in which coal was used.

[1] These data are derived from the author's work on *Railway Machinery*, 1855, page 156; and *Railway Locomotives*, page 33*. Reference is made to these works for information on the details of the boilers.

Table No. 280.—LOCOMOTIVE-BOILERS:—PROPORTIONS AND RESULTS OF EVAPORATIVE PERFORMANCE.

The Fuel used was Coke, except when Coal is specifically stated.

No.	Name of Locomotive.	Area of Fire-grate.	Heating Surface (Tubes measured on the Outside).	Ratio of Heating Surface to Grate.	Coke consumed per Square Foot of Grate per hour.	Water Consumed per Square Foot of Grate per hour.	Water Evaporated per Pound of Coke, from and at 212° F.
		sq. ft.	sq. ft.	ratio.	lbs.	cu. ft.	lbs.
	EARLIEST LOCOMOTIVES.						
1	Killingworth,...............	7.0	41.25	6	44 (coal)	2.3	4.02
2	Do. improved,...	10.9	124	11.4	57 (coal)	4	5.32
3	Rocket,.......................	6	138	23	35.5	3	6.27
4	Phœnix,.......................	6	326	55	54	5.7	7.86
5	Atlas,.........................	9.20	275	30	60	5.14	6.35
6	Star,...........................	7.76	359	46	92	8.22	6.53
7	Average of 4 locomotives,	6.5	348	53.5	90	9.8	8.04
8	Soho, ...	8.44	412	35	100	10	7.42
9					130	13.03	7.38
10					92	11	8.87
11	Hecla, ...	8.34	418	49	125	11.3	6.65
12	Bury's goods locomotives,	9.2	461	50	111	9.24	6.15
13	Bury's passenger ,,	9.2	387	42	112	8.15	4.93
	GT. WESTERN RAILWAY.						
14	Ixion,.........................	13.4	699	52	138	15	8.33
15	Hercules,......................	13.6	699	51.4	105	15	10.70
16	Etna, Capricornus,.........	11.4	467	41	97	10.7	8.21
17	Giraffe,........................	12.5	608	48.6	76	8.8	8.61
18	Mentor, Cyclops,...........	13.6	699	51.4	69	8	8.67
19	Royal Star,..................	11.7	822	70	91	10.8	8.85
20	Pyracmon Class, ...	18.44	1363	74	69	8.4	9.09
21	Ajax,...........................	13.67	1067	78	84	11.2	9.90
22	Great Britain, Iron Duke,	21	1938	92	82	11	9.95
23	Great Britain Variety, ...	21	1938	92	90	11	9.17
24	Courier Variety,............	23.62	1866	79	75	8.6	8.60
	LONDON AND NORTH-WESTERN RAILWAY, &c.						
25	A, York & North-Midland Railway,.........	9.6	903	94	132	17	10.52
26	Hercules, York&North-Midland Railway, ...	9.6	828	86	105	15	10.70
27	Sphynx, Man., Sheffield, & Lincoln Railway,	10.56	1056	100	157	22.1	10.41
	(Later engines.)						
28	Heron, L. & N.-W. Ry.,	10.5	782	74.5	90	11.1	9.29
29	No. 291, ,, ,,	19	1449	76.26	56.5	6.2	8.23
30	No. 300, ,, ,,	22	1263	57.41	50.7	6.6	9.28
	SOUTH-EASTERN RAILWAY						
31	No. 142,......................	14.7	1158.2	78.8	62.25 (coal)	8.77	10.15
32	No. 118,......................	26.25	963.5	36.7	30.86 (coal)	4.54	10.60
33	No. 58,......................	12.25	705.7	57.6	61.22 (coal)	8.60	10.13
34	No. ,, ...	,,	,,	,,	44.49 (coal)	7.35	11.91
35	No. 142,......................	14.7	1158.2	78.8	55.71	7.73	9.77
36	No. 105,......................	10.5	623.1	59.3	55.91	9.43	11.68
37	No. 9,......................	10.5	623.1	59.3	66.19	10.00	10.96

Table No. 280 (*continued*).

No.	Name of Locomotive.	Area of Fire-grate.	Heating Surface (Tubes measured on the Outside).	Ratio of Heating Surface to Grate.	Coke Consumed Per Square Foot of Grate.	Water Consumed per Square Foot of Grate.	Water Evaporated per Pound of Coke, from and at 212° F.
		sq. ft.	sq. ft.	ratio.	lbs.	cu. ft.	lbs.
	LONDON AND SOUTH-WESTERN RAILWAY.						
38	Snake, ...	12.4	985	79	87	12.26	10.59
	Canute (coal burning locomotive) :—						
39	Canute, feed-water heated, tiles, ...	16	871	54.4	35 (coal)	6.18	11.02
40	Canute, feed-water heated, tiles, ...	,,	,,	,,	57 (coal)	9.65	10.57
41	Canute, cold feed-water, tiles, ...	,,	,,	,,	49 (coal)	7.77	9.90
42	Canute, feed-water heated, no tiles, ...	,,	,,	,,	42 (coal)	6.42	9.54
43	Canute, cold water, no tiles, ...	,,	,,	,,	58 (coal)	8.89	9.56
44	Canute, feed-water heated, tiles, ...	,,	,,	,,	46 (coke)	6.46	8.76
45	Canute, feed-water heated, no tiles, ...	,,	,,	,,	49 (coke)	7.17	9.13
46	Canute, cold water, no tiles, ...	,,	,,	,,	54 (coke)	8.69	10.04
	CALEDONIAN RAILWAY, &c.						
47	No. 33, Caledonian Ry.,	10.5	831	79	42	7	12.46
48	No. 42, ,,	10.5	788	75	57	7.8	10.11
49	No. 43, ,,	10.5	788	75	61	9.2	11.31
50	No. 51, ,,	10.5	788	75	45	6.7	11.04
51	No. 13, ,,	10.5	788	75	108	11.6	8.09
52		10.5	788	75	57	8.2	10.71
53	No. 13, ,,	9.0	788	87.6	102	14.7	9.52
54	Nos. 125, 127, ,,	11.37	1050	92	66	8.66	9.72
55	No. 102, ,,	11.8	974	82.5	94	10.3	8.15
56	Orion, Sirius, E. & G. Ry.,	12.23	758	62	44	6.29	10.71
57	America, Nile, ,,	11.10	736	66.3	70	8.8	9.31
58	Pallas, ,,	16.04	818	51	38	6	10.47
59	Brindley, ,,	9.15	802	87.65	54	7.2	9.94
60	Orion, G. & S.-W. Ry.,	9.24	495	53.6	84	9.4	8.28
61	Queen, ,,	10.5	688	65.5	87	10	8.57

EVAPORATIVE PERFORMANCE OF PORTABLE STEAM-ENGINE BOILERS WITH COAL. 1872.

The results of the excellently conducted trials of portable steam-engines exhibited at the show of the Royal Agricultural Society, at Cardiff, in 1872, were fully reported by the judges, Mr. F. J. Bramwell and Mr. W. Menelaus.[1] To this report, with the valuable tables appended to it, prepared by the consulting engineers, Messrs. Eastons & Anderson, the author is indebted for the data with which he has formed the table No. 281. The fuel used was Llangennech (Welsh) coal; an analysis of it, by Mr. G. J. Snelus is given at page 415, *ante*. The quantity of ash and clinker averaged, so far as it was observed, about 6 per cent. of the fuel. The boiler was of the ordinary pattern, having a firebox and multitubular flues; but Messrs. Davey, Paxman, & Co.'s boiler contained, in addition, ten circulating wrought-iron bent water-tubes, 2¼ inches in diameter, in the firebox, rising from the sides to the top.

Table No. 281.—PORTABLE STEAM-ENGINE BOILERS.—PROPORTIONS AND RESULTS OF EVAPORATIVE PERFORMANCE. 1872.

(Compiled and reduced from the Report of the Judges, Royal Agricultural Society's Show, Cardiff.)

Fuel:—Llangennech (Welsh) Coal.

No.	Constructors.	Area of Fire-grate.		Heating Surface (Tubes measured on the Outside).	Ratio of Heating Surface to Trial Fire-grate.	Coal Consumed per Sq. Foot of Trial-grate per Hour.	Equivalent Water Evaporated from and at 212° F. per Square Foot of Grate, per Hour.		Equivalent Water Evaporated per Pound of Coal.
		Normal.	As Reduced for Trial.						
		sq. ft.	sq. ft.	sq. ft.	ratio.	lbs.	lbs.	cu. ft.	lbs.
1	Marshall, Sons, & Co.	4.4	3.0	283.5	94.5	15.7	161	2.58	10.23
2	Clayton & Shuttleworth	5.3	3.2	220.0	69	12.8	151	2.42	11.83
,,	Clayton & Shuttleworth	,,	,,	,,	,,	12.5	148	2.36	11.81
3	Hayes............	5.1	5.1	170.6	33	14.8	66.5	1.06	4.59
4	Davey, Parman, & Co.	3.75	3.75	168.4	45	10.3	114	1.83	11.02
5	Tuxford & Sons	6.13	—	193.0	—	—	—	—	—
6	Brown & May...	3.2	3.2	159.1	50	9.53	104	1.66	10.89
7	Tasker & Sons...	4.7	4.7	158.0	34	13.0	119	1.91	9.33
8	Reading Iron-Works........	7.2	2.37	211.0	89	20.4	214	3.43	10.49
9	Lewin............	4.3	1.6	151.6	—	—	—	—	—
10	E. R. & F. Turner......	3.5	3.5	187.8	54	20.7	204	3.26	9.93
11	Barrows & Stewart.....	5.0	5.0	129.8	26	13.6	120	1.93	8.97
12	Ashbey, Jeffery, & Luke	5.5	2.0	204.5	102	31.1	319	5.10	9.27

[1] *The Trials of Portable Steam-Engines at Cardiff; Report by the Judges.* 1872.

RELATIONS OF GRATE-AREA AND HEATING SURFACE TO EVAPORATIVE PERFORMANCE.

Special Experiments on the Relative Value of the Different Parts of Heating Surface.

Mr. Graham's Experiments, 1858.—Mr. John Graham published, in 1858,[1] an account of his experiments on the proportional evaporative value of the different parts of the heating surfaces of boilers.

1st Series. Four open tin pans, 12 inches square, in a row, set in brickwork. A grate 12 inches square was set directly under the first pan, 29 inches below it, from which a flash-flue 3 inches deep conducted the gaseous products under the other pans towards a chimney. The first pan showed "the direct heating effect of fire;" the second, the effect of an "equal surface of blaze," the third and fourth, the effect of heated air only. With a "moderately strong draught" the quantities of water evaporated per hour were proportionally as follows:—

		Percentage of Evaporative Duty.
1st pan,	as 100	67.6
2d „	„ 27	18.2
3d „	„ 13	8.8
4th „	„ 8	5.4
		100.0

Showing that two-thirds of the whole evaporation was effected from the first pan, and only a twentieth from the last pan.

2d Series. Three cylinders of ¼-inch plate, 3 feet in diameter, and 3 feet long, open to the atmosphere, in a row end to end, were set in brickwork. A grate was placed under the first cylinder, 3 feet long and 2 feet wide, and 9½ inches below the cylinder; with a flash-flue under the second and third cylinders, concentric with them, of 4 inches radial width, and carried up on each side to the level of the centre of the cylinders. The average results of eleven trials for evaporation, with the calculated heating surfaces, were as follows:—

Area of grate,	6	square feet.
Heating surface of 1st cylinder,	10.53	„
Do. 2d do.	14.13	„
Do. 3d do.	14.13	„
	38.79	„

Worsley coal consumed, 72 lbs. per hour, or 12 lbs. per square foot of grate per hour. Water evaporated from 60° F., 4.55 lbs. per pound of coal; the duty was proportionally as follows:—

		Percentage of Duty. For Whole Surface.		Percentage of Duty. Per Square Foot.
1st boiler,	as 100	66.4,	or	73 per cent.
2d „	„ 34.7	23.0,	„	18.5 „
3d „	„ 16	10.6,	„	8.5 „
		100.0		100.0

[1] *Transactions of the Literary and Philosophical Society of Manchester*, vol. xv., 1858.

Showing that about three-fourths of the evaporative work per square foot of surface was done by the first cylinder, and only a twelfth by the third cylinder.

Experiments of Messrs. Woods and Dewrance, 1842.[1]—Mr. Edward Woods and Mr. John Dewrance, in 1842, tested the evaporative duty of successive portions of the flue-tubes of a locomotive boiler, 5 feet 6 inches long, divided into six compartments by vertical diaphragms. The first compartment was 6 inches long, and each of the others 12 inches. It was found that the evaporative duty of the first compartment was about the same per square foot as that of the fire-box; that of the second compartment about a third of that value; that of the remaining compartments very small; and that the first 6 inches did more work than the remaining 60 inches of tube.

Experimental Deductions of M. Paul Havrez, 1874.—The important deduction, that the evaporative performance of similar boilers per unit of grate-area, increases with the square of the surface-ratio, is confirmed by the deduction made by M. Paul Havrez of the following law, from the performances of locomotive boilers:[2]—That the quantities of water evaporated by consecutive equal lengths of flue-tubes decrease in geometrical progression, whilst the distances from the commencement of the series increase in arithmetical progression. The point, he adds, at which the law begins to prevail, is that at which the radiation of heat from the fuel ceases, and heat is communicated by conduction alone. One of the experiments of which the results were investigated by M. Havrez, was made by M. Pétiet, of the Northern Railway of France, who repeated the experiment of Mr. Woods and Mr. Dewrance, and tested the evaporative value of the different parts of a locomotive boiler having tubes of a length of 12 feet 3 inches divided into five compartments. The first compartment consisted of the fire-box, with 3 inches of length of the tubes; and the four tube-sections were 3.02 feet long. Using coke and briquettes as fuel, the average results were as follows:—

	Fire-box Section.	1st Tube Section.	2d Tube Section.	3d Tube Section.	4th Tube Section.
Surface	60.28 box 16.15 tubes. 76.43	179	179	179	179 sq. ft.
Water evaporated per square foot per hour, with coke	24.5	8.72	4.42	2.52	1.68 lbs.
Water evaporated per square foot per hour, with briquettes	36.9	11.44	5.72	3.52	2.31 lbs.

M. Havrez's law of progression is traceable here, and whether it be exact, or only approximately true, the rapidly diminishing evaporations are corroborative of the results of previous experiments. If the successive evapora-

[1] *The Engineer*, March, 1858.

[2] "Evaporation in Steam-boilers decreasing in Geometrical Progression," by M. Paul Havrez, *Annales du Génie Civil*, August and September, 1874; abstracted in the *Proceedings of the Institution of Civil Engineers*, vol. xxxix., page 398, 1874–75.

tions be set off as ordinates to a base line representing the advance of the heating surface, and contoured, the area of the figure is a measure of the total evaporation. The area would bulk largely at the first part, and taper down quickly towards the end; and it is easily comprehended that such areas of evaporation for boilers of different total lengths or quantities of surface, may increase practically as the squares of the total surfaces,—supposing that the final temperatures of the gases in leaving the boilers were the same.

FORMULAS FOR THE RELATIONS OF GRATE-AREA, HEATING SURFACE, WATER, AND FUEL.

It is well known that, in a given boiler, in which the grate and the heating surface are constant—and, of course, also the ratio of the surface to the grate-area,—the greater the quantity of fuel consumed per hour the greater also is the quantity of water evaporated; but that the production of steam increases at a less rate than the combustion, in other words, that the quantity of water evaporated per pound of fuel is diminished. But it has remained a question:—At what rate does this diminution of efficiency take place? The answer is supplied by the fact, generalized from the experimental observations on stationary, portable, marine, and locomotive boilers, detailed or noticed in preceding pages, that the total quantity of water evaporated per square foot of grate is expressed by a constant quantity, A, plus a constant multiple, B c, of the fuel consumed per square foot of grate, or by the general formula

$$w = A + B\,c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

The sense of this equation is, that though the water evaporated per square foot of grate does not keep pace with the fuel consumed, yet that the quantity of water increases by equal increments for equal increments of fuel per square foot of grate.

Again, on the inverse supposition, that the efficiency of the fuel remains constant, how is the performance of a boiler affected by the proportions of the grate-area and the heating surface? The author, in 1852, investigated this question by the aid of the observations already noticed, of the evaporative performance of locomotive-boilers, using coke; and he deduced from them, that, assuming throughout a constant efficiency of the fuel, or proportion of water evaporated to the fuel, the evaporative performance of a locomotive boiler, or the quantity of water which it was capable of evaporating per hour, *decreases* directly as the grate-area is increased; that is to say, the larger the grate the smaller is the evaporation of water, at the same rate of efficiency of fuel, even with the same heating surface. 2d. That the evaporative performance *increases* directly as the square of the heating surface, with the same area of grate and efficiency of fuel. 3d. The necessary heating surface *increases* directly as the square root of the performance; that is to say, for example, for four times the performance, with the same efficiency, twice the heating surface only is required. 4th. The necessary heating surface *increases* directly as the square root of the grate, with the same efficiency; that is to say, for instance, if the grate be enlarged to four times its first area, twice the heating surface would be

required, and would be sufficient, to evaporate the same quantity of water per hour with the same efficiency of fuel.

Let W be the quantity of water evaporated per hour, and C the weight of coke consumed per hour, W and C varying so as to preserve a constant ratio to each other; let h = the heating surface, and g = the area of grate, in square feet; then

$$W = m\frac{h^2}{g}; \quad (2)$$

in which m is a constant. When the water, W, is expressed in cubic feet, and 9 lbs. of water is evaporated per pound of fuel, the value of m, deduced from the results of forty experiments, was found to be .00222, and

$$W = .00222\frac{h^2}{g} \quad (3)$$

Reduced to the standard of one square foot of grate, let w and c be the weights of water and fuel respectively, per square foot of grate, in constant ratio to each other; then, dividing the above formulas respectively by g,

$$w = m\left(\frac{h}{g}\right)^2, \quad (4)$$

$$\text{and } w \text{ (cubic feet)} = .00222\left(\frac{h}{g}\right)^2 \quad (5)$$

Showing that, when the ratio of water to fuel is constant, the performance of the boiler, per square foot of grate, increases as the square of the ratio of the heating surface to the grate-area. The following table of examples, extracted from *Railway Machinery*,[1] shows how closely the evaporation proceeded according to the square of the surface-ratio, when 9 lbs. of water, at the ordinary temperatures and pressures, was evaporated per pound of coke.

Table No. 282.—Of Relative Heating Surfaces and Rates of Consumption of Water in Locomotive Boilers.

(Railway Machinery.)

Classified Groups of Locomotives.	Surface-ratio.	Consumption of Water per Hour per Sq. Foot of Grate.	Water per Pound of Coke.	Number of Experiments.
	ratio.	cubic feet.	lbs.	
Orion, Sirius, Pallas, E. & G. Ry, ...	52	6.15	9	13
C. R. Passenger Engines, ...	66	8	9.1	17
Snake, L. & S. W. Ry.	72	12	8.9	2
Sphynx, A, Hercules, ...	90	18	8.92	8

The quantities of water are thrown into the parabolic curve, A C, Fig. 328, next page, being ordinates to the base-line, A B, on which the relative surface-ratios are measured.

It was thus found, that, practically, there can never be too much heating surface, as regards economical evaporation, but there may be too little; and

[1] *Railway Machinery*, page 158.

that, on the contrary, there may be too much grate-area for economical evaporation, but there cannot be too little, so long as the required rate of combustion per square foot, does not exceed the limits imposed by physical conditions.

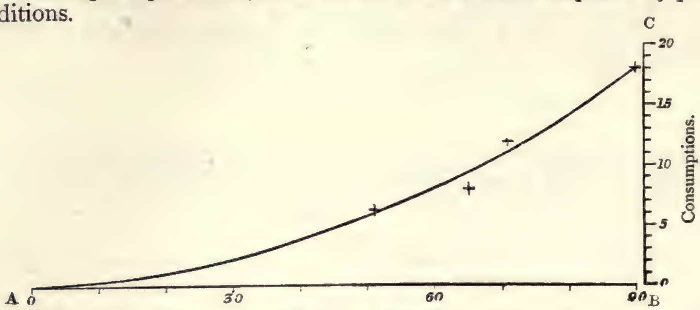


Fig. 328.—Diagram to show Rate of Economical Consumption of Water per hour per foot of Grate, for given Surface-ratios.

To co-relate the formula (1), in which the surface-ratio is constant, with the formula (4), in which the evaporative efficiency of the fuel is constant, it may suffice for the present to observe that the quantity Bc is constant for all surface-ratios, and that the quantity A varies as the square of the surface-ratio. Let the surface-ratio $\frac{h}{g}=r$, then $A=ar^2$, in which a is a constant which is specific for each kind of boiler; and

$$w=ar^2+Bc \qquad (6)$$

w = the water evaporated in pounds per square foot of grate per hour.
c = the fuel consumed in pounds per foot per hour.
$E=\frac{w}{c}$ = the efficiency of the fuel, or the weight of water evaporated per pound of fuel.
$A=ar^2$ = a constant, which is specific for each kind of boiler.
B = a constant multiplier, specific for each kind of boiler.
$r=\frac{h}{g}$ = the ratio of the heating surface to the grate-area.
a = a constant, specific for each kind of boiler.

When the water and fuel per foot of grate per hour are given, the value of the required surface-ratio is found from the above formula, for $ar^2=w-Bc$, and

$$r=\sqrt{\frac{w-Bc}{a}} \qquad (7)$$

When the water per foot of grate per hour, and the surface-ratio, are given, to find the fuel per foot of grate per hour required to evaporate the water: $Bc=w-ar^2$, and

$$c=\frac{w-ar^2}{B} \qquad (8)$$

When the efficiency $E=\frac{w}{c}$, of the fuel is given, that is, the weight of water evaporated per pound of fuel; also, the surface-ratio; to find the fuel that

may be consumed per square foot of grate per hour corresponding to that efficiency. As $\frac{w}{c} = E = \frac{ar^2 + Bc}{c} = B + \frac{ar^2}{c}$; then $ar^2 = c\,(E - B)$; and

$$c = \frac{ar^2}{E - B} \quad \text{......................... (9)}$$

When the efficiency E or $\frac{w}{c}$, and the fuel consumed per foot of grate per hour, are given, to find the surface-ratio required to effect that evaporation. As already found, $ar^2 = c\,(E - B)$, and $r^2 = \frac{c\,(E - B)}{a}$; whence,

$$r = \sqrt{\frac{c\,(E - B)}{a}} \quad \text{...................... (10)}$$

Newcastle Marine Boiler, page 785.

Select for comparison, from tables Nos. 274 and 275, pages 786 and 788, the performance of this boiler with a grate-area of 22 square feet, and 749 square feet of heating surface, 34.05 times the grate, with increasing rates of combustion of coal per square foot per hour. Find the corresponding weights of water evaporated per square foot, and plot them to a vertical scale,

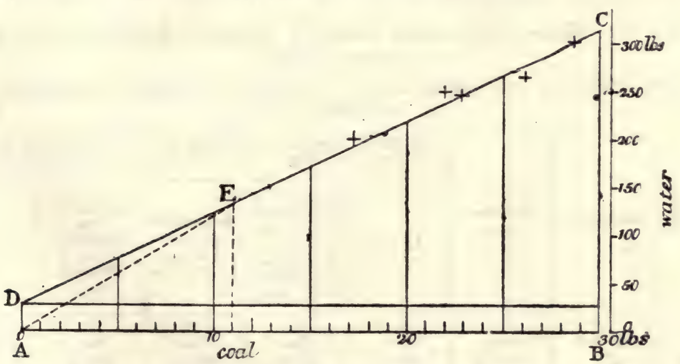


Fig. 329.—Newcastle Marine Boiler.—Diagram to show Relation of Water and Coal per square foot of Grate-area, 22 square feet. Surface-ratio, 34.05.

upon a base-line A B, Fig. 329, measuring the weights of coal consumed. They are found to lie in, or close to, a straight line, D C, drawn obliquely upwards from a point, D, in the ordinate of zero, at a level which is 25 lbs. above the base-line, and the general formula (6) becomes

$$w = 25 + 9{\cdot}71\,c; \quad \text{.......................... (11)}$$

in which $ar^2 = 25$, and $B = 9.71$. The annexed table, No. 283, shows the correspondence of the actual quantities of water evaporated, with those which are calculated from the coal consumed, by this formula (11).[1]

[1] The diagonal line C D, in Fig. 329, does not exactly strike the average of the results for the grate of 22 square feet alone; but it is the average for the results obtained from the various sizes of grate taken together. For reference to the line A E, see page 817.

Table No. 283.—NEWCASTLE MARINE BOILER—RELATIONS OF COAL AND WATER.

Grate 22 square feet. Surface-ratio 34.05.

Nos. of Experiments in Tables No. 274 and 275.	Coal per Foot of Grate per Hour.	Water per Pound of Coal, from and at 212° F.	Total Water per Foot of Grate per Hour.			Water per Pound of Coal, according to Formula.
			Observed.	By Formula (11).	Difference by Formula.	
	lbs.	lbs.	lbs.	lbs.	per cent.	lbs.
5	17.27	11.70	202.0	192.7	−4.6	11.16
14	22.08	11.41	251.9	239.4	−5.0	10.84
15	23.04	10.62	244.7	248.7	+1.6	10.79
16	25.97	11.17	290.1	277.2	−4.4	10.67
17	26.05	10.33	269.1	277.9	+3.3	10.67
6	26.98	10.80	291.4	287.0	−1.5	10.64
18	28.51	10.58	301.6	301.8	0.0	10.58

Since $A = ar^2 = 25$, in the present instance; $a = \frac{25}{r^2} = \frac{25}{34.05^2} = .02156$, and $ar^2 = .02156r^2$. By substitution, the following formula is obtained, which applies to all surface-ratios in the Newcastle boiler:—

$$w = .02156r^2 + 9.71c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

Table No. 284.—NEWCASTLE MARINE BOILER—RELATIONS OF COAL AND WATER.

Varying grate-area and surface-ratio. Calculations for normal surface-ratio 34.05 by formula (11).

No. of Experiment.	Grate-area.	Surface-ratio.	Coal per Square Foot of Grate per Hour.		Water per Sq. Foot of Grate per Hour, for Normal Surface-ratio, 34.05.		
			Actual.	Reduced in the Ratio of the Squares of the Surface-ratios, for Normal Ratio, 34.05.	Reduced in the same Ratio as for the Coal.	Calculated from Column 5 by Formula (11).	Difference by Formula.
(1)	(2) sq. feet.	(3) ratio.	(4) lbs.	(5) lbs.	(6) lbs.	(7) lbs.	(8) per cent.
10	42	17.83	16.0	58.35	563.1	591.6	+ 5.1
11	"	"	17.6	63.82	583.3	644.7	+10.5
12	"	"	18.13	66.12	592.4	667.0	+12.6
13	33	22.7	20.36	45.81	423.7	469.8	+10.9
2	28.5	26.28	19.0	31.90	355.0	334.8	− 5.7
5	22	34.05	17.27	17.27	202.0	192.7	− 4.6
14	"	"	22.08	22.08	251.9	239.4	− 5.0
15	"	"	23.04	23.04	244.7	248.7	+ 1.6
16	"	"	25.97	25.97	290.1	277.2	− 4.4
17	"	"	26.05	26.05	269.1	277.9	+ 3.3
6	"	"	26.98	26.98	291.4	287.0	− 1.5
18	"	"	28.51	28.51	301.6	301.8	0.0
4	19.25	38.91	17.25	13.21	165.5	153.3	− 7.4
21	18	41.61	18.67	12.50	139.7	146.4	+ 4.8
22	"	"	24.89	16.67	182.7	186.9	+ 2.3
7	"	"	27.36	18.32	208.3	202.9	− 2.6
8	15.5	48.32	37.40	18.57	197.4	205.3	+ 4.0

The results of the other experiments with the Newcastle boiler, made with different areas of grate, may be reduced for direct comparison with those made with the 22-feet grate, by reducing both the coal and the water per square foot per hour, in the ratio of the squares of the respective surface-ratios, whilst the ratio of the coal and water, or the efficiency, remains constant. The table No. 284 shows the reduced water (column 6) corresponding to the reduced coal (column 5), for the normal surface-ratio 34.05. In column 7, the reduced waters are given as calculated by the formula (11); and the differences by the formula, which are, upon the whole, inconsiderable, are given in the last column.

To show the suitability of the formula (12) for the calculation of water evaporated, from the given surface-ratios, as they are, the annexed table, No. 285, shows, by comparison (columns 5 and 6), the actual and calculated quantities of water evaporated by the coals (column 4), with the ratios in column 3. The percentages of differences are identical with those already exhibited in the previous table.

Table No. 285.—Newcastle Marine Boiler—Relations of Coal and Water.

Varying grate-areas and surface-ratios. Calculations for the actual ratios, by formula (12).

Number of Experiment.	Grate-area.	Surface-ratio.	Coal per Square Foot of Grate per Hour.	Water per Square Foot of Grate per Hour, for the given Surface-ratios.		
				Actual, as from and at 212° Fahr.	Calculated by Formula (12)	Difference by Formula.
(1)	(2) square feet.	(3) ratio.	(4) lbs.	(5) lbs.	(6) lbs.	(7) per cent.
10	42	17.83	16.0	154.4	162.2	+ 5.1
11	„	„	17.6	160.9	177.7	+ 10.5
12	„	„	18.13	162.5	182.8	+ 12.6
13	33	22.7	20.36	188.3	231.5	+ 10.9
2	28.5	26.28	19.0	211.5	199.4	− 5.7
5	22	34.05	17.27	202.0	192.7	− 4.6
14	„	„	22.08	251.9	239.4	− 5.0
15	„	„	23.04	244.7	248.7	+ 1.6
16	„	„	25.97	290.1	277.2	− 4.4
17	„	„	26.05	269.1	277.9	+ 3.3
6	„	„	26.98	291.4	287.0	− 1.5
18	„	„	28.51	301.6	301.8	0.0
4	19.25	38.91	17.25	216.1	200.1	− 7.4
21	18	41.61	18.67	208.5	218.6	+ 4.8
22	„	„	24.89	272.8	279.0	+ 2.3
7	„	„	27.36	311.1	303.0	− 2.6
8	15.5	48.32	37.40	397.6	413.5	+ 4.0

The consistency of the results of the application of the formula under widely varying proportions of boiler, and varying rates of combustion, affords evidence of the correctness of the principles on which it is based.

Wigan Marine Boiler, page 781.

The trials of this boiler were made with a constant grate of 10.3 square feet area, and a constant surface of 508 square feet, giving a surface-ratio

of 50. The average results of the trials selected for the present purpose, are placed in the following table, together with the quantities of water evaporated, as calculated by the following formula deduced from the plotting of the results:—

$$w = 25 + 10.75\, c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

Showing a smaller constant and a greater multiple than the formula of the Newcastle boiler. Substituting for 25 the general expression $a\,r^2$, and reducing for the value of w, the general formula is,

$$w = .01\, r^2 + 10.75\, c; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

which may be employed for different surface-ratios.

Table No. 286.—Wigan Marine Boiler—Relations of Coal and Water.

Grate 10.3 square feet; surface-ratio 50.

Description of Coals.	Coal per Foot of Grate per Hour.	Water per Pound of Coal, from and at 212° Fahr.	Total Water per Square Foot of Grate per Hour. Observed.	By Formula (13).	Difference by Formula.
	lbs.	lbs.	lbs.	lbs.	per cent.
South Lancashire and Cheshire coals—Mr. Fletcher's trials......	27.63	11.54	318.8	322.1	+ 1.0
South Lancashire and Cheshire coals—Messrs. Nicol & Lynn...	27.50	11.92	327.8	320.6	− 2.2
	41.25	11.36	468.6	468.6	0.0
Hartley's (Newcastle) coals	28.83	11.95	344.5	334.9	− 2.8
Welsh coals	26.20	12.44	325.9	306.6	− 6.0

It appears from this table that the South Lancashire and Cheshire coals, and the Newcastle coals, were equally efficient; and that the Welsh coals had a slightly greater evaporative action than the others.

Experimental Marine Boiler, Navy Yard, New York, U.S., page 795.

This boiler affords examples of very low surface-ratios. With its normal proportions, 10.8 square feet of grate and 150.3 square feet of surface, the surface-ratio is 14. When the flue-tubes were stopped off, the surface-ratio was only 4.21. By the plotting of the experimental results, reduced for a uniform surface-ratio of 14, the following formula was derived:—

$$w = .0204\, r^2 + 7.624\, c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

It is seen in the following table, that the calculated evaporation is considerably in excess of the actual reduced evaporation, in the extreme instances of the flash-flue and the small surface-ratio, 4.21. It is obvious that such dissimilar cases as those of a flash-flue and a multitubular boiler, are not directly comparable.

Table No. 287.—EXPERIMENTAL MARINE BOILER, NAVY YARD, NEW YORK—RELATIONS OF COAL AND WATER.

Varying grate-area and surface-ratio. Calculations for normal surface-ratio 14.

Index to Experiment.	Grate-area.	Surface-ratio.	Coal per Square Foot of Grate per Hour.		Water per Square Foot of Grate per Hour, for Normal Ratio 14.		
			Actual.	Reduced in the Ratio of the Squares of the Surface-ratios, for Normal Ratio 14.	Reduced in the Same Ratio.	Calculated from Column 5 by Formula (15).	Difference by Formula.
	square feet.	ratio.	lbs.	lbs.	lbs.	lbs.	per cent.
X	10.8	4.21	11.77	130.2	865	996.1	+15
V	"	"	16.57	183.2	1082	1400	+29.4
W	"	"	16.58	183.3	1119	1401	+25.2
A	"	14	5.57	5.57	51.63	46.4	−10.1
B	"	"	10.99	10.99	98.39	87.7	−10.9
C	"	"	16.57	16.57	131.7	130.3	− 1.0
D	"	"	22.10	22.10	172.5	172.4	0.0
E	"	"	27.76	27.76	205.4	215.5	+ 4.9
I	8.64	17.24	15	9.88	84.80	79.3	− 6.5
L	"	"	20.73	13.66	109.60	108.1	− 1.4
J	6.48	22.84	15	5.64	46.67	47.0	+ 0.7
M	"	"	22.84	10.30	76.54	82.5	+ 7.1
K	4.32	34.03	15	2.54	22.67	23.3	+ 2.8
N	"	"	27.58	4.67	33.81	39.6	+17

Wigan Stationary Boilers, page 771.

The data afforded by these typical boilers are specially useful, as they represent classes of boilers in general use in England. The several experimental results, required for the present purpose, are collected in the annexed table. The first two are the results for flash-draughts, for which the side and bottom flues were cut off, and the gases were conducted direct to the chimney after having passed through the tubes. By plotting the coal and water reduced according to the squares of the surface-ratios, for a uniform ratio of 30, this formula was obtained,—

$$w = 20 + 9.56c \qquad (16)$$

And in the general form, for various ratios,—

$$w = .0222r^2 + 9.56c \qquad (17)$$

By the formula (16), the quantities of water in column 6 of the table No. 288 were calculated from the reduced coals in column 5.

The agreement of the reduced and the calculated quantities of water (columns 6 and 7) is very close, excepting for the flash-draught.

Table No. 288.—Wigan Stationary Boilers—Relation of Coal and Water.

Varying grate-area and surface-ratio. Calculations for ratio 30.

Boilers (Without Economizer).	Grate-area.	Surface-ratio.	Coal per Square Foot of Grate per Hour. Actual.	Coal per Square Foot of Grate per Hour. Reduced in the Ratio of the Squares of the Surface-ratios, for Ratio 30.	Water per Square Foot of Grate per Hour for Ratio 30. Reduced in the same Ratio.	Water per Square Foot of Grate per Hour for Ratio 30. Calculated from Column 5 by Formula (16).	Water per Square Foot of Grate per Hour for Ratio 30. Difference by Formula.
(1)	(2) sq. feet.	(3) ratio.	(4) lbs.	(5) lbs.	(6) lbs.	(7) lbs.	(8) per cent.
Galloway, flue-tubes only	31.5	13.70	18.58	89.10	757.3	871.8	+ 15.0
Lancashire, flue-tubes only	”	14.74	19.91	82.47	678.8	808.4	+ 19.0
Galloway, complete	”	22.8	18.3	31.68	322.9	322.9	0.0
Lancashire and Galloway	”	23.5	14.0	22.82	230.4	238.2	+ 3.4
Lancashire	”	24.4	17.26	26.03	271.5	268.8	− 1.0
Do.	”	”	18.6	28.12	290.2	288.8	− 0.5
Do.	”	”	19.1	28.87	293.7	296.0	+ 0.8
Do. with water tubes	”	25.4	16.71	23.31	251.0	242.8	− 3.3
Galloway	21	34.3	21.8	16.68	179.6	179.5	0.0
Lancashire and Galloway	”	35.5	23.0	16.43	179.2	177.1	− 1.2
Lancashire	”	36.5	21.5	14.52	158.0	158.8	+ 0.5
Do.	”	”	22.7	15.33	165.1	166.6	+ 0.9

Stationary Boilers in France, page 796.

The proportions and the results of performance are treated in the following table. The following special formulas have been deduced for the three boilers respectively, and for the three collectively:—

"Fairbairn" .... $w = .01143 r^2 + 7.7 c$ .... (18)
Lancashire............. $w = .01126 r^2 + 8.0 c$ .... (19)
French.................. $w = .01126 r^2 + 8.0 c$ .... (20)
All the boilers......... $w = .0111 r^2 + 7.82 c$ .... (21)

It is seen that the same formula applies to the Lancashire and the French boilers; and that, therefore, the reporters of the trials were justified in asserting that these boilers were equally efficient. The comparatively inferior quantity evaporated in the first trial in the table, resulted probably from an excessively large surplus of air admitted into the furnace: the total quantity of air in that instance, amounted to 261 cubic feet per pound of coal.

Table No. 289.—STATIONARY BOILERS IN FRANCE—RELATIONS OF COAL AND WATER.

Calculations of evaporative performance for surface-ratio 30. Ronchamp coal.

BOILERS.	Grate-area.	Surface-ratio.	Coal per Square Foot of Grate per Hour. Actual.	Coal per Square Foot of Grate per Hour. Reduced in the Ratio of the Squares of the Surface-ratios for Ratio 30.	Water per Square Foot of Grate per Hour, for Surface-ratio 30. Reduced in the same Ratio.	Water per Square Foot of Grate per Hour, for Surface-ratio 30. Calculated from Column 5, by Formulas (18), (19), (20).	Water per Square Foot of Grate per Hour, for Surface-ratio 30. Difference by Formulas.
	sq. feet.	ratio.	lbs.	lbs.	lbs.	lbs.	per cent.
"Fairbairn"...	20.5	49.5	10.70	3.93	34.8	40.7	+ 17
"	"	"	18.53	6.81	62.7	63.3	+ 0.9
Lancashire.....	"	29.8	10.41	10.55	94.1	92.5	− 1.7
"	"	"	19.15	19.41	165.0	161.8	− 1.9
"	"	"	19.50	19.76	166.8	164.5	− 1.4
French..........	20.1	30.3	11.36	11.14	95.5	97.1	+ 1.7
"	"	"	19.87	19.48	165.4	162.3	− 1.9
"	"	"	20.57	20.16	166.6	167.6	+ 0.6

Locomotive-Boilers, page 798.

The experimental trials from which the evaporative performances of locomotives have been tabulated, have, of course, been conducted under various conditions. There is, nevertheless, a remarkable degree of harmony amongst them, for, when plotted, they are seen, with a very few exceptions of early date, to follow the laws of evaporative performance already enunciated. Even the performance of the boiler of the primitive Killingworth engine, when the evaporative efficiency is increased by one-half to represent the value of coke compared with coal as imperfectly burned in that boiler,—range as well as should have been expected, with those of other locomotives. In fact, the improved Killingworth boiler exhibits a performance above the general average.

Using good coke as fuel, the evaporative performance of locomotive-boilers in which the flue-tubes are spaced sufficiently apart to admit of a free circulation of water around them, is substantially embraced by the following formula when the surface-ratio is 75, which is a good practical ratio:—

$$w = 100 + 7.94c \text{ (coke)} \qquad (22)$$

For any given surface-ratio, the general formula is,—

$$w = .0178r^2 + 7.94c \text{ (coke)} \qquad (23)$$

Using good coal as fuel, the formulas for the coal-burning locomotive boilers in table No. 280, page 799; namely, Nos. 31–34, and Nos. 39–41, are:—

	Nos. 31–34, S. E. R.	Nos. 39–41, L. & S. W. R.	
For surface-ratio 75,	$w = 50 + 9.6c$......	$w = 50 + 9.82c$......	(24)
For any surface-ratio,	$w = .009r^2 + 9.6c$	$w = .009r^2 + 9.82c$...	(25)

Portable-Engine Boilers, page 801.

These boilers are arranged in the table No. 290, in the order of the surface-ratios. The coal and the water per square foot of grate are reduced for the ratio 50 (columns 5, 6), from which has been deduced, by plotting, the formula,—

$$w = 20 + 8.6c \qquad (24)$$

For any given surface-ratio, the general formula is,—

$$w = .008r^2 + 8.6c \qquad (25)$$

The calculated quantities of water (column 7), by formula (24), follow closely the reduced quantities (column 6), except in the first three instances, Nos. 12, 1, and 8, where they are much in excess. In these instances, the excessive reduction of the grate has involved a material departure from the normal disposition of a firebox, especially for No. 8, in which the grate was reduced to a third of its normal area. The surface-ratios were driven up to 102, 94.5, and 89. The first two boilers, Nos. 12 and 1, have the greatest numbers and the smallest diameters of tubes. The drift of the evidence goes to show that fewer tubes, of larger diameter, do better for the combustion of coal, the circulation of water, and the absorption of heat.

There is another exception, No. 3, with a surface-ratio 33, in which the calculated quantity of water is twice as much as the reduced actual quantity. The excess in this case is satisfactorily accounted for by causes which were pointed out by the judges in their report.[1]

Table No. 290.—PORTABLE-ENGINE BOILERS:—RELATIONS OF COAL AND WATER.

Calculations of evaporative performance for surface-ratio 50.

No. of Boiler.	Grate-area as Reduced for Trial.	Surface-ratios.	Coal per Square Foot of Grate per Hour. Actual.	Coal per Square Foot of Grate per Hour. Reduced in the Ratio of the Squares of the Surface-ratios for Ratio 50.	Water per Square Foot of Grate per Hour for Surface-ratio 50. Reduced in the same Ratio.	Water per Square Foot of Grate per Hour for Surface-ratio 50. Calculated from Column 5, by Formula (24).	Water per Square Foot of Grate per Hour for Surface-ratio 50. Difference by Formula.
(1)	(2) sq. feet.	(3) ratio.	(4) lbs.	(5) lbs.	(6) lbs.	(7) lbs.	(8) per cent.
12	2.0	102	31.1	7.473	69.28	84.27	+ 21.6
1	3.0	94.5	15.7	4.395	44.96	57.80	+ 28.5
8	2.37	89	20.4	5.73	60.11	69.28	+ 15.2
2	3.2	69	12.8	6.721	79.51	77.80	− 2.1
"	"	"	12.5	6.564	77.52	76.45	− 1.4
10	3.5	54	20.7	17.75	176.2	172.6	− 2.0
6	3.2	50	9.53	9.53	103.8	102.0	− 1.7
4	3.75	45	10.32	12.72	140.1	129.4	− 7.6
7	4.7	34	13.0	28.11	262.3	261.7	− 0.2
3	5.1	33	14.8	33.97	155.9	312.10	+ 100.0
11	5.0	26	13.6	50.30	451.1	452.6	+ 0.3

[1] "The engine was indifferently managed." . . . "It would appear that the boiler did about one-half, or rather less than one-half, its duty in making steam."—*Report of the Judges*, page 17.

Looking to the evaporating capabilities of the portable-engine boilers in their ordinary condition, with unrestricted grates, it may be useful to show at what rates they are capable of evaporating water from and at 212°, in the ratio of 10 lbs. of water per pound of coal consumed. For the calculation of these rates, the formula (9), page 807, may be employed. It is,

$$c = \frac{a r^2}{E - B}. \qquad (26)$$

The value of E is 10, of B is 8.6, and of a is .008; and, by substitution, $c = \frac{.008 r^2}{10 - 8.6}$; or

$$c = \frac{.008 r^2}{1.4}. \qquad (27)$$

By this formula, the value of c, the quantity of coal consumed per square foot of grate per hour, is found, when the surface-ratio r is given, for each boiler. Thence, multiplying by the grate-area, is found the total quantity of coal per hour; and ten times the coal is the quantity of water. In this way the following table, No. 291, is calculated; in which the boilers are placed in the order of their surface-ratios. It is seen that No. 2 boiler is capable of evaporating at the given rate of efficiency, 8.15, say 8, cubic feet of water per hour. This is just 1 cubic foot per nominal horse-power:—all the boilers having been designated of 8 horse-power. No. 11 boiler would only evaporate 3 cubic feet per hour, at the given rate of efficiency, whilst No. 1 is capable, by calculation, of evaporating 16½ cubic feet per hour. It has already been seen that there is reason, in the design of its tube-surface, for doubting whether No. 1 is capable of so good a performance.

Standard Average Practice for Portable-Engine Boilers.—The last line of the table No. 291 may be assumed as a standard result of average practice for portable-engine boilers of 8 nominal horse-power. The following data may be taken, in round numbers:—

Nominal horse-power,..	8 H.P.
Area of fire-grate, ...	5.5 square feet.
Area of heating-surface,..	220.0 „
Ratio of heating-surface to grate-area, or surface-ratio,..	40 to 1.
Coal of good quality consumed per hour, ...	50 lbs.
Do. per horse-power per hour,......	6.25 lbs.
Do. per square foot of grate,...say	9 lbs.
Water evaporated from and at 212° F. per hour, at the rate of 10 lbs. per pound of coal,........................	500 lbs., or 8 cubic feet.
Do. per horse-power,......	62.4 „ or 1 „
Do. per square foot of grate, 91 lbs.,.................. say	90 „ or 1.45 „

Table No. 291.—PORTABLE-ENGINE BOILERS:—CALCULATED EVAPORATIVE PERFORMANCE.

From and at 212° F., at the rate of 10 lbs. of water per pound of coal.

No. of Boiler.	Surface-ratio.	Grate-area.	Coal Consumed per Hour.		Total Water Evaporated per Hour.	
			Per Square Foot of Grate.	Total.		
	ratio.	square feet.	lbs.	lbs.	lbs.	cubic feet.
1	64	4.4	23.40	102.96	1029.6	16.50
10	54	3.5	16.66	58.31	583.1	9.34
6	50	3.2	14.30	45.76	457.6	7.33
4	45	3.75	11.57	43.24	432.4	6.93
2	41	5.3	9.60	50.88	508.8	8.15
12	37	5.5	7.82	43.01	430.1	6.89
9	35	4.3	7.00	30.10	301.0	4.82
7	34	4.7	6.60	31.02	310.2	4.97
3	33	5.1	6.22	31.72	317.2	5.08
5	31	6.13	5.50	33.71	337.1	5.40
8	29	7.2	4.23	30.45	304.5	4.88
11	26	5.0	3.86	19.30	193.0	3.09
Averages,	40	4.84	9.14	44.24	442.4	7.09
To evaporate 8 cubic feet of water per hour, ...	40	5.46	9.14	49.92	499.2	8.0

GENERAL FORMULAS FOR PRACTICAL USE.

By the French experiments with stationary boilers, the Lancashire and French boilers were, by the formulas, page 812, identical in performance; and the so-called "Fairbairn" boiler was nearly as effective as these,—within 3½ per cent. The three forms of boiler may, therefore, be accepted as equally efficient; and they may be classed with the Wigan boiler, as of equal efficiency, with the same coal, and with the same management.

The performance of the Howard boiler, likewise, is conformable to the formula for the Wigan boiler; and the Howard boiler is a type of the "sectional" kind of boilers.

The formula for the Wigan boiler is, therefore, applicable to all stationary boilers, other than multitubular, with good coal and good management.

The performances of the Newcastle and the Wigan marine boilers, are nearly alike. Thus, for a surface-ratio 30, the corresponding quantities of water, w, for different rates of coal, c, per square foot of grate per hour, are as follows:—

Coal, ...	$c=$	10	20	30	40	lbs.
Newcastle boiler,.......	$w=$	116.5	213.6	310.7	407.8	„
Wigan boiler,............	$w=$	116.5	224.0	331.5	439.0	„
Differences,...........	$w=$	0.0	10.4	20.8	31.2	„
Less than Wigan,...		0.0	4.6	6.3	7.1	per cent.

Halve the difference, and take a mean of the formulas; the mean will be a satisfactory general formula for marine boilers:—

Newcastle,...... $w = .02156\, r^2 + 9.71\, c$
Wigan,.......... $w = .01\, r^2 + 10.75\, c$

Mean,........ $w = .016\, r^2 + 10.25\, c$ .... (28)

For coal-burning locomotive boilers a mean of the two formulas adduced, page 813, which are nearly identical, will be a satisfactory formula:—

S. E. Railway,....... $w = .009\, r^2 + 9.6\, c$
L. & S. W. Railway, $w = .009\, r^2 + 9.82\, c$

Mean,.............. $w = .009\, r^2 + 9.7\, c$ .... (29)

The general formulas which have been deduced are here collected together:—

Formulas for the Relation of Coal and Water consumed in Steam-boilers per square foot of grate-area per hour, and the ratio of the heating-surface to the area of the fire-grate.

	Water taken as evaporated from and at 212° F.	
Stationary Boilers,........	$w = .0222\, r^2 + 9.56\, c$	(30)
Marine Boilers,............	$w = .016\, r^2 + 10.25\, c$	(31)
Portable-engine Boilers,..	$w = .008\, r^2 + 8.6\, c$	(32)
Locomotive Boilers (coal-burning), ...	$w = .009\, r^2 + 9.7\, c$	(33)
Locomotive Boilers (coke-burning), ...	$w = .0178\, r^2 + 7.94\, c$...........	(34)

Limits to the Application of the Formulas (30) *to* (34).

There are minimum rates of consumption of fuel below which these formulas are not applicable. The limit varies for each kind of boiler, and it varies with the surface-ratio. It is imposed by the fact that the maximum evaporative power of fuel is a fixed quantity, and is naturally at that point of the scale, say E in Fig. 329, page 807, where the reduction of the rate of combustion for a given ratio, procures the absorption into the boiler of the whole of the proportion of the heat which is available for evaporation. In the combustion of good coal the limit of evaporative efficiency may be taken as measured by 12½ lbs. of water from and at 212° F.; and in that of good coke by 12 lbs. of water from and at 212° F. The dotted line E A, Fig. 329, represents the correct course of the diagram towards the zero point, indicating a constant proportion of $w = 12.5\, c$, for coal; or $w = 12\, c$, for coke.

To ascertain the minimum rates of combustion of coal for stationary boilers, to which the formula (30) applies:—The limit is reached when w becomes equal to $12.5c$; or when $12.5c = .0222r^2 + 9.56c$, or $.0222r^2 = (12.5 - 9.56)\, c = 2.94c$. By reduction, $c = \frac{.0222}{2.94} r^2 = .00755\, r^2$. For a given surface-

ratio r, the limiting value of c is found by multiplying the square of the ratio by .00755.

For the other kinds of boiler, the limiting values of c are found in the same way. They are here placed all together:—

Stationary boilers, ...	limiting value of c	$=.00755\,r^2$.
Marine boilers,.........................	,, ,,	$=.007\,r^2$.
Portable-engine boilers,.............	,, ,,	$=.002\,r^2$.
Locomotive boilers (coal-burning),	,, ,,	$=.00325\,r^2$.
,, ,, (coke-burning),	,, ,,	$=.0044\,r^2$.

For lower values of c, or consumptions of fuel per square foot of grate per hour, the values of w, the corresponding quantities of water, are simply $12.5\,c$ for coal, and $12\,c$ for coke.

The annexed table, No. 292, contains the limiting values of c for given surface-ratios r.

Table No. 292.—Minimum Values of c, or Minimum Quantities of Fuel Consumed per Square Foot of Grate per Hour, for given Surface-Ratios, to which the Formulas (30) to (34) are Applicable.

	Surface-ratios.						
	5	10	15	20	30	40	50
	Minimum Consumption of Fuel per Square Foot of Grate per Hour.						
	lb.	lb.	lb.	lbs.	lbs.	lbs.	lbs.
Stationary, ...	.2	.7	1.7	3.0	6.8	12.1	18.9
Marine,.............................	.17	.7	1.6	2.8	6.3	11.2	17.5
Portable,...........................	.05	.2	.4	.8	1.8	3.2	5.0
Locomotive (coal-burning),.	.1	.3	.7	1.3	2.9	5.2	8.1
Do. (coke-burning),	.1	.4	1.0	1.8	4.0	7.0	11.0
	Surface-ratios (*continued*).						
	60	70	75	80	90	100	
Locomotive (coal-burning),.	11.7	15.9	18.3	20.8	26.3	32.5	
Do. (coke-burning),	16	21	25	28	36	44	

The only limit to the application of the formulas (30) to (34), to ascending values of c, or quantities of fuel per square foot per hour, is the limit of endurance of the fuel itself under the action of the draught:—from 100 lbs. to 120 lbs. per square foot per hour, for ordinary hard coal or coke. Beyond this limit, the fuel is liable to be shaken and partly dispersed, unconsumed, by the force of the draught; although coke has been known to withstand the draught of a locomotive when consumed at the rate of 130 lbs. per square foot per hour.

Table No. 293.—Evaporative Performance of Steam-Boilers, for Increasing Rates of Combustion and Different Surface-Ratios.

For best coal and best coke; surface-ratio 30.

Kind of Boiler, and Fuel.	Water from and at 212° F. per Hour.	Fuel per Square Foot of Grate per Hour, in pounds.						
		5	10	15	20	30	40	50
		lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Stationary, coal, formula (30).	Per square foot	62.5*	116	163	211	307	402	498
	Per lb. of coal	12.5	11.56	10.89	10.56	10.23	10.06	9.96
Marine, coal, formula (31).	Per square foot	62.5*	117	168	219	322	424	527
	Per lb. of coal	12.5	11.69	11.25	10.95	10.69	10.61	10.54
Portable, coal, formula (32).	Per square foot	50	93	136	179	265	351	437
	Per lb. of coal	10	9.3	9.01	8.95	8.83	8.77	8.74
Locomotive (coal-burning), formula (33).	Per square foot	57	105	154	202	299	396	493
	Per lb. of coal	11.4	10.5	10.26	10.10	9.97	9.90	9.86
Locomotive (coke-burning), formula (34).	Per square foot	56	95	135	175	254	334	413
	Per lb. of coke	11.14	9.54	9.02	8.75	8.47	8.35	8.03

Surface-ratio 50.

Kind of Boiler, and Fuel.	Water from and at 212° per Hour.	Fuel per Square Foot of Grate per Hour, in pounds.						
		5	10	15	20	30	40	50
		lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Stationary, coal, formula (30).	Per square foot	62.5*	125*	187.5*	247	342	438	534
	Per lb. of coal	12.5	12.5	12.5	12.33	11.41	10.95	10.67
Marine, coal, formula (31).	Per square foot	62.5*	125*	187.5*	245	348	450	552
	Per lb. of coal	12.5	12.5	12.5	12.25	11.58	11.25	11.05
Portable, coal, formula (32).	Per square foot	62.5*	106	149	192	278	364	450
	Per lb. of coal	12.5	10.6	9.93	9.6	9.27	9.10	9.00
Locomotive (coal-burning), formula (33).	Per square foot	62.5*	120	168	217	314	411	508
	Per lb. of coal	12.5	11.95	11.20	10.85	10.45	10.26	10.15
Locomotive (coke-burning), formula (34).	Per square foot	60*	120*	164	203	283	362	442
	Per lb. of coke	12.0	12.0	10.91	10.16	9.42	9.05	8.83

Surface-ratios 75.

Kind of Boiler, and Fuel.	Water from and at 212° per Hour.	Fuel per Square Foot of Grate per Hour, in pounds.						
		30	40	50	60	75	90	100
		lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Locomotive (coal-burning), formula (33).	Per square foot	342	439	536	633	778	927	1020
	Per lb. of coal	11.39	10.97	10.71	10.65	10.37	10.26	10.20
Locomotive (coke-burning), formula (34).	Per square foot	338	418	497	576	695	815	894
	Per lb. of coke	11.27	10.44	9.94	9.61	9.26	9.05	8.94

[1] These quantities fall below the scope of the formulas for the water, as explained in the text.

APPLICATIONS OF THE GENERAL FORMULAS FOR THE EVAPORATIVE PERFORMANCE OF STEAM-BOILERS.

The table No. 293, preceding page, contains the relative quantities of fuel consumed and water evaporated, for surface-ratios and rates of combustion per square foot of grate per hour, within the range of ordinary practice. It is seen that, with the surface-ratios 30 and 50, the boilers are in the order of evaporative efficiency as follows:—

SURFACE-RATIO 30.	SURFACE-RATIO 50.
Marine.	Marine.
Stationary.	Stationary.
Locomotive (coal-burning).	Locomotive (coal-burning).
Portable.	Do. (coke-burning).
Locomotive (coke-burning).	Portable.

Table No. 294.—EQUIVALENT WEIGHTS OF BEST COAL AND INFERIOR FUELS.

To be used with formulas (30) to (34), page 817.

Relative Heating Power.	Equivalent Weight of Best Coal.	Equivalent Weight of Inferior Fuel.	Relative Heating Power.	Equivalent Weight of Best Coal.	Equivalent Weight of Inferior Fuel.	Relative Heating Power.	Equivalent Weight of Best Coal.	Equivalent Weight of Inferior Fuel.
	best coal=1.	best coal=1.		best coal=1.	best coal=1.		best coal=1.	best coal=1.
100	1	1.	70	.70	1.43	40	.40	2.50
99	.99	1.01	69	.69	1.45	39	.39	2.56
98	.98	1.02	68	.68	1.47	38	.38	2.63
97	.97	1.03	67	.67	1.49	37	.37	2.70
96	.96	1.04	66	.66	1.52	36	.36	2.78
95	.95	1.05	65	.65	1.54	35	.35	2.86
94	.94	1.06	64	.64	1.56	34	.34	2.94
93	.93	1.08	63	.63	1.59	33	.33	3.03
92	.92	1.09	62	.62	1.61	32	.32	3.13
91	.91	1.10	61	.61	1.64	31	.31	3.23
90	.90	1.11	60	.60	1.67	30	.30	3.33
89	.89	1.12	59	.59	1.69	29	.29	3.45
88	.88	1.14	58	.58	1.72	28	.28	3.57
87	.87	1.15	57	.57	1.75	27	.27	3.70
86	.86	1.16	56	.56	1.79	26	.26	3.85
85	.85	1.18	55	.55	1.82	25	.25	4.00
84	.84	1.19	54	.54	1.85	24	.24	4.17
83	.83	1.20	53	.53	1.89	23	.23	4.35
82	.82	1.22	52	.52	1.92	22	.22	4.55
81	.81	1.23	51	.51	1.96	21	.21	4.76
80	.80	1.25	50	.50	2.00	20	.20	5.00
79	.79	1.27	49	.49	2.04	19	.19	5.27
78	.78	1.28	48	.48	2.08	18	.18	5.56
77	.77	1.30	47	.47	2.13	17	.17	5.88
76	.76	1.32	46	.46	2.17	16	.16	6.25
75	.75	1.33	45	.45	2.22	15	.15	6.67
74	.74	1.35	44	.44	2.27	14	.14	7.14
73	.73	1.37	43	.43	2.33	13	.13	7.69
72	.72	1.39	42	.42	2.38	12	.12	8.33
71	.71	1.41	41	.41	2.44	11	.11	9.09
						10	.10	10.0

Portable-engine boilers are clearly inferior in efficiency to coal-burning locomotive boilers, and they may be constructed like these with sensible advantage.

Employment of the Formulas (30) *to* (34) *for Fuels of Inferior Heating Power.*—1st. To find the evaporative performance of a given weight of inferior fuel, per square foot of grate per hour. Substitute, for the given weight of inferior fuel, the equivalent weight of best coal, and find by the formula the water evaporated.

The equivalent weight of best coal is found by multiplying the weight of inferior fuel by the number in column 2 of the table No. 294, opposite the relative heating power of the inferior fuel.

2d. To find the weight of an inferior fuel required for a given evaporative performance. Find, by the formula, in its inverted form, on the model of the equation (9), page 807, the weight of best coal required, and substitute for this weight the equivalent weight of the inferior fuel.

The equivalent weight of inferior fuel is found by multiplying the weight of best coal by the number in column 3 of the table, opposite the relative heating power of the inferior fuel.

A table of relative heating powers of fuels is given at page 769.

STEAM ENGINE.

ACTION OF STEAM IN A SINGLE CYLINDER.

Pressure of Steam during Expansion in a Cylinder.

When steam is admitted to a cylinder during a portion of the stroke, then cut off, and expanded in the cylinder, upon the piston, for the remainder of the stroke, the pressure on the piston, during the period of admission, is or ought to be uniform, whilst the pressure during the period of expansion falls as the piston advances and the steam expands. In engines in good working order, the expansion follows substantially the law of Boyle, or Mariotte, according to which the pressure falls in the inverse ratio of the expansion. Substantially, it is said, for the actual changes of pressure seldom follow the law exactly. The pressure usually falls more rapidly in the first portion of the expansion, and less rapidly in the last portion, than is indicated by the law of the inverse ratio; and thus, the final pressure may be, and it usually is, greater than that which would be deduced from the ratio of expansion. But the fulness of the expansion-curve depicted on the indicator-diagram, near the end, compensates for the hollowness near the beginning; and, sinking details, it is found that, practically, the area bounded by the curve is equal to that which would be bounded by a hyperbolic curve formed according to Mariotte's law.[1]

It is, therefore, assumed, for purposes of illustration and the calculation of power, that the expansion of steam in the cylinder takes place according to Mariotte's law: the curve representing the diminishing pressures due to the increasing volume being a portion of a hyperbola.

To formulate the method of describing a hyperbolic curve of expansion over a given base-line:—

Let L = the length of the stroke, in feet, supposing that there is not any clearance.

l = the period of admission, or the cut-off, in feet.

s = any greater part of the stroke measured from the commencement, in feet.

P = the total initial pressure, in pounds per square inch.

P' = the total pressure in pounds per square inch, at the end of a given part of the stroke s.

P'' = the total final pressure, or the pressure at the end of the stroke, in pounds per square inch.

[1] Mr. David Thomson arrived at the same conclusion in his excellent paper "On Compound Engines," in the *Proceedings of the London Association of Foremen Engineers*, September, 1873.

$$\text{Then } P' = \frac{P \times l}{s}; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

expressed by the following rule:—

RULE 1. *To find the pressure at any point of the period of expansion when the initial pressure is given.*—Multiply the initial pressure in pounds per square inch by the period of admission in feet, and divide the product by the distance of the given point from the beginning of the stroke. The quotient is the pressure in pounds per square inch.

The pressure P' may also be found from the final pressure P'' by the formula

$$P' = \frac{P'' \times L}{s}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

giving the rule:—

RULE 2. *To find the pressure at any point of the period of expansion when the final pressure is given.*—Multiply the final pressure in pcunds per square inch by the length of the stroke, and divide the product by the distance of the given point from the beginning of the stroke. The quotient is the pressure in pounds per square inch.

Note.—When there is clearance, it is to be reckoned in parts of the stroke, and added to the values of L, l, and s, before using these for calculation.

Let the base-line mn, be the length of the stroke, say 6 feet; mc the initial pressure, say 63 lbs.; cd the period of admission, say one-third of the stroke. Suppose, for simplicity, that there is not any clearance. Draw the perpendicular dd', from the point of cut-off, and divide the period of expansion $d'n$ into any suitable number of parts, say 10 parts, at the points 1, 2, 3, &c. Calculate by the rule the several pressures at the points 1, 2, 3, &c., and set them off by the scale of pressure on vertical ordinates from the points; the curve dg traced through the ends of the ordinates is the hyperbolic curve of expansion. At the successive points of the base of the expansion-line, which are,—

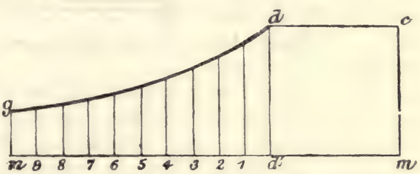


Fig. 330.—Construction of a Hyberbolic Curve.

d', 1, 2, 3, 4, 5, 6, 7, 8, 9, n,

the values of the ordinates, or pressures, are—

63, 52.5, 45, 39.4, 35, 31.5, 28.6, 26.1, 24.2, 22.5, 21 lbs. per sq. in.;

or, putting the initial pressure = 1, they are relatively as

1, .833, .714, .625, .555, .500, .455, .417, .385, .357, .333.

The extreme ordinate ng is thus found to be a third of dd', or 21 lbs., and the ordinate No. 5 is a half, or 31.5 lbs. As an example of the calculations for an ordinate, take No. 2. The period of admission is 2 feet, the divisions of the base of expansion are $\frac{4}{10}$ foot, and the length m 2 is 2.8 feet; then, by the rule, the pressure measured by No. 2 ordinate, is,—

$$\frac{63 \text{ lbs.} \times 2 \text{ feet}}{2.8 \text{ feet}} = 45 \text{ lbs.}$$

The calculation may generally be simplified by taking, as a datum, the length of stroke = 1. In this instance, the period of admission would be = .333, the period of expansion = .666, and each tenth division = .0666. By the rule,

$$\frac{63 \text{ lbs.} \times .333}{.333 + (.0666 \times 2)} = \frac{21}{.467} = 45 \text{ lbs.},$$

as before.

To illustrate generally the application of the hyperbolic law of expansion, showing that the product of the pressure and the volume at any point of the expansion-curve is constant, let the base-line A B represent the course of a piston in a cylinder, and the volume described by it. Supposing that there is no clearance, let steam of 10 lbs. total pressure A C, be admitted for a space 1 foot in length, A D. The rectangle A E is the product of the pressure and volume of the steam admitted. If expanded to the double volume A *d*, and to half the pressure *de*, the area of the elongated rectangle A*e* is equal to that of the initial rectangle A E. Expanding further, to four volumes A *d′*, and to the fourth part of the initial pressure, *d′ e′*, the new rectangle A *e′* is equal to each of the others A*e* and A E. Similarly, the rectangles A *e″* and A *e‴*, for a fifth and a sixth of the initial pressure, and five times and six times the initial volume, are each equal to the initial rectangle A E. The hyperbolic curve containing these rectangles may be indefinitely extended at either end, to embrace, on the one part, intense pressures and small volumes, and, on the other part, very low pressures and large volumes.

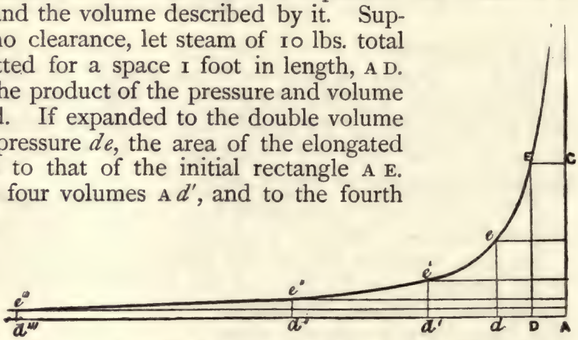


Fig. 331—To Illustrate the Hyperbolic Law of the Expansion of Steam.

The Work of Steam by Expansion.

Proceeding, now, to a consideration of the area of the diagram, Fig. 331; —as the area of the rectangle A E, is the product of the pressure and volume, and expresses the work done upon the piston by the steam in entering and occupying the cylinder, so, likewise, the hyperbolic area, D E *d‴ e‴*, expresses the work done by the steam by expansion within the cylinder after it is shut in. This area, and consequently the quantity of work done, may be computed by means of the known relations of hyperbolic superficies with their base-lines:—according to which, if the base-lines A D, A *d*, A *d′*, &c., extend in a geometrical ratio, or as 1, 2, 4, 8, 16, &c., the successive areas D *e*, D *e′*, &c., increase in an arithmetical ratio, or as 1, 2, 3, 4, &c. On the principles of logarithms, which represent, in arithmetical ratio, natural numbers in geometrical ratio, special tables of so-called hyperbolic logarithms are compiled, to facilitate the calculation of the areas of work due to various degrees of expansion. The hyperbolic numbers consist, in fact, of the multiples of common logarithms by 2.302585, which, thus modified, become direct expressions of the proportions borne by the work by expansion

pertaining to different degrees of expansion, to the initial work done by the steam during its admission into the cylinder; but they are not employed as logarithms. For example, the initial volume being expressed by 1, and the total volumes by expansion, by the following numbers in geometrical ratio,

1, 2, 4, 8, 16,

the hyperbolic logarithms of these numbers are, in arithmetical ratio,

	.000,	.693,	1.386,	2.079,	2.772,
being as	0,	1,	2,	3,	4,

and these logarithms express the actual ratio of the whole work by expansion, for different degrees of expansion, to the initial work of the steam, expressed as 1. The total work done by a quantity of steam expanded successively from the initial volume,

1 to 2, 4, 8, 16 volumes,

will therefore be in the proportions of

	1,	1 + .693,	1 + 1.386,	1 + 2.079,	1 + 2.772,
or	1,	1.693,	2.386,	3.079,	3.772,

showing that, for an expansion of 16 times, the initial work done by the steam during its admission is nearly quadrupled.

But it is necessary to make a deduction for the back pressure from the condenser, to find the effective work of the steam. Suppose a cylinder of 5 feet stroke, represented by A B, Fig. 331, with a piston having an area of 1 square inch, into which steam of 10 lbs. pressure per square inch is admitted for 1 foot of the stroke, A D, against a uniform back pressure of, say, 2 lbs. per square inch, for the whole stroke. Let the steam be expanded through the remaining four-fifths of the stroke, and construct the diagram of work, Fig. 332, in which the 2-lb. zone of resistance or back pressure is shaded. Then,

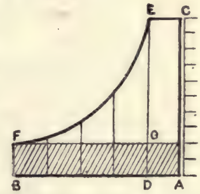


Fig. 332.—Work of Steam worked expansively.

At the end of the............	1st,	2d,	3d,	4th,	5th foot of stroke,
The total pressures are....	10	5	3 $^1/_3$	2½	2 lbs. per sq. inch;
The back pressures are...	2	2	2	2	2 lbs. do. do.;
The effective pressures....	8	3	1 $^1/_3$	½	0 lbs. do. do.

The total work done by expansion up to the end of each foot of stroke, is represented by the hyperbolic logarithm of the ratio of expansion, the initial work being = 1. Thus,—

At the end of the............	1st,	2d,	3d,	4th,	5th foot of stroke,
The steam is expanded into	—	2	3	4	5 volumes,
Of which the hyperbolic logarithms are..........	—	.69	1.10	1.39	1.61
The initial duty being as...	1	1	1	1	1 (unity),
And the total duty as........	1	1.69	2.10	2.39	2.61

As the initial work, represented by 1, is 10 foot-pounds, being 10 lbs. exerted through 1 foot, and the resistance is 2 foot-pounds for each foot of the stroke,—

At the end of the	1st,	2d,	3d,	4th,	5th	foot of the stroke,
The work by expansion is	0.0	6.9	11.0	13.9	16.1	foot-pounds;
The total work done is	10	16.9	21.0	23.9	26.1	do.
The total resistance is	2	4	6	8	10	do.
The total effective work is	8	12.9	15.0	15.9	16.1	do.
And the gain by expansion is	0	61	87	99	101	per cent.

From the foregoing particulars, it appears that the total work of the steam, by expanding it to five times the initial volume, is fully 2½ times the initial work done without expansion. When the back pressure is allowed for, the effective work, 16.1 foot-pounds, is only twice the initial work, 8 foot-pounds; making a gain of 101 per cent., when the expansion is extended to the extreme limit, where the positive pressure becomes equal to the back pressure.

It further appears that the effective work of the steam expanded down to the back pressure from the condenser, is just equal to the work developed by expansion alone. The initial work is balanced in amount by the resistance, each of them being 10 foot-pounds.

The same conclusions apply to a non-condensing cylinder discharging the steam into the atmosphere. Let the total initial pressure, A C, Fig. 332, be 75 lbs. per square inch, and suppose the steam to be expanded five times, as before, down to a pressure of 15 lbs. per square inch, and then exhausted into the atmosphere, maintaining a back pressure of 15 lbs. per square inch throughout the stroke, represented by the shaded zone. On a piston of 1 square inch area, the proportions of work will be as follows:—

At the end of the............	1st,	2d,	3d,	4th,	5th	foot of stroke,
The total work done is as	1	1.69	2.10	2.39	2.61	
The total work done is actually..................	75	126.7	157.5	179.2	195.7	foot-pounds.
The total resistance is......	15	30	45	60	75	do.
The total effective work is	60	96.7	112.5	119.2	120.7	do.
The gain by expansion is	0	61	87	99	101	per cent.

In this case, where steam of five atmospheres is expanded five times, and exhausted into the atmosphere at a pressure of one atmosphere, the proportions of work done are the same as when steam of 10 lbs. pressure per square inch is expanded five times and exhausted at a pressure of one-fifth, or 2 lbs. per inch; and they indicate equal degrees of efficiency of the steam in the way it is applied.

It may be concluded, generally, that when the steam is expanded down to the back pressure in the cylinder, whether from the condenser or from the atmosphere, the effective work done in the cylinder is just equal to the total work done by expansion, the total initial work being just balanced and neutralized in amount by the resistance of back pressure.

And the utmost useful ratio of expansion, looking to the operations within the cylinder, is measured by the number of times which the total back pressure is contained in the total initial pressure of the steam in the cylinder. Indeed, it may be affirmed that four-fifths of this measure of expansion is sufficient as a limit, for it has been shown that whilst the gain by expansion to four times is 99 per cent., that of a fivefold expansion is 101 per cent., which is only 2 per cent. more.

Another reason usually advanced for arresting the fall of pressure, in expanding, at a higher limit than the back pressure, is based on the frictional or passive resistance of the engine. This resistance is to be opposed by the steam in the cylinder; and the total pressure, it is said, should not fall below that which is equivalent to the back pressure, plus the frictional resistance, since, it is argued, if the pressure at any part of the stroke do fall below the sum of these resistances, the excess of these above the positive pressure is so much dead resistance, and is so much in reduction of the useful efficiency of the steam. This argument is plausible, but fallacious; and it would be valid only on the supposition that the engine could move without, at the same time, doing its proper duty in driving shafting and machinery. The supposition is, of course, impossible. But, why draw the line of so-called useless resistance at the fly-wheel shaft? The shafting for driving the machinery also opposes dead resistance, and before the engine can move at all, the resistance of the shafting must be overcome. The resistance of all the machinery must likewise be overcome. The useful work to be done must likewise be overcome; in fact, the whole of the work, dead and alive, must be overcome. So the argument leads to the absurd conclusion that the pressure in the cylinder should not fall below the total mean pressure exerted; and as it is not to fall below, neither can it reach above the mean pressure, for that would imply an additional initial force, which would render a greater mean pressure, which is absurd. If the argument had any truth in it, it would lead necessarily to the abandonment of all expansive working, and to the employment of a uniform pressure, with the admission of steam throughout the whole of the stroke.

Clearance in Steam-Cylinders.

The clearance, or free space, between the piston when at the beginning of a stroke, and the slide-valve, is filled with steam of the initial pressure at the commencement of each stroke; and this padding, as it may be called, does no work directly, and is entirely non-effective in non-expansive engines. But in expansive-working cylinders, the clearance-steam does its proper quota of work, in conjunction with the other steam, during the period of expansion.

The volume of the clearance may be measured in parts of the stroke supposed to be multiplied into the area of the piston; and it is here taken, for purposes of discussion, at 7 per cent. of the stroke.

Formulas for the Work of Steam in the Cylinder.

Now, let L = the length of stroke, in feet,

l = the period of admission, or the cut-off, in feet, excluding clearance,

c = the total clearance at one end of the cylinder, the volume being measured in feet of the stroke,

L' = the length of the stroke, plus the clearance, or $L + c$,

l' = the period of admission, plus the clearance, or $l + c$,

R = the nominal ratio of expansion, or $L \div l$,

R' = the actual ratio of expansion, or $L' \div l'$,

a = the area of the piston in square inches,

P = the total initial pressure in lbs. per square inch, supposed to be uniform during admission,

p = the average total pressure, in lbs. per square inch, for the whole stroke,

p' = the average back pressure, in lbs. per square inch, for the whole stroke,

w = the whole work done in one stroke, in foot-pounds,

w' = the work of back pressure for one stroke, in foot-pounds,

W = the net work done in foot-pounds.

The actual ratio of expansion is

$$\frac{L+c}{l+c} = \frac{L'}{l'} = R'.$$

The work done during admission is equal to the total pressure on the piston, $a \times P$, multiplied by the period of admission, or $a\,P\,l$, which is the work in foot-pounds, and this work is done by a volume of steam measured by the period of admission, plus the clearance, or by $l + c = l'$; and as $l = l' - c$, then

$$\text{whole work done during admission} = a\,P\,l = a\,P\,(l' - c). \quad\ldots\ldots \quad (3)$$

To find the work done by expansion to the end of the stroke, the total pressure on the piston, $a\,P$, is to be multiplied by l', the period of admission plus the clearance, and by the hyperbolic logarithm of R', the actual ratio of expansion, or

$$\text{whole work done during expansion} = a\,P\,l' \times \text{hyp log } R', \quad\ldots \quad (4)$$

which is the work done by expansion, in foot-pounds. Add together these two quantities of work, (3) and (4), and reduce; then, for the total work, w, done by the steam in one stroke of the piston,

$$w = a\,P\,[l'\,(1 + \text{hyp log } R') - c]. \quad\ldots\ldots\ldots\ldots \quad (5)$$

The work of back pressure for one stroke is

$$w' = a\,p'\,L; \quad\ldots\ldots\ldots\ldots \quad (6)$$

and the net work, such as may be measured by an indicator-diagram, is $w - w'$; or,

$$W = a\,[P\,(l'\,(1 + \text{hyp log } R') - c) - p'\,L]. \quad\ldots\ldots \quad (7)$$

RULE 3. *To find the net work done by steam in the cylinder for one stroke of the piston, with a given cut-off.*—1. To the hyperbolic logarithm of the actual ratio of expansion, allowing for clearance, add 1; multiply the sum by the period of admission, plus the clearance, in feet; from the product subtract the clearance, and multiply the remainder by the total initial pressure in lbs. per square inch. The product is the total work done in foot-pounds per square inch on the piston. 2. Multiply the average back pressure in lbs. per square inch by the length of the stroke; the product is the negative work of back pressure in foot-pounds per square inch. 3. Subtract the second product from the first product; the remainder is the net work in foot-pounds per square inch on the piston. 4. Multiply the area of the piston by the net work per square inch; the product is the net work in foot-pounds done in the cylinder for one stroke.

Note.—When the period of admission and the clearance are expressed as percentages of the stroke, the percentages are to be converted into feet of the stroke. The actual ratio of expansion is found by dividing 100 plus the percentage of clearance, by the sum of the percentages of admission and clearance.

To exemplify the application of the rule, take a non-condensing steam-cylinder 3 feet in diameter with a stroke of 5 feet, and initial steam of a total pressure of 70 lbs. per square inch on the piston, cut off at one-fourth of the stroke, and expanded during the remaining three-fourths. The average back pressure is 17 lbs. per square inch, and the clearance is 5 per cent. of the stroke. What is the whole work done in one stroke? The steam is cut off at 15 inches, to which the clearance, which is 5 per cent. of the stroke, or 3 inches, is to be added. The sum is 18 inches, or 1.5 feet, and the actual ratio of expansion is $\frac{5+.25}{1.5} = 3.5$, of which the hyperbolic logarithm is 1.204; to this add 1, making 2.204, to be multiplied by 1.5, making 3.306. From this product subtract the clearance .25 feet, leaving 3.056. Then 3.056×70 lbs. $= 213.92$ foot-pounds of total work per square inch of piston; and 213.92×1017.87 square inches area of piston $=$ 217,750 foot-pounds, the total work done in one stroke. The back pressure 17 lbs. per square inch $\times 5 = 85$ foot-pounds per square inch for the whole stroke; and $85 \times 1017.87 = 8653$ foot-pounds, the negative work of back-pressure. Finally—

	foot-pounds.
Total work done on the piston, for one stroke,.......	217,750
Negative work of back pressure, for one stroke,......	8,653
Difference, or net work for one stroke,........	209,097

INITIAL PRESSURE IN THE CYLINDER.

Inverting formula (7), the required initial pressure *for a given net quantity of work* in one stroke, is as follows:—

$$P = \frac{W + a p' L}{a\,[l' \,(1 + \text{hyp log } R') - c]} \quad \ldots\ldots\ldots\ldots\ldots\ldots \;(\;8\;)$$

The initial pressure required *to produce a given average total pressure per square inch* for a given actual ratio of expansion, is found by sub-

stituting, for W, its equivalent $a\,L\,(p-p')$, in formula (8); and reducing. Then

$$P=\frac{p\,L}{l'\,(1+\text{hyp log }R')-c}\;\ldots\ldots\ldots\ldots\ldots\ldots\ (9)$$

Average Total Pressure in the Cylinder.

The average total pressure, p, in the cylinder, *in terms of the initial pressure*, for a given actual ratio of expansion, is found by dividing the second member of the equation (5), by the area of the piston and by the length of the stroke; or by a simple inversion of equation (9):—

$$p=\frac{P\,[l'\,(1+\text{hyp log }R')-c]}{L}\;\ldots\ldots\ldots\ldots\ (10)$$

The average total pressure, p, *in terms of the total work done* for one stroke, is also,

$$p=\frac{w}{a\,L}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (11)$$

Average Effective Pressure in the Cylinder.

The average effective pressure is found by subtracting the average back pressure from either of the above values of p, formula (10) or (11), or it is found by dividing the second member of equation (7) by the area of the piston and by the length of the stroke: giving, by reduction,

$$(p-p')=\frac{P\,[l'\,(1+\text{hyp log }R')-c]}{L}-p'\;\ldots\ldots\ldots\ (12)$$

The Period of Admission and the Actual Ratio of Expansion.

The actual rate of expansion required for the production of a given average total pressure from a given initial total pressure may be found tentatively by inverting the formula (10), for initial pressure, and reducing, by which the following formula is obtained:—

$$\text{hyp log }R'=\frac{\frac{p}{P}L+c}{l'}-1\ldots\ldots\ldots\ldots\ldots\ldots\ (13)$$

Here, there are two unknown quantities, namely, hyp log R' and l'.

Rule.—Multiply the length of stroke by the mean pressure, and divide by the initial pressure; and to the quotient add the clearance, making a sum A. Assume a period of admission, and add to it the clearance, to make a value for the divisor l', and find the corresponding value for hyp log R', the hyperbolic logarithm of a ratio of expansion. Find the ratio in a table of hyperbolic logarithms, and by it divide the sum of the stroke and the clearance. If the quotient be equal to the assumed period of admission plus the clearance, it follows that the assumed period is the required period of admission, and the ratio of expansion is the required actual ratio. But if the quotient be greater than the sum of the assumed period and the clearance, then the assumed period of admission is too long. If the quotient, on the contrary,

be less, the assumed period is too short. Try again, and assume a shorter or a longer period of admission, as the case may require, until the required period of admission and ratio of expansion have been arrived at.

This is a long rule, but the operation of it is less tedious than may be imagined. For example, reverting to previous data, take the stroke = 5 feet; clearance .25 feet, total initial pressure = 70 lbs., and average total pressure = 42.78 lbs. per square inch; to find the required period of admission. Then

$$\frac{42.78 \times 5}{70} + .25 = 3.306 \text{ .... (Sum A)}$$

Assume a period of admission, 1.75 feet; then

$$1.75 + .25 = 2.00 \text{ .... (Sum B)}$$

And, $3.306 \div 2 = 1.653$, from which deduct 1; the remainder .653 is the hyperbolic logarithm of the ratio of expansion, 1.92. Now, the stroke plus the clearance is 5.25, and

$$\frac{5.25}{1.92} = 2.73 \text{ feet, as a period of admission plus clearance;}$$

and $2.73 - .25 = 2.48$ feet. But this is greater than the assumed period namely, 1.75 feet. Try, therefore, a smaller period to begin with, say 1 foot then

$$1 + .25 = 1.25 \text{ .... (Sum B)}$$

$3.306 \div 1.25 = 2.61$; and $2.61 - 1 = 1.61$, which is the hyperbolic logarithm of the ratio 5; then

$$\frac{5.25}{5} = 1.05 \text{ feet; and } 1.05 - .25 = .80 \text{ foot.}$$

But .80 foot is less than the assumed period, namely, 1 foot; and 1 foot is too short. The required period must be less than 1.75, and more than 1 foot; and nearer to 1 foot than to 1.75 feet. Try 1.25 feet, then

$$1.25 + .25 = 1.50 \text{ .... (Sum B)}$$

$3.306 \div 1.50 = 2.2040$; and $2.2040 - 1 = 1.2040$, which is the hyperbolic logarithm of the ratio 3.5; then

$$\frac{5.25}{3.5} = 1.5 \text{ feet; and } 1.5 - .25 = 1.25 \text{ feet,}$$

which is equal to the period last assumed. The required period of admission is, therefore, 1.25 feet; and the ratio of expansion is 3.5.

Note.—Calculation for this rule may be shortened by using the following table (No. 295), page 836, particularly when the clearance is 7 per cent. of the stroke, as is assumed in the composition of that table. When the clearance deviates by 1 or 2 per cent. from the standard of the table, suitable allowances may be made on the results drawn from the table, by which near approximations may be made. Take the last example, in which the clearance is 5 per cent. of the stroke. Reduce the given mean pressure to the expression .611, which is its relative value when the initial

pressure is taken as 1, thus $42.78 \div 70 = .611$. Looking down the fourth column of the table, the nearest values are .619 and .608, corresponding to the ratios of expansion 3.5 and 3.6, the exact ratio being 3.5. The corresponding periods of admission in column 3 are 23.6 and 22.7 per cent. of the stroke, and adding to these 2 per cent., to compensate for the difference of clearance—5 per cent. in the example, as against 7 per cent. in the table—the sums average about 25 per cent., which is the correct admission.

RULE 4. *To find the Period of Admission required for a given Actual Ratio of Expansion.*—Divide the length of stroke plus the clearance by the actual ratio of expansion; and deduct the clearance from the quotient. The remainder is the period of admission.

2. *When the Quantities are given as Percentages of the Stroke.*—Add the percentage of clearance to 100, and divide the sum by the actual ratio of expansion; and deduct the percentage of clearance from the quotient. The remainder is the period of admission as a percentage of the stroke.

The Period of Admission required for a given Actual Ratio of Expansion is

$$l = \frac{L'}{R'} - c \qquad (14)$$

RULE 5. *The Pressure of Steam expanded in the Cylinder, at the end of the Stroke, or at any other point of the Expansion*, is found by dividing the initial pressure by the ratio of actual expansion calculated to the given point of the stroke. The quotient is the pressure at that point.

Or, multiply the initial pressure by the period of admission plus the clearance, and divide the product by the length of the part of the stroke described up to the given point, plus the clearance. The quotient is the pressure at that point.

THE RELATIVE PERFORMANCE OF EQUAL WEIGHTS OF STEAM WORKED EXPANSIVELY.

The steam may be said to be measured off for each stroke of the piston, a cylinder-full at a time, of expanded steam; whilst the final pressure is a measure of the density, and therefore of the weight, of this steam. The mean pressures, again, are measures of the total performance of the same body of steam. It follows, that the relative total performance is directly as the mean pressure, and inversely as the weight of steam condensed or as the final pressure, and that, if the former be divided by the latter, the quotients will show the relative total performance of a given weight of the steam, as admitted and cut off at different points, and expanded to the end of the stroke, with a clearance of 7 per cent. of the stroke, as follows:—

When the steam is cut off at

1, ¾, ½, ¼, $^1/_5$, ⅛, $^1/_{10}$, $^1/_{15}$ of stroke,

the relative total performance per unit of steam is directly as the average pressures,

1.000, .969, .860, .637, .567, .457, .413, .348,... (A)

and is inversely as the final pressures,

1.000, .769, .532, .298, .250, .182, .159, .128.

The relative, or proportional, total performance of given equal weights of steam are therefore in the ratio of the second last row of figures divided by the last row of figures; the total performance for steam admitted for the whole stroke, without any expansion, being taken as 1. Thus,

$$\frac{1}{1}, \quad \frac{.969}{.769}, \quad \frac{.860}{.532}, \quad \frac{.637}{.298}, \quad \frac{.567}{.250}, \quad \frac{.457}{.182}, \quad \frac{.413}{.159}, \quad \frac{.348}{.128}$$

or the quotients,

1.00, 1.26, 1.62, 2.13, 2.27, 2.51, 2.60, 2.72.... (B)

These quotients may be found, otherwise, from the actual ratios of expansion, which are inversely as the final pressures, by multiplying the average pressures by the respective ratios. For example, when the steam is cut off at ¾, the actual ratio of expansion is 1.3, and the mean pressure $.969 \times 1.3 = 1.26$, which is the relative efficiency, as already found above.

It is seen that the total work or performance of a given weight of steam is fully doubled by cutting off and expanding at a fourth of the stroke, as compared with the admission of steam for the whole of the stroke.

In these comparisons of the relative performance of steam worked expansively, the opposition of back pressure has, for simplicity, been omitted from the calculations. Taking the back pressure as constant with all ratios of expansion, it would constitute a uniform quantity to be deducted from each of the total mean pressures, of which the ratios are given in line A; and as the remainders would thus decrease more rapidly than the total pressures, it would follow that the quotients, line B, would increase less rapidly than as they are there shown to increase.

Proportional Work done by Admission and by Expansion.

To ascertain in what proportions the whole work for the stroke is done by admission and by expansion, leaving unconsidered the back pressure: the work by admission is in proportion to the period of admission, and if this be subtracted from the proportional mean pressure, the remainder is the proportional work by expansion. Thus, when the steam is cut off at

1, ¾, ½, ⅓, ¼, 1/5, ⅛, 1/10, 1/15 of stroke,

these fractions are the periods of admission, and are proportional to the work by admission, and are decimally as follows:—

1.000, .750, .500, .333, .250, .200, .125, .100, .066,

which being subtracted from the relative total average pressures, the remainders are the relative work by expansion:—

.000, .219, .360, .393, .387, .367, .332, .313, .282;

the sum of the last two rows, or the total average pressures, being as

1.000, .969, .860, .726, .637, .567, .457, .413, .348;

which are the same as the values in line A, page 832.

Here it appears that the quantity of work done by expansion, arrives at a maximum when the period of admission is about one-third of the stroke.

With a greater or a less admission it is reduced. But the proportion of work by expansion, relative to the work by admission, increases regularly as the admission is reduced. Thus, taking the work for the periods of admission successively, as

1, 1, 1, 1, 1, 1, 1, 1, 1,

the corresponding proportions of work done by expansion, are successively

as 0, .29, .72, 1.31, 1.55, 1.83, 2.66, 3.13 4.27.

The loss by clearance-space neutralizes a considerable proportion of the gain by expansion, as appears from the following examples.

The Influence of Clearance in reducing the Performance of Steam in the Cylinder.

To note the effect of clearance in reducing the efficiency of steam in the cylinder, let the steam be admitted for one-fourth of the stroke, and let *cdgnm* be the indicator-diagram described, with a perfect vacuum, of which the base *m n* is the length of the stroke = 100, and the extension of the base, *mm'*, is the length of the clearance = 7. The average pressure, *pp*, is, by the formula (10), .637, when the initial pressure is 1. The loss of pressure by clearance, is represented by the initial area *mm'c'c*, the pressure being = 1, and the volume = 7 per cent. of that of the stroke. Averaged for the whole stroke, that is, multiplying 1 by 7 and dividing by 100, the average loss of pressure for the whole stroke is $1 \times \frac{7}{100} = .070$; and if this average loss be added to the average pressure, the sum, $.637 + .070 = .707$, expresses the relative efficiency with which a given weight of steam would be worked if there were no loss by clearance. It shows that there would be a gain of 11 per cent. This greater relative efficiency is represented on the diagram by the upper line *p'p'*.

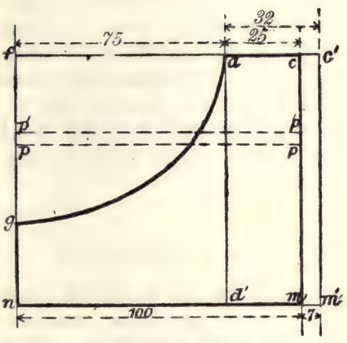


Fig. 333.—Diagram to show Influence of Clearance on the Work of Steam.

The relative efficiency may be otherwise found by means of formula (10), for the average total pressure, the item of clearance being eliminated from it. Suppose the clearance in the diagram, Fig. 333, to be included as part of the stroke, then the period of admission becomes 32 per cent., and the length of stroke 107 per cent.; and, when the initial pressure is 1,

$$\frac{32 \left(1 + \text{hyp log } \frac{107}{32} \text{ or } 3.35\right)}{107} = \frac{70.7}{107} = .661,$$

the average pressure, as against .637 the average pressure with clearance. But, as the strokes are different, the average pressures are to be multiplied

by their respective strokes, to give the proportion of the efficiencies; thus,

$.637 \times 100 = 63.7$	relative efficiency, for	25 %	admission, with	7 % clearance;
$.661 \times 107 = 70.7$	do.	32	do.	without clearance;

being in the same ratio to each other, as the values .637 and .707 already found.

The comparison is extended for other periods of admission by simply adding the average loss .070, to the corresponding average pressures in the 4th column of the table, No. 295. Thus,

When the steam is cut off at

full stroke,	¾,	½,	⅓,	¼,	⅛,	$^1/_{10}$,	$^1/_{15}$ of stroke,

the average pressures representing the relative work, when the pressure during admission = 1, are

1.000,	.969,	.860,	.726,	.637,	.457,	.413,	.348,

and, adding the loss by 7 per cent. of clearance, .070, the increased relative work done by a given weight of steam, if there were no clearance, would be

1.070,	1.039,	.930,	.796,	.707,	.527,	.483,	.418,

showing that the gain would be

7,	7.2,	8.1,	9.6,	11.0,	15.3,	17,	20 per cent.,

which is lost by clearance.

Table No. 295.—Ratios of Expansion of Steam, with Relative Periods of Admission, Pressures, and Total Performance.

To facilitate calculations about steam expanded in cylinders, the table No. 295 has been composed. The actual ratios of expansion, column 1, range from 1.0 to 8.0, for which the hyperbolic logarithms are given, for ready reference, in column 2. The 3d column contains the periods of admission relative to the actual ratios of expansion, as percentages of the stroke, calculated by Rule 4. The 4th column gives the values of the mean pressures relative to the initial pressures, the latter being taken as 1, calculated by formula (10). The 5th column gives the values of the initial pressures relative to the mean pressures, when the latter are taken as 1. These values are the reciprocals of those of the 4th column; at the same time they may be calculated by formula (9). In the calculation of these last three columns, 3, 4, and 5, clearance is taken into account, and its amount is assumed at 7 per cent. of the stroke. In the 6th column, of final pressures, they are such as would be arrived at by the continued expansion of the whole of the steam to the end of the stroke, the initial pressure being equal to 1. They are the reciprocals of the ratios of expansion, column 1, as indicated by Rule 5.

The 7th column contains the relative total performance of equal weights of steam worked with the various actual ratios of expansion: the total performance when steam is admitted for the whole of the stroke without expansion being equal to 1. They are calculated on the principle exemplified at page 832.

Table No. 295.—Expansive Working of Steam:—Actual Ratios of Expansion; with the Relative Periods of Admission, Pressures, and Performance.

Clearance at each end of the Cylinder, 7 per cent. of the stroke.

(Single Cylinder.)

1 Actual Ratio of Expansion; Or Number of Volumes to which the Initial Volume is Expanded.	2 Hyperbolic Logarithm of Actual Ratio of Expansion.	3 Corresponding Period of Admission, or Cut-off. Clearance, 7 per cent. of the Stroke.	4 Average Total Pressure.	5 Total Initial Pressure.	6 Total Final Pressure.	7 Ratio of Total Performance of Equal Weights of Steam. (Col. 4÷Col. 6.)
initial volume =1.		stroke = 100.	initial pressure=1.	mean pressure=1.	initial pressure=1.	with 100 % of admission =1.000.
1.0	.0000	100	1.000	1.000	1.000	1.000
1.05	.0488	95.0	.9997	1.003	.952	1.050
1.1	.0953	90.3 or 9/10	.996	1.004	.909	1.096
1.15	.1398	86.0	.990	1.010	.870	1.138
1.18	.1698	83.3 or 5/6	.986	1.014	.847	1.164
1.2	.1823	82.1	.983	1.017	.833	1.180
1.23	.2070	80.0 or 4/5	.980	1.020	.813	1.206
1.25	.2231	78.6	.977	1.024	.800	1.221
1.3	.2624	75.3 or ¾	.969	1.032	.769	1.261
1.35	.3000	72.3	.961	1.041	.741	1.297
1.39	.3293	70.0 or 7/10	.953	1.049	.719	1.325
1.4	.3365	69.4	.951	1.052	.714	1.332
1.45	.3716	66.8 or 2/3	.942	1.062	.690	1.365
1.5	.4055	64.3	.932	1.073	.666	1.399
1.54	.4317	62.5 or ⅝	.925	1.081	.649	1.425
1.55	.4382	62.0	.922	1.085	.645	1.429
1.6	.4700	59.9 or 3/5	.913	1.095	.625	1.461
1.65	.5008	57.9	.903	1.107	.606	1.490
1.7	.5306	56.0	.894	1.119	.588	1.520
1.75	.5595	54.1	.883	1.132	.571	1.546
1.8	.5878	52.4	.873	1.145	.555	1.573
1.85	.6153	50.8	.864	1.157	.541	1.597
1.88	.6314	50.0 or ½	.860	1.163	.532	1.616
1.9	.6419	49.3	.854	1.171	.526	1.624
1.95	.6678	47.9	.846	1.182	.513	1.649
2.0	.6931	46.5	.836	1.196	.500	1.672
2.1	.7419	44.0	.818	1.222	.476	1.718
2.2	.7885	41.6	.799	1.251	.455	1.756
2.28	.8241	40.0 or 2/5	.787	1.271	.439	1.793
2.3	.8329	39.5	.782	1.279	.435	1.798
2.4	.8755	37.6 or ⅜	.766	1.305	.417	1.837
2.5	.9163	35.8	.750	1.333	.400	1.875
2.6	.9555	34.2	.736	1.359	.385	1.912
2.65	.9745	33.3 or 1/3	.726	1.377	.377	1.925
2.7	.9933	32.6	.719	1.391	.370	1.943
2.8	1.030	31.2	.706	1.416	.357	1.978
2.9	1.065	29.9 or 3/10	.692	1.445	.345	2.006
3.0	1.099	28.7	.679	1.473	.333	2.039
3.1	1.131	27.5	.665	1.504	.323	2.059
3.2	1.163	26.4	.652	1.534	.313	2.083
3.3	1.194	25.4	.641	1.560	.303	2.115

Table No. 295 (*continued*).

1 Actual Ratio of Expansion; Or Number of Volumes to which the Initial Volume is expanded.	2 Hyperbolic Logarithm of Actual Ratio of Expansion.	3 Corresponding Period of Admission, or Cut-off. Clearance, 7 per cent. of the stroke.	4 Average Total Pressure.	5 Total Initial Pressure.	6 Total Final Pressure.	7 Ratio of Total Performance of equal Weights of Steam. (Col. 4÷Col. 6.)
initial volume =1.		stroke = 100.	initial pressure=1.	mean pressure=1.	initial pressure=1.	with 100 % of admission =1.000.
3.35	1.209	25.0 or ¼	.637	1.570	.298	2.129
3.4	1.224	24.5	.631	1.585	.294	2.146
3.5	1.253	23.6	.619	1.615	.286	2.164
3.6	1.281	22.7	.608	1.645	.278	2.187
3.7	1.308	21.9	.597	1.675	.270	2.211
3.8	1.335	21.2	.589	1.698	.263	2.240
3.9	1.361	20.4	.579	1.727	.256	2.262
4.0	1.386	19.7 or 1/5	.567	1.764	.250	2.278
4.1	1.411	19.1	.559	1.789	.244	2.291
4.2	1.435	18.5	.551	1.815	.238	2.315
4.3	1.459	17.9	.542	1.845	.233	2.326
4.4	1.482	17.3	.533	1.876	.227	2.348
4.5	1.504	16.8 or 1/6	.526	1.901	.222	2.370
4.6	1.526	16.3	.518	1.930	.217	2.387
4.7	1.548	15.8	.511	1.957	.213	2.399
4.8	1.569	15.3	.503	1.988	.208	2.418
4.9	1.589	14.8	.494	2.024	.204	2.422
5.0	1.609	14.4 or 1/7	.488	2.049	.200	2.440
5.2	1.649	13.6	.476	2.101	.193	2.466
5.4	1.686	12.8	.462	2.164	.185	2.497
5.5	1.705	12.5 or 1/8	.457	2.188	.182	2.511
5.6	1.723	12.1	.450	2.222	.178	2.528
5.8	1.758	11.4	.438	2.283	.172	2.547
5.9	1.775	11.1 or 1/9	.432	2.315	.169	2.556
6.0	1.792	10.8	.427	2.342	.167	2.567
6.2	1.825	10.3	.419	2.387	.161	2.585
6.3	1.841	10.0 or 1/10	.413	2.421	.159	2.597
6.4	1.856	9.7	.407	2.457	.156	2.609
6.6	1.887	9.2 or 1/11	.398	2.513	.152	2.619
6.8	1.917	8.7	.388	2.577	.147	2.639
7.0	1.946	8.3 or 1/12	.381	2.625	.143	2.664
7.2	1.974	7.9	.373	2.681	.139	2.683
7.3	1.988	7.7 or 1/13	.369	2.710	.137	2.693
7.4	2.001	7.5	.365	2.740	.135	2.703
7.6	2.028	7.1 or 1/14	.357	2.801	.132	2.711
7.8	2.054	6.7 or 1/15	.348	2.874	.128	2.719
8.0	2.079	6.4 or 1/16	.342	2.924	.125	2.736

The pressures have been calculated on the supposition that the pressure of steam, during its admission into the cylinder, is uniform up to the point of cutting off, and that the expansion is continued regularly to the end of the stroke. In practice, of course, there are deviations from these ideal conditions. Wiredrawing action occasionally causes a fall of pressure during admission, and the opening of the exhaust before the piston arrives at the end of the stroke causes the expansion-line to fall away towards the end.

The allowances necessary to be made for these deviations, as well as for the back pressure of the air in non-condensing engines, and that from the condenser in condensing engines, and for compression of exhaust steam towards the end of the return stroke, will be considered at a subsequent stage. The calculations have been made for periods of admission ranging from 100 per cent., or the whole of the stroke, to 6.4 per cent. or $^1/_{16}$th of the stroke. And though, nominally, the expansion is 16 times in the last instance, it is actually only 8 times, as given in the first column. The great difference between the nominal and the actual ratios of expansion is caused by the clearance, which is equal to 7 per cent. of the stroke, and causes the nominal volume of steam admitted, namely, 6.4 per cent., to be augmented to $6.4 + 7 = 13.4$ per cent. of the stroke, or more than double, for expansion. When the steam is cut off at $^1/_9$ th, the actual expansion is only 6 times; when cut off at $^1/_5$ th, the expansion is 4 times; when cut off at ⅓d, the expansion is 2⅔ times; and to effect an actual expansion to twice the initial volume, the steam is cut off at 46½ per cent. of the stroke, not at half stroke.

Though a uniform clearance of 7 per cent. at each end of the stroke has been assumed as a fair average proportion for the purpose of compiling the table, the clearance of cylinders with ordinary slides varies considerably—say, from 5 to 8 or 9 per cent. With the mean clearance, 7 per cent., that has been assumed, the table gives approximate results sufficient for most practical purposes; they will economize calculation, and they are certainly more trustworthy than such as can be deduced by calculations based on simple tables of hyperbolic logarithms, where clearance is neglected.

It has already been exemplified at page 831, how the table may serve in making approximate calculations when the clearance is other than 7 per cent.

Total Work done by One Pound of Steam Expanded in a Cylinder.

If 1 lb. of water be converted into steam of atmospheric pressure—14.7 lbs. per square inch, or 2116.8 lbs. per square foot—it gradually occupies a volume equal to 26.36 cubic feet; and the work done in acquiring this volume under one atmosphere is equal to 2116.8 lbs. × 26.36 feet = 55,799 foot-pounds. The equivalent quantity of heat expended is 1 unit per 772 foot-pounds, or, altogether, $55,799 \div 772 = 72.3$ units. This is precisely the work of 1 lb. of steam of one atmosphere, acting on a piston without expansion.

The gross work thus done on a piston by 1 lb. of steam, generated at total pressures varying from 15 lbs. to 100 lbs. per square inch, varies, in round numbers, from 56,000 to 62,000 foot-pounds, equivalent to from 72 to 80 units of heat.

The simple work of a pound of steam, without expansion, thus exemplified, is reduced by clearance according to the proportion it bears to the net capacity of the cylinder. If the clearance be 7 per cent. of the stroke, then 107 parts of steam are consumed in doing the work of a stroke, which is represented by 100 parts, and the work of a given weight of steam without expansion, admitted for the whole of the stroke, is reduced in the ratio of 107 to 100. Having determined, by this ratio, the quantity of work by 1 lb. of steam without expansion, as reduced by clearance, the work for various ratios of expansion may be deduced from that, in terms of the

relative performance of equal weights of steam, as exemplified, page 835, and given in the 7th column of table No. 295.

To find the total actual work of 1 lb. of steam, for any ratio of expansion, it is only necessary to multiply the simple work, without expansion, as reduced by clearance, by the ratio or relative performance just referred to. The simple work of a pound of steam does not greatly vary with the pressure; and, for present purposes, the work of steam of a total pressure of 100 lbs. per square inch will be calculated and tabulated. This pressure corresponds to a net pressure, above the atmosphere, of 85 lbs. per square inch—a convenient average standard of pressure. The volume of 1 lb. of saturated steam of 100 lbs. per square inch is 4.33 cubic feet, and the pressure per square foot is $144 \times 100 = 14{,}400$ lbs.; then, the total simple work—or total initial work, as it may be called—is,

$$14{,}400 \times 4.33 = 62{,}352 \text{ foot-pounds.}$$

This amount is to be reduced for a clearance of, say, 7 per cent., thus:—

$$62{,}352 \times \frac{100}{107} = 58{,}273 \text{ foot-pounds,}$$

which is the total simple work of 1 lb. of steam of 100 lbs. total pressure per square inch, after the loss by clearance is deducted; and, divided by Joule's equivalent, 772, it is equal to 75.5 units of heat. Now, the total or constituent heat of 1 lb. of 100-lb. steam, reckoned from a temperature of 212° F., is 1001.4 units; reckoned from 102° F., the temperature of water from the condenser under a pressure of 1 lb. per square inch, the constituent heat is 1111.4 units. The equivalent of the net simple work, 75.5 units, is, then, 7.5 per cent. of the total heat reckoned from 212° F., or 6.7 per cent., if reckoned from 102° F. For shorter admissions, with complementary expansion, the work is increased as in the following examples:—

When the steam is cut off at

1, ¾, ½, ¼, 1/5, ⅛, 1/10, 1/15 of stroke,

the actual ratios of expansion are,

1, 1.3, 1.88, 3.35, 4.0, 5.5, 6.3, 7.8 times;

the comparative performances of 1 lb. of steam are as

1, 1.261, 1.616, 2.129, 2.278, 2.511, 2.597, 2.719,

and the total actual work of 1 lb. of 100-lb. steam is in the same proportion,

58,273, 73,513, 94,200, 124,066, 132,770, 146,325, 151,370, 158,414 foot-pounds.

The equivalents, as heat, of the actual work done, are

75.5, 95.2, 122.0, 160.7, 171.9, 189.5, 196.1, 205.2 units,

which are, in parts of the constituent heat reckoned from 102° F., equal to

6.7, 8.5, 11.0, 14.5, 15.5, 17.0, 17.6, 18.5 per cent.

From these examples, it appears that the total work done by 1 lb. of steam, without making any allowance for back pressure or other contingencies, varies from about 60,000 foot-pounds when applied without expansion, to about double that, or about 120,000 foot-pounds, when expanded three times, cutting off at about 27 per cent. of the stroke; and to about

150,000 foot-pounds, or $2\frac{1}{2}$ times the first performance, when expanded about six times, cutting off at about 10 per cent. of the stroke.

Also, that, of the heat consumed in the formation of steam, not 7 per cent. is converted into total work when there is no expansive action; that substantially with an expansion of six times there is only $17\frac{1}{2}$ per cent. converted; and that even with an expansion of eight times, when the steam is cut off at $^1/_{15}$th, less than 20 per cent., or one-fifth of the heat consumed, is converted into work. The remainder of the heat is lost, as for the purpose of the steam-engine.

Consumption of Steam Worked Expansively per Horse-power of Total Work per Hour.

The measure of a horse-power is the performance of 33,000 foot-pounds per minute, or of $33{,}000 \times 60 = 1{,}980{,}000$ foot-pounds per hour. This work is to be divided by the work of 1 lb. of steam, and the quotient is the weight of steam or water required per horse-power per hour. For example, the total actual work done in the cylinder by 1 lb. of 100-lb. steam, without expansion, and with 7 per cent. of clearance, is 58,273 foot-pounds; and $\frac{1{,}980{,}000}{58{,}273} = 34$ lbs. of steam, is the weight of steam consumed for the total work done in the cylinder per horse-power per hour. For any shorter period of admission, with expansion, the weight of steam per horse-power is less, as the total work by 1 lb. of steam is more, and may be found by dividing 1,980,000 foot-pounds by the respective total work done; or by dividing 34 lbs. by the ratio of performance, column 7, table No. 295. In this way it is found that, when the steam is cut off at

1, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, $^1/_5$, $\frac{1}{8}$, $^1/_{10}$, $^1/_{15}$ of stroke,

the quantities of steam, or water as steam, consumed per horse-power of total work per hour, are

34.0, 26.9, 21.0, 16.0, 14.9, 13.5, 13.1, 12.5 lbs.

Further, allowing that 10 lbs. of steam are generated by the combustion of 1 lb. of coal, the fuel consumed per horse-power of total work per hour is,

3.40, 2.69, 2.10, 1.60, 1.49, 1.35, 1.31, 1.25 lbs.

Table (No. 296) of the Total Work done by 1 pound of Steam of 100 lbs. Total Pressure per Square Inch.

The table No. 296, which follows, is compiled on the basis of the conditions above laid down, which are repeated under the heading of the table, for ready reference. The 1st, 2d, and 3d columns are repeated from table No. 295. The 4th column, of total actual work done by 1 lb. of steam of 100 lbs. total pressure, is calculated by multiplying the work without expansion, namely, 58,273 foot-pounds, by the ratios in column 3, for the proportional work when expanded. The 5th column contains the equivalent of heat converted into work, which is found by dividing the work in foot-pounds by Joule's equivalent, 772; and the 6th and 7th columns give these values as percentages of the total heat of steam raised from 212° and 102° F. respectively. The 8th column contains the quantity of steam consumed for the total work done per horse-power per hour.

Table No. 296.—Total Work done by One Pound of Steam of 100 lbs. total Pressure per Square Inch.

Assumptions.—That the initial pressure is uniform; that the expansion is complete to the end of the stroke; that substantially the pressure in expansion varies inversely as the volume; that there is no back pressure; and that there is no compression.

Volume of 1 lb. of steam of 100 lbs. pressure per square inch, or 14,400 lbs. per square foot, ...4.33 cubic feet.

Product of initial pressure and volume, ...62,352 foot-pounds.

Constituent heat of 1 lb. of this steam—
Reckoned from 212° F., ...1001.4 units.
Reckoned from 102° F., ...1111.4 units.

Clearance at each end of the cylinder, ...7 per cent. of the stroke.

Actual Ratio of Expansion.	Corresponding Period of Admission or Cut-off, in percentage of Stroke.	Total Actual Work done by 1 lb. of 100-lb. Steam.		Equivalent of Heat converted into Work.			Quantity of Steam consumed per Horse-power of actual Work done per Hour.
		Ratio of Work done (col. 7, table No. 295).	Actual Work done.	Heat converted.	Percentage of Constituent Heat converted, as calculated from 212° F. and 102° F.		
(1) initial vol. = 1.	(2) per cent.	(3)	(4) foot-pounds.	(5) units.	(6) % from 212° F.	(7) % from 102° F.	(8) lbs.
1.0	100	1.000	58,273	75.5	7.5	6.7	34.0
1.05	95	1.050	61,193	79.3	7.9	7.1	32.4
1.1	90.3 or 9/10	1.096	63,850	82.7	8.3	7.5	31.0
1.15	86.0	1.138	66,310	85.9	8.6	7.8	29.9
1.18	83.3 or 5/6	1.164	67,836	87.9	8.8	7.9	29.2
1.2	82.1	1.180	68,766	89.1	8.9	8.0	28.8
1.23	80.0 or 4/5	1.206	70,246	91.0	9.1	8.2	28.2
1.25	78.6	1.221	71,151	92.2	9.2	8.3	27.8
1.3	75.3 or ¾	1.261	73,513	95.2	9.5	8.5	26.9
1.35	72.3	1.297	75,575	97.9	9.8	8.8	26.2
1.39	70.0 or 7/10	1.325	77,242	100.1	10.0	9.0	25.6
1.4	69.4	1.332	77,616	100.6	10.1	9.1	25.5
1.45	66.8 or 2/3	1.365	79,555	102.9	10.3	9.3	24.9
1.5	64.3	1.399	81,546	105.6	10.6	9.5	24.3
1.54	62.5 or ⅝	1.425	83,055	107.6	10.8	9.7	23.8
1.55	62.0	1.429	83,299	107.9	10.8	9.7	23.7
1.6	59.9 or 3/5	1.461	85,125	110.3	11.0	9.9	23.3
1.65	57.9	1.490	86,828	112.5	11.3	10.2	22.8
1.7	56.0	1.520	88,598	114.8	11.5	10.4	22.4
1.75	54.1	1.546	90,115	116.7	11.7	10.5	22.0
1.8	52.4	1.573	91,680	118.7	11.9	10.7	21.6
1.85	50.8	1.597	93,065	120.5	12.0	10.8	21.3
1.88	50.0 or ½	1.616	94,200	122.0	12.2	11.0	21.0
1.9	49.3	1.624	94,610	122.5	12.3	11.1	20.9
1.95	47.9	1.649	96,100	124.5	12.4	11.2	20.6
2.0	46.5	1.672	97,432	126.2	12.6	11.3	20.3
2.1	44.0	1.718	100,266	129.7	13.0	11.7	19.8
2.2	41.6	1.756	102,366	132.5	13.2	11.9	19.4
2.28	40.0 or 2/5	1.793	104,466	135.3	13.5	12.2	19.0
2.3	39.5	1.798	104,855	135.7	13.6	12.3	18.9
2.4	37.6 or ⅜	1.837	107,050	138.6	13.9	12.5	18.5
2.5	35.8	1.875	109,266	141.5	14.1	12.7	18.1
2.6	34.2	1.912	111,400	144.3	14.4	13.0	17.8

Table No. 296 (*continued*).

Actual Ratio of Expansion.	Corresponding Period of Admission or Cut-off, in percentage of Stroke.	Total Actual Work done by 1 lb. of 100-lb. Steam.		Equivalent of Heat converted into Work.			Quantity of Steam consumed per Horse-power of actual Work done per Hour.
		Ratio of Work done (col. 7, table No. 295).	Actual Work done.	Heat converted.	Percentage of Constituent Heat converted, as calculated from 212° F. and 102° F.		
(1) initial vol. = 1.	(2) per cent.	(3)	(4) foot-pounds.	(5) units.	(6) % from 212° F.	(7) % from 102° F.	(8) lbs.
2.65	33.3 or 1/3	1.925	112,220	145.4	14.5	13.1	17.7
2.7	32.6	1.943	113,244	146.7	14.7	13.2	17.6
2.8	30.2	1.978	115,244	149.2	14.9	13.4	17.2
2.9	29.9 or 3/10	2.006	116,885	151.4	15.1	13.6	16.9
3.0	28.7	2.039	118,820	153.9	15.4	13.9	16.7
3.1	27.5	2.059	119,970	155.4	15.5	13.9	16.5
3.2	26.4	2.083	121,386	157.2	15.7	14.1	16.3
3.3	25.4	2.115	123,278	159.6	16.0	14.4	16.1
3.35	25.0 or ¼	2.129	124,066	160.7	16.1	14.5	16.0
3.4	24.5	2.146	125,066	162.0	16.2	14.6	15.8
3.5	23.6	2.164	126,125	163.4	16.3	14.7	15.7
3.6	22.7	2.187	127,450	165.1	16.5	14.9	15.5
3.7	21.9	2.211	128,860	166.9	16.7	15.0	15.4
3.8	21.2	2.240	130,533	169.1	16.9	15.2	15.2
3.9	20.4	2.262	131,800	170.7	17.1	15.4	15.0
4.0	19.7 or 1/5	2.278	132,770	171.9	17.2	15.5	14.9
4.1	19.1	2.291	133,500	172.9	17.3	15.6	14.8
4.2	18.5	2.315	134,900	174.8	17.5	15.8	14.7
4.3	17.9	2.326	135,555	175.6	17.6	15.8	14.6
4.4	17.3	2.348	136,825	177.2	17.7	15.9	14.5
4.5	16.8 or 1/6	2.370	138,130	178.8	17.9	16.1	14.34
4.6	16.3	2.387	139,100	180.2	18.0	16.2	14.23
4.7	15.8	2.399	139,800	181.1	18.1	16.3	14.16
4.8	15.3	2.418	140,920	182.5	18.2	16.4	14.05
4.9	14.8	2.422	141,210	182.8	18.3	16.5	14.03
5.0	14.4 or 1/7	2.440	142,180	184.2	18.4	16.6	13.92
5.2	13.6	2.466	143,720	186.2	18.6	16.9	13.78
5.4	12.8	2.497	145,525	188.5	18.8	16.9	13.60
5.5	12.5 or ⅛	2.511	146,325	189.5	18.9	17.0	13.53
5.6	12.1	2.528	147,320	190.8	19.1	17.2	13.44
5.8	11.4	2.547	148,390	192.2	19.2	17.3	13.34
5.9	11.1 or 1/9	2.556	148,940	192.9	19.3	17.4	13.29
6.0	10.8	2.567	149,586	193.7	19.4	17.5	13.23
6.2	10.3	2.585	150,630	195.1	19.5	17.6	13.14
6.3	10.0 or 1/10	2.597	151,370	196.1	19.6	17.6	13.08
6.4	9.7	2.609	152,033	196.9	19.7	17.7	13.02
6.6	9.2 or 1/11	2.619	152,595	197.7	19.8	17.8	12.98
6.8	8.7	2.629	153,810	199.2	19.9	17.9	12.87
7.0	8.3 or 1/12	2.664	155,200	201.1	20.1	18.1	12.75
7.2	7.9	2.683	156,330	202.6	20.3	18.3	12.66
7.3	7.7 or 1/13	2.693	156,960	203.3	20.3	18.3	12.61
7.4	7.5	2.703	157,560	204.1	20.4	18.4	12.57
7.6	7.1 or 1/14	2.711	157,975	204.6	20.4	18.4	12.53
7.8	6.7 or 1/15	2.719	158,414	205.2	20.5	18.5	12.50
8.0	6.4 or 1/16	2.736	159,433	206.5	20.7	18.6	11.83

APPENDIX TO TABLE No. 296.

TABLET OF MULTIPLIERS for the total Work done by 1 lb. of Steam of other Pressures than 100 lbs. per Square Inch, to be applied to the total actual Work as given in the table. See explanatory notice of the table, page 840.

Total Pressures below 100 lbs. per Square Inch.		Total Pressures above 100 lbs. per Square Inch.	
lbs. per square inch.	multiplier.	lbs. per square inch.	multiplier.
65	.975	100	1.000
70	.981	110	1.009
75	.986	120	1.011
80	.988	130	1.015
85	.991	140	1.022
90	.995	150	1.025
95	.998	160	1.031

An initial total pressure of 100 lbs. per square inch has been adopted for the table, as an average pressure in ordinary good practice, and the contents of the table are good as approximate values for other pressures considerably different from 100 lbs., more or less. A tablet is, however, appended to the table No. 296, containing multipliers for various other total pressures, which may be applied to the total actual work given in the table for the purpose of determining the correct total quantities of work for steam of the respective pressures. These multipliers are arrived at by multiplying the total pressure of any other given steam per square foot, by the volume in cubic feet of 1 lb. of such steam, and dividing the product by 62,352, which is the product in foot-pounds for steam of 100 lbs. pressure. The quotient is the multiplier for the given pressure. From the tablet it appears that, between the extremes of 65 lbs. and 160 lbs. per square inch, the deviation from the work done as given for 100 lbs. pressure does not exceed 2½ and 3 per cent.

NET CYLINDER-CAPACITY RELATIVE TO THE STEAM EXPENDED AND WORK DONE IN ONE STROKE.

The quantity of cylinder-capacity required for the performance of a given weight of steam admitted for one stroke depends on the volume of the steam, and on the ratio of expansion. If the given weight admitted be multiplied by the volume of 1 lb. of the steam and by the actual ratio of expansion, the product is the gross cylinder-capacity, including clearance. For example, if 1 lb. of steam of 100 lbs. pressure per square inch be admitted for the whole stroke, without expansion, the gross capacity is the volume of 1 lb. of such steam, namely, 4.33 cubic feet; and the net capacity, supposing the clearance to be 7 per cent. of the stroke, is

$$4.33 \times \frac{100}{100+7} = 4.047 \text{ cubic feet.}$$

If, again, 2 lbs. of steam of 100 lbs. pressure be admitted and expanded into three times its initial volume, the gross capacity is,

$$4.33 \times 2 \times 3 = 25.98 \text{ cubic feet;}$$

and the net capacity is

$$25.98 \times \frac{100}{107} = 24.28 \text{ cubic feet.}$$

From this is derived following rule for net capacity:—

RULE 6. *To find the net Capacity of Cylinder for a given Weight of Steam admitted for one stroke, and a given actual ratio of expansion.*—Multiply the volume of 1 lb. of the steam by the given weight in pounds, and by the actual ratio of expansion. Multiply the product by 100, and divide by 100 plus the percentage of clearance. The quotient is the net capacity of cylinder.

Again, the quantity of cylinder-capacity required for the performance of a given amount of total actual work, in one stroke, depends on the initial pressure, and the actual ratio of expansion, according to the following rule:—

RULE 7. *To find the net Capacity of Cylinder for the performance of a given amount of total Actual Work, in one stroke, with a given initial pressure, and actual ratio of expansion.*—Divide the given work by the total actual work done by 1 lb. of steam of the same pressure, and with the same actual ratio of expansion; the quotient is the weight of steam necessary to do the given work, for which the net capacity is found by Rule 6, preceding.

Conversely, the weight of steam admitted per cubic foot of net capacity, for one stroke, is the reciprocal of the cylinder-capacity per pound of steam, as obtained by Rule 6.

Likewise, the total actual work done per cubic foot of net capacity, for one stroke, is the reciprocal of the cylinder-capacity per foot-pound of work done, as obtained by Rule 7.

Finally, the total actual work done per square inch of piston, per foot of the stroke, is $^1/_{144}$th part of the work done per cubic foot; a prism 1 inch square and 1 foot long being $^1/_{144}$th part of a cubic foot. The work, in either measure, is in direct proportion to the mean total pressure per square inch.

Table of Relations of Net Capacity of Cylinder to Steam Admitted and Work done.

The table No. 297 gives the net capacity of cylinder required in relation to the quantity of steam of 100 lbs. total pressure per square inch consumed, and of work done, in one stroke. Columns 1, 2, and 3, are the ratios of expansion, periods of admission, and total actual work done by 1 lb. of steam of 100 lbs. pressure, repeated from columns 1, 2, and 4, in the previous table, No. 296. In the 4th column are the net capacities of cylinder required for each pound of steam admitted for one stroke, found by Rule 6; and in the 5th column are the net capacities of cylinder for 100,000 foot-pounds of total actual work done in one stroke, found by Rule 7. The 6th column contains the weights of steam of 100 lbs. pressure admitted to the cylinder for one stroke, per cubic foot of

net capacity. These values are simply the reciprocals of those in column 4. The 7th column contains the total actual work done by steam of 100 lbs. initial pressure, for one stroke, per cubic foot of net capacity. These values are the products of the reciprocals of those in column 5, by 100,000, since this is the number of foot-pounds for which the values in column 5 are calculated. The 8th column gives the total actual work per square inch of piston-area, per foot of stroke, in foot-pounds; the initial pressure is 100 lbs., and the following pressures, for the different ratios of expansion, may also be read as percentages of the initial pressure. The total actual works are directly proportional to the mean total pressures per square inch, as given in column 4, table No. 295, page 836, where the initial pressure is taken as 1; and the former have been found by multiplying the latter respectively by 100.

The contents of the table No. 297 are calculated for steam of 100 lbs. per square inch, total initial pressure; but they are available, with the aid of a set of multipliers, for other pressures. For column 3, the total actual work done, the multipliers have already been given in the tablet appended to the table at page 848, for pressures of from 65 lbs. to 160 lbs. per square inch. For column 4, of the net capacity of cylinder per pound of steam expended in one stroke, the multipliers are simply the ratios of the volume of 1 pound of 100-lb. steam to the respective volumes of 1 pound of steam of other initial pressures. Thus, for steam of 65 lbs. total pressure, of which the volume of 1 pound is 6.49 cubic feet, to be compared with 4.33 cubic feet, which is the volume of a pound of 100-lb. steam, the multiplier is

$$\frac{6.49}{4.33} = 1.5;$$

and the net capacity of cylinder per pound of 65-lb. steam, when the steam is admitted for the whole of the stroke, is

$$4.05 \text{ cubic feet (for 100-lb. steam)} \times 1.5 = 6.07 \text{ cubic feet.}$$

For column 5, of the net capacity of cylinder per 100,000 foot-pounds of total actual work done in one stroke, the capacity for other pressures is modified, in the first place, in the inverse ratio of the multipliers, as found for tablet, page 848, for 100-lb. steam, and steam of the given pressure; secondly, in the ratio of the volume of 1 pound of 100-lb. steam, to that of a pound of the given steam. For example, for steam of 65 lbs. total pressure per square inch, the weight of 100-lb. steam to do a given total work, as compared with the weight of 65-lb. steam, is as .975 to 1.000, and the volumes of a pound each of the two steams are respectively 4.33 and 6.49 cubic feet, or as 1 to 1.5 as already found. The values in column 5 are therefore to be increased in the compound ratio of

.975 to 1.000
and 1 to 1.5

or, combined, as .975 to 1.5,
or as 1 to 1.54.

The multiplier for 65-lb. steam is thus 1.54.

Table No. 297.—NET CYLINDER-CAPACITY, WITH RELATION TO STEAM ADMITTED, AND TOTAL ACTUAL WORK DONE.

For Steam of 100 lbs. total pressure per square inch.

Clearance at each end of the Cylinder, 7 per cent. of the stroke.

ACTUAL RATIO OF EXPANSION.	PERIOD OF ADMISSION, or CUT-OFF, as a Percentage of Stroke.	TOTAL ACTUAL WORK DONE by 1 lb. of 100-lb. Steam.	Net Capacity of Cylinder. Per pound of 100-lb. Steam, admitted in one Stroke.	Net Capacity of Cylinder. Per 100,000 Foot-pounds of Total Actual Work done by Steam of 100 lbs. pressure, in one Stroke.	Per Cubic Foot of Net Capacity of Cylinder. WEIGHT OF STEAM of 100 lbs., Total Pressure admitted for one Stroke, per Cubic Foot.	Per Cubic Foot of Net Capacity of Cylinder. TOTAL ACTUAL WORK DONE by Steam of 100 lbs., Total Initial Pressure, in one Stroke, per Cubic Foot.	TOTAL ACTUAL WORK DONE per Sq. Inch of Piston, per Foot of Stroke, by 100 lbs. Steam.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
initial volume = 1.	per cent.	foot-pounds.	cubic feet.	cu. feet.	pound.	foot-pounds.	foot-pounds.
1.0	100	58,273	4.05	6.94	.247	14,400	100
1.05	95.0	61,193	4.25	6.95	.235	14,388	99.97
1.1	90.3 or 9/10	63,850	4.45	6.97	.225	14,347	99.6
1.15	86.0	66,310	4.65	7.02	.215	14,245	99.0
1.18	83.3 or 5/6	67,836	4.78	7.04	.209	14,204	98.6
1.2	82.1	68,766	4.86	7.06	.206	14,164	98.3
1.23	80.0 or 4/5	70,246	4.98	7.09	.201	14,104	98.0
1.25	78.6	71,151	5.06	7.11	.198	14,065	97.7
1.3	75.3 or ¾	73,513	5.26	7.16	.190	13,966	96.9
1.35	72.3	75,575	5.46	7.23	.183	13,831	96.1
1.39	70.0 or 7/10	77,242	5.63	7.28	.178	13,736	95.3
1.4	69.4	77,616	5.67	7.30	.176	13,699	95.1
1.45	66.8 or 2/3	79,555	5.87	7.38	.170	13,550	94.2
1.5	64.3	81,546	6.07	7.45	.165	13,423	93.2
1.54	62.5 or ⅝	83,055	6.23	7.50	.161	13,333	92.5
1.55	62.0	83,299	6.27	7.53	.159	13,280	92.2
1.6	59.9 or 3/5	85,125	6.47	7.61	.155	13,141	91.3
1.65	57.9	86,828	6.68	7.69	.150	13,004	90.3
1.7	56.0	88,598	6.88	7.77	.145	12,870	89.4
1.75	54.1	90,115	7.08	7.92	.141	12,626	88.3
1.8	52.4	91,680	7.30	7.95	.137	12,579	87.3
1.85	50.8	93,065	7.49	8.04	.134	12,438	86.4
1.88	50.0 or ½	94,200	7.61	8.08	.131	12,376	86.0
1.9	49.3	94,610	7.69	8.13	.130	12,300	85.4
1.95	47.9	96,100	7.89	8.22	.127	12,165	84.6
2.0	46.5	97,432	8.09	8.31	.124	12,034	83.6
2.1	44.0	100,266	8.50	8.49	.118	11,778	81.8
2.2	41.6	102,366	8.90	8.70	.112	11,494	79.9
2.28	40.0 or 2/5	104,466	9.23	8.83	.108	11,325	78.7
2.3	39.5	104,855	9.31	8.89	.107	11,249	78.2
2.4	37.6 or ⅜	107,050	9.71	9.07	.103	11,025	76.6
2.5	35.8	109,266	10.12	9.26	.099	10,799	75.0
2.6	34.2	111,400	10.52	9.45	.095	10,582	73.6
2.65	33.3 or 1/3	112,220	10.72	9.56	.093	10,460	72.6
2.7	32.6	113,244	10.93	9.65	.091	10,363	71.9
2.8	30.2	115,244	11.33	9.83	.088	10,173	70.6
2.9	29.9 or 3/10	116,855	11.74	10.04	.085	9,960	69.2

Table No. 297 (*continued*).

Actual Ratio of Expansion.	Period of Admission, or Cut-off, as a Percentage of Stroke.	Total Actual Work done by 1 lb. of 100-lb. Steam.	Net Capacity of Cylinder. Per pound of 100-lb. Steam, admitted in one Stroke.	Net Capacity of Cylinder. Per 100,000 Foot-pounds of Total Actual Work done by Steam of 100 lbs. pressure, in one Stroke.	Per Cubic Foot of Net Capacity of Cylinder. Weight of Steam of 100 lbs., Total Pressure admitted for one Stroke, per Cubic Foot.	Per Cubic Foot of Net Capacity of Cylinder. Total Actual Work done by Steam of 100 lbs., Total Initial Pressure, in one Stroke, per Cubic Foot.	Total Actual Work done per Sq. Inch of Piston, per Foot of Stroke, by 100 lbs. Steam.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
initial volume=1.	per cent.	foot-pounds.	cubic feet.	cu. feet.	pound.	foot-pounds.	foot-pounds.
3.0	28.7	118,820	12.14	10.22	.082	9,785	67.9
3.1	27.5	109,970	12.55	10.46	.080	9,560	66.5
3.2	26.4	121,386	12.95	10.67	.077	9,372	65.2
3.3	25.4	123,278	13.35	10.83	.075	9,234	64.1
3.35	25.0 or ¼	124,066	13.56	10.93	.074	9,149	63.7
3.4	24.5	125,066	13.76	11.00	.073	9,091	63.1
3.5	23.6	126,125	14.16	11.21	.071	8,961	61.9
3.6	22.7	127,450	14.57	11.43	.069	8,749	60.8
3.7	21.9	128,860	14.97	11.60	.067	8,621	59.7
3.8	21.2	130,533	15.38	11.78	.065	8,489	58.9
3.9	20.4	131,800	15.78	11.98	.063	8,347	57.9
4.0	19.7 or 1/5	132,770	16.19	12.19	.062	8,203	56.7
4.1	19.1	133,500	16.59	12.39	.060	8,071	55.9
4.2	18.5	134,900	17.00	12.60	.059	7,936	55.1
4.3	17.9	135,555	17.40	12.80	.058	7,812	54.2
4.4	17.3	136,825	17.81	13.01	.056	7,686	53.3
4.5	16.8 or 1/6	138,130	18.21	13.19	.055	7,581	52.6
4.6	10.3	139,100	18.62	13.38	.054	7,474	51.8
4.7	15.8	139,800	19.02	13.58	.053	7,364	51.1
4.8	15.3	140,920	19.43	13.79	.051	7,252	50.3
4.9	14.8	141,210	19.83	14.01	.050	7,138	49.4
5.0	14.4 or 1/7	142,180	20.23	14.23	.049	7,027	48.8
5.2	13.6	143,720	21.04	14.64	.047	6,831	47.6
5.4	12.8	145,525	21.85	15.02	.046	6,658	46.2
5.5	12.5 or ⅛	146,325	22.25	15.20	.045	6,579	45.7
5.6	12.1	147,320	22.66	15.38	.044	6,502	45.0
5.8	11.4	148,390	23.47	15.80	.043	6,329	43.8
5.9	11.1 or 1/9	148,940	23.87	16.01	.042	6,246	43.2
6.0	10.8	149,586	24.28	16.23	.041	6,161	42.7
6.2	10.3	150,630	25.09	16.63	.040	6,013	41.9
6.3	10.0 or 1/10	151,370	25.49	16.83	.039	5,942	41.3
6.4	9.7	152,033	25.90	17.04	.038	5,868	40.7
6.6	9.2 or 1/11	152,595	26.71	17.47	.037	5,724	39.8
6.8	8.7	153,810	27.52	17.89	.036	5,590	38.8
7.0	8.3 or 1/12	155,200	28.33	18.27	.035	5,473	38.1
7.2	7.9	156,330	29.14	18.64	.0343	5,365	37.3
7.3	7.7 or 1/13	156,960	29.54	18.83	.0339	5,311	36.9
7.4	7.5	157,560	29.95	19.01	.0334	5,260	36.5
7.6	7.1 or 1/14	157,975	30.76	19.47	.0325	5,136	35.7
7.8	6.7 or 1/15	158,414	31.57	19.93	.0317	5,018	34.8
8.0	6.4 or 1/16	159,433	32.38	20.31	.0309	4,923	34.2

APPENDIX TO TABLE No. 297.

TABLET OF MULTIPLIERS FOR NET CYLINDER-CAPACITY, STEAM ADMITTED, AND TOTAL WORK DONE.

For Steam of other pressures than 100 lbs. per square inch.

(See explanatory notice of the table, page 844.)

Total Pressures per Square Inch.	Multipliers.				
	For Column 3. Total Work.	For Column 4. Capacity.	For Column 5. Capacity.	For Column 6. Weight of Steam.	For Columns 7 and 8. Work.
lbs.					
65	.975	1.50	1.54	.666	.65
70	.981	1.40	1.43	.714	.70
75	.986	1.31	1.33	.763	.75
80	.988	1.24	1.25	.806	.80
85	.991	1.17	1.18	.855	.85
90	.995	1.11	1.11	.901	.90
95	.998	1.05	1.05	.952	.95
100	1.000	1.00	1.00	1.00	1.00
110	1.009	.917	.909	1.09	1.10
120	1.011	.843	.833	1.17	1.20
130	1.015	.781	.769	1.28	1.30
140	1.022	.730	.714	1.37	1.40
150	1.025	.683	.667	1.46	1.50
160	1.031	.644	.625	1.55	1.60

For column 6, of the weights of steam per cubic foot of net capacity for one stroke, the weights given for 100-lb. steam are to be multiplied by the reciprocals of the multipliers for column 4, corresponding to other pressures. Thus, for 65-lb. steam, the multiplier is the reciprocal of 1.5, or .666. For column 7, of the total actual work done, per cubic foot of net capacity, for one stroke, the work given for 100-lb. steam is to be multiplied by the reciprocal of the multiplier for column 5, as determined for the given other pressure. Thus, for 65-lb. steam, the multiplier is the reciprocal of 1.54, or .65.

For the total actual work done per square inch of piston per foot of stroke, by steam of any other pressure, the values given in column 8, for 100-lb. steam, are to be multiplied by the given pressure and divided by 100. For 65-lb. steam, for example, the multiplier is, in fact, $\frac{65}{100} = .65$, the same as for column 7. Otherwise regarded, the values in column 8 may be taken as percentages of the work for steam as admitted for the whole of the stroke, whatever the initial pressure may be.

It is apparent that the multipliers for columns 7 and 8, are simply equal to the respective total pressures divided by 100, since the multiplier for 100-lb. pressure is taken as 1. These multipliers must obviously be in the direct ratio of the pressures; and, of course, the multipliers for column 5 must be in the inverse ratio of the pressures.

COMPOUND STEAM-ENGINE.

The compound steam-engine, consisting of two cylinders, is reducible to two forms, in which, first, the steam from the first cylinder is exhausted direct into the second cylinder, as in the Woolf engine; and, second, the steam from the first cylinder is exhausted into an intermediate reservoir, whence the steam is supplied to, and expanded in, the second cylinder, as in the "receiver-engine." In the Woolf engine, according to the original type, the pistons of the two cylinders move together, and make the same strokes simultaneously, the steam from the top and the bottom of the first cylinder, being exhausted into the bottom and the top, respectively, of the second cylinder. In the receiver-engine, the pistons are connected to cranks on one shaft, at right angles to each other.

It is, in the first place, assumed that there is no clearance at either end of either cylinder; that there is no frictional resistance nor other hindrance in the engine to the flow of steam; that the pressure of expanding steam is inversely as the volume; that the expansion of the steam is continued in both cylinders to the end of the stroke; and that in the second cylinder the steam acts against a perfect vacuum on the other face of the piston. It is assumed, further, that no intermediate fall, or "drop" of pressure takes place between the first and second cylinders; that is, that the initial pressure in the second cylinder is equal to the final pressure in the first cylinder; also, that there is no loss of steam by condensation within the cylinders.

Taken generally, the work done by expansion into the second cylinder, is that due to the increase of volume of the steam in this, the second stage.

Woolf Engine—Ideal Diagrams.

The diagrams of pressure of the Woolf engine are produced in Fig. 334, as they would be described by the indicator, according to the arrows. In these ideal diagrams, pq is the atmospheric line, mn the straight line of perfect vacuum, cd the straight line of admission; the terminal lines, mc and ng, straight and perpendicular to the atmospheric line, dg the hyperbolic curve of expansion in the first cylinder, and gh the consecutive expansion-line of back pressure for the return stroke of the first piston, and of positive pressure for the steam-stroke of the second piston. At the point h, at the end of the stroke of the second piston, the steam is exhausted into the condenser, and the pressure falls to the level of perfect vacuum, mn.

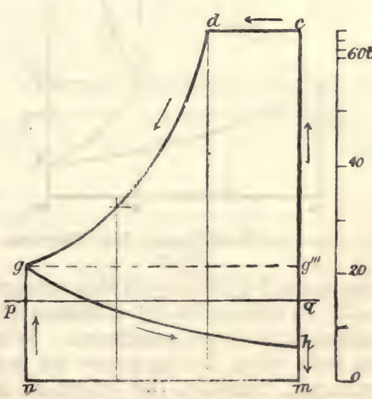


Fig. 334.—Woolf Engine:—Ideal Indicator Diagrams.

The diagram pertaining to the second cylinder, below the curve gh, is characterized by the absence of any specific period of admission; the whole of the steam-line gh being expansive, and generated by the expansion of the initial body of steam contained in the first cylinder into the second. The

initial volume—the volume of the first cylinder—is, however, gradually reduced by the advancing movement of the first piston, by which the initial steam is gradually driven into the second cylinder; until, when the stroke is completed, the whole of the steam is transferred from the first, and is shut into the second cylinder. The first cylinder, then, acts as a collapsible head to the second cylinder for each steam-stroke of the latter; the second cylinder being, at the beginning of the stroke, augmented by the capacity of the first; and at the end of the stroke, reduced to its normal dimensions. The final pressure and volume of the steam in the second cylinder are, consequently, the same as if the whole of the initial steam had been admitted at once into the second cylinder, and then expanded to the end of the stroke; the hypothetical period of admission being such a fraction of the stroke of the second cylinder as would represent the ratio of the volume of the first cylinder, which is the volume of the steam admitted, to that of the second cylinder.

The net work of the steam, according to the supposed distribution in one cylinder only, would be the same as in the two cylinders compounded. Construct a combined diagram, with a continuous expansion-line, by piecing the first upon the second of the two diagrams, Fig. 334, page 849, representing the work as if it were done in one cylinder equal in capacity to the second of the compound cylinders. For this purpose, the first diagram is to be contracted to a length bearing the same proportion to the length of the second diagram, as the volume of the first cylinder to that of

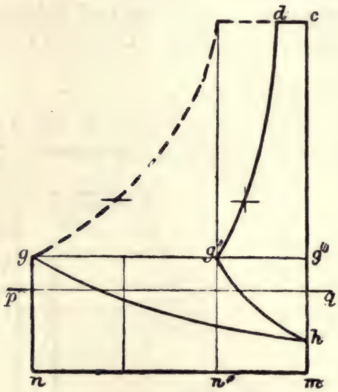


Fig. 335.—Woolf Engine:—Ideal Diagrams, reduced.

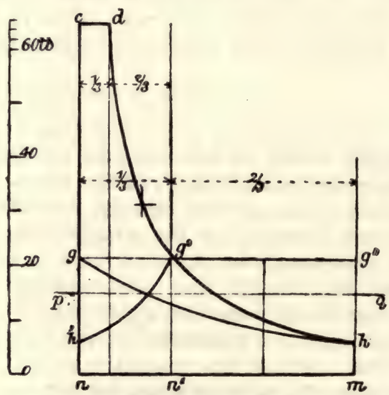


Fig. 336.—Woolf Engine:—Ideal Diagrams, combined.

the second cylinder—in this example, one-third. It is necessary to do so in order to reduce the two elements of the combined diagram to the same horizontal scale for measurement. When the strokes of the cylinders are equal to each other, the capacities are in the simple ratio of their areas. In this example, therefore, the length of the first diagram is to be reduced to one-third of the length of the second. For this purpose, let *ghmn*, Fig. 335, annexed, be the diagram from the second cylinder, over which the original diagram from the second cylinder is shown in dotting. Draw gg''' parallel to the base, and *mhc* perpendicular to the base, equal to the height of the first diagram; set off $g'''g''$ equal to one-third of the base, and upon this

reduced base $g'''g''$ complete the first diagram cdg'', by drawing cd parallel to the base, and equal to one-third of it, and the expansion-curve dg''. For the lower part of the figure, the line of back pressure $g''h$ may be described by the method of ordinates, repeated from the curve gh. Thus the contracted diagram $cdg''h$ is completed. To combine the first diagram, thus reduced to uniformity of scale, with the second diagram, let again $ghmn$, Fig. 336, annexed, be the second diagram, and describe the first diagram as reduced, in a reversed position $cdg''h'$, at the head of the second diagram, the same letters of reference being used. Finally, continue the hyperbolic expansion-line dg'' to the end of the stroke at h. Then the area $gg''h$, is equal to the area $gg''h'$, and, when substituted for it, completes the regular indicator diagram $cdhmn$, with a continuous expansion-line $dg''h$. The substitution is, in fact, necessary, since the lower part of the first diagram partly overlaps the second diagram.

According to this combination, the upper part of the diagram, Fig. 336, above the curve gh, namely, $cdhg$, represents the diagram, as contracted, for the first cylinder, modified in form, but unaltered in area; and the lower part, below the curve gh, remains unaltered, both in form and in area, as the diagram for the second cylinder. The combined diagram, as a whole, exactly measures the whole net work done in both cylinders, and is such as would be formed by admitting and expanding the same quantity of steam in one cylinder having the dimensions of the second cylinder, with the period of admission, cd, equal to one-third of the capacity of the first cylinder, or one-ninth of the capacity or the stroke of the second cylinder.

It follows further, that the work effected by expansion into the second cylinder of the Woolf engine,—that is, the total work arising from expansion against the second piston, plus the gain of work in the first cylinder by the gradual reduction of back pressure in accordance with the expansion,—is equal to that which would be effected by delivering the whole of the steam into the second cylinder before expansion is commenced, as in the receiver-engine. By this distribution, the upper part of the combined diagram, Fig. 336, cut off by the horizontal line gg'', would measure the net work of the first cylinder, as there would be a uniform back pressure equal to ng on the piston; and the lower part of the diagram, below gg'', would measure the work of the second cylinder, with a period of admission equal to gg'',—the capacity of the first cylinder at the pressure ng,—and with expansion to the end of the stroke.

To exemplify the foregoing conclusions, under the conditions originally stated, suppose that the steam is admitted to the first cylinder at a total initial pressure of 63 lbs. per square inch; that the areas of the first and second cylinders are respectively 1 and 3 square inches, and that the common length of stroke is 6 feet. The steam being cut off in the first cylinder at one-third of the stroke for the period of admission, cd, Fig. 335, page 850, it is expanded to three times its initial volume, and to one-third of the initial pressure, namely, ng, equal to 21 lbs., at the end of the stroke. The steam is admitted to the second cylinder at the same pressure, ng, and is expanded there to three times the volume it acquired in the first cylinder, or to $3 \times 3 = 9$ times the initial volume in the first cylinder. At the same time, the pressure is reduced in the second cylinder to one-third of the final pressure in the first cylinder, or to one-ninth of the initial pressure there, namely, to 7 lbs. per square inch, measured by mh, Fig. 335.

The work of the compound engine may be calculated from the combined diagram, Fig. 336, page 850; regarding the upper part of the figure, above the line gg'', as the net work of the first cylinder, according to the equivalent distribution mentioned at page 851, where the action is compared to that of a receiver-engine; and the lower part, below gg'', as the work of the second cylinder, with a period of admission equal to gg'', and an expansion to the end of the stroke. For the first section, the total work, over the base nn'', is calculated, and the work of the pressure, ng, as back pressure, on the same base, is deducted from it to give the net work. Now, the total pressure, nc, calculated on the area of the second cylinder, is 63 lbs. × 3 square inches = 189 lbs.; and the period of admission, cd, is one-ninth of the stroke, or $^2/_3$ foot. The total initial work is, then, $189 \times \frac{2}{3} = 126$ foot-pounds, and the total work, with an expansion of three times for the stroke, gg'', 2 feet, is

$$126 \times (1 + \text{hyp log } 3) = 264.4236 \text{ foot-pounds.}$$

The work of the back pressure, ng, or $21 \times 3 = 63$ lbs., into the stroke, gg'', or 2 feet, is 63 lbs. × 2 feet = 126 foot-pounds, and the net work, above the line gg'', is $264.4236 - 126 = 138.4236$ foot-pounds.

For the second section, according to the equivalent distribution, the initial work is equal to that of the back pressure on the first piston, which has just been calculated, namely, 126 foot-pounds, and the work for an expansion of three times, through the stroke, nm, is found by what is only a repetition of the calculation for the upper section, to be 264.4236 foot-pounds.

The sum of the two sections is the total net work of the two cylinders; thus:—

Upper section,................	138.4236	foot-pounds.
Lower section,................	264.4236	"
Total net work,.........	402.8472	"

Otherwise, the combined diagram, Fig. 336, represents the whole of the work as if it were done in one cylinder equal in capacity to the second cylinder—assumed, in this instance, to have the same diameter and stroke. The period of admission, cd, is one-ninth of the stroke, or $6 \div 9 = \frac{2}{3}$ foot; the initial pressure being 63 lbs. × 31 = 89 lbs., and the initial work $189 \times \frac{2}{3} = 126$ foot-pounds. The whole work of the stroke is, therefore,

$$126 \times (1 + \text{hyp log } 9) = 126 \times 3.1972 = 402.8472 \text{ foot-pounds;}$$

as was calculated before.

Receiver-Engine—Ideal Diagrams.

The hypothetical distribution which has been described for the Woolf engine, according to which all the steam with which the second cylinder is charged, is supposed to be admitted into the second cylinder before expansion begins, is that which actually takes place in the receiver-engine,—the second general combination of the compound engine,—in which the pistons of the two cylinders are connected to cranks at right angles to each other, on the same shaft, with an intermediate receiver. The receiver is occupied by steam exhausted from the first cylinder, and it supplies steam to the

second cylinder, in which it is cut off and then expanded to the end of the stroke. On the assumption that the initial pressure in the second cylinder is equal to the final pressure in the first cylinder, and, of course, equal to the pressure in the receiver, the volume cut off in the second cylinder must be equal to the volume of the first cylinder, for the second cylinder must admit as much at each stroke as is discharged from the first cylinder.

For illustration, suppose again that the areas and capacities of the first and second cylinders, with the same length of stroke, are as 1 to 3, and that the steam is cut off at one-third of the stroke, and equally expanded in both cylinders, the ratio of expansion in each cylinder being thus equal to the ratio of the capacities of the cylinders. With this distribution, the volume admitted to the second cylinder is equal to the volume discharged from the first cylinder, and there is no intermediate fall of pressure. The ideal diagrams of pressure which would thus be formed are shown in juxtaposition in Fig. 337. Here, *pq* is the atmospheric line, *cd* is the line of admission, and *hg* the exhaust-line for the first cylinder, both of them being

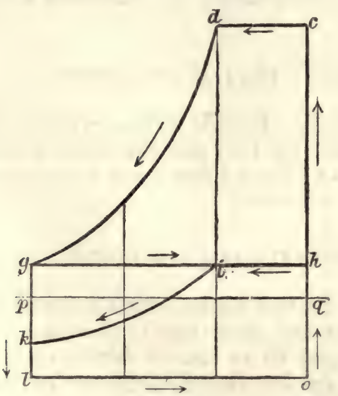


Fig. 337.—Receiver-Engine:—Ideal Indicator Diagrams.

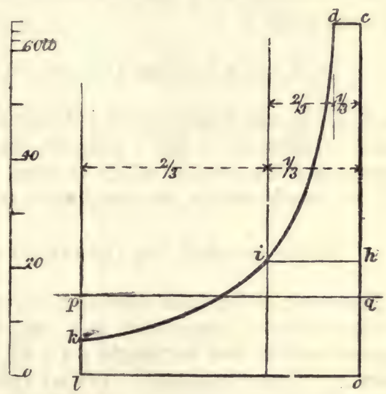


Fig. 338.—Receiver-Engine:—Ideal Diagrams, reduced and combined.

parallel to the atmospheric line; and *dg* is the expansion-curve. In the region below the exhaust-line of the first cylinder, between it and the line of perfect vacuum, the diagram of the second cylinder is formed; *hi*, the second line of admission, coincides with the exhaust-line *hg* of the first cylinder, and thus shows that, in the ideal diagrams, there is no intermediate fall of pressure. The line of perfect vacuum, *ol*, is parallel to the atmospheric line, and *ik* is the expansion-curve. The arrows indicate the order in which the diagrams are formed.

The expansive working of the steam, though clearly divided into two consecutive stages, is, as in the Woolf engine, essentially continuous from the point of cut-off in the first cylinder to the end of the stroke of the second cylinder, where it is delivered to the condenser; and the first and second diagrams may be placed together and combined to form a continuous diagram. For this purpose, take, as was done for the Woolf engine, the second diagram as the basis of the combined diagram, namely, *hiklo*, Fig. 338, adding the atmospheric line, *pq*. The period of admission, *hi*, is one-third of the stroke, and as the ratios of the cylinders are as 1 to 3, *hi*

is also the proportional length of the first diagram as applied to the second. Produce *oh* upwards, and set off *oc* equal to the total height of the first diagram above the vacuum line; and, upon the shortened base *hi*, and the height *hc*, complete the first diagram with the steam-line *cd*, and the expansion-line, *di*.

By this construction, the regular indicator diagram *cdklo* is formed, as applied to the second cylinder, and it measures the whole net work done in both cylinders:—the upper section, *cdih*, being the measure of the net work done in the first cylinder, and the lower section, *hiklo*, being the measure of the work of the second cylinder.

Resuming the data supplied for exemplifying the Woolf engine, with reference to the ideal diagrams for the receiver-engine, Fig. 337, let the areas of the first and second cylinders be respectively 1 and 3 square inches, the stroke 6 feet, and the initial pressure in the first cylinder 63 lbs. per square inch. The steam being cut off at one-third of the stroke of the first cylinder, the final pressure is 21 lbs., and the total initial work therein is equal to $oc \times cd = 63$ lbs. $\times$ 2 feet $= 126$ foot-pounds; for the whole stroke the total work is

$$126 \times (1 + \text{hyp log } 3) = 126 \times 2.0986 = 264.4236 \text{ foot-pounds,}$$

the same as was found for the Woolf engine. For the second cylinder, the initial pressure is 21 lbs. $\times$ 3 square inches $= 63$ lbs., and the initial work is represented by $oh \times hi$, which is equal to 63 lbs. $\times$ 2 feet $= 126$ foot-pounds. For the whole stroke, the total work is

$$126 \times (1 + \text{hyp log } 3) = 126 \times 2.0986 = 264.4236 \text{ foot-pounds.}$$

The work of the back pressure, *oh*, on the first piston, which is continued for the whole of the stroke, is to be deducted from this total work; it is represented by the rectangle $oh \times hg$, equal to 21 lbs. $\times$ 6 feet $= 126$ foot-pounds. Then $(264.4236 - 126 =)$ 138.4236 foot-pounds is the net or effective work for the second cylinder for one stroke. This, the work for the second cylinder, is to be added to the work for the first cylinder, and the sum, 402.8472 foot-pounds, is the united work for one stroke of the two cylinders.

This is the same quantity of work as was calculated from the Woolf diagrams.

It is obvious that the two combined diagrams, Figs. 336 and 338, pages 850 and 853, are identical in form and development.

Work of Steam as affected by Intermediate Expansion.

That the work of expanding steam is to be calculated from the expansion upon a moving piston only, is obvious enough when it is considered that the steam may expand into an intermediate receiver, and into intermediate passages, without doing any work on a piston, whilst at the same time the pressure falls or "drops" as the volume is enlarged. Under these circumstances, the second cylinder receives the steam at a lower pressure and in larger volume than it has when there is no intermediate expansion and fall of pressure; and there is less work done, whilst the ratio of active expan-

sion is necessarily reduced. If the second cylinder, however, be enlarged in capacity, in proportion to the enlargement of the volume of the steam and the fall of pressure, by intermediate expansion, the ratio of expansion, and the work done in it, would remain the same.

Whilst, then, there is no reduction of work consequent on intermediate expansion of the steam, provided that the ratio of expansion originally designed be maintained by means of a second cylinder of suitably large capacity; there is actually a reduction of work, or loss of effect, by such intermediate expansion, when the capacity of the second cylinder remains the same.

Intermediate Expansion in the Woolf Engine.

To proceed with the investigation of such loss by intermediate expansion as is suffered in the Woolf engine, take the example of Woolf engine already treated, with the same proportions and dimensions, and suppose that the total capacity of the passages from the first to the second cylinder is one-third, or $33\,^1/_3$ per cent., of the capacity of the first cylinder. The ideal diagrams Fig. 335, and the combined diagram Fig. 336, page 850, are reproduced, partly in dot-lining, in Figs. 339 and 340 annexed, the same letters of reference being applied. To these are added the modifications introduced by the intermediate fall of pressure. The admission and expansion of the steam in the first cylinder are indicated, as

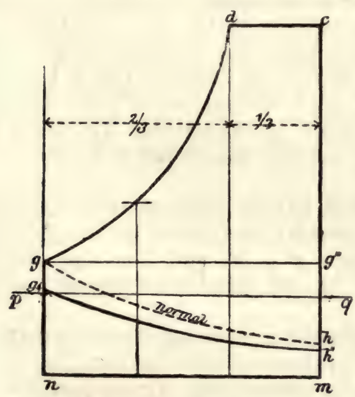


Fig. 339.—Woolf Engine:—Diagrams showing intermediate fall of pressure.

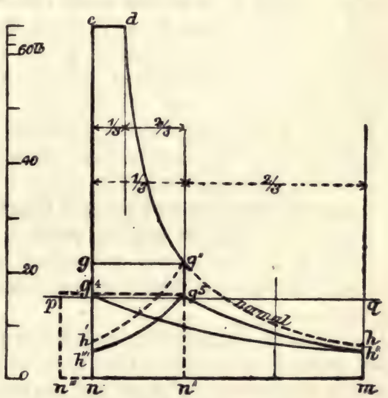


Fig. 340.—Woolf Engine:—The same diagrams reduced and combined.

before, by the straight line *cd*, representing an initial pressure of 63 lbs. per square inch, and the curve *dg*, or *dg″*, with a terminal pressure, *ng*, or *n″g″*, of 21 lbs. But when the exhaust is opened, at the end of the stroke, *g* or *g″*, the steam expands into the intermediate space, and occupies a total volume equal to $1\,^1/_3$ or $^4/_3$ times the capacity of the first cylinder, before the second piston commences its stroke. The final pressure is, at the same time, reduced, in the inverse ratio, to $\frac{3}{4}$ths of 21 lbs., or 15.75 lbs. from *ng* to ng^4, or from $n''g''$ to $n''g^5$, Fig. 340. With this lower pressure, and the augmented initial volume, $1\,^1/_3$ times the capacity of the first cylinder, the

steam expands into the second cylinder, and acquires a final volume by expansion equal to the capacity of the second cylinder plus the intermediate space, or $3\,^1/_3$ times the capacity of the first cylinder. Hence the ratio of expansion into the second cylinder is $\frac{3\,^1/_3}{1\,^1/_3} = 2.5$; and the final pressure mh'', is (15.75 lbs. $\div 2.5 =$) 6.3 lbs. per square inch. The actual curve of expansion, $g^4 h''$, is, like the normal curve gh, an elongated hyperbolic curve—an elongation of the hypothetical expansion-curve, $g^5 h''$, which flows from the augmented initial volume, as illustrated in the annexed Fig. 341, in which those curves are reproduced, and in which the augmented initial volume is measured by the extension, $nn'''n'$, of the base-line; this extension comprises $n'n'''$, the capacity of the first cylinder, and $n'''n$, the capacity of the intermediate space, one-third of $n'n'''$; making together $1\,^1/_3$ times the capacity of the first cylinder. At the end of the stroke of the second cylinder, when the piston has arrived at m, the first piston has arrived at n''', leaving the clear interval of intermediate space, $n'''n$, open to the second cylinder, which, added to the capacity nm of this cylinder, makes $n'''nm$, the final volume of the steam expanded into the second cylinder, equal to $3\,^1/_3$ times the capacity of the first cylinder. Of this total volume, the section $n'''n''$, $1\,^1/_3$ times the capacity of the first cylinder, is the hypothetical period of admission, composed of the intermediate space $n'''n$, and the capacity of the first cylinder nn''; the remaining section $n''m$ being the hypothetical period of expansion. The curve of back-pressure, $g'\,h'''$, on the first piston, has the same initial and final pressures as the expansion-curve $g^4\,h''$.

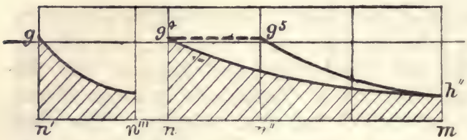


Fig. 341.—Woolf Engine:—Expansion Curves for first and second cylinders.

In piecing the first and second diagrams, to form the combined diagram, Fig. 340, the triangular area of positive pressure on the first piston, $g^4 g^5 h'''$, is replaced by the equal triangular area $g^4\,g^5\,h''$, and thus the united work of the two cylinders is indicated by the seven-sided diagram cdg'' $g^5\,h''\,m\,n$.

The effect of the intermediate fall of pressure in reducing the performance of the expanding steam, is clearly shown by the combined diagram, Fig. 340, in which the section $g''\,h$ of the normal expansion-curve $d\,g''\,h$, is replaced by the lower expansion-curve $g^5\,h''$, with the vertical line $g''g^5$ denoting the intermediate fall of pressure. The four-sided area $g''g^5h''h$ expresses the net loss of useful expansive work caused by the intermediate fall: being the balance of loss after deducting the gain by the reduction of back-pressure on the first piston, measured by the area $g''g^5h'''\,h'$, from the loss of pressure on the second piston, measured by the area, $gg^4\,h''\,h$, shown in both the Figs. 339 and 340.

The loss of work by expansion of steam in the Woolf engine, into an intermediate space between the first and second cylinders, may be shown and calculated in the same way for other volumes of intermediate space—say, one-half more, and as much more as the capacity of the first cylinder. Take all four cases, as follows:—

Intermediate Space.	Ratios of Expansion.		Combined Ratio.
1st case:—Nil	1st cylinder	1 to 3	
	2d "	1 to 3	1 to 9
2d case:— $^1/_3$ capacity of 1st cylinder	1st cylinder	1 to 3	
	2d "	1 to 2.5	1 to 7.5
3d case:—½ " "	1st cylinder	1 to 3	
	2d "	1 to 2 $^1/_3$	1 to 7
4th case:—1 " "	1st cylinder	1 to 3	
	2d "	1 to 2	1 to 6

Applying these ratios to the initial steam admitted to the first cylinder, the total initial work is, as before, 126 foot-pounds, and the total net work for one stroke of the two cylinders is as follows:[1]—

	Foot-pounds.
1st case:—$126 \times (1 + \text{hyp log } 9)$ or $3.1972 =$	402.8472
2d case:—$126 \times (1 + \text{hyp log } 7.5)$ or $3.0149 =$	379.8774
3d case:—$126 \times (1 + \text{hyp log } 7)$ or $2.9459 =$	371.1834
4th case:—$126 \times (1 + \text{hyp log } 6)$ or $2.7918 =$	351.7668

Intermediate Expansion in the Receiver-Engine.

With respect to the loss by intermediate expansion and fall of pressure in the receiver-engine, take examples based on the same data as have been applied to the discussion of the Woolf engine; and suppose, in the first instance, that the steam is expanded in the receiver into $1^1/_3$ times, or four-thirds of, its volume when exhausted from the first cylinder, the pressure being proportionally reduced to three-fourths of the final pressure in the first cylinder, prior to its being admitted into the second cylinder. With this modification, the action of the steam is represented diagrammatically in Figs. 342 and 343 annexed; the first for the two cylinders separately, the second being the combined diagram. These diagrams are fundamentally the same as the ideal or normal diagrams, Figs. 337 and 338, page 853, constructed for the receiver-engine without any intermediate fall of pressure, and the same letters of reference apply to the same parts. The admission-line *c d*, one-third of the stroke, and the expansion-curve *d g*, for the first cylinder, Fig. 337, are the same as in the normal diagram, showing an expansion of three times; the initial pressure, *o c*, being 63 lbs. per square inch, and the final pressure, *l g*, being equal to 21 lbs. on the square inch area of piston. At the end of the stroke, the pressure of the steam, as it is exhausted into the receiver, falls one-fourth, to 15.75 lbs., measured by *l g'*, which gives the level of the back pressure on the first piston, *g' h'*, parallel to *g h*. The steam exhausted from the first cylinder is at the same time expanded to $1^1/_3$ times the capacity of the cylinder. A volume of steam $1^1/_3$ times the first cylinder must, therefore, be admitted

[1] These calculations, as well as others which are quoted in this section on compound engines, are detailed at length in a work on the steam engine, by the author, now in course of preparation.

to the second cylinder, of the reduced pressure 15.75 lbs. per square inch. The length *h′ i′*, set off on the line *h′g′*, the measure of the enlarged volume, is the period of admission into the second cylinder, and it is $1^1/_3$ times *h i*, or $1^1/_3$ thirds of the length of the stroke *h′g′*, or 2⅔ feet. The point *i′*, in fact, obviously lies in the normal curve of expansion *i k;* and from this point to the end of the stroke, at *k*, the two curves of expansion, namely, the normal curve *i k*, and the new curve *i′ k*, so far as it extends, are identical, with a terminal pressure, *l k*, of 7 lbs. per square inch. The outline of the combined diagram is, then, *c d i n i′ k l o.*

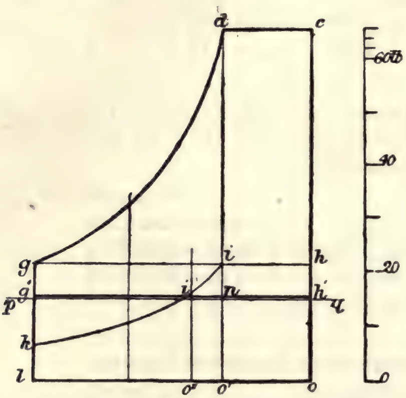


Fig. 342.—Receiver-engine:—Diagrams showing Intermediate Fall of Pressure.

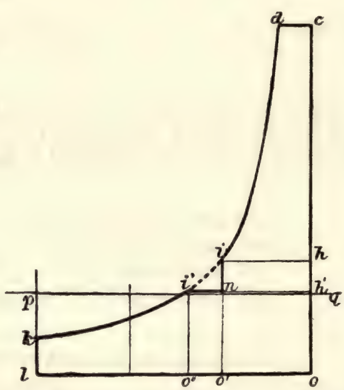


Fig. 343.—The same diagrams reduced and combined.

The rate of expansion in the second cylinder, according to this distribution, is not so high as in the first cylinder—the initial volume being $1^1/_3$ times the first cylinder, and the final volume being the capacity of the second cylinder, or 3 times the first cylinder. The ratio of expansion is, therefore, $\frac{3}{1^1/_3} = 2.25$.

On the first diagram, Fig. 342, it appears that, whilst there is a gain of net work to the first cylinder when compared with the normal performance, measured by the drop of pressure, *g g′*, for the whole stroke *g h;* there is, on the contrary, a loss of work to the second cylinder, measured by the same drop of pressure, *h h′*, for the area *h i i′ h′*. The net loss is directly indicated on the combined diagram, Fig. 343, in which the gain in the first cylinder is measured by the rectangle *h i n h′*, and the loss in the second cylinder by the trapezoid *h i i′ h′*. The difference of these, the small triangular area *i i′ n*, is the measure of the net loss.

Taking four cases for comparison, corresponding to those calculated for the Woolf engine, the results of calculation are as follows:[1]—The augmented initial volumes for expansion in the second cylinder, and the actual ratios of expansion in the two cylinders, are,—

[1] These calculations, as well as others which are quoted in this section on compound engines, are detailed at length in a work on the steam engine, by the author, now in course of preparation.

	Augmented Initial Volume in Parts of the First Cylinder.	Ratios of Expansion.	Combined Ratio.
1st case.........	1	1st cylinder,... 1 to 3 2d do. ... 1 to 3	1 to 9
2d case.........	1⅓	1st cylinder,... 1 to 3 2d do. ... 1 to 2.25	1 to 6.75
3d case.........	1½	1st cylinder,... 1 to 3 2d do. ... 1 to 2	1 to 6
4th case.........	2	1st cylinder,... 1 to 3 2d do. ... 1 to 1.5	1 to 4.5

The net works are calculated in terms of the initial work, 126 foot-pounds, and the combined ratios of expansion, with an allowance for the net work acquired by the first cylinder due to the intermediate fall of pressure:—

		foot-pounds.
1st case:— $126 \times (1 + \text{hyp log } 9)$	or $3.1972 =$	402.8472
2d case:— $126 \times (1\frac{1}{4} + \text{hyp log } 6.75)$	or $3.1595 =$	398.0970
3d case:— $126 \times (1\frac{1}{3} + \text{hyp log } 6)$	or $3.1251 =$	393.7542
4th case:— $126 \times (1\frac{1}{2} + \text{hyp log } 4.5)$	or $3.0041 =$	378.5166

By comparison, it appears that the loss of work by intermediate fall of pressure, is less in the receiver-engine than in the Woolf engine; being only 6 per cent. of loss in the former, as against 12.7 per cent. in the latter, when the pressure falls to half the final pressure in the first cylinder.

Work of the Woolf Engine, with Clearance.

Let Figs. 344 and 345 represent the diagrams of pressure from the first and second cylinders of a Woolf engine having the same dimensions, pro-

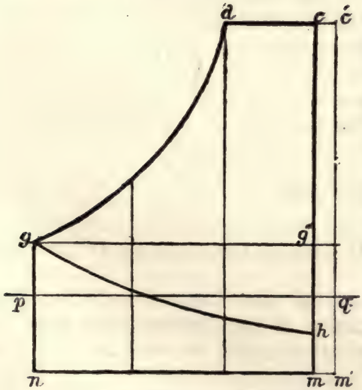


Fig. 344.—Woolf Engine:—Diagrams with Clearance.

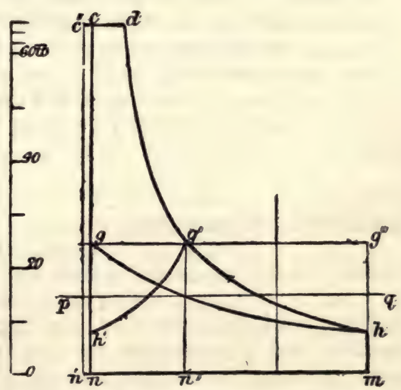


Fig. 345.—Woolf Engine:—The same diagrams reduced and combined.

portions, and letters of reference, as in the preceding examples, with the addition of a clearance at each end of the first cylinder, measured by cc'

or *m m'*, equal to 7 per cent. of the stroke, or .42 foot. The rectangular clearance space *c c' m' m*, measures the passive work of the clearance, or the product of the pressure *m c* by the period of the clearance *c c'*.

As the steam is cut off at a third, or $33^1/_3$ per cent., of the stroke, the actual ratio of expansion is (see page 828) $\frac{100+7}{33^1/_3+7} = 2.653$. The initial pressure being 63 lbs., as before, the final pressure, *n g*, is $\frac{63}{2.653} = 23.75$ lbs.; and the final volume, taking the working capacity of the first cylinder as 1, is 1 + (7 per cent.), or 1.07. If there be no more clearance, or no intermediate space, between the first and second cylinders, the initial volume for expansion into the second cylinder is 1.07; and the final volume is 3. The ratio of expansion in it is, therefore, $\frac{3}{1.07} = 2.804$; and the final pressure, *m h*, is $\frac{23.75}{2.804} = 8.47$ lbs. per square inch.

In view of these pressures, it is apparent that, whilst the work of admission during the period *c d* is the same as it was when there was no clearance, namely, 126 foot-pounds, the work by expansion is greater, for the final pressures are respectively as follows:—

	With no Clearance.	With Clearance.
In the first cylinder,......	21 lbs. per sq. inch.	23.75 lbs. per sq. inch.
In the second cylinder,...	7 „ „	8.47 „ „

If it were practicable to construct the compound cylinder so that there should not be any intermediate clearance, the employment of a smaller cylinder, as a prefix to a given cylinder, for receiving the charge of steam to be expanded through both cylinders, would have the economical effect of reducing the percentage of end-clearance, measured in parts of the larger cylinder. But it is not practicable to do so; and it remains to trace the influence of intermediate space combined with the initial clearance of the first cylinder, on the action and work of the steam in the second cylinder.

Take, as before, four cases, and suppose that the volume of the intermediate space, including what is technically the clearance of the second cylinder, is a simple fraction of the capacity of the first cylinder plus its clearance, 7 per cent., or of 1.07 times the capacity of the first cylinder, as follows:—

for the 1st, 2d, 3d, 4th case,
the intermediate spaces are,
0, $^1/_3$, ½, 1, part of the capacity of the first cylinder plus its clearance; or they are,
0, .357, .535, 1.07 of the capacity of the first cylinder. Add to these 1.07, the capacity of the first cylinder plus its clearance; and the sums are the total initial volumes for expansion in the second cylinder,
1.07, 1.427, 1.605, 2.14, times the capacity of the first cylinder. Again, to the same values of the intermediate space, add 3, the capacity of the second cylinder; and the sums are the final volumes by expansion in the second cylinder,
3.0, 3.357, 3.535, 4.07 times the capacity of the first

cylinder. The ratios of expansion in the second cylinder are the quotients of the final by the initial volumes:—

2.804, 2.352, 2.202, 1.902, ratios of expansion. The intermediate falls of pressure are, in parts of the final pressure in the first cylinder,

0, ¼, ⅓, ½ of the final pressure; or, putting the final pressure equal to 23.75 lbs., as was found, they are

0, 5.94, 7.92, 11.87 lbs. per square inch. The initial pressures for expansion in the second cylinder are,

1, ¾, ⅔, ½ of the final pressure in the first cylinder; or

23.75, 17.81, 15.83, 11.87 lbs. per square inch; and the final pressures in the second cylinder are,

8.47, 7.57, 7.19, 6.24 lbs. per square inch.

The combined ratios in the four cases are as follows:—

			Combined Ratio.
1st case:—	1st ratio of expansion,.....	1 to 2.653	
	2d do.	1 to 2.804	1 to 7.438
2d case:—	1st ratio of expansion,.....	1 to 2.653	
	2d do.	1 to 2.352	1 to 6.241
3d case:—	1st ratio of expansion,.....	1 to 2.653	
	2d do.	1 to 2.202	1 to 5.843
4th case:—	1st ratio of expansion,.....	1 to 2.653	
	2d do.	1 to 1.902	1 to 5.046

The initial work of the steam of 63 lbs. total pressure, admitted into the first cylinder, for 2 feet of the stroke, and with a clearance of 7 per cent., or .42 feet, is as follows:—

Work done on the piston, ... 63 lbs. × 2 feet = 126 foot-pounds.
Work done in the clearance,... 63 lbs. × .42 foot = 26.46 „

Total initial work of the steam, 63 lbs. × 2.42 feet = 152.46 „

This sum is the initial work on which the work by expansion is calculated; whilst it is 26.46 foot-pounds in excess of the initial work done on the piston. The total work is, then, calculated as follows:—

		Net Work in Foot-pounds.
1st case:—152.46 × (1 + hyp log 7.44) or 3.0069 =	458.27	
less, work in initial clearance,..........	26.46	431.81
2d case:—152.46 × (1 + hyp log 6.24) or 2.8310 =	431.47	
less	26.36	405.11
3d case:—152.46 × (1 + hyp log 5.84) or 2.7647 =	421.35	
less	26.36	394.99
4th case:—152.46 × (1 + hyp log 5.05) or 2.6194 =	399.29	
less	26.36	372.93

The calculations have been made in this form for the sake of comparison with those that were made for the work when there was no clearance (page 857). They can be made more directly by means of the formula (7) at page 828. The reductions of net work, in the 2d, 3d, and 4th cases, are successively 6.2, 8.6, and 13.7 per cent. of the work in the 1st case.

Work of the Receiver-Engine, with Clearance.

For the work of receiver-engines, with clearance, taken at 7 per cent., at each end of the stroke of each cylinder, the annexed Fig. 346 shows the diagrams of pressure, using the same data and letters of reference as in Fig. 342, p. 858, with the clearance measured by cc', hh', or oo', equal to 7 per cent. of ol. The steam being cut off at $^1/_3$d, the actual ratio of expansion in the first cylinder is, as for the Woolf engine, p. 860, $\frac{100+7}{33^1/_3+7} = 2.653$; and the final pressure, lg, is $\frac{63}{2.653} = 23.75$ lbs., which is also the pressure in the receiver, when there is no intermediate fall of pressure. The same is the initial pressure oh, in the second cylinder, with the clearance oo'. The volume admitted into the second cylinder is equal to the capacity of the

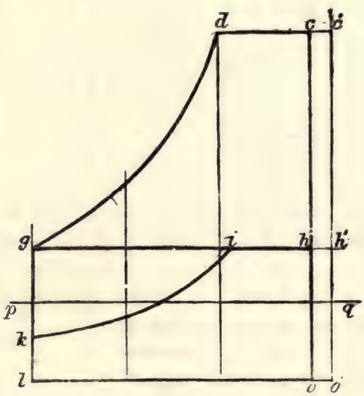


Fig. 346.—Receiver-engine:—Diagrams with Clearance.

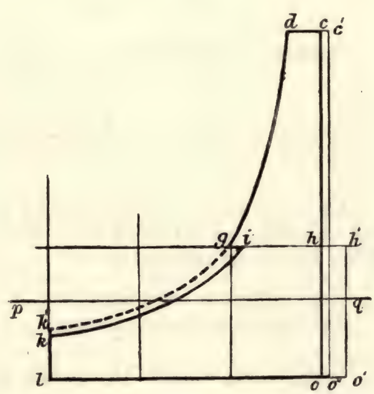


Fig. 347.—Receiver-engine:—The same diagrams reduced and combined.

first cylinder plus its clearance, or to one-third of the capacity of the second cylinder plus its clearance; that is, to one-third of 107 per cent., or $35^2/_3$ per cent., which consists of the clearance, 7 per cent., and ($35^2/_3 - 7 =$) $28^2/_3$ per cent. of the stroke of the second cylinder. The steam admitted into the second cylinder thus occupies less than one-third of the stroke, by $4^2/_3$ per cent., as indicated by the length of the period of admission, hi, in the diagram. As the steam is expanded from the capacity of the first cylinder plus its clearance, to that of the second cylinder plus its clearance, the ratio of expansion in the second cylinder, is necessarily equal to the ratio of the capacities of the two cylinders, which is 3; and $\frac{100+7}{28^2/_3+7} = 3$; and the final pressure, lk, is $\frac{23.75}{3} = 7.92$ lbs. per square inch.

The combined diagram, Fig. 347, shows a dislocated expansion-line, in two parts: *dg* for the first cylinder, and *ik* for the second cylinder. The first part, *dg*, is extended continuously to the end of the stroke at *k'*, and shows the loss of work caused by the excess of the volume of clearance of the second cylinder over that for the first, as measured by the area of the strip *igk'k*.

For the other three cases, of intermediate falls of pressure, respectively ¼th, ⅓d, and ½ of the final pressure in the first cylinder, the relations are as follows:—

For the 1st, 2d, 3d, 4th case
the augmented initial volumes for expansion in the second cylinder are,
1, 1⅓, 1½, 2 times the capacity of the first cylinder plus the clearance; or they are
1.07, 1.427, 1.605, 2.14 times the capacity of the first cylinder. The final volumes by expansion in the second cylinder are equal to the capacity of the second cylinder plus its clearance, or to
3.21, 3.21, 3.21, 3.21 times the capacity of the first cylinder; and the ratios of expansion in the second cylinder are
3.00, 2.25, 2.00, 1.50.
The intermediate falls of pressure are
0, ¼, ⅓, ½ the final pressures in the first cylinder; and they are actually
0, 5.94, 7.92, 11.87 lbs. per square inch. The pressure in the reservoir, and the initial pressure in the second cylinder, are
23.75, 17.81, 15.83, 11.87 lbs. per square inch; and the final pressures in the second cylinder are
7.92, 7.92, 7.92, 7.92 lbs. per square inch.

To calculate the works done in the four cases:—First, the normal net work of the first cylinder, above the level of the terminal pressure, *hig*, common to all the cases. The total initial work done on the piston is, as was found (p. 861), 126 foot-pounds, and the work of the clearance 26.46 foot-pounds. The sum, 152.46 foot-pounds, is the initial work on which the work by expansion 2.653 times is calculated; thus,

	Foot-pounds.
152.46 × (1 + hyp log 2.653) or 1.9757 =	301.21
Less work in initial clearance,.......	26.46
Total work on piston, above the vacuum line,	274.75
Deduct work of back pressure, *oh* × *hg*, or 23.75 foot-pounds × 3 inches × 2 feet,.......	142.50
Normal net work on the first piston,.............	132.25

Next, the works gained to the first piston by the intermediate falls of pressure.

For the 1st, 2d, 3d, 4th cases,
the work is expressed by the rectangles,

	0,	5.94 lbs. × 3 × 2,	7.92 lbs. × 3 × 2,	11.87 lbs. × 3 × 2,
or,	0,	35.64,	47.52,	71.22 foot-pounds.

Third, for the work done in the second cylinder, the initial work for expansion must be the same as that for the first, namely, 152.46 foot-pounds; there being the same quantity of steam. The clearance is .42 foot, and the passive work in the clearance,

for the	1st,	2d,	3d,	4th case,
is	23.75 lbs. $\times 3 \times .42$,	$17.81 \times 3 \times .42$,	$15.83 \times 3 \times .42$,	$11.87 \times 3 \times .42$,
or	29.92,	22.44,	19.95,	14.95 foot-pounds.

The works in the second cylinder are calculated from these data, with the ratios of expansion, as follows:—

		Foot-pounds.
1st case:—$152.46 \times (1 + \text{hyp log } 3) =$	319.93	
less the work in clearance,	29.92	290.01
2d case:—$152.46 \times (1 + \text{hyp log } 2.25) =$	276.10	
less the work in clearance,	22.44	253.66
3d case:—$152.46 \times (1 + \text{hyp log } 2) =$	258.13	
less the work in clearance,	19.95	238.18
4th case:—$152.46 \times (1 + \text{hyp log } 1.5) =$	214.28	
less the work in clearance,	14.95	199.33

The total net work in both cylinders for one stroke, is found by adding together the three portions of work for each case:—

		Net Work in Foot-pounds.	Ratio of Net Work.
1st case:—first cylinder above final pressure,	132.25		
intermediate,	0.00		
	132.25		
second cylinder,	290.01	422.26,	as 100
2d case:—first cylinder above final pressure,	132.25		
intermediate,	35.64		
	167.89		
second cylinder,	253.66	421.55,	as 99.8
3d case:—first cylinder above final pressure,	132.25		
intermediate,	47.52		
	179.77		
second cylinder,	238.18	417.95,	as 99.0
4th case:—first cylinder above final pressure,	132.25		
intermediate,	71.22		
	203.47		
second cylinder,	199.33	402.80,	as 95.4

Here it is seen that the reduction of the quantity of work performed for one stroke of the pistons, by intermediate falls of pressure, does not exceed 1 per cent. when the fall amounts to one-third of the final pressure in the first cylinder; and it is less than 5 per cent. even when the fall amounts to half the final pressure. The proportional reduction of work is something less with clearance,—as in this instance,—than without clearance, as exemplified at page 859.

The work for one stroke may be calculated in terms of the combined ratios of expansion for the two cylinders; making allowances for the loss by clearance, and the gain to the first cylinder by the intermediate fall of pressure. The total initial work for expansion is, as was found (page 863), 152.46 foot-pounds; and the ratios of expansion are as follows:—

			Compound Ratio.
1st case:—first ratio of expansion,	....	1 to 2.653	
second do.	....	1 to 3.000	1 to 7.959
2d case:—first ratio of expansion,	....	1 to 2.653	
second do.	....	1 to 2.250	1 to 5.969
3d case:—first ratio of expansion,	....	1 to 2.653	
second do.	....	1 to 2.000	1 to 5.306
4th case:—first ratio of expansion,	....	1 to 2.653	
second do.	....	1 to 1.500	1 to 3.979

With respect to the first case, it is obvious from an inspection of the combined diagram, Fig. 347, page 862, that the calculation of the work in terms of the initial work for expansion, and the total ratio of expansion, covers the whole area of the diagram, including the clearance-areas, thus:—

$$152.46 \times (1 + \text{hyp log } 7.959) \text{ or } 3.0743 = 468.71 \text{ foot-pounds.}$$

From this is to be deducted the work of the initial clearance in the first cylinder, 26.46 foot-pounds, represented by the area *cc′o″o*; and also the work of the excess of clearance in the second cylinder, over and above that of the clearance in the first cylinder, calculated on the pressure in the receiver. As the clearance of the second cylinder is, like that of the first, 7 per cent. of the stroke, or .42 feet, the volumes of the respective clearances are in the ratio of the capacities of the cylinders, or as 3 to 1, and are measured by the spaces *oo′* and *oo″* on the diagram. The clearance steam of the first cylinder, therefore, when transferred to the second cylinder, fills only one-third of its clearance space, measured by *oo″*, and of the pressure *oh*; and additional steam from the receiver, of the same pressure, is required to fill the remaining two-thirds of the clearance of the second cylinder, or $.42 \times \frac{2}{3} = .28$ foot, measured by *o″o′*. The work of the two clearances, to be deducted, is, then, as follows:—

Work of clearance of first cylinder, *cc′o″o*, 63 lbs. × 3 in. × .14 foot, or 63 lbs. × 1 in. × .42 foot.....	= 26.46	foot-pounds.
Excess of clearance of second cylinder, *h′o′o″*, 23.75 lbs. × 3 in. × .28 foot........................	= 19.95	do.
Work of clearances,............................	46.41	do.

The gross work of the diagram being, as above,..........	468.71	foot-pounds.
The work of clearances to be deducted is.................	46.41	do.
Net work for one stroke,................................	422.30	do.

For the 2d case, the gross work is—

152.46 × (1 + hyp log 5.969) or 2.7866 =	424.84	foot-pounds.
Deduct the work of the clearances, as above,...	49.41	do.
	378.43	do.

To this is to be added the compensatory gain by the fall of the pressure in the reservoir, which is equal to 5.94 lbs. per square inch. It is to be multiplied into the length of the stroke of the first cylinder, for the reduction of back-pressure, and the clearance of the second cylinder for the saving of passive work in the clearance. The stroke is, as reduced, 2 feet; the clearance is .42 foot, and the sum of these is 2.42 feet. Then the work of the gain is,

5.94 lbs. × 3 in. × 2.42 feet =	43.12	foot-pounds,
which is to be added to the remainder, above,...	378.43	do.
making the net work for one stroke..............	421.55	do.

The calculations of net work are similarly performed for the 3d and 4th cases, and they are all brought together for the four cases for comparison, as follows:—

		Foot-pounds.
1st case:—152.46 × (1 + hyp log 7.959) or 3.0743	= 468.71	
deduct for clearances,..........................	46.41	422.30
2d case:—152.46 × (1 + hyp log 5.969) or 2.7866	= 424.84	
deduct for clearances,..........................	46.41	
	378.43	
add for fall of receiver-pressure 5.94 lbs. × 3 in. × 2.42 feet,.........................	43.12	421.55
3d case:—152.46 × (1 + hyp log 5.306) or 2.6688	= 406.87	
deduct for clearances,..........................	46.41	
	360.46	
add 7.92 lbs. × 3 in. × 2.42 feet................	57.50	417.96
4th case:—152.46 × (1 + hyp log 3.979) or 2.3810	= 363.01	
deduct for clearances,..........................	46.41	
	316.60	
add 11.87 lbs. × 3 in. × 2.42 feet,..............	86.18	402.78

The net works thus obtained are the same as those that were deduced from the diagrams treated separately (page 864).

Comparative Work of Steam in the Woolf Engine and the Receiver-Engine.

It has been shown that the work of steam in the compound engine, when there is no clearance and no intermediate fall of pressure, is the same in amount, whether performed on the Woolf system or the receiver-system; but that, when there is an intermediate fall of pressure, with the enlargement of volume by which it is accompanied, the work done on the receiver-system is greater than that on the Woolf system; that is to say, the reduction of work by fall of pressure is less rapid with the receiver than on the Woolf system. This is apparent in the following comparative note of the performances, from which it also appears that, whilst the receiver-engine does more work, it expands the steam to a less number of times than the Woolf engine:—

	Woolf Engine (no clearance).		Receiver-Engine (no clearance).	
	Ratio of Expansion.	Net Work.	Ratio of Expansion.	Net Work.
1st case	9.0	402.85 ft.-pds.	9.0	402.85 ft.-pds.
2d case	7.5	379.88	6.75	398.10
3d case	7.0	371.18	6.0	393.75
4th case	6.0	351.77	4.5	378.52

In fact, it was found that the reduction of work in the 4th case, when the pressure fell to one-half intermediately, was about 13 per cent. in the Woolf engine, and only 6 per cent. in the receiver-engine. The apparent anomaly that the engine in which the greater expansion of steam takes place, performs a less net work, is explained by the fact that in the former,—the Woolf engine,—much of the initial work of the steam for the second cylinder is lost in the intermediate space; whilst, in the latter,—the receiver-engine,—there is no loss of this kind.

By the addition of clearance to each cylinder, equal to 7 per cent. of the stroke at each end, the actual ratios of expansion are sensibly reduced as compared with the ratios without clearance,—in the Woolf engine, from 9 to 7.4 when there was no intermediate fall of pressure, and from 6 to 5 when there was a fall of one-half. In the receiver-engine, the reduction of ratio is less than in the Woolf engine:—it is from 9 to 8 when there is no fall, and from 4.5 to 4 when there is a fall of one-half. Thus, the effect of the addition of clearance is clearly to reduce the net expansion. At the same time, it increases the net work done, as appears from the following statement:—

	Woolf Engine—7 % clearance.		Receiver-Engine—7 % clearance.	
	Ratio of Expansion.	Net Work.	Ratio of Expansion.	Net Work.
1st case	7.44	431.71 ft.-pds.	7.96	422.30 ft.-pds.
2d case	6.24	405.11 „	5.97	421.55 „
3d case	5.84	394.99 „	5.31	417.96 „
4th case	5.05	372.93 „	3.98	402.78 „

Taking only the 4th case:—in the Woolf engine, the net work is raised, by the addition of clearance, from 352 to 373 foot-pounds; and, in the receiver-engine, from 378.5 to 403 foot-pounds.

With clearance, as without clearance, it is found that the reduction of net work, by intermediate fall of pressure, is less in the receiver-engine,

where it is only 4½ per cent., with clearance, than in the Woolf engine, where it amounts to about 14 per cent., when the pressure falls intermediately one-half.

As the combined ratios of expansion in the receiver-engine are less, for each case, than in the Woolf engine; so the terminal pressures of the expanded steam in the second cylinder, on passing to the condenser, are greater in the receiver-engine than in the Woolf engine:—

For the 1st, 2d, 3d, 4th case, with 7 per cent. clearance, the terminal pressures in the second cylinder are, for the Woolf engine,

8.47, 7.57, 7.19, 6.24 lbs. per square inch,

and for the receiver-engine,

7.92, 7.92, 7.92, 7.92.

In the first case, the terminal pressure in the Woolf engine is greater than in the receiver-engine; for there was no intermediate space assumed in the former, whilst clearance-space for the second cylinder was assumed in the latter; but, in the other cases, the terminal pressures in the former fall consecutively below that of the first case. They also fall below those of the latter, which remain constant for all the cases. This constancy of terminal pressure in the second cylinder of the receiver-engine, simply follows from the fact that the terminal volume of the expanded steam is always the same,—that of the second cylinder plus the clearance,—whatever be the intermediate fall of pressure; whilst in the Woolf engine, on the contrary, the terminal volume is equal to that of the second cylinder, increased by the volume of the intermediate space, and the terminal pressure must be less as the terminal volume is increased.

As the terminal pressure in the receiver-engine is thus shown to be, in all practical cases, greater than in the Woolf engine, other conditions being the same, it directly follows that the work performed in expanding from a given initial pressure to the several terminal pressures, must be greater in the receiver-engine than in the Woolf engine.

It may be gathered from these arithmetical deductions that the receiver-engine is an elastic system of compound engine, in which considerable latitude is afforded for adapting the pressure in the receiver to the demands of the second cylinder, without considerably diminishing the effective work of the engine. In the Woolf engine, on the contrary, it is clearly of much importance that the intermediate volume of space between the first and second cylinders, which is the cause of an intermediate fall of pressure, should be reduced to the lowest practicable amount.

Supposing that there is no loss of steam in passing though the engine, by cooling and condensation, it is obvious that whatever steam passes through the first cylinder, must also find its way through the second cylinder, neither more nor less. By varying, therefore, in the receiver-engine, the period of admission in the second cylinder, and thus also the volume of steam admitted for each stroke, the steam will be measured into it at a higher pressure and of a less bulk, or at a lower pressure and of a greater bulk: the pressure and density naturally adjusting themselves to the volume permitted to escape from the receiver into the cylinder. With a sufficiently restricted admission, the pressure in the receiver may be maintained at the pressure of the steam as exhausted from the first cylinder. On the contrary, with a wider admission, the pressure in the receiver may

fall or "drop" to three-fourths, or even one-half of the pressure of the exhaust steam from the first cylinder.

There is a means of counterbalancing the loss of performance by intermediate fall of pressure, by so enlarging the second cylinder as to effect the same ultimate ratio of expansion behind the pistons, as would be effected in the originally designed engine if there were no intermediate fall. For example, when the capacities of the first and second cylinders are as 1 to 3, and the steam is cut off in each at one-third of the stroke, without any intermediate fall, the steam, if there be no clearance, is expanded into nine times its initial volume. But, when there is an intermediate fall of pressure, of, say, one-fourth of the final pressure in the first cylinder, involving an increase of volume of steam in the ratio of 3 to 4, the second cylinder must be correspondingly enlarged in the ratio of 3 to 4, in order to contain the charge of steam for expansion, when cut off, as before, at one-third of the stroke. By such enlargement of the second cylinder, in the ratio of the intermediate enlargement of the steam, the same ultimate ratio of expansion is secured, and an equivalent performance is effected. Such a remedy, when specially applied for the purpose of counterbalancing ineffective expansion of steam, involves the employment of enlarged cylinders, and entails the objections of increased weight, bulk, and cost of machinery. It would be more useful as a remedy, when applied to the Woolf engine, than to the receiver-engine.

Formulas and Rules for Calculating the Expansion and the Work of Steam in Compound Engines.

In view of the preceding discussions of the expansive working of steam in compound cylinders, the following algebraic symbols are used:—

a = the area of the first cylinder in square inches.
a' = the area of the second cylinder in square inches.
r = the ratio of the area of the second cylinder to that of the first cylinder.
L = the length of the stroke in feet, supposed to be the same for both cylinders.
l = the period of admission to the first cylinder, in feet, excluding clearance.
c = the clearance at each end of the cylinders, in parts of the stroke, in feet.
L′ = the length of the stroke plus the clearance, in feet.
l' = the period of admission plus the clearance, in feet.
s = the length of a given part of the stroke of the second cylinder, in feet.
P = the total initial pressure in the first cylinder, in pounds per square inch, supposed to be uniform during admission.
P′ = the total pressure at the end of the given part of the stroke, s.
p = the average total pressure for the whole stroke.
R = the nominal ratio of expansion in the first cylinder, or $L \div l$.
R′ = the actual ratio of expansion in the first cylinder, or $L' \div l'$.
R″ = the actual combined ratio of expansion behind the pistons, in the first and second cylinders together.
R‴ = the actual ratio of expansion, or number of volumes into which the steam occupying the first cylinder at the end of the stroke, is expanded in the second cylinder at the end of any part of the return stroke, s:—the special initial volume, or the capacity of the first cylinder, being = 1.
n = the ratio of the final pressure in the first cylinder to any intermediate fall of pressure between the first and second cylinders.
N = the ratio of the volume of the intermediate space in the Woolf engine, reckoned up to, and including the clearance of, the second piston, to the capacity of the first cylinder plus its clearance.
w = the whole net work in one stroke, in foot-pounds.

Formulas and rules may be constructed on the basis of the combined ratios of expansion behind the two pistons:—the combined ratio being the product of the actual ratios of expansion in the first and the second cylinders. When, as usually happens in practice, intermediate expansion takes place between the cylinders, if the ratio of this expansion be multiplied into the combined ratio of expansion behind the pistons; or, if the three individual ratios of expansion in the first and second cylinders, and in the intermediate space, be multiplied together, the product is the ratio of total expansion of the steam within the engine, to the end of the stroke of the second cylinder, when it is discharged into the condenser. For example, if the steam be expanded to three times its volume in the first cylinder, twice in the second cylinder, and one-and-a-half times in the intermediate space; the combined ratio of expansion behind the pistons is the product of the first and second of these; that is,—

In first cylinder,	1 to 3,	.... or 3
In second cylinder,	1 to 2,	.... or 2
Combined ratio of expansion behind pistons,	1 to 6,	.... or 6
Intermediate expansion,	1 to 1.5,	.... or 1.5
Ratio of total expansion,	1 to 9,	.... or 9

Or, the individual ratios may be placed consecutively thus:—

In first cylinder,	1 to 3,	.... or 3
Intermediate expansion,	1 to 1.5,	.... or 1.5
In second cylinder,	1 to 2,	.... or 2
Ratio of total expansion,	1 to 9,	.... or 9

Conversely, when the total expansion is given, the expansion behind the pistons may be calculated by dividing the total ratio by the ratio of intermediate expansion. Thus, if the total ratio be 9, and the intermediate ratio be 1.5, the combined ratio behind the pistons is $\frac{9}{1.5} = 6$; or 1 to 6. Generally, if the total ratio be divided by any one of the individual ratios, the quotient is the product or combination of the two others.

Further, if two ratios be equal to each other, the combined ratio is equal to the square of one of them; and, conversely, the square root of a given ratio is the value of two elementary ratios, which when combined yield the given ratio. Thus, if there be two ratios, each equal to 3, then 3×3, or $3^2 = 9$, the combined ratio formed by those two. Conversely, $(\sqrt{9} =)\ 3$ is the value of two elementary ratios which, if combined, form the ratio 9. Similarly, the two equal ratios which, when combined, form the ratio 7, are each equal to $\sqrt{7} = 2.65$; the square of 2.65, or 2.65×2.65, being equal to 7.

To find the actual ratio of expansion in the first cylinder.—This is found by the formula, page 828, when the stroke, the period of admission, and the clearance are given. It is equal to

$$\frac{L'}{l'} = R'. \quad \text{(15)}$$

That is to say, the actual ratio of expansion in the first cylinder is equal to the quotient of the length of stroke plus the clearance divided by the period of admission plus the clearance. For example, if the steam be cut off at one-third of the stroke, and the clearance be 7 per cent., the length of stroke being equal to 1; then the stroke plus the clearance is equal to 1.07, and the period of admission is equal to $.3333 + .07 = .4033$; and $\frac{1.07}{.4033} = 2.653$, the actual ratio of expansion.

To find the ratio of Intermediate Expansion.—According to the assumption that the volume of a given weight of steam is inversely as its elasticity, or its pressure per square inch, the enlargement of volume, or expansion, may be deduced from the pressures before and after expansion. Thus, if the pressure be reduced from 20 lbs. to 15 lbs. per square inch, the volume, inversely, is enlarged in the ratio of 15 to 20; and, if the initial volume be taken as 1, then, by proportion, $15 : 20 :: 1 : 1.33$; or $1 \times \frac{20}{15} = 1.33$. Thus, in reducing the pressure ¼th, the volume is enlarged ⅓d, and the ratio of expansion is 1.33.

By the notation, n is the ratio of the final pressure in the first cylinder to the intermediate fall of pressure between the first and second cylinders; or, it is the denominator of the fractional part of the final pressure, expressing the fall of pressure. When the fall is ¼th, therefore, $n = 4$; and the remaining pressure is ¾ths, and is as 3, or $n - 1$. The pressures, then, before and after the fall, are as $n : n - 1$; and, inversely, the volumes are as $n - 1 : n$. Taking the capacity of the first cylinder with its clearance, as 1, the expanded volume is found by the proportion $n - 1 : n :: 1 : \frac{n}{n-1}$; and the ratio of intermediate expansion is equal to

$$\frac{n}{n-1}. \quad \text{(16)}$$

Substituting 4 for n in this expression, the ratio of expansion in the preceding example, is $\frac{4}{4-1} = \frac{4}{3} = 1.33$, as already found.

It is necessary, in the receiver-engine, thus to reckon backwards,—from the observed pressures to the volumes,—in order to find the intermediate ratio of expansion, since the volume of the receiver affords no evidence whatever of the amount of expansion between the first and second cylinders.

The same process may, of course, be applied in the Woolf engine, to find the intermediate expansion; but the ratio of this expansion is, otherwise, exactly and directly determinable by the volume of the intermediate space. The ratio of the capacity of the first cylinder plus the clearance, to the intermediate space, is, by the notation, as 1 to N; and the sum of these,

or the enlarged volume, is as $(1+N)$. The ratio of intermediate expansion is, therefore, as 1 to $(1+N)$; or it is

$$(1+N). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

To find the value of N in terms of the intermediate fall of pressure:—The intermediate ratio was found, in terms of the ratio of pressure n, to be $\frac{n}{n-1}$; and $1+N=\frac{n}{n-1}$; so that

$$N=\frac{n}{n-1}-1. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

That is, the volume of the intermediate space relative to that of the first cylinder plus its clearance, is equal to the quotient of the final pressure in the first cylinder, divided by the reduced pressure after the fall, minus 1. For example, the final and reduced pressures being 20 lbs. and 15 lbs. respectively, $20 \div 15 = 1.33$; and $1.33 - 1 = .33$, which is the value of N, the intermediate space, relative to the capacity of the first cylinder plus its clearance, taken as 1. In this calculation, the actual values of the pressures have been used, instead of their relative values as indicated in the above expression (18); but the result is the same, for, putting for n the ratio 4, then, $\frac{4}{4-1}=\frac{4}{3}=1.33$; and $1.33-1=.33$, as before.

The capacity of the intermediate space in the Woolf engine, is found by multiplying that of the first cylinder plus its clearance by the ratio N.

To find the Ratio of Expansion in the second cylinder.—In the Woolf engine. This would be expressed by the ratio of the capacity of the first cylinder to that of the second cylinder, if there were no clearances nor other intermediate space. With clearances and intermediate space, the ratio of expansion in the second cylinder is less than that, and is equal to the ratio of the capacity of the first cylinder plus its clearance plus the intermediate space, to the capacity of the second cylinder plus the intermediate space, this last being taken to include the clearance of the first and second cylinders. Taking the capacity of the first cylinder plus its clearance as 1, that of the intermediate space is N. The capacity of the second cylinder, with its clearance, is expressed by the ratio r; without clearance, it is less than r by as much in proportion as the capacity of the cylinder is less than the cylinder plus the clearance, or as L is less than $(L+c)$, or L′. The reduced ratio is, then, $r \times \frac{L}{L'}$; and the ratio of expansion in the second cylinder is as $(1+N)$ is to $(r \times \frac{L}{L'})+N$; or

$$\text{Ratio of expansion in second cylinder} = \frac{(r \times \frac{L}{L'})+N}{1+N} \quad \ldots\ldots\ldots\ldots \quad (19)$$

That is, the ratio of the first to the second cylinder is multiplied by the length of stroke, and divided by this length plus the clearance; and the ratio of the intermediate space is added to the quotient, making a sum, say, A.

To the ratio of the intermediate space is added 1, making a sum, say, B. Sum A is divided by sum B, and the quotient is the ratio of expansion in the second cylinder. For example, let $r = 3$, $N = .333$, $L = 6$ feet, and $L' = 6$ plus 7 per cent. of 6, or 6.42. Then

$$\frac{(3 \times \frac{6}{6.42}) + .333}{1 + .333} = 2.353,$$

the ratio of expansion in the second cylinder.

In the receiver-engine. The actual ratio of expansion, in the second cylinder, is not affected by clearance, assuming, of course, that the percentage of clearance is the same as in the first cylinder. When there is no intermediate fall of pressure, the ratio of expansion is simply that of the first and second cylinders, or r. But, with an intermediate fall, this ratio is reduced as the ratio of intermediate expansion is increased, namely $\frac{n}{n-1}$; and it is as this ratio inversely, or,

$$\text{Ratio of expansion in second cylinder} = r \times \frac{n-1}{n} = \frac{(n-1)\,r}{n} \quad \ldots\ldots \ (20)$$

For example, putting the ratio of the cylinders, $r = 3$, and the ratio, n, of the intermediate fall to the final pressure in the first cylinder, $= 4$, as before; then, $\frac{(4-1)\,3}{4} = \frac{3 \times 3}{4} = 2.25$, the actual ratio of expansion in the second cylinder.

To find the total actual Ratio of Expansion as well as the combined actual Ratio of Expansion behind the two pistons. The total actual ratio of expansion is, as was stated (page 870), the product of the ratios of the three consecutive expansions:—in the first cylinder, in the intermediate space, and in the second cylinder.

For the Woolf engine. The expressions of these expansions are numbered (15), (17), and (19), and their product is as follows:—

$$\frac{L'}{l'} \times (1 + N) \times \frac{(r \times \frac{L}{L'}) + N}{1 + N}; \text{ or,}$$

$$\text{Total actual ratio of expansion} = \frac{L'}{l'} \times (r\frac{L}{L'} + N), \quad \text{or } R'(r\frac{L}{L'} + N) \ldots \ (21)$$

That is to say, the ratio, r, of the first to the second cylinder is to be multiplied by the length of stroke, and divided by this length plus the clearance; and the ratio-value of the intermediate space, N, is added to the quotient. The sum is then multiplied by the actual ratio of expansion in the first cylinder, and the product is the total actual ratio of expansion. For example, let the steam be cut off at a third of the stroke of the first cylinder, with a clearance of 7 per cent.; let the ratio, r, of the cylinders be 3, and the ratio-value, N, of the intermediate space, .333 or ⅓d. Then, the stroke of the first cylinder being = 1, the actual ratio of expansion in

it, R′, as was exemplified at page 871, is $1.07 \div .4033 = 2.653$. The modified ratio of the cylinders is $3 \times \frac{1}{1.07} = 2.804$; and $2.804 + .333 = 3.137$. Finally, $2.653 \times 3.137 = 8.322$, the total actual ratio of expansion. It may be observed, that the fraction, $\frac{1}{1.07}$, above employed, is equivalent to the fraction, $\frac{6}{6.42}$, employed for the same purpose, in the example, page 873.

The combined actual ratio of expansion behind the pistons, in the Woolf engine, is the product of the first and third of the above-cited expressions, namely (15) and (19), or,

$$\frac{L'}{l'} \times \frac{(r \times \frac{L}{L'}) + N}{1 + N} = R' \frac{(r\frac{L}{L'} + N)}{1 + N} \qquad (22)$$

That is to say, the product, as above found, for the total expansion, is to be divided by the ratio-value of the intermediate space plus 1; the quotient is the combined actual ratio of expansion behind the pistons. For example, resuming the data of the preceding example, the final product expressing the total actual ratio of expansion, was found to be 8.322; and the divisor to be applied to it, is $1 + .333 = 1.333$. Then, $\frac{8.322}{1.333} = 6.242$, the combined actual ratio of expansion behind the pistons.

For the Receiver-engine. The total actual ratio of expansion is the product of the expressions of the three consecutive expansions, numbered (15), (16), and (20); their product is as follows:—

$$\frac{L'}{l'} \times \frac{n}{n-1} \times \frac{(n-1)\,r}{n} = r\,\frac{L'}{l'} = r\,R' \qquad (23)$$

That is to say, the ratio, r, of the first and second cylinders is to be multiplied by the actual ratio of expansion, R′, in the first cylinder. The product is the total actual ratio of expansion. For example, making, as before, $r = 3$, and $R' = 2.653$, the product $(3 \times 2.653 =)\ 7.959$, is the total actual ratio of expansion.

The product of the first and third of the above three expressions, namely (15) and (20), gives the value of the combined actual ratios of expansion behind the pistons; thus,

$$\frac{L'}{l'} \times \frac{(n-1)\,r}{n} = \frac{n-1}{n}\,r\,R' \qquad (24)$$

That is to say, the ratio of the first and second cylinders is multiplied by the actual ratio of expansion in the first cylinder, and by the ratio of the intermediate fall of pressure to the final pressure in the first cylinder minus 1; and the final product is divided by this ratio simply. The quotient is the combined actual ratio of expansion behind the pistons. For example, resuming the product in the last preceding example, and taking n, the ratio of the intermediate fall of pressure $= 4$; then $3 \times 2.653 \times \frac{4-1}{4} = 7.959 \times \frac{3}{4}$ $= 5.969$, the required ratio.

To find the Work done in the two cylinders of compound engines—The Woolf engine. It has already been stated that the formula (5), page 828, for the work of steam expanded in one cylinder, applies also to the work of steam in the Woolf engine, when the combined actual ratio of expansion behind the pistons in the two cylinders, is given. Thus, the net total work for one stroke of the two pistons, quoting that formula, is,

$$w = a\,\mathrm{P}\,[l'\,(1 + \text{hyp log R}'') - c], \qquad (25)$$

RULE 1. *To find the net work done by steam in the two cylinders of a Woolf engine, for one stroke, with a given combined actual ratio of expansion.*—To the hyperbolic logarithm of the combined actual ratio of expansion behind the two pistons, add 1; multiply the sum by the period of admission to the first cylinder plus the clearance, in feet; and from the product subtract the clearance. Multiply the area of the first piston, in square inches, by the initial pressure in pounds per square inch, and by this remainder. The product is the net work in foot-pounds.

For example, let the 2d case, pages 860, 861, be calculated by this rule:—a P $= 1 \times 63 = 63$ lbs., $l' = 2.42$ feet, $c = .42$ foot, and R$'' = 6.24$. Then,

$$\begin{aligned} w &= 63\,[2.42\,(1 + \text{hyp log } 6.24) - .42] \\ &= 63\,[(2.42 \times 2.8310) - .42] \\ &= 63\,(6.851 - .42) = 405.20 \text{ foot-pounds,} \end{aligned}$$

as was before calculated, allowing for small errors of approximation.

The Receiver-engine.—A complete formula for the work of the receiver-engine necessarily comprises three elements:—First, the expression of the gross work, including the work of the clearances; second, the deduction for the passive work of the clearances; third, the addition for the gain of work by the reduction of the back pressure on the first piston when there is an intermediate fall of pressure. Beginning with the first case, pages 863, 864, in which there is no intermediate fall of pressure, the total initial work of the steam admitted to the first cylinder is expressed by $a\,\mathrm{P}\,l'$; whence the total work with expansion is

$$a\,\mathrm{P}\,l'\,(1 + \text{hyp log R}''). \qquad (26)$$

This measures the total area of the diagram, Fig. 347, page 862, including the clearances. The work of the clearance of the first cylinder, $cc'o''o$, is

$$a\,\mathrm{P}\,c.$$

The work of the clearance of the second cylinder is the rectangle $hh'o'o$, which includes the section hoo'' of the first clearance; and, deducting this, the remainder, which is the rectangle $h'o'o''$, is to be added to the first clearance. To express this remainder algebraically, the volumes of the first and second clearances, oo'' and oo', are in the ratio of the areas of the cylinders, or as 1 to r, and the volume of the difference, $o''o'$, is as $c\,(r - 1)$. The height, $o'h'$, is the final pressure in the first cylinder, and is equal to the

initial pressure divided by R′, the actual ratio of expansion in the first cylinder; or,

$$\frac{P}{R'}.$$

Therefore the work of the excess, $o''o'$, of the second clearance is,

$$a\,P\,c\frac{r-1}{R'};$$

and the two works of the clearances are together,

$$a\,P\,c\,(1+\frac{r-1}{R'}),$$

to be deducted from the gross work by expansion (26). Whence the equation for the net work, in the first case:—

$$w = a\,P\,l'\,(1+\text{hyp log }R'') - a\,P\,c\,(1+\frac{r-1}{R'});\text{ or}$$

$$w = a\,P\,[l'\,(1+\text{hyp log }R'') - c\,(1+\frac{r-1}{R'})\,],\text{.........}\quad(27)$$

when there is no intermediate fall of pressure.

Before reducing this formula to a rule, it may be remarked that it gives values which approximate closely to the true values, for cases in which there are intermediate falls of pressure—such cases as usually occur in practice;—and, for ordinary practical purposes, the results of the application of this formula will be sufficiently near to exactness. It was found, in fact (page 864), that the reductions of work by intermediate falls, as compared with the work done when there was no fall, were as follows:—When the pressure falls to

¾, ⅔, ½ of the final pressure in the first cylinder,

the reduction of work is,

0.2, 1.0, 4.6 per cent. of that in the first case.

The intermediate fall of pressure is rarely so much as two-thirds; and even with this fall the reduction of work, it is seen, only amounts to 1 per cent. The slightness of the reduction results from the fact, as was before explained, that though the actual ratio of expansion, with intermediate falls, is less than when there is no intermediate fall, yet the loss of work by such reduction of expansion is practically compensated by the gain of net work on the first piston by the fall of back pressure against it.

Adopting, then, the formula (27) as applicable for all cases of receiver-engines arising in practice, it is required only to give the actual ratio of expansion in the first cylinder, and to multiply this ratio by the ratio of the capacities of the two cylinders, to arrive at the ratio of expansion to be employed in the formula. This is literally the actual combined ratio of expansion for the first case, without intermediate fall of pressure, as was found (page 865), represented by R″ in the formula (27).

RULE 2. *To find the net work done by steam in the two cylinders of a receiver-engine for one stroke, with a given actual ratio of expansion in the first cylinder.*—Multiply the actual ratio of expansion in the first cylinder by the ratio of the two cylinders, and to the hyperbolic logarithm of the compound ratio add 1; multiply the sum by the initial period of admission to the first cylinder, plus the clearance, in feet (product A). Divide the ratio of the two cylinders, minus 1, by the actual ratio of expansion in the first cylinder; add 1 to the quotient, and multiply the sum by the initial clearance in feet (product B). Subtract product B from product A, giving the remainder C. Multiply the area of the first cylinder, in square inches, by the total initial pressure in pounds per square inch, and by the remainder C. The product is the net work in foot-pounds for one stroke.

This rule is applicable to any of the four cases, page 865: $a = 1$ square inch, $P = 63$ lbs. per square inch, $c = .42$ foot, $l' = 2.42$ feet, $R' = 2.653$, $R'' = 7.959$, $r = 3$, and hyp log $R'' = 2.0743$. Then, on the model of the given formula (27),

$$w = 63\left[2.42\,(1 + 2.0743) - .42\,\left(1 + \frac{3-1}{2.653}\right)\right]$$

$$= 63\,(7.440 - .737) = 422.29 \text{ foot-pounds,}$$

as was before calculated for the first case. Or, following the wording of the rule:—The combined actual ratio of expansion is 7.959, of which the hyperbolic logarithm is 2.0743; adding 1 to this, the sum, 3.0743, is multiplied by 2.42, the initial period of admission plus the clearance, and $3.0743 \times 2.42 = 7.440$ (product A). Again, the ratio of the cylinders is 3, and $3 - 1 = 2$; the actual ratio of expansion in the first cylinder is 2.653, and $2 \div 2.653 = .754$. Adding 1 to this quotient, the sum is multiplied by the initial clearance .42, or $1.754 \times .42 = .737$ (product B). The difference of products A and B is $(7.440 - .737 =)$ 6.703, and this, multiplied by 63 lbs., the initial pressure per square inch, and by 1, the area of the piston in square inches, gives

$$6.703 \times 63 \times 1 = 422.29 \text{ foot-pounds,}$$

the work of one stroke.

COMPRESSION OF STEAM IN THE CYLINDER.

The work expended in compressing such exhaust steam as is not permitted to escape during the return-stroke of the piston, and is shut into the cylinder against the retiring piston, is to be reckoned against the quantity of steam thus reclaimed. For every phase of the distribution there is a particular period of compression, by the adoption of which the resulting efficiency of the steam, for a given distribution, is raised to a maximum. The method of determining the best period of compression will be given in the author's work on *The Steam Engine.* The following table, No. 298, contains the best periods of compression for several periods of admission, with 7 per cent. clearance, and for several back exhaust-pressures. It is seen, by the table, that, the more expansively the steam is worked, the greater should be the period of compression—that is, the exhaust port should be closed the earlier in the course of the return-stroke; and that the greater the proportion of back-pressure to initial pressure, the less should be the period of compression.

Table No. 298.—COMPRESSION OF STEAM IN THE CYLINDER.
BEST PERIODS OF COMPRESSION:—Clearance 7 per cent.

Cut-off in Percentages of the Stroke.	Total Back-pressure, in Percentages of the Total Initial Pressures.							
	2½	5	10	15	20	25	30	35
	Periods of Compression, in parts of the Stroke.							
per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.
10	65	57	44	32	—	—	—	—
15	58	52	40	29	23	—	—	—
20	52	47	37	27	22	—	—	—
25	47	42	34	26	21	17	—	—
30	42	39	32	25	20	16	14	12
35	39	35	29	23	19	15	13	11
40	36	32	27	21	18	14	13	11
45	33	30	25	20	17	14	12	10
50	30	27	23	18	16	13	12	10
55	27	24	21	17	15	13	11	9
60	24	22	19	15	14	12	11	9
65	22	20	17	15	14	12	10	8
70	19	17	16	14	14	12	10	8
75	17	16	14	13	12	11	9	8

NOTES TO TABLE.—1. For periods of admission, or percentages of back-pressure, other than those given, the periods of compression may be readily found by interpolation.

2. For other clearances, the values of the tabulated periods of compression are to be altered in the ratio of 7 to the given percentage of clearance.

PRACTICE OF EXPANSIVE WORKING OF STEAM.

ACTUAL PERFORMANCE OF STEAM IN THE STEAM-ENGINE.

In working steam expansively, the practical performance is affected by several circumstances. There is the influence of the wire-drawing of the steam during its admission into the cylinders; of the needful opening of the exhaust passages before the end of the stroke, for the escape of the steam from the cylinder; of the back exhaust pressure on the piston, and the closing of the exhaust passage before the end of the return-stroke, with the consequent shutting in and compression by the piston of a portion of the exhausting steam. These influences have been analyzed and measured by the author. He concluded that, when the cylinders were liberally proportioned, first, the possible loss by early exhaust was of no importance, and that the early release was, on the contrary, beneficial, in facilitating a complete exhaust during the return-stroke; second, that the loss by wire-drawing was of little or no moment, and that, as wire-drawing was, to some degree, equivalent to an earlier cut-off, it might even prove advantageous in point of economy; third, that the loss by back exhaust pressure in excess of the atmospheric resistance in non-condensing engines, in good practice, is of little or no importance. These conclusions were based upon the performance of locomotives, fitted with the link-motion, and worked with steam of 100 lbs. effective pressure per square inch in the boiler; but they are applicable to all classes of steam-engine.[1]

The only obstacle to the working of steam advantageously to a high degree of expansion in one cylinder, in general practice, is the condensation to which it is subjected, when it is admitted into the cylinder at the beginning of the stroke, by the less hot surfaces of the cylinder and the piston; the proportion of which is increased with the ratio of expansion, so that the economy of steam by expansive working ceases to increase when the period of admission is reduced down to a certain fraction of the stroke, and that, on the contrary, the efficiency of the steam is diminished as the period of admission is reduced below that fraction. The initial condensation here pointed out, is succeeded by the re-evaporation of a portion of the condensed steam during the later portion of the period of expansion; because, as the pressure falls, the temperature of the steam, and of the water which it contains, also falls, until it ultimately descends below the actual temperature of the cylinder, when the heat of the cylinder is absorbed by the water, and

[1] See *Railway Machinery*, 1855, pp. 69–99; also a paper on "The Expansive Working of Steam in Locomotives," in the *Proceedings of the Institution of Mechanical Engineers*, 1852, pp. 60–82, and 109–128.

evaporation takes place. The author, in 1851, experimentally demonstrated the existence of this condensation in the cylinders of locomotives. Its reality and importance are now thoroughly understood and admitted.[1] He deduced from his experiments that, in jacketless cylinders, imperfectly protected, the quantity of steam condensed amounted to from 11 to 42 per cent. of the whole of the steam admitted to the cylinders, according to the period of admission, ranging from 75 to 12 per cent. of the stroke.[2]

The author also deduced that, on the contrary, when the cylinders of locomotives were thoroughly protected and heated in the smoke-box, there was no evidence to prove that initial condensation took place in the cylinders, to any important extent, within the limits of the expansive-working that was practised. By the application of a jacket of steam from the boiler, to the cylinder, a material increase in the efficiency of the steam has, in most circumstances, been effected. But, it is incontestable that the jacket, though it diminishes, does not wholly prevent initial condensation of the steam admitted.

By the compounding of cylinders, steam may be worked more expansively, and with a greater degree of efficiency, than in a single cylinder; for, obviously, the fluctuations of temperature which give rise to the condensation that interferes with the action of steam worked expansively, are divided and reduced to one-half, in each cylinder, of what they amount to when the whole of the expansive action is confined to one cylinder.

Data of the Practical Performance of Steam.

Single-Cylinder Condensing Engines:—Steam-jacketted and covered.—The following data are reduced from the recorded performances of the engines:[3]—

[1] The author was the first, so far as he is aware, to discover and demonstrate the existence of initial condensation in steam-cylinders, and to prove that it increases rapidly and to a formidable extent as the ratio of expansion is increased. See his paper on "Expansive Working of Steam in Locomotives," in the *Proceedings of the Institution of Mechanical Engineers*, 1852, page 109. See also *Railway Machinery*, 1855:—"When steam is admitted to the cylinder while the latter is comparatively cold, a very sensible condensation of the steam takes place during admission, which continues to a certain extent during expansion. The heat thereby separated is absorbed by the material of the cylinder, and raises its temperature. A portion of this heat passes off, and is irrecoverably lost; the remainder is re-absorbed by the precipitated steam during the expansion of the existing steam, if the expansion be long enough continued—that is, until the temperature of the latter has fallen below that of the cylinder. This is clearly proved by indicator-diagrams taken at very slow speeds, on which occasions, the cylinder is cold enough to exhibit these operations in high relief."—page 84.

[2] In condensing engines, the loss by initial condensation may be much greater than 40 per cent., for which examples will here be given. Mr. Sutcliffe has followed the same method of analysis in stationary engines, and in the seventh edition of *Hopkinson on the Indicator*, published in 1875, he appears to have precisely adopted the conclusions and even the language of the author. "The initial condensation," he says, page 298, "relatively to the initial measure of steam used, and the pressure of steam found at the end of the stroke, is greater as the cut-off is earlier; by the diagrams referred to, and others from the same engines [referring to the Corliss engines at Saltaire], we find the initial condensation, relatively to the terminal vario-thermal line, to be as follows:—

"At 7.4 expansions = 27.0 per cent.
9.04 ,, = 36.67 per cent.
11.4 ,, = 46.67 per cent."

[3] For Nos. 1, 2, 3, 4, *Proceedings of the Institution of Mechanical Engineers*, 1862, 1867, 1868. For No. 5, *Report of the American Commission on the Vienna Exhibition*, vol. iii., page 23.

Table No. 299.—WORK OF EXPANDED STEAM:—SINGLE-CYLINDER CONDENSING ENGINES.

		Actual Ratio of Expansion.	Weight of Steam per I.H.P. as per Indicator.		Coal Consumed per I.H.P.	Total Initial Pressure at Cut-off.
			As cut off.	As expanded		
			lbs.	lbs.	lbs.	lbs.
1	Corliss, Saltaire,..........	5.20	14.51	16.03	2.5	34½
2	Allen Engine,..............	6.62	15.43	20.78	—	55
	Do.	4.08	17.28	20.97	—	51
	Do.	3.31	17.83	19.05	—	50
3	Crossness Pumping Engine,..............	6.60	13.12	17.20	—	44
		6.30	14.66	16.73	—	43
	Do.	6.50	14.00	18.28	—	50
	Do.	4.90	14.28	15.58	—	47
	Crossness averages,......	6.07	14.27	16.95	2.2	46
4	East London Pumping Engines:—					
	72-inch cylinder, ...	1.93	16.24	15.74	—	15
	Do.	2.80	14.25	15.22	—	19.75
	Do.	3.62	12.92	14.91	—	23.25
	Do.	4.38	12.25	13.58	—	27.25
	Do.	5.10	11.60	13.77	—	30.5
	80-inch cylinder, ...	2.81	13.95	16.93	—	17.5
	90 do.	3.66	11.33	14.44	—	23
	100 do.	3.65	14.81	14.83	—	29
5	Sulzer Engine (Corliss gear),..................	10.0	—	—	3.3	50

With regard to No. 1, the Corliss engine at Saltaire, it is stated by Mr. Hopkinson that 6.96 lbs. of water were evaporated per pound of coal. For No. 3, with good boilers, allow, say, 8½ lbs. of steam per pound of coal. For Nos. 5 and 6, the quantities consumed were observed. Then, the following are the actual quantities of steam consumed, compared with the quantities indicated:—

	Steam Consumed per I.H.P. per Hour.	More than Sensible Steam cut off.
1. Saltaire, ...	17.4 lbs.	20 per cent.
3. Crossness (estimated)	18.7 „	31 „
4. East London:—		
72-inch cylinder (ratio 3.62),......	20.72 „	60 „
80 „	21.38 „	53 „
90 „	18.82 „	66 „
100 „	20.08 „	35 „
5. Sulzer,..	19.6 „	—

Compound Condensing Engines, with and without steam-jackets.—The following data are deduced from particulars supplied by the constructors:—

Table No. 300.—Work of Expanded Steam:—Compound Condensing Engines.

	Actual Ratio of Expansion.	Weight of Steam per I.H.P. as per Indicator.		Coal Consumed per I.H.P.	Total Initial Pressure at Cut-off.
		As cut off.	As expanded.		
		lbs.	lbs.	lbs.	lbs.
4*a*. Day, Summers, & Co., Receiver, Marine, steam-jacketted,..........	1st cyl. 1.715 2d cyl. 2.220	14.74 10.75	17.00 11.99	2.10	49½
	Both 3.807				
5*a*. John Elder & Co., Receiver, Marine, steam-jacketted,.........	1st cyl. 1.850 2d cyl. 1.852	14.45 13.21	14.85 14.85	1.61	56
	Both 3.426				
6. J. & E. Wood, Receiver, Stationary, no jackets, ...	1st cyl. 4.010 2d cyl. 1.857	10.94 13.34	10.77 12.03	2.14[1]	85½
	Both 7.446				
7. Donkin, Woolf, Stationary, 2d cylinder only steam-jacketted, ...	1st cyl. 5.269 2d cyl. 2.590	10.09 11.12	— 19.16	1.9	51½
	Both 13.650				
8. Donkin, Woolf, Stationary, steam-jacketted,.....................	1st cyl. 2.486 2d cyl. 3.221	13.18 13.87	— 17.85	—	50½
	Both 8.007				
9. Donkin, Woolf, Stationary, without steam in jackets,...............	1st cyl. 3.165 2d cyl. 3.221	15.59 18.73	— 19.01	—	48½
	Both 10.200				
10. Thomson, Woolf, Stationary, steam-jacketted, ...	1st cyl. 2.985 2d cyl. 3.384	10.84 12.71	— 15.27	—	36½
	Both 10.100				

[1] This quantity is the result of an estimate. The actual quantity of coal consumed per indicator horse-power was 2.67 lbs., from which one-fifth, estimated as for general heating purposes, was deducted, leaving 2.14 lbs. per indicator horse-power, as consumed by the engine.

The quantities of steam consumed from the boiler were observed in each trial, except for the first three, and were as follows:—

Woolf Engines.	Steam consumed per I.H.P.		More than Sensible Steam cut off.
7. Donkin, 2d cylinder, steam-jacketted,...	20.55 lbs.	......	103 per cent. more.
8. " steam-jacketted,.....................	22.51 "	......	71 "
9. " no steam in jacket,................	32.72 "	......	110 "
10. Thomson, steam-jacketted,...............	20.93 "	......	93 "

Single-Cylinder Engines, steam-jacketted, non-condensing.—The trials of

portable engines at Cardiff, in 1872,[1] afford various examples, from which the following deductions are made:—

Table No. 301.—Work of Expanded Steam:—Portable Engines.

	Constructors.	Actual Ratio of Expansion.	Weight of Steam per I.H.P., as per Indicator.		Total Initial Pressure at Cut-off.
			As cut off.	As expanded	
			lbs.	lbs.	lbs.
11	Marshall, Sons, & Co.,......	4.8	16.87	29.82	74 to 80
12	Davey, Paxman, & Co.,....	5.0	14.93	26.45	73
13	Brown & May,................	{7, 2.6} 3.8	20.52	28.87	73
14	Tasker,.........................	2.4	25.32	30.27	52
15	Reading Engine Works,...	3.8	18.54	23.93	72.5
16	Turner,..........................	2.7	20.08	22.13	77.2
17	Ashby & Co.,..................	5.0	16.28	29.98	63.0

The quantities of water, as steam, actually consumed per indicator horse-power, are subjoined; together with the effective mean pressures in the cylinders. And, to make a comparison with what the performance would amount to if the atmospheric pressure were removed, as if the steam were condensed, one atmosphere, or 15 lbs. per square inch, is added to the effective mean pressure, as given in the second last column, with the weight of steam per total indicator horse-power accruing, in the last column:—

	Effective Mean Pressure.	Steam Consumed per I.H.P.	More than Sensible Steam cut off.	Effective Mean Pressure plus 15 lbs.	Steam Consumed per Total I.H.P.
	lbs.	lbs.	per cent.	lbs.	lbs.
11	31.25	25.9	54	46.25	17.4
12	33.9	29.6	99	48.9	20.7
13	29.2	31.8	55	44.2	21.2
14	29.7	37.9	50	44.7	25.2
15	37.0	24.1	30	52.0	17.2
16	36.24	27.6	37	51.2	19.3
17	20.4	43.2	165	35.4	24.7

Single-Cylinder Engines, completely covered and heated; non-condensing.— Average results of the trials of the "Great Britain" locomotive made in 1850, analyzed by the author in 1852, and published in 1856,[2] are given in the following table, No. 302. The cylinders are "inside," being placed

[1] *The Trials of Portable Steam-engines at Cardiff. Report by the Judges*, 1872.

[2] *Railway Machinery*, 1856, page 80. See also a paper "On the Behaviour of Steam in the Cylinders of Locomotives during Expansion," by D. K. Clark, in the *Minutes of Proceedings of the Institution of Civil Engineers*, vol. lxxii. 1882–83; page 275.

within the smoke-box, and totally surrounded by the atmosphere of hot burnt gases. The cylinders are 18 inches in diameter, with a stroke of 24 inches; and 8 feet driving wheels. The ratios of expansion are here reckoned in terms of the whole of the stroke, though they were not so reckoned in the original investigation.

Table No. 302.—WORK OF EXPANDED STEAM—"GREAT BRITAIN" LOCOMOTIVE.

Notch.	Cut-off.	Actual Ratio of Expansion.	Weight of Steam per Indicator Horse-power, as per Indicator.	Total Initial Pressure per Square Inch at Cut-off.
No.	per cent.	ratio.	lbs.	lbs.
1st	67	1.45	28.97	79.4
3d	50	1.90	24.52	72.6
5th	29	2.94	19.74	67.4

The general effect of the observations was that there was no material degree of initial condensation of the steam in the cylinders of the "Great Britain," confirmatory of the results of the author's experiments with the well-protected and heated inside cylinders of locomotives on the Edinburgh and Glasgow Railway.[1] Here follows a statement of steam consumed per indicator horse-power, taking the initial quantities cut off for the quantities actually consumed; showing the relative quantities that would have been due if the atmospheric pressure had been removed, as in a condensing engine:—

Notch.	Effective Mean Pressure per Square Inch.	Steam Consumed per I.H.P.	Effective Mean Pressure plus 15 lbs. per Square Inch.	Steam Consumed per Total I.H.P.
	lbs.	lbs.	lbs.	lbs.
1st	68.1	28.97	83.1	23.7
3d	53.5	24.52	68.5	19.2
5th	32.1	19.74	47.1	13.5

American Marine Engines.[2]—Mr. Emery tested the condensing engines of the U.S. steamers *Bache* and *Rush*, having compound cylinders fully steam-jacketted and lagged, and the *Dexter* and *Dallas*, having single cylinders felted and lagged only. The *Bache* was tried in four ways:—with and without steam in the jackets, using both cylinders, and using only the second cylinder. The following are the best results of performance for each series of trials:—

[1] *Railway Machinery*, page 82, &c.; and the table in the same book, page 151.

[2] *Journal of the Franklin Institute*, February and March, 1875. See also notices of the trials in the *Proceedings of the Institution of Civil Engineers*, vol. xl. page 292, and vol. xli. page 296.

Table No. 303.—WORK OF EXPANDED STEAM:—AMERICAN MARINE ENGINES—CONDENSING.

	STEAMER.	Actual Ratio of Expansion.	Indicator Horse-power.	Weight of Steam per I.H.P. per Hour. By Indicator, First Cylinder.	Weight of Steam per I.H.P. per Hour. Actually Consumed.	Total Initial Pressure per Square Inch.
	Bache (Woolf).			lbs.	lbs.	lbs.
18	Without steam in jackets:—					
	First cylinder,	3.75	55.93	15.41	23.76	about 90
	Both cylinders,	9.15				
	First cylinder,	2.73	77.06	15.67	23.04	90
	Both cylinders,	6.66				
	First cylinder,	2.31	46.40	15.37	23.21	90
	Both cylinders,	5.63				
19	With steam in jackets:—					
	First cylinder,	6.91	46.40	14.55	25.11	90
	Both cylinders,	16.85				
	First cylinder,	3.77	77.45	14.10	20.71	90
	Both cylinders,	9.19				
	First cylinder,	2.86	99.18	14.97	20.33	90
	Both cylinders,	6.98				
	First cylinder,	2.35	110.50	15.85	20.37	90
	Both cylinders,	5.73				
	First cylinder,	2.34	104.03	15.85	21.97	90
	Both cylinders,	5.71				
	First cylinder,	2.09	102.26	14.85	22.38	90
	Both cylinders,	5.10				
	First cylinder,	1.74	134.53	17.27	21.17	90
	Both cylinders,	4.24				
20	Without steam in jacket:—					
	Second cylinder only,	11.82	47.24	21.03	35.08	90
	Do. do.	7.62	71.75	17.76	29.62	90
	Do. do.	5.32	89.14	17.35	26.25	90
21	With steam in jacket:—					
	Second cylinder only,	12.62	54.84	16.42	27.11	90
	Do. do.	8.57	74.62	15.58	24.09	90
	Do. do.	5.11	116.01	16.25	23.15	90
	Do. do.	2.18	66.74	24.05	34.03	27
	Rush (Receiver).					
22	Steam-jacketted:—					
	First cylinder,	2.46	266.5	17.1	18.4	82.3
	Both cylinders,	6.22				
	First cylinder,	1.60	168.7	19.7	22.1	50.2
	Both cylinders,	4.03				
	Dexter (single cylinder).					
23	Felted and lagged,	4.46	185.9	16.2	23.9	80.4
	Do. do.	3.49	292.4	16.3	23.9	—
	Do. do.	2.08	196.2	20.3	31.8	53.6
	Dallas (single cylinder).					
24	Felted and lagged,	5.07	138.0	19.2	26.7	47
	Do. do.	3.13	221.4	20.1	26.9	—
	Do. do.	2.32	234.3	22.8	31.0	39.4

French Stationary Engines.[1]—M. Hallauer reports the results of experiments on a 24-inch single-cylinder engine, worked by steam superheated 150 degrees, and lagged and felted—yielding 135 indicator horse-power; and his own experiments on a Woolf engine, with steam-jacketted cylinders:—

FRENCH STATIONARY ENGINES—CONDENSING.

		Actual Ratio of Expansion.	Steam Consumed per I.H.P. per Hour.	Total Initial Pressure in Cylinder.
			lbs.	lbs.
25	Hirn (superheated steam in single cylinder)............	4	15.5	60
26	Leloutre (Woolf engine, steam-jacketted)..........	12	24.83	—

GENERAL DEDUCTIONS FROM THE DATA OF THE ACTUAL PERFORMANCE OF STEAM.

Single Cylinders, with steam-jackets; condensing.—The analysis of diagrams from the Allen engine, No. 2, page 881, indicates that the expansion-ratio 6.62 was better than the ratios 4.08 and 3.31. Mr. C. T. Porter maintains that the ratio 8 is best for the Allen engine. For the Crossness engines, No. 3, the ratio 6 appears to be the best; though perhaps the Corliss, No. 1, with the ratio 5.2, is fully as good as the Crossness. The Sulzer (Corliss gear), No. 5, with the ratio 10, is not so efficient as these others. Of the East London engines, No. 4, the 72-inch engine appears to have greater efficiency when expanding 5.10 times than for less ratios, and of the expansion-ratios for the four observed consumptions of water per indicator horse-power, the highest ratio, 3.66, gives the greatest efficiency. Again, the *Bache* marine engine, No. 21, page 885, yielded the highest observed efficiency with a ratio 5.11, but, by plotting, it is easily shown that the efficiency is practically the same for a ratio of 6.

Non-Condensing.—The portable engines, page 883, supply examples:—

For initial pressures between 70 lbs. and 80 lbs., omitting No. 13 as unequal.

No.	Total Maximum Pressure.	Expansion-ratio.	Water per Normal I.H.P. per Hour.	Water per Total I.H.P., if Atmosphere were removed.
	lbs.		lbs.	lbs.
16	77.2	2.7	27.6	19.3
15	72.5	3.8	24.1	17.2
11	74 to 80	4.8	25.9	17.4
12	73	5.0	29.6	20.7
For lower initial pressures.				
14	52	2.4	37.9	25.2
17	63	5.0	43.2	24.7

[1] "Recherches Expérimentales Sur les Machines à Vapeur." By MM. Hallauer and Dwelshauver-Déry; *Bulletin de la Société Industrielle de Mulhouse*, 1877, page 190.

It is seen that the highest efficiency is attained with an expansion-ratio of 3.8, whether against or without atmospheric resistance.

Single Cylinders without steam-jackets; condensing.—The most favourable results of Nos. 20, 23, 24, and 25, are here abstracted:—

No.	Expansion-ratio.	Water Consumed per I.H.P. per Hour.
20	5.32	26.25 lbs.
23	4.46	23.9
"	3.49	23.9
24	5.07	26.7
"	3.13	26.9
25	4.13	18.62—steam is superheated.

It appears that, using ordinary steam, expansion-ratios within the limits of 3½ and 4½ are practically of equal efficiency, and that they give the highest efficiency. The same ratios probably apply to the use of superheated steam, of which there is only one result, No. 25.

Non-Condensing.—The results from the cylinder of the "Great Britain" locomotive, page 884, show that the highest ratio of expansion that was tried, namely 2.94, gave the highest efficiency. A greater ratio of expansion would probably have given a still greater efficiency.

Compound Cylinders with steam-jackets; condensing.—Receiver-engines.—Comparing the marine engines, Nos. 4*a* and 5*a*, it appears that the ratio of expansion, 3.426, gave more efficiency than 3.807. But in the *Bache* and the *Rush*, Nos. 19 and 22, it appears that a ratio of from 6 to 7 was most efficient. With Nos. 4*a* and 5*a*, the total initial pressure was 49½ lbs. and 56 lbs. absolute per square inch; but in Nos. 19 and 22, it was 90 lbs. and 82 lbs.

Woolf Engines.—The ratio 13.65 for No. 7 was better than the ratio 8.007 for No. 8, requiring 20.55 lbs. and 22.51 lbs. of water per indicator horse-power respectively. No. 10, with a ratio 10.1, required 20.93 lbs. of water; whilst No. 26, with a ratio of 12, required 24.83 lbs. The most efficient ratio of expansion is most probably about 10.

Proportional Ratios of Expansion in the first and second cylinders—Receiver-engines.—Comparing Nos. 4*a*, 5*a*, and 22, the best action is seen to be obtained with equal ranges of expansion; for No. 5*a* is better than No. 4*a*, and No. 22, in which the ratios are equal, being each 2½, is the best.

Woolf Engines.—Nos. 7, 8, and 10 form a curious series:—

No.	Expansion-ratio, in First Cylinder.	Total Expansion-ratio.	Steam consumed per I.H.P.
8	2.486	8.007	22.51 lbs. or 71 per cent. more than per diagram.
10	2.985	10.100	20.93 " or 93 "
7	5.269	13.650	20.55 " or 103 "

It appears that, as the initial proportion of steam condensed in the first cylinder increases,—as shown by the percentages in the last column,—the efficiency increases. The best performances of the *Bache*, No. 19, are made with a total expansion of from 6 to 9, having from 2⅓ to 3¾ expan-

sion-ratios in the first cylinders, and 2½ in the second. No. 26, with a total expansion-ratio 12, consumed 24.83 lbs. of steam per indicator horse-power per hour; and the *Bache*, No. 19, for an expansion-ratio of 16.85, consumed 25.11 lbs.,—the highest rate of consumption recorded for Woolf engines.

Compound Engines without steam in jackets; condensing.—The only example of receiver-engine is No. 6, which is not provided with steam-jackets. Nos. 9 and 18 are examples of Woolf engines. The results are here brought together:—

No.	Total Initial Pressure per Square Inch.	Actual Ratios of Expansion.	Steam Consumed per I.H.P. per Hour.	Coal per I.H.P. per Hour.
	lbs.			
6. Receiver	85½	4.010 1.857 7.446	—	2.14 lbs. (estimated, see note to table, page 882).
9. Woolf	48½	3.165 3.221 10.200	32.72	—
18. "	90	3.75 2.44 9.15	23.76	—
"	90	2.73 2.44 6.66	23.04	—
"	90	2.31 2.44 5.63	23.21	—

In the Woolf engines, it appears from the results, that the highest efficiency is obtained by an expansion-ratio of about 3 in the first cylinder, with a total ratio of 7. In the receiver-engine No. 6, the ratio in the first cylinder was carried to 4, with a total ratio of 7½, with an apparently excellent result.

Conclusions on the Actual Performance of Steam.

For the development of the highest efficiencies of steam, as used in the steam-engine, the steam-jacket or other means for protecting the steam from the cooling and condensing action of the cylinder, must be employed. The superheating of steam prior to its introduction into the cylinder is probably the most efficient means that may be employed for this purpose. The application, to the cylinder, of hot gases—hotter than the steam—is probably the next best means; and next comes the steam-jacket.

The importance of sustaining the temperature of steam expanded in a cylinder,—preventing its falling low and leading to the cooling of the cylinder,—is strikingly proved by the foregoing hypothetical calculations of the consumption of steam per indicator horse-power in non-condensing cylinders, on the assumption that the resistance of the atmosphere is removed,—likening the conditions to those of condensing engines. In the cases of the portable engines and the locomotive, the consumptions, on this supposition, amounted only to 17.2 lbs. per indicator horse-power per hour, for the portable engine (No. 15), with an expansion-ratio of 3.8; and to 14.8 lbs. for the locomotive (No. 18), with an expansion-ratio of 2.94. These results are below anything that has been recorded of single-cylinder condensing engines for the same ratios of expansion; and their superiority is due doubtless to the fact that the temperature of non-condensing cylinders never falls below 212°.

It is deducible from the results, that the compound steam-engine is more efficient than the single-cylinder engine, and that, of the two kinds of compound engines, the receiver-engine is more efficient than the Woolf engine. The reasons for the superiority of the receiver-engine have been partly pointed out in the comparative analysis, page 867. There is another reason in the fact that whilst the temperature of the first cylinder of the receiver-engine never falls below, nor even down to, that of the receiver, which stands at a constant pressure and temperature; in the Woolf engine, on the contrary, the average temperature in the first cylinder must be that of the steam expanding into the second cylinder, which falls continuously with the expansion.

The most efficient ratios of expansion, together with the quantities of steam, or water, from the boiler, consumed per indicator horse-power per hour,—deduced from the foregoing analyses,—are placed for comparison in the table No. 304.

It is scarcely necessary to observe, that the evidence of the initial condensation of steam during the period of its admission into the cylinder, is of great importance, and that, clearly, there is a wide margin for economy in the employment of steam for the production of power. Mr. Bramwell, in an excellent and interesting paper on marine engines, in 1872,[1] showed that the average consumption of coal per indicator horse-power per hour, by steam-ships with compound engines in long sea voyages, varied from 2.8 lbs. to 1.7 lb. in nineteen steamers, for which the average consumption amounted to 2.11 lbs. The foregoing deductions are consistent with, and are corroborated by, these facts.

In the same paper,[2] Mr. Bramwell states that in H.M.S. *Briton*, fitted with compound engines on the system of Mr. E. A. Cowper, the steam was heated within a steam-jacket, on its passage from the first to the second cylinder, and that the consumption of coal, at nearly maximum power, was 1.98 lbs. per indicator horse-power per hour, and that, at a third of the power, the consumption of coal was as low as 1.30 lbs. This evidence is confirmatory of the conclusion that the work of steam is most efficiently developed when it is previously superheated.

[1] "On the Progress effected in the Economy of Fuel in Steam Navigation, considered in Relation to Compound-Cylinder Engines and High-Pressure Steam," in the *Proceedings of the Institution of Mechanical Engineers*, 1872.

[2] Page 153.

From the foregoing and other evidence discussed in the author's work on the *Steam-Engine*, the following summary table has been prepared, showing the most economical results of performance of single-cylinder and compound-cylinder steam-engines under various conditions. These results are not put forward as final; but simply to indicate the directions in which the best action of the steam-engine may be obtained.

Table No. 304.—Practical Performance of Steam-Engines:—The most Efficient Ratios of Expansion, and the Quantities of Water Consumed from the Boiler per Indicator Horse-Power.

Description of Cylinders.	Most Efficient Ratio of Expansion.	Steam, or Water from the Boiler, consumed per I.H.P. per Hour.
	initial volume =1.	pounds.
Single cylinder, with steam-jacket, condensing:—		
Thoroughly steam-jacketted — Long stroke.....	4	21
Thoroughly steam-jacketted — Short stroke....	6	20.6
Only side-jacketted............ — Long stroke....	3.2	21.7
Only side-jacketted............ — Short stroke....	5	23
Single cylinder, with steam-jacket, non-condensing,	4	25
Single cylinder, without jacket, condensing:—		
Long stroke.....	4.5	20
Short stroke....	4.25	25
Single cylinder, without jacket, condensing, steam superheated..	4	15½
Single cylinder, without jacket, non-condensing, cylinder well protected..............................	4	18½
Compound cylinder, steam-jacketted, condensing:—		
Receiver	10	15 to 16
Woolf..	12	14 to 18
Compound cylinder, no jackets, condensing:—		
Receiver	6½	23
Woolf..	9½	21

Frictional Resistance of Steam Engines.

See page 951.

FLOW OF AIR AND OTHER GASES.

DISCHARGE OF GASES THROUGH ORIFICES.

AIR.

Gases and vapours act like liquids in flowing through orifices and tubes, in virtue of the difference of the inside and outside pressures; and the velocity of flow is regulated with respect to the fundamental formula for gravity, page 279,

$$v = 8\sqrt{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

For liquids, the height through which the water falls, to the orifice of flow, can be ascertained by direct measurement; whilst, for gases, it is necessary to find the height for calculation, thus:—The head due to the difference of pressures per square inch of the gas or vapour, is equal to the height of a column of the gas inside, 1 inch square, of which the weight is equal to the difference of pressure; and if this net pressure per square inch be divided by the weight of a prism of the gas, 1 inch square and 1 foot high, the quotient is the height in feet, of the equivalent column of gas, from which the velocity of flow is to be calculated.

Flow of Air through an orifice due to small differences of pressure.

The velocities of flow due to small differences of pressure measured by a water-gauge, are given by the expression,—

$$V = \sqrt{\frac{2gh}{12} \times 773.2 \times \left(1 + \frac{t-32}{493}\right) \times \frac{29.92}{p}}, \quad \ldots\ldots\ldots \quad (2)$$

in which V = the velocity in feet per second, $2g = 64.4$, h = the height of the column of water, in inches, measuring the difference of pressure, t = the temperature, and p = the barometric pressure in inches of mercury. The quantity 773.2 is the volume of air at 32°, and under a pressure of 29.92 inches, when that of an equal weight of water at 32° F. is taken as 1. The expression may be reduced to the simpler form—

Velocity of Flow of Air through an orifice.

For small differences of pressure.

$$V = 352\sqrt{(1 + .00203\,(t-32)) \times \frac{h}{p}} \quad \ldots\ldots\ldots\ldots \quad (3)$$

For the temperature $t = 62°$ F., the formula becomes, by substitution,

$$V = 363\sqrt{\frac{h}{p}} \quad \ldots\ldots\ldots\ldots \quad (4)$$

For the temperature $t = 62°$ F., and the pressure $p = 29.92$ inches of mercury—the most usual values—the formula becomes,

$$V = 66.35\sqrt{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

These values must be multiplied by the coefficients pertaining to differently formed orifices, which are given by Weisbach, as follows:—

Orifice.	Coefficient of Efflux.
Conoidal mouth-piece, of the form of the contracted vein, with effective pressure of from .23 to 1.1 atmospheres,	.97 to .99
Circular orifices in thin plates,..	.56 to .79
Short cylindrical mouth-pieces,..	.81 to .84
The same, rounded at the inner end,................................	.92 to .93
Conical converging mouth-pieces,....................................	.90 to .99

Anemometer.

When a current of air flows through a tube, which is restricted, or reduced to a smaller diameter at some portion of its length, the velocity of the current is accelerated in passing into the restricted portion, and is retarded in passing out of it into the tube of normal diameter; and, if the restriction be so formed as to accelerate and retard the current without shock, there is no loss of head in the operation. M. Arson[1] employs this principle in his anemometer; but he modifies the form of the restriction, in so far that whilst the approach to the restriction is gradually contracted, the exit from it is square, so that the current passes abruptly from the orifice into the tube of full bore. The pressure, or rather the degree of "vacuum" at the exit, is measured by a water-gauge attached at the entering angle or corner; and the difference of the heights $(h_v - h)$ due respectively to the external or barometrical pressure, and the internal pressure, is equal to the difference of the heights $\left(\frac{v^2}{2g} - \frac{v_v^{\,2}}{2g}\right)$ due to the normal and the maximum velocities; that is,

$$h_v - h = \frac{v^2}{2g} - \frac{v_v^{\,2}}{2g}.$$

M. Arson so adjusts, by preference, the sectional area of the restriction, that $\frac{v^2}{2g} = 2\frac{v_v^{\,2}}{2g}$; whence the difference $(h_v - h)$, becomes equal to $\frac{v_v^{\,2}}{2g}$. The difference $(h_v - h)$ is measured directly by the water-gauge; and thus, by simple computation, the normal velocity, that is, the velocity of the wind blowing through the tube when fairly directed towards it, may be determined. That the quantity $\frac{v^2}{2g}$ may be equal to $2\frac{v_v^{\,2}}{2g}$, the sectional areas of the tube and the restriction must be as 1 to $\sqrt{2}$, or 1 to 1.414. From direct observation, it appears that the results obtained by the formula, say formula (2), page 891, are to be reduced by the coefficient 0.94, to give the actual velocities.

[1] *Compte Rendu de la Société des Ingénieurs Civils*, 1876, page 505.

Outflow of Steam through an Orifice.[1]

The flow of steam of a greater pressure into an atmosphere of a less pressure, increases as the difference of pressure is increased, until the outside pressure is reduced to 58 per cent. of the absolute pressure in the boiler. The flow of steam is neither increased nor diminished by reducing the outside pressure below 58 per cent. of the inside pressure, even to the extent of a perfect vacuum. In flowing through a nozzle of the best form, the steam expands to the outside pressure, and to the volume due to this pressure, so long as it is not less than 58 per cent. of the inside pressure. For an outside pressure of 58 per cent., and for lower pressures, the ratio of expansion is 1 to 1.624. The following table is selected from Mr. Brownlee's data in the "Report on Safety Valves," to exemplify the varying discharges under a constant initial pressure in the boiler, into various outside pressures. The formulas by means of which the results of the table were calculated are given by Mr. Brownlee at page 30 of the "Report."

Table No. 305.—Outflow of Steam:—From a given Absolute Initial Pressure into Various Lower Pressures.

Initial Pressure in Boiler, 75 lbs. per square inch.

Absolute Pressure in Boiler, per Square Inch.	Outside Pressure, per Square Inch.	Ratio of Expansion in Nozzle.	Velocity of Efflux at Constant Density.	Actual velocity of Efflux, Expanded.	Weight discharged per Square Inch of Orifice per Minute.
lbs.	lbs.	ratio.	feet per second.	feet per second.	pounds.
75	74	1.012	227.5	230	16.68
75	72	1.037	386.7	401	28.35
75	70	1.063	490	521	35.93
75	65	1.136	660	749	48.38
75	61.62	1.198	736	876	53.97
75	60	1.219	765	933	56.12
75	50	1.434	873	1252	64
75	45	1.575	890	1401	65.24
75	43.46 (58 p. cent.)	1.624	890.6	1446.5	65.3
75	15	1.624	890.6	1446.5	65.3
75	0	1.624	890.6	1446.5	65.3

When, on the contrary, steam of varying initial pressures is discharged into the atmosphere,—pressures of which the atmospheric pressure is not

[1] See on the subject of the efflux of steam, Mr. Wm. Froude's paper on the "Discharge of Elastic Fluids under Pressure," in the *Proceedings of the Institution of Civil Engineers*, vol. vi., 1847, page 356; also, Mr. R. D. Napier's account of his experiments, 1866; Dr. Rankine, in *The Engineer*, November and December, 1869; and the "Report on Safety-Valves," in the *Transactions of the Institution of Engineers and Ship-Builders in Scotland*, vol. xviii., 1874–75, page 13; from the last of which the particulars given in the text are derived; also Eli W. Blake, in *The Engineer*, December, 1869, page 418; Wilson on Elastic Fluids, in *Engineering*, vol. xiii., page 35, &c., 1872.

more than 58 per cent.,—the velocity of efflux, at constant density, that is, supposing the initial density to be maintained, is given by the formula,—

$$v = 3{\cdot}5953 \sqrt{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

v = the velocity of outflow in feet per minute, as for steam of the initial density.
h = the height in feet of a column of steam of the given absolute initial pressure, of uniform density, the weight of which is equal to the pressure on the unit of base.

The lowest initial pressure to which the formula applies, when the steam is discharged into the atmosphere at 14.7 lbs. per square inch, is $(14.7 \times \frac{100}{58} =)$ 25.37 lbs. per square inch. A number of examples of the application of the formula are given in table No. 306, for initial absolute pressures of from 25.37 lbs. to 100 lbs. per square inch.

The truth of the formulas is confirmed with a surprising degree of exactness by the experiments of Mr. Brownlee.

Table No. 306.—Velocity of Efflux of Steam into the Atmosphere.

Absolute Initial Pressure per Square Inch.	Outside Pressure per Square Inch.	Ratio of Expansion in Nozzle.	Velocity of Efflux, as at Constant Density.	Actual Velocity of Efflux, Expanded.	Weight of Steam discharged per Minute, per Square Inch.
lbs.	lbs.	ratio.	feet per second.	feet per second.	pounds.
25.37	14.7	1.624	863	1401	22.81
30	14.7	1.624	867	1408	26.84
40	14.7	1.624	874	1419	35.18
45	14.7	1.624	877	1424	39.78
50	14.7	1.624	880	1429	44.06
60	14.7	1.624	885	1437	52.59
70	14.7	1.624	889	1444	61.07
75	14.7	1.624	891	1447	65.30
90	14.7	1.624	895	1454	77.94
100	14.7	1.624	898	1459	86.34

Flow of Air through Pipes and other Conduits.

Mr. Hawksley[1] states, as the result of varied experience, that the formula put forward by him for the flow of water in pipes, given at page 933, may be employed also for the flow of air in pipes. It is,

$$v = 48 \sqrt{\frac{h\,d}{l}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

in which v is the velocity in feet per second, h is the head in feet of air, d is the diameter in feet, and l is the length in feet. But, it is convenient to express the head in inches of water. Taking the density of water as

[1] *Proceedings of the Institution of Civil Engineers*, vol. xxxiii., page 55.

815 times that of air, the multiplier $(\frac{815}{12}=)$ 68 is to be placed under the sign, when $v=48\sqrt{\frac{68\,h\,d}{l}}$, and by reduction,—

Flow of Air through pipes.

$$v=396\sqrt{\frac{h\,d}{l}} \quad \text{.............................} \quad (8)$$

$$h=\frac{l\,v^2}{156{,}800\,d} \quad \text{.............................} \quad (9)$$

v = velocity, in feet per second.
h = the head, in inches of water.
d = the diameter in feet.
l = the length in feet.
c = the perimeter in feet.
a = the sectional area in square feet.
$Q = v \times a$ = the quantity of air discharged, in cubic feet per second.
H = the horse-power required.

For passages or conduits of irregular forms, as shafts and air-ways in mines and tunnels, the perimeter and the sectional area become factors in the formula:[1]—

Flow of Air through passages of any form of section.

$$v=796\sqrt{\frac{a\,h}{c\,l}} \quad \text{.............................} \quad (10)$$

$$h=\frac{v^2\,c\,l}{633{,}000\,a} \quad \text{.............................} \quad (11)$$

The quantity of air discharged is expressed by the product of the velocity in formulas (8) and (10), and the sectional area, or by $(v \times a)$; whence, by reduction,

Quantity of Air discharged

from a pipe,............................. $Q=311\sqrt{\frac{h\,d^5}{l}}$.......... (12)

from a passage of any form of section, $Q=796\sqrt{\frac{a^3\,h}{c\,l}}$. .... (13)

The effective horse-power expended on the net work done in drawing air through a pipe or other passage, is expressed by the product of the sectional area by the velocity in feet per second, and by the head or "drag" in pounds per square foot, divided by 550. That is to say, $H=\frac{v \times a \times h \times 5.20}{550}$, in which 1 inch of water is taken as equivalent to a pres-

[1] These formulas have been worked out from Mr. Hawksley's fundamental formula, by the author, in the *Proceedings of the Institution of Civil Engineers*, vol. xliv., page 90, in their application to tunnels; and also in *Simms' Practical Tunnelling*, 3d edition. Mr. Hawksley states that his formulas apply with exactness to the shafts and air-ways of mines.

sure of 5.20 lbs. per square foot, for any passage. By substituting $.7854\, d^2$ for a, the expression is adapted for pipes. Reducing, the formulas are,—

Effective horse-power in net work of discharge of air from straight passages:—

from a passage of any form of section, $H = \dfrac{v\,a\,h}{106} = \dfrac{Q\,h}{106}$ .... (14)

from a pipe, ... $H = \dfrac{v\,d^2\,h}{135}$ .... (15)

Substitute in these formulas, the values of h, (9) and (11), and the horse-power is given in terms of the velocity, perimeter, and length:—

from a passage of any form of section, $H = \dfrac{v^3\,c\,l}{67{,}000{,}000}$ .... (16)

from a pipe, ... $H = \dfrac{v^3\,d^2\,l}{21{,}200{,}000}$ .... (17)

Flow of Compressed Air through pipes.—According to the experiments of d'Aubuisson, and those of a Sardinian commission, on the resistance of air through long conduits or pipes, the diminution of pressure is very nearly directly as the length, and as the square of the velocity, and inversely as the diameter. The resistance is not varied by the density. A table of the loss of pressure in pipes, for a length of 1000 metres, and for diameters of from 10 to 35 centimetres, at velocities of from 1 to 6 metres per second, is given by Mr. Cornut,[1] and is here reproduced in English measures. The absolute pressure is not stated.

Table No. 307.—Loss of Pressure by Flow of Air in Pipes.

Length of pipe, 1000 metres, or 3280 feet.

Velocity at the Entrance to the Pipe.		Diameter of Pipe, in Inches.					
		4	6	8	10	12	14
		Loss of Pressure in lbs. per Square Inch.					
metres per second.	feet per second.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1	3.28	.114	.076	.057	.057	.038	.038
2	6.56	.500	.343	.250	.210	.172	.153
3	9.84	1.183	.800	.592	.477	.394	.343
4	13.12	2.060	1.374	1.030	.840	.687	.600
5	16.40	3.200	2.160	1.610	1.290	1.100	.923
6	19.68	4.446	2.964	2.223	1.778	1.482	1.280

At the works for excavating the Mont Cenis Tunnel, the supply of compressed air was conveyed in a cast-iron pipe 7⅝ inches in diameter. The loss of pressure, and leakage of air, from the supply pipes, in a length of 1 mile 15 yards, was only 3½ per cent. of the head:—the absolute initial pressure was 5.70 atmospheres, and it was reduced to 5.50 atmospheres,

[1] See M. Cornut's paper on "Compressed-air Machinery," *Bulletin de la Société Industrielle Minerale*, 1866–67, page 201.

whilst there was an expenditure at the rate of 64 cubic feet of compressed air per minute. In the middle of the tunnel, through a length of pipe of, say, 20,000 feet, or 3.80 miles, the absolute pressure only fell from 6 atmospheres to 5.7 atmospheres, or to 95 per cent. of the original pressure.

Resistance of Air to the Motion of Flat Surfaces.

Mr. James C. Fairweather[1] found by experiment that the law of the resistance of air to flat vanes moving through it perpendicularly to their planes, was correctly expressed by Dr. Hutton:—"In the case of slow motion, nearly as the square of the velocity; but gradually increasing more and more above that proportion as the velocity increases." From the results given in table No. 307A, it may be inferred that the resistance per square foot increases with the total area of surface exposed. These results are considerably in excess of those of Colonel Beaufoy, heretofore accepted.

Table No. 307A.—Resistance of Air to the Motion of Flat Vanes.
In lbs. pressure on the given surface.
(Reduced from Mr. Fairweather's Experiments.)

Lineal Dimension.	Area.		Speed, in feet per second.					
			5	10	15	20	25	30
side of square.	sq. inches.	sq. feet.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
7.41	54.85	.38	.55	1.4	3.25	5.7	9.4	14.0
12.9	166.3	1.155	1.3	5.5	13.6	—	—	—
18.58	345.2	2.40	3.25	15.0	—	—	—	—
diameter of circle.								
7.24	41.15	.286	.30	1.15	2.6	4.6	7.4	10.9
12.65	125.8	.875	.85	3.85	9.1	16.4	—	—
18.36	264.8	1.840	2.4	10.0	—	—	—	—

Ascension of Air by Difference of Temperature.

Mr. Hawksley[2] gives the following formula for the velocity of air in the up-cast shaft of a mine, due to the different weights of the columns of air, of different temperatures, in the up-cast and down-cast shafts, comprising allowance for frictional resistances.

$$v = 96\sqrt{\frac{\frac{(T-t)}{(T+448)}Ds}{ml+368s}} \qquad (18)$$

T = the temperature of the air in the up-cast shaft, in Fahrenheit degrees.
t = the temperature of the air in the down-cast shaft.
D = the depth of the shaft, in feet.
m = the periphery of the air-course, in feet.
s = the section of the air-course, in square feet.
l = the length traversed by the current, in feet.
v = the velocity of the current, in feet per second.

[1] "Resistance of Air," *Proceedings of the Royal Society of Edinburgh*, 1872–75; vol. viii., page 351.

[2] *Proceedings of the Institution of Civil Engineers*, 1847, vol. vi., page 192. Mr. Hawksley states, vol. xxx., page 304, that the formula tallies exactly with the results of Mr. Nicholas Wood's experiments on the ventilation of collieries.

WORK OF DRY AIR OR OTHER GAS,
COMPRESSED OR EXPANDED.

In this investigation of the work of air by compression and by expansion, the following symbols are employed:—

t = the temperature of the gas in Fahrenheit degrees.
T = the absolute temperature of the gas $= t + 461°$.
h = the specific heat of the gas, under constant pressure.
h' = the specific heat of the gas under constant volume.
K = the specific heat of 1 pound of the gas, under constant pressure, in foot-pounds.
K′ = the specific heat of 1 pound of the gas, under constant volume, in foot-pounds.
γ = the ratio of the specific heats of a gas $= \frac{h}{h'} = \frac{K}{K'}$.
J = the mechanical equivalent of a unit of heat = 772 foot-pounds.
P = the total pressure of the gas, in pounds per square foot.
p = the total pressure of the gas, in pounds per square inch.
V = the volume of the gas, in cubic feet.
v = the volume of clearance at each end of the cylinder.
W = work done in foot-pounds.

It was stated, page 349, that the product of the volume and the pressure of 1 pound of a gas, is equal to the product of the absolute temperature and a constant coefficient; or,

$$V\,P = a\,T, \quad \text{.......} \quad (1)$$
$$V\,p = a'\,T, \quad \text{.......} \quad (2)$$

in which a and a' are the constants to be used respectively for the pressures P and p. The values of a' have already been given for several gases, in table No. 114, page 349, at pressures, p, per square inch. For the values, a, due to pressures, P, per square foot, those values of a' are to be multiplied by 144; or the values, a, may be deduced, independently, from the respective densities of the gases. The two series of values of the constants a and a', are here annexed for several gases:—

One Pound of Gas.	Constant a, Formula (1).	Constant a', Formula (2).
Hydrogen,	767.4	5.33200
Gaseous steam,	85.4	.59372
Nitrogen,	54.72	.38027
Olefiant gas,	53.98	.37506
Air,	53.15	.36935
Oxygen,	48.07	.33406
Carbonic acid (ideal),	35.00	.24322
Do. (actual),	34.76	.24155
Sulphuric ether vapour (ideal),	20.49	.14246
Vapour of mercury (ideal),	7.62	.05296

It follows from the general equations, as was stated at page 345, that the pressure of a gas varies inversely as the volume, when the temperature is constant; and that the product of the pressure and the volume is proportional to the absolute temperature. When the temperature is uniform, therefore, the product of the pressure and volume of a given weight of a gas is also uniform; and, if the gas be either compressed or expanded, it follows the hyperbolic ratio. The resulting hyperbolic curve of expansion or of compression is called an *isothermal curve*, or curve for constant temperature.

When a mass of a gas is either compressed by or expanded on a piston within a cylinder, of non-conducting materials, so that heat is neither received nor given out by the gas, the curve of compression or of expansion is called an *adiabatic-curve*. The work of compression, as internal work, is converted into heat, which pervades the gas and raises its temperature; and, reversely, a portion of the heat of the gas is converted into the work of expansion, as internal work, and the temperature is lowered.

The general equation for air, with the values of the constants a and a', are:—

$$V\,P = 53.15\ T \qquad (3)$$
$$V\,p = .36935\ T \qquad (4)$$

WORK OF DRY AIR AT CONSTANT TEMPERATURE, OR ISOTHERMALLY.

When the temperature is constant, say, at 62° F., the absolute temperature is $461° + 62° = 523°$, and the value of the constant products in formulas (3) and (4), are,—

$$V\,P = 53.15\ T = 27{,}800 \qquad (5)$$
$$V\,p = .36935\ T = 193.2 \qquad (6)$$

Since $V\,P = V'\,P'$, in which V' and P' are any other corresponding volume and pressure, at the same temperature, the relation stands as follows:—

Isothermal Compression or Expansion of Air.

$$\frac{P'}{P} = \frac{V}{V'}. \qquad (7)$$

The hyperbolic ratio of expansion, or of compression, followed by air at a constant temperature, has already, page 822, been taken as, practically, the ratio according to which steam is expanded in the cylinder; and the nature of the relation, and the deductions from it, which have there been considered, apply also to the case of air.

WORK OF COMPRESSION OF AIR AT CONSTANT TEMPERATURE, WITHOUT CLEARANCE.

If there be no clearance, and if there were no back-pressure, the work of simple compression in one stroke, calculated in terms of the initial work $P'V'$, or the equivalent PV:—

$$W = P\,V \text{ hyp log} \frac{V}{V'}, \qquad (8)$$

represented by the area E F B D, Fig. 348. But there is an assisting pressure P′,—the initial atmospheric pressure,—the work of which is $(P' \times V)$, represented by the rectangle D F, which is equal to $(P - P')\,V'$, the rectangle C G; and the net work for simple compression, is

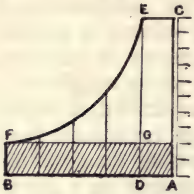


Fig. 348.—Compression of Air at Constant Temperature, or Isothermally.

$$W = P\,V \text{ hyp log}\frac{V}{V'} - (P' - P)\,V'. \quad \ldots\ldots\ldots \quad (9)$$

Total Work for One Stroke of an Air-compressor.—There is to be added the work of expelling the air from the cylinder into a reservoir, equal to,

$$(P' - P)\,V', \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

and the sum of (9) and (10), reduced, is:—

Total Net Work for One Stroke of the Air-compressor, isothermally, without Clearance.

$$W = P\,V \text{ hyp log}\frac{V}{V'}; \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$\text{or, } W = P\,V \text{ hyp log}\frac{P'}{P}. \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

Work of Compression of Air at Constant Temperature, with Clearance.

The work of compression in one stroke, leaving back-pressure out of calculation, is,

$$W = P\,(V + v) \text{ hyp log}\frac{V + v}{V' + v}. \quad \ldots\ldots\ldots\ldots \quad (13)$$

Deduct the work of atmospheric back-pressure, or $(P' - P)\,(V' + v)$; and add the work expended in expelling the air from the cylinder into a reservoir, equal to,

$$(P' - P)\,V' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

The resulting total net work is,

Total Net Work for One Stroke of the Air-compressor, isothermally, with Clearance.

$$W = P\,(V + v) \text{ hyp log}\frac{V + v}{V' + v} - (P' - P)\,v \ldots\ldots\ldots \quad (15)$$

Work of Expansion of Air, at a Constant Temperature.

When the pressure of the air at the end of the stroke is reduced by expansion to an equality with the back-pressure, the equations (8) to (15), for compression, express also the relations of pressure, volume, and work, for expansion—represented also by Fig. 348.

Total Net Work for One Stroke of a Compressed-air Engine, when the air is expanded down to the back-pressure:—

Without clearance... $W = P\,V \text{ hyp log} \dfrac{V'}{V}$ .... (16)

With clearance .... $W = P\,(V + v) \text{ hyp log} \dfrac{V' + v}{V + v} - (P - P')\,v$... (17)

When the back-pressure P'' is less than P', the lower limit of the positive pressure, there is a sudden fall of pressure at the end of the stroke, from P' to P'', and the expression for the total net work is:—

$$W = P\,(V + v) \text{ hyp log} \frac{V' + v}{V + v} - P''V' + P\,V \qquad (18)$$

WORK OF DRY AIR IN A NON-CONDUCTING CYLINDER, ADIABATICALLY.

The specific heat of 1 lb. of air in foot-pounds of work, is

	Unit.		Foot-pounds.
At constant pressure,...............	.2377 × 772	=	183.45 = K.
At constant volume,.................	.1688 × 772	=	130.3 = K′.
Difference,..			53.15 = a = K – K′.

The difference, 53.15 foot-pounds, is equal to the value of the constant a, formula (1), for air, as given in formula (3), page 899.

The ratio of the specific heats at constant pressure and constant volume is as 1.408 to 1, or 1.408, whether they are expressed in heat-units or in foot-pounds.

Adiabatic Compression of a Gas.

Suppose that the gas, having the initial pressure P, volume V, and temperature T, is compressed adiabatically, and attains the pressure P′, volume V′, and temperature T′; the relations of the pressure, volume, and temperature are as follows:—

Adiabatic Compression.

$$\frac{P'}{P} = \left(\frac{V}{V'}\right)^{\gamma}; \qquad \text{for air, } \frac{P'}{P} = \left(\frac{V}{V'}\right)^{1.408} \qquad (19)$$

$$\frac{T'}{T} = \left(\frac{V}{V'}\right)^{\gamma - 1}; \qquad \text{for air, } \frac{T'}{T} = \left(\frac{V}{V'}\right)^{.408} \qquad (20)$$

$$\frac{T'}{T} = \left(\frac{P'}{P}\right)^{\frac{\gamma - 1}{\gamma}}; \qquad \text{for air, } \frac{T'}{T} = \left(\frac{P'}{P}\right)^{.29} \qquad (21)$$

Showing that, for air, the pressure according to the adiabatic curve, varies inversely as the 1.408 power of the volume:—that, in fact, the product $P\,V^{\gamma}$ is a constant quantity; and that the absolute temperature varies as

the .29 power of the pressure, and inversely as the .408 power of the volume. For instance,

when the pressure is doubled, or as .... 1 to 2
the volumes are inversely as 1 to 1.636, or directly as, 1 to .611
the absolute temperatures are as......................... 1 to 1.222

Table No. 308 contains the relative values of the ratios of the initial and final temperatures $\frac{T'}{T}$, and volumes $\frac{V}{V'}$, for given ratios $\frac{P'}{P}$, of initial and final pressures, 1.2 to 10 times, calculated by means of formulas (21) and (19), with columns of differences to facilitate the calculation of interpolations when required.[1]

Table No. 308.—COMPRESSION OR EXPANSION OF AIR WITHOUT RECEIVING OR GIVING OUT HEAT.

Corresponding ratios of pressure, temperature, and volume, according to equations (19) and (21).

Ratio of Greater to Less Pressures.	Ratio of Greater to Less Absolute Temperatures.		Inverse of these Ratios. Ratio of Less to Greater Absolute Temperatures.		Ratio of Greater to Less Volume.		Inverse of these Ratios. Ratio of Less to Greater Volumes.	
	numbers.	differ.	numbers.	differ.	numbers.	differ.	numbers.	differ.
1.2	1.054	48	.948	41	1.138	132	.879	91
1.4	1.102	44	.907	34	1.270	126	.788	72
1.6	1.146	40	.873	30	1.396	122	.716	57
1.8	1.186	36	.843	25	1.518	118	.659	48
2	1.222	35	.818	22	1.636	114	.611	40
2.2	1.257	32	.796	20	1.750	112	.571	34
2.4	1.289	30	.776	18	1.862	109	.537	30
2.6	1.319	29	.758	16	1.971	106	.507	26
2.8	1.348	27	.742	15	2.077	105	.481	23
3	1.375	26	.727	13	2.182	102	.458	20
3.2	1.401	25	.714	13	2.284	100	.438	19
3.4	1.426	24	.701	11	2.384	99	.419	16
3.6	1.450	23	.690	11	2.483	97	.403	15
3.8	1.473	22	.679	10	2.580	96	.388	14
4	1.495	21	.669	9	2.676	94	.374	13
4.2	1.516	21	.660	9	2.770	93	.361	12
4.4	1.537	20	.651	9	2.863	93	.349	11
4.6	1.557	19	.642	7	2.955	91	.338	10
4.8	1.576	19	.635	8	3.046	89	.328	9
5	1.595	86	.627	32	3.135	434	.319	39
6	1.681	77	.595	26	3.569	412	.280	29
7	1.758	70	.569	22	3.981	396	.251	23
8	1.828	63	.547	18	4.377	382	.228	18
9	1.891	59	.529	16	4.759	370	.210	15
10	1.950		.513		5.129		.195	
1	2		3		4		5	

[1] This table is abstracted from a masterly paper, "Étude Théorique sur les Machines à Air Comprimé;" by M. Mallard (*Bulletin de la Société de l'Industrie Minérale*, 1866–67).

Let mn be the length of a cylinder filled with 1 lb. of gas, having the initial pressure P, volume V, and temperature T. Let the gas be compressed adiabatically by a piston into the volume V′, to the pressure P′, and the temperature T′. The work of compression is measured by the area $dgnd'$, and as shown by Mr. J. H. Cotterill,[1] it is expressed by

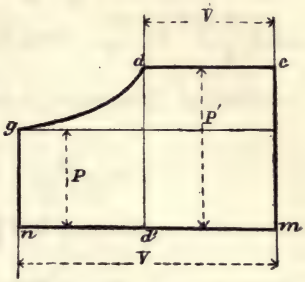


Fig. 349.—Compression of Air adiabatically.

$$\left.\begin{array}{l} Jh'\,(T'-T)=K'(T'-T); \\ \text{for air, } 130.3\,(T'-T) \end{array}\right\} \ldots\ldots \quad (22)$$

Add the work of driving the compressed air out of the cylinder into the reservoir, measured by the rectangle dm, equal to V′P′; and subtract the work contributed by the air from the initial source,—the atmosphere, for instance,—which presses on the other face of the piston, measured by the rectangle gm, equal to V P. The net work expended is,—

$$\left.\begin{array}{r} W=K'\,(T'-T)+V'P'-V\,P. \\ \text{For air } W=130.3\,(T'-T)+V'P'-V\,P. \end{array}\right\} \ldots\ldots\ldots\ldots\ldots \quad (23)$$

By formula (1), $V'P'=(K-K')\,T'$, and $V\,P=(K-K')\,T$, (a being $=K-K'$):—

$$\left.\begin{array}{r} V'P'-V\,P=(K-K')\,(T'-T); \\ \text{for air, } V'P'-V\,P=53.15\,(T'-T) \end{array}\right\} \ldots\ldots\ldots\ldots\ldots \quad (23a)$$

Substituting the value of $V'P'-V\,P$, in equation (23), and reducing, it becomes,—

Work expended in Compressing 1 pound of Dry Gas, in terms of the temperatures.

$$W=K\,(T'-T); \quad \text{for air, } W=183.45\,(T'-T)\ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

That is, the net work expended in compressing 1 pound of gas, is equal to the increase of temperature, or the difference of the initial and final temperatures, in Fahrenheit degrees, multiplied by the specific heat in foot-pounds at constant pressure.

When the initial temperature only is given, $T'-T=T\,(\frac{T'}{T}-1)$, and by substitution in formula (24), the final temperature may be found when the pressures are given:—

Work of Compressing 1 pound of Dry Gas (formula to aid in finding the final temperature).

$$W=KT\,(\frac{T'}{T}-1); \quad \text{for air, } W=183.45\,T\,(\frac{T'}{T}-1)\ldots\ldots\ldots\ldots \quad (25)$$

Corresponding to the ratio of the pressures $\frac{P}{P'}$, the value of $\frac{T'}{T}$ is found for air, in the table No. 308; thence the work, and also the final temperature.

[1] *Notes on the Theory of the Steam-Engine.* 1871.

To express the work in terms of the pressures, $PV = (K - K')T$, by formula (1), and $T = \frac{PV}{K - K'}$. Substitute this value of T in formula (24); and also $\left(\frac{P'}{P}\right)^{.29}$ for $\frac{T'}{T}$; then, by reduction,—

Work of Compressing 1 *pound of Dry Gas, in terms of pressures and initial volume.*

$$\left.\begin{aligned} W &= \frac{K}{K - K'} \times PV\left(\left(\frac{P'}{P}\right)^{.29} - 1\right); \\ \text{for air, } W &= 3.45\ PV\left(\left(\frac{P'}{P}\right)^{.29} - 1\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ (26)$$

The value of V, the volume of a pound of air, may be found for various pressures and temperatures by the formulas (1), (2), page 898.

Again, substitute the value of $T = \frac{PV}{K - K'}$, and $T' = \frac{P'V'}{K - K'}$, in equation (24); and reduce:—

Work of Compressing 1 *pound of Dry Gas, in terms of pressures and volumes.*

$$W = \frac{K}{K - K'}(P'V' - PV); \quad \text{for air, } W = 3.45\,(P'V' - PV) \ldots\ldots\ (27)$$

To exemplify the rise of temperature by adiabatic compression, take atmospheric air at 62° F., or (461 + 62 =) 523° F. absolute temperature. In doubling the pressure, the ratio $\frac{P'}{P} = 2$, and by the table No. 308, the corresponding ratio of the absolute temperatures is 1.222; whence, 523° × 1.222 = 639°, the increased absolute temperature, and 639 − 461 = 178° F., the final temperature.

For ratios of pressure, 2, 3, 4, 5, 10,
the ratios of the initial and final absolute temperatures are,—

1.222, 1.375, 1.495, 1.595, 1.950,

and when the initial temperature is 62° F., the final temperatures are,—

178° 258° 321° 373° 559°.

It may be noted that, in this example, for the ratios of pressure, 2, 3, 4, 5, and 10, the final temperatures are, very roughly, 3, 4, 5, 6, and 9 times the initial temperature 62°.

Adiabatic Expansion of Gases.

Adiabatic expansion is a duplicate, in reverse, of the adiabatic compression of a gas against a piston, and the primary formula (1), page 898,—

$$\left.\begin{aligned} PV &= aT, \\ PV &= (K - K')T, \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (28)$$

with its derivatives (19) to (27), are applicable, by reversing the order of the symbols of initial and final pressures, volumes, and temperatures, defined at page 898.

The compressed-air engine differs from the compressing engine, in being controlled by a valve by which the supply of air to the cylinder is cut off at any point of the stroke, and any degree of expansion is effected. The air may thus be worked in three ways:—1st, when it is completely expanded down to atmospheric pressure before it is exhausted; 2d, when it is admitted for the whole of the stroke, and exhausted at full pressure; 3d, when it is only partially expanded, and exhausted at a pressure above atmospheric pressure.

Referring for explanations to the discussion of adiabatic compression, it is sufficient now to repeat the formulas for compression, as adapted for adiabatic expansive-working.

When a gas is completely expanded behind a piston from the pressure P, volume V, and temperature T, to P′, V′, and T′, the relations are as follows:—

Adiabatic Expansion of a Gas.

$$\frac{P}{P'} = \left(\frac{V'}{V}\right)^{\gamma}; \qquad \text{for air, } \frac{P}{P'} = \left(\frac{V'}{V}\right)^{1.408} \text{ .... } (29)$$

$$\frac{T}{T'} = \left(\frac{V'}{V}\right)^{\gamma-1}; \qquad \text{for air, } \frac{T}{T'} = \left(\frac{V'}{V}\right)^{.408} \text{ .... } (30)$$

$$\frac{T}{T'} = \left(\frac{P}{P'}\right)^{\frac{\gamma-1}{\gamma}}; \qquad \text{for air, } \frac{T}{T'} = \left(\frac{P}{P'}\right)^{.29} \text{ .... } (31)$$

The table, No. 308, contains corresponding values of ratios of pressures, volumes, and temperatures, to save calculation.

1st. When the Gas is Completely Expanded down to an Equality with the Back-Pressure.

In the diagram, Fig. 349, let *m n* be the length of the stroke of a cylinder, into which 1 pound of a gas of the pressure P is admitted, occupying the portion of the stroke *c d*, or the volume V; and let the gas be expanded to the end of the stroke, and the volume V′, and the pressure P′, equal to the pressure of the surrounding medium, constituting back-pressure. The initial work, during admission, is measured by the rectangle *d m*, equal to V P, and the back-pressure by the rectangle *g m*, equal to V′ P′. The work of expansion between the initial and final temperatures T and T′, is measured by the area *d g n d′*, and is expressed by,

$$J\,h'\,(T - T') = K'\,(T - T'); \quad \text{for air, } 130.3\,(T - T') \text{...... } (32)$$

That is, the work by simple expansion is equal to the fall of temperature, or the *difference* of the initial and final temperatures in Fahrenheit degrees, multiplied by the specific heat in foot-pounds at constant volume.

Add the initial work, and deduct the work of back-pressure, and the net total work expended is,

$$\left.\begin{array}{r} W = K'\,(T' - T) + V\,P - V'\,P', \\ \text{for air, } W = 130.3\,(T' - T) + V\,P - V'\,P' \end{array}\right\} \text{ .... } (33)$$

By substitution and reduction, as was done for compression, page 903:—

Work performed by One Pound of Dry Gas expanded down to the back-pressure, in terms of the temperatures.

$$W = K\,(T - T'); \quad \text{for air, } W = 183.45\,(T - T') \;\ldots\ldots\ldots\; (34)$$

That is, *the net work performed is equal to the fall of temperature, or the difference of the initial and final temperatures, in Fahrenheit degrees, multiplied by the specific heat in foot-pounds at constant pressure.*

When the initial temperature only is given, $T - T' = T\,(1 - \frac{T'}{T})$; and by substitution in formula (34):—

Work performed by One Pound of Dry Gas expanded down to the back-pressure (formula to aid in finding the final temperature).

$$W = K\,T\,(1 - \frac{T'}{T}); \quad \text{for air, } W = 183.45\,T\,(1 - \frac{T'}{T}) \;\ldots\ldots\; (35)$$

The value of $\frac{T'}{T}$ corresponds in table No. 308, column 3, to the ratio of the initial and final pressures, $\frac{P}{P'}$. Thence the work may be found; also the final temperature.

To express the work in terms of the pressures, $P\,V = (K - K')\,T$, by formula (28), and $T = \frac{P\,V}{K - K'}$. Substitute this value for T in formula (35); and also $\left(\frac{P'}{P}\right)^{.29}$ for $\frac{T'}{T}$; then, by reduction,—

Work performed by One Pound of Dry Gas, in terms of pressure and initial volume.

$$\left.\begin{array}{l} W = \dfrac{K}{K - K'} \times P\,V\left(1 - \left(\dfrac{P'}{P}\right)^{.29}\right); \\ \text{for air, } W = 3.45\;P\,V\left(1 - \left(\dfrac{P'}{P}\right)^{.29}\right) \end{array}\right\} \;\ldots\ldots\ldots\ldots\ldots\ldots\; (36)$$

Again, substitute the value of $T = \frac{P\,V}{K - K'}$, and $T' = \frac{P'\,V'}{K - K'}$ in equation (34); and reduce,—

Work performed by One Pound of Dry Gas, in terms of pressures and volumes.

$$W = \frac{K}{K - K'}\,(P\,V - P'\,V'); \quad \text{for air, } W = 3.45\,(P\,V - P'\,V') \ldots.\; (38)$$

To exemplify the fall of temperature by adiabatic expansion, take atmospheric air at 62° F., or (461 + 62 =) 523° F. absolute temperature. In reducing the pressure to a half, the inverse ratio $\frac{P}{P'} = 2$, and the corresponding ratio of temperatures, column 3, table No. 308, is .818; whence 523° ×

.818 = 428°, the final absolute temperature, and 461 − 428 = −33° F., the final temperature. Similarly,

for inverse ratios of pressure, 2, 3, 4, 5, 10,
the ratios of the initial and final absolute temperatures are,
.818, .727, .669, .627, .513,
and when the initial temperature is 62° F., the final temperatures are,
−33°, −81°, −111°, −133°, −193° F.

These instances illustrate the limitless possibilities of producing cold by the expansion of air. It is clearly as impracticable to work a compressed-air engine in such low temperatures, when every particle of moisture and lubricant would be frozen, as amongst the high temperatures previously noticed.

2D. When the Gas is Admitted to the Cylinder for the Whole of the Stroke.

In this case, there is no expansive working, and the gas is exhausted at full pressure. The work done by 1 pound of dry gas is—

$$W = V\,(P - P'), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

in which P and P′ are the initial and the exhaust pressures. $P\,V = (K - K')\,T$, by formula (28), page 904, and, by inversion,

$$V = \frac{(K - K')\,T}{P}; \quad \text{for air, } V = \frac{53.15\,T}{P}; \quad \ldots\ldots\ldots\ldots \quad (40)$$

and, by substitution and reduction,

$$W = (K - K')\,T\,(1 - \frac{P'}{P}); \quad \text{for air, } W = 53.15\,T\,(1 - \frac{P'}{P}). \ \ldots \quad (41)$$

Again, the general equation for the work done by 1 pound of dry gas is (formula (34), page 906),

$$W = K\,(T - T'); \quad \text{and } W = K\,T\,(1 - \frac{T'}{T}), \quad \ldots\ldots\ldots \quad (42)$$

in which T and T′ are the initial and the final temperatures.

Equating these expressions for W, (41) and (42), and, reducing,[1]

$$\frac{T'}{T} = (1 - \frac{K - K'}{K}) + (\frac{K - K'}{K} \times \frac{P'}{P}); \text{ or,}$$

$$\frac{T'}{T} = \frac{K'}{K} + (\frac{K - K'}{K} \times \frac{P'}{P}); \qquad \text{for air, } \frac{T'}{T} = .71 + .29\,\frac{P'}{P}; \ \ldots\ldots \quad (43)$$

$$T' = T\,(\frac{K'}{K} + (\frac{K - K'}{K} \times \frac{P'}{P})); \quad \text{for air, } T' = T\,(.71 + .29\,\frac{P'}{P}) \ldots \quad (44)$$

By either of these formulas, (43, 44), the final temperature T′ is found, when the initial temperature T is given. For a ratio, for air, $\frac{P'}{P} = \frac{1}{2}$, or $\frac{P}{P'} = 2$, for instance, with the initial temperature 62° F., or absolute tempera-

[1] This method of finding the final temperature, by equating the two expressions for W, is borrowed from M. Mallard. See the preceding note, page 902.

ture 523°, the final temperature $T' = 523\ (.71 + .29 \times \frac{1}{2}) = 523 \times .855 = 447°$; and $461 - 447 = -14°$ F.

To facilitate calculation, by means of formula (43), the values of the ratios of the absolute temperatures corresponding to given ratios of the pressures, are given in table No. 309.

Table No. 309.—COMPRESSED-AIR ENGINE:—AIR ADMITTED FOR THE WHOLE OF THE STROKE.—CORRESPONDING RATIOS OF PRESSURES AND TEMPERATURES.

Ratio of the Final to the Initial Pressure.	Ratio of the Initial to the Final Pressure.	Ratio of the Final to the Initial Absolute Temperatures.	Ratio of the Final to the Initial Pressure.	Ratio of the Initial to the Final Pressure.	Ratio of the Final to the Initial Absolute Temperatures.
1	1	1	1/6	6	.758
½	2	.855	1/7	7	.751
1/3	3	.806	⅛	8	.746
¼	4	.782	1/9	9	.742
1/5	5	.768	1/10	10	.739

The final temperatures of air under adiabatic expansion, and also when exhausted at full pressure, without expansion, due to given ratios of pressure, are detailed, for comparison, in table No. 310, in the second and third columns. The reduced efficiency by adiabatic expansion, supposing the initial temperature to fall to 62° F., given at page 910, is here given in column 4. The same, for full pressure, without expansion, is given in column 5. It is calculated thus, in the first instance, for example:—The final temperature, column 3, is $-14°$ F., and is $(62 + 14 =)$ 76° below 62°,—being the range of the temperature in doing work. But the range of temperature in compressing the air adiabatically to twice the initial pressure is (178° (as at page 904) $- 62 =$) 116°; and $(\frac{76}{116} \times 100 =)$ 66 per cent. is the reduced efficiency without expansion, as in column 5. The ratios of these reduced efficiencies,

Table No. 310.—COMPRESSED-AIR ENGINE:—AIR EXPANDED ADIABATICALLY, AND AIR ADMITTED FOR THE WHOLE STROKE.—COMPARATIVE FINAL TEMPERATURES, AND REDUCED EFFICIENCIES.

Initial temperature = 62° F.

Ratio of the Initial to the Final Pressure.	Final Temperature.		Reduced Efficiency.		Ratio of Reduced Efficiencies:—Without Expansion and with Complete Expansion.
	With Adiabatic Expansion.	Without Expansion.	With Adiabatic Expansion.	Without Expansion.	
	Fahr.	Fahr.	per cent.	per cent.	per cent.
2	−33°	−14°	82	66	80
3	−81	−40	73	52	71
4	−111	−52	67	44	66
5	−133	−60	63	39	62
10	−193	−75	51	27.5	54

in columns 4 and 5, are given in the last column; found thus, in the first example, for instance:—$(\frac{66}{82} \times 100 =)$ 80 per cent. These ratios may also be calculated as the ratios of the ranges of temperature in the two cases. In the first instance, for example, $(-33+62=)$ 95°, and $(-14+62=)$ 76°, are the ranges for adiabatic expansion, and without expansion; and $(\frac{76}{95} \times 100 =)$ 80 per cent. is the ratio of the reduced efficiencies. The table indicates, generally, the economical disadvantage of working compressed air without expansion.

3D. When the Gas is but partially Expanded.

The absolute temperature of the gas, when expanded, falls from T to T′ at the end of the stroke. Here, it is suddenly exhausted into the surrounding medium, and the temperature falls still further, to T″. The work done by 1 pound of gas, in terms of the extreme temperatures, is, by the general formula (34), page 906,

$$W = K\,(T - T''); \qquad \text{for air, } W = 183.45\,(T - T'') \;\ldots\ldots\ldots\ldots\; (45)$$

whence, as in (35),

$$W = K\,T\,\left(1 - \frac{T''}{T}\right); \qquad \text{for air, } W = 183.45\,T\,\left(1 - \frac{T''}{T}\right) \;\ldots\ldots\ldots\; (46)$$

$$\text{or, } W = K\,T\left(1 - \left(\frac{T''}{T'} \times \frac{T'}{T}\right)\right); \quad \text{for air, } W = 183.45\,T\left(1 - \left(\frac{T''}{T'} \times \frac{T'}{T}\right)\right). \quad (47)$$

When the successive pressures, P, P′, P″, are known, the ratios of the temperatures in these last two formulas are easily found in the table No. 308, page 902, from the ratios of the pressures $\frac{P''}{P}$, or $\frac{P''}{P'}$, and $\frac{P'}{P}$; when the calculation for the work may be completed.

The temperatures T′ and T″ may be found from T; first, for T′, by inverting equation (31), page 905,

$$T' = T\,\left(\frac{P'}{P}\right)^{\frac{\gamma-1}{\gamma}}; \qquad \text{for air, } T' = T\,\left(\frac{P'}{P}\right)^{.29} \;\ldots\ldots\ldots\; (48)$$

Thence, the value of T″, the ultimate temperature, is found according to formula (44), page 907, to be,

$$T'' = T'\left(\frac{K'}{K} + \left(\frac{K-K'}{K} \times \frac{P''}{P'}\right)\right); \quad \text{for air, } T'' = T'\,(.71 + .29\,\frac{P''}{P'}) \ldots\ldots (49)$$

EFFICIENCY OF COMPRESSED-AIR ENGINES.

The work by expansion would be an exact duplicate, in reverse, of the work expended for compression, and the two works would be equal to each other, if the reverse actions took place between the same temperatures, pressures, and volumes. The efficiency of the combined compressor and motor would be equal to 100 per cent., irrespective of losses by friction and clearance. But, under practical conditions, the initial temperature for

expansion is not more than that of the surrounding atmosphere; and, in working, by expansion, back to atmospheric pressure, even between the same extremes of pressure, the volumes are smaller, since the temperatures are lower; and the efficiency must, of course, be less than 100 per cent.

In working air, under these conditions, between two given pressures, first compressively, and, second, expansively, let the ratios of the pressures, which are the same in both actions, be $\frac{P}{P'}$, P being the higher pressure, and P′ the lower, or atmospheric pressure. Put T″ for the higher temperature by adiabatic compression, whilst T is, as before, the atmospheric temperature, and T′ the final temperature by expansion. Then, according to formula (31), $\frac{T''}{T} = \frac{T}{T'} = \left(\frac{P}{P'}\right)^{.29}$; that is to say, the ratios of the absolute temperatures are equal to each other, since they are each equal to $\left(\frac{P}{P'}\right)^{.29}$. It follows that,

$$T'' : T :: T : T'; \text{ and that } T'' - T : T - T' :: T'' : T;$$

that is to say, the range or difference of the temperatures for compression, (T″ – T), is to the range for expansion (T – T′), in the ratio of the higher absolute temperatures, T″ and T, for compression and for expansion respectively; and the loss of efficiency by the intermediate fall of the temperature of the compressed air from that, T″, due to the compression, to T, the atmospheric temperature, is simply the proportion which this fall, T″ – T, bears to the maximum temperature T″.

It is so, because the volume is as the absolute temperature T″, and the loss of temperature T″ – T, indicates the loss of volume by contraction, under the same pressure. For instance, in compressing dry air at 62° F., to two atmospheres of pressure, in a non-conducting vessel, the temperature is raised to 178°, and the fall in reverting to 62° is (178 – 62 =) 116°. The loss of efficiency is the proportion of 116° to (461 + 178 =) 639°, the maximum absolute temperature, thus:—

(461 + 178 =) 639°
(461 + 62 =) 523

Difference, or loss,.... 116° = 18 per cent. of the maximum absolute temperature.
Leaving .... 523 = 82 " "

For ratios of pressure, or atmospheres	2,	3,	4,	5,	10,
the final temperatures for compression are,	178°,	258°,	321°,	373°,	559° Fahr.;
and the reduced efficiency, supposing the initial temperature for expansion becomes 62° F., is	82,	73,	67,	63,	51 per cent.,
whilst the loss of efficiency is	18,	27,	33,	37,	49 "

Here it is obvious that the lower the degree of compression applied to the air, the less is the rise of temperature, the less is the loss of heat by dissipation, and the greater is the efficiency of the machine. When an initial temperature can be maintained for the expansion-engine, higher than that of the surrounding atmosphere, the range of temperature within which

the air may be expanded before it arrives at the freezing-point, as a lower limit, is greater than if it commence at atmospheric temperature; and the performance is also greater in the same proportion.

When the compression is carried to 10 atmospheres, the efficiency for working in a compressed-air engine, above indicated, is only 51 per cent.

Add, that the efficiencies of the machines themselves,—the compressor and the power-engine,—are factors for the calculation of their resultant efficiency; and if the efficiency of each machine be taken at 80 per cent., the combined percentage of the two machines is $\left(\frac{80 \times 80}{100} = \right)$ 64 per cent., or two-thirds; and 64 per cent of 51 per cent., is 33 per cent., the resultant efficiency of the combined compressor and engine, working to 10 atmospheres. Similarly, it is found that the resultant efficiency, working to 2 atmospheres, is 52 per cent. The less the degree of compression, the greater is the efficiency; because the less is the proportional loss from the intermediate reduction of temperature. In general practice, the resultant efficiency rarely exceeds 30 per cent.

M. Piccard's illustration.—M. Piccard[1] happily illustrates by examples the difference of the conditions and the efficiency of the work of compressed air, in three cases, for which he adopts the initial temperature 32° F. He supposes that, in the 1st and 2d cases, the pressure and temperature are raised adiabatically during compression; and that the temperature relapses to the normal point, 32° F., before the air is applied to work, illustrated by Figs. 350 and 351; and, in the 3d case, that the temperature is constant at 32° F., whilst the air undergoes compression isothermally; Fig. 352.

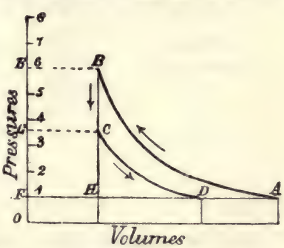


Fig. 350.—1st case—Compression Adiabatically.

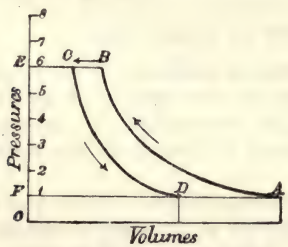


Fig. 351.—2d case—Compression Adiabatically.

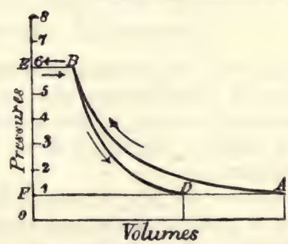


Fig. 352.—3d case—Compression Isothermally.

Illustrations of Adiabatic and Isothermal Compression of Air; with Adiabatic Expansion.

1st case:—Air compressed, cooled, and expanded within the same cylinder, without any reservoir. 2d case, Compressed air cooled in a reservoir. 3d case, Air cooled during compression. The pressure to which the air is compressed is, in each case, 6 atmospheres; whilst the final volumes to which it is compressed, taking the initial volume as 1, are,—

[1] "Du Rendement de l'Air Comprimé," *Bulletin de la Société Vaudoise des Ingénieurs et des Architectes*, June, 1876, page 10; abstracted in the *Proceedings of the Institution of Civil Engineers*, vol. xlv., 1875–76, page 273.

	Final Volume.		Fraction of Initial Volume.	During the Interval
1st case,...	0.281	or	$^1/_{3.5}$	pressure falls to 3.56 atmospheres.
2d case,...	0.281	or	$^1/_{3.5}$	final volume reduced to $^1/_6$ initial.
3d case,...	0.167	or	$^1/_6$	pressure and volume stationary.

The final pressures, volumes, and temperatures are subjoined; and to these are added the efficiency for each case, or the ratio of the useful work done to the work expended in producing the supply of compressed air:—

	Final Pressure. Atmospheres.	Final Volume. Total=1.	Final Temperature. Initial = 32° F.	Efficiency. Compression = 1.
1st case,.........	1	.69	– 119° F.	36.4 per cent.
2d case,.........	1	.595	– 168	59.2 „
3d case,.........	1	.595	– 168	78.0 „

Ordinary practical conditions oscillate between cases 2 and 3; and it is clear that, the more the air is cooled during the process of compression, the less is the expenditure of work on compression, and the greater is the resultant efficiency. M. Piccard gives the following for the respective efficiencies for various pressures:—in the 3d case, and in the case when the air is admitted for the whole of the stroke, without expansion:—

Pressures. atmospheres.	Efficiency in the 3d Case. per cent.	Efficiency, without Expansion. per cent.
1	100	100
2	90.6	72.1
4	82.4	54.1
6	78.0	46.0
8	75.2	42.1
10	72.9	39.1

It may be inferred that, under every condition, the efficiency is reduced as the pressure is multiplied.

Compression and Expansion of Moist Air.

M. Mallard has investigated the influence of moisture in air upon the variations of temperature, and on the work of compression or expansion. The principal results of the investigation are here given. It is assumed that the vapour generated from the moisture is always in the condition of saturation.

Temperature in Compression.—The rise of temperature is much less when moisture is present in the air, than when the air is dry, and is compressed adiabatically. Atmospheric air at 68° F. initial temperature, when compressed to 7½ atmospheres, rises, if dry, to 490° F.; and, if sufficiently moist, to 194° F. only.

Work for Compression.—The work is the same for dry air and moist air at 68° F. when compressed to 1½ atmospheres. For a less degree of compression, it is rather less for dry air; but for higher compressions, it is less for moist air. For 7½ atmospheres, it is 14 per cent. less.

Proportion of Moisture in Saturation.—The weight of saturated vapour in moist air at 68° F., compressed to from 1½ to 7½ atmospheres, is from 2½ to 6½ per cent. of the weight of the air.

Particulars of the compression of air, dry and moist, are given in table No. 311:—

Table No. 311.—Compression of Air, Dry and Moist.—Temperature and Work.

(Deduced from M. Mallard's data.)

Final Pressures. — Initial Pressure = 1 Atmosphere.	Final Temperatures for Compression. — Initial Temperature=68° Fahr.		Work Expended in Compressing 1 pound of Air.		Moisture Required to Produce Saturation in parts of the Weight of the Air Compressed.
	Dry Air.	Air with Sufficient Moisture.	Dry Air.	Air with Sufficient Moisture.	
atmospheres.	Fahr.	Fahr.	foot-pounds.	foot-pounds.	per cent.
1½	133°	94°	13,300	13,200	2.4
2	185	111	23,500	22,500	3.0
2½	229	124.5	30,500	29,000	3.6
3	266	135.5	37,000	35,000	4.0
3½	300	145.4	43,200	40,500	4.4
4	330	153.5	48,500	45,000	4.8
4½	357	161.6	53,600	49,000	5.1
5	383	167	58,500	52,500	5.4
5½	407	173	63,200	56,500	5.7
6	428	179	67,000	60,000	6.0
6½	440	184	71,000	63,000	6.2
7	470	190	75,000	66,000	6.4
7½	490	194	78,300	68,300	6.6

Work in Expansion.—There is a slight gain in work done, by the presence of vapour in the air, in a state of saturation; but it may be neglected in ordinary calculations.

Table No. 312.—Expansion of Air, Dry and Moist.—Temperatures.

(Reduced from M. Mallard's data.)

Temperatures.		Ratio of Expansion.	
Final.	Initial.	Dry Air.	Air with Sufficient Moisture.
Fahr.	Fahr.	ratio.	ratio.
32°	40°	1.05	1.10
32	50	1.13	1.24
32	60	1.22	1.38
32	62	1.23	1.41
32	68	1.28	1.50
32	70	1.30	1.56
32	80	1.37	1.75
32	90	1.47	2.00
32	100	1.57	2.28
32	110	1.67	2.63
32	120	1.76	3.00
32	130	1.88	3.45
32	140	2.00	4.00

Temperature in Expansion.—When moisture is present in air in the condition of saturation, the fall of temperature during expansion, is greatly less than what takes place when dry air is expanded. That a compressed-air engine may work without the freezing of any moisture or vapour in the air, it should not exhaust at a temperature lower than the freezing-point. Table No. 312, page 913, shows a few examples of the maximum ratio of expansion that may be practised, with given initial temperatures, when the final temperature is to be 32° F.:—

The table shows that air at 120° F. may be introduced into the cylinder at a pressure of 3 atmospheres, and expanded to atmospheric pressure, without risk of interference from the freezing of moisture; whilst with dry air, the maximum pressure, under the same condition, is only 1.76 atmospheres.

AIR MACHINERY.

MACHINERY FOR COMPRESSING AIR, AND FOR WORKING BY COMPRESSED AIR.

Compression of Air by Water at Mont Cenis Tunnel Works (Compresseurs à colonne d'eau).[1]

The motive power was derived from the fall of a column of water, having a head of 85¼ feet, acting on the principle of a hydraulic ram,—the water, by the power of its fall, compressing a given quantity of air at each stroke. There were 11 rams, to each of which the water was conducted from the reservoir by a 24-inch pipe. Each ram made from 2½ to 3 strokes per minute, and the air was compressed to 6 atmospheres of total pressure. The volume of air at atmospheric pressure, shut in and compressed for service at each stroke of the ram, was measured by a column in the air-limb of the pipe, 2.04 feet in diameter and 14.1 feet high, making a volume of 46.1 cubic feet of atmospheric air, or $(46.1 \div 6 =)$ 7.68 cubic feet of compressed air. The volume of compressed air for 2½ strokes per minute was 19.2 cubic feet per minute. The net horse-power is

$$[(6 \times 15) \times 144 \times 19.2 \times \text{hyp log } 6] \div 33000 = 13.51 \text{ horse-power.}$$

The total expenditure of power in the water for generating compressed air was,—

$$\frac{2.04^2 \times .7854 \times 14.1 \times 62\tfrac{1}{2} \text{ lbs.} \times 85\tfrac{1}{4} \text{ feet} \times 2\tfrac{1}{2} \text{ strokes}}{33000} = 18.5 \text{ H.P.}$$

The efficiency was, thus, equal to 73 per cent.

Compression of Air by Direct-action Steam-Pumps.

In the temporary machines used at the works for the St. Gothard tunnel, the steam-piston, 19.7 inches in diameter, was fixed to the same rod with the air-piston of 17.73 inches, with a stroke of 4 feet. The air-pumps worked in water. The minimum number of double strokes per minute was 5, but the machine could make 20 per minute. In compressing air to 3 atmospheres, the efficiency, according to the indicator-diagrams, was 84 per cent.

These pumps have been replaced by others on Colladon's system, in which the air-cylinder is kept cool by exposing every piece that is in contact with the air when undergoing compression, to currents of cold water. The pump makes 90 revolutions per minute, and is maintained sufficiently cool in compressing air to 8 atmospheres of pressure.

[1] *Simms' Practical Tunnelling*, 3d edition, 1877, page 261.

Compressed-Air Machinery at Powell Duffryn Collieries.[1]

This machinery was constructed by Messrs. J. Fowler & Co., for Sir George Elliott. There is a pair of horizontal air-compressing engines connected to one shaft, the steam-cylinder and the air-cylinder being in one line, on the same rod. The steam-cylinders are 34 inches, and the air-cylinders 40 inches in diameter, with a stroke of 6 feet. The engine is worked with steam of 70 lbs. effective pressure, cut off at one-fourth, and is fitted with Cornish steam- and exhaust-valves, 8 inches and 9 inches in diameter. The engines make 20 turns per minute, giving 240 feet of piston per minute, to indicate 482 horse-power, against a pressure of air of 40 lbs. per square inch above the atmosphere. The air-cylinders are immersed each in a cold-water bath, open at the upper side.

Experiments were made with a double-cylinder air-compressing engine, similar in arrangement to the above, having 16-inch cylinders for steam and for air, of 30 inches stroke, with an air-receiver 5 feet in diameter and 24 feet long. The steam was cut off at 80 per cent. The air-engine was an ordinary semi-portable engine, having two 10-inch cylinders of 12 inches stroke, cutting off at three-fourths. The air from the receiver was led into and passed through the boiler of the portable engine, and was thereby cooled down to within 5° of the atmospheric temperature before it passed into the cylinder. The principal results of the trials are quoted from the paper and given in table No. 313; in which the two lines, 7 and 12, have been calculated and added by the author.

Table No. 313.—Air-Compressing Engines, and Compressed-Air Engines, at Powell Duffryn Colliery—Results of Trials.

Pressure of Air in Receiver, Effective,..... lbs.	40.0	34.0	28.5	24	19
1. Effective mean pressure in steam-cylinders, lbs.	26.3	25.1	21.5	19.7	16.6
2. Do. do. air-cylinders,....lbs.	24.0	22.7	19.5	16.5	14.5
3. Speed of piston, per minute, ...feet	190	155	140	110	60
4. Effective mean pressure in air-engine,.....lbs.	35.6	29.8	24.7	21.0	17.0
5. Speed of piston, per minute, ...feet	108	104	104	108	88
Air-compressing engine—					
6. In steam-cylinder (A), ...I.H.P.	59.4	46.2	35.8	25.8	11.8
7. In air-cylinder (B),...................I.H.P.	52.6	40.7	32.2	21.7	10.1
8. Air-engine, cylinder (C),.................I.H.P.	18.3	14.7	12.2	10.8	7.1
9. Do., brake (D),..................... H.P.	15.3	12.5	10.2	9.0	5.4
10. Efficiency of D in parts of A,.........per cent.	25.8	27.1	28.5	34.9	45.8
11. Do. C ,, A,.........per cent.	30.8	31.8	34.1	41.9	60.2
12. Do. B ,, A,.........per cent.	87.7	88.0	89.8	84.3	85.4
13. Total Pressure in receiver,........atmospheres	3.72	3.31	2.94	2.63	2.29
14. Actual final volume in air—cylinder of compressing engine,..............initial vol. = 1	.380	.425	.470	.518	.575
15. Final volume according to the adiabatic curve ....initial vol. = 1	.393	.427	.465	.503	.555
16. Final volume according to the hyperbolic curve, ...initial vol. = 1	.269	.302	.340	.380	.437
Actual mean pressure:—					
17. From indicator-diagram,..................lbs.	24.0	22.7	19.5	16.5	14.5
18. By the adiabatic-curve,....................lbs.	23.5	21.1	18.6	16.4	13.8
19. By the hyperbolic-curve, ...lbs.	19.3	17.6	15.9	14.2	12.2

[1] *Proceedings of the Institution of Mechanical Engineers*, 1874.

HOT-AIR ENGINES.

Engines worked by heated air are of two classes:—1st. Those in which the air is heated and cooled alternately by contact with hot and cold surfaces; and, 2d, those in which the air is mixed with the hot products of combustion when heating surface is not used.

1st Class.—Rider's Hot-Air Engine.

In this engine, which is called a compression-engine, two single-acting cylinders are placed vertically, a little apart, connected at the upper part by a regenerator composed of thin plates. One of these is the working or hot cylinder, under which a fire is maintained, the other is the air-pump, or cold cylinder, surrounded by water to cool the air which is drawn into it, and which is pumped back into the hot cylinder. The plungers of these cylinders are worked by cranks placed at an angle of 95° on a shaft overhead. The working plunger of the 1 horse-power engine has a diameter of 6¾ inches, with a stroke of 9½ inches; the pump-plunger is 6¾ inches in diameter, with a stroke of 8.6 inches.

"The compression (pump) piston first compresses the cold air in the lower part of the compression-cylinder into about one-third of its normal volume, when, by the advancing or upward motion of the power (working) piston, and the completion of the down-stroke of the compression-piston, the air is transferred from the compression-cylinder, through the regenerator, and into the heater, without any appreciable change of volume. The result is a greater increase of pressure, corresponding to the increase of temperature, and this impels the power-piston up to the end of its stroke. The pressure still remaining in the power-cylinder, and reacting on the compression-piston, forces the latter upward till it reaches nearly to the top of its stroke, when, by the cooling of the charge of air, the pressure falls to its minimum [about atmospheric pressure], the power-piston descends, and the compression again begins. In the meantime the heated air, in passing through the regenerator, has left the greater portion of its heat in the regenerator-plates, to be picked up and utilized on the return of the air towards the heater."

From indicator diagrams, taken at 120 turns per minute, it appears that the effective mean pressure in the working cylinder was 16.8 lbs., and that in the pump was 7.15 lbs. per square inch. Reducing the pump-pressure in the ratio of the strokes, it becomes $7.15 \times \frac{8.6}{9.5} = 6.47$ lbs.; then $(16.8 - 6.47 =)$ 10.33 lbs. per square inch is the net effective pressure on the working plunger, from which the power is to be calculated. The area of the plunger is 35.78 square inches, and the net indicator horse-power is—

$$\frac{35.78 \text{ lbs.} \times 10\tfrac{1}{3} \text{ lbs.} \times .80 \text{ foot} \times 120}{33{,}000} = 1.076 \text{ horse-power.}$$

It is stated that the quantity of coal consumed is from 2 to 3 lbs. per net indicator horse-power.

An engine of ½ horse-power was tested to deliver from 650 to 700 gallons

of water per hour, 90 feet high, with a consumption of 4 lbs. of coal per hour.[1] Taking a mean of 675 gallons, the performance is equivalent to (675×10 lbs. $\times$ 90 feet $\div 60 =$) 10,125 foot-pounds per minute, or to ($10,125 \div 33,000 =$) .307 horse-power of net duty, for which (4 lbs. $\div .307 =$) 13 lbs. of coal was consumed per horse-power.

2D CLASS.—BELOU'S HOT-AIR ENGINE AT CUSSET.

The air is supplied by a feeding cylinder, 1 metre in diameter, with 1½ metres of stroke, in which it is compressed, and from which it is discharged into a close furnace, where it is heated by the combustion produced by it. Thence, it is passed to the working cylinder, 1.4 metres in diameter, with 1½ metres of stroke, where it acts with full pressure and expansively, after which it is exhausted into the atmosphere. These cylinders are double-acting. The feeding cylinder draws 1 cubic metre of air at each stroke. The furnace is inclosed in a horizontal cast-iron cylinder; the grate is inclined, and has an area of .80 square metre, or 8.6 square feet. The greater portion of the air passes through the grate. The engine makes 23 turns per minute, giving a speed of pistons of 225 feet per minute. The temperature in the chimney is 480° F.

The absolute pressure in the feeding cylinder, is raised to 1.94 atmospheres, for which the period of compression is 51.5 per cent. of the stroke. In the working cylinder, the initial pressure is 1.68 atmospheres; the air is cut off on the upper side at 39 per cent. of the stroke, and expanded exactly to atmospheric pressure at the end of the stroke; on the lower side, the admission is longer, to compensate for the weight of the piston—about 2 tons. The difference of the pressures, ($1.94 - 1.68 =$) .26 atmosphere, or 3.8 lbs. per square inch, represents the resistance in the furnace and the passages. The average effective pressures are, in the feeding cylinder, 9.4 lbs., and in the working cylinder 7.13 lbs. per square inch; yielding respectively 80.62 and 119.74 indicator horse-power. Thus, it is seen that two-thirds of the working indicator power is expended in supplying air to the working cylinder. Allowing only 10 per cent. of the indicator power for general resistances, and so reducing it to 107.77 horse-power, the net useful work is $107.77 - 80.62 = 27.15$ horse-power, which is 22.67 per cent. of the indicator power.

The quantity of coal consumed is 88 lbs. per hour, being at the rate of .735 lbs. per indicator horse-power, or 3.24 lbs. per net horse-power, as at the brake.[2]

GAS-ENGINES.

Gas-engines are worked by the explosion of a mixture of coal-gas and air, which acts on a piston within a cylinder. They may be double-acting or single-acting, and the explosion may be effected by means of an electric battery, or of lighted jets of gas placed in communication with the mixture.

[1] At the meeting of the Royal Agricultural Society at Birmingham; Messrs. Eastons and Anderson, Engineers.

[2] See the *Annales du Conservatoire des Arts et Métiers*, vol. vii., for full particulars of Belou's engines. The data above given are drawn from this source.

Lenoir's Double-acting Gas-Engine.

Two horizontal engines of this kind, fired by electricity, were tested by M. Tresca.[1] During a part of the stroke, the gas and air, in fixed proportions, are admitted into the cylinder, and then exploded by an electric spark. By the explosion, heat and pressure are generated, and the pressure acts on the piston during the remainder of the stroke. During the return-stroke, the gaseous products are exhausted into the atmosphere; whilst the explosive action takes place on the other face of the piston. The heat of the cylinder is reduced by a continuous current of cold water applied on the outside.

In the first engine, the cylinder was 7.1 inches in diameter, with a stroke of 4 inches. The mixture of gas and air was cut off at half-stroke, and the maximum absolute pressure in the cylinder was a little less than 6 atmospheres. The average speed of the engine was 129 turns per minute, giving a speed of piston of 153 feet per minute. The power measured by the brake was .57 horse-power, and the quantity of gas consumed amounted to 112 cubic feet per brake horse-power per hour. The gas and air were mixed in proportions of 1 to 10. Fifty-three per cent. of the heat generated in the cylinder was carried off by the water outside. The combustion of the gases was very nearly complete.

For the second trial, the engine had a cylinder 9½ inches in diameter, with a stroke of 4¾ inches. The weight of the engine complete was 14 cwts. The speed of the engine was 100 turns per minute, giving 158 feet of piston per minute. The period of admission was a little more than half-stroke, and the maximum absolute pressure was 5.36 atmospheres. The quantity of gas consumed amounted to 97 cubic feet per brake horse-power per hour; the power developed at the brake being about 1 horse-power. The gas and air were mixed in the proportion of 1 to 11½; and the volume of gas admitted for each stroke was 24½ cubic inches, the heat of combustion of which is, according to M. Tresca, 96 English units. It is not surprising that the temperature and the pressure after explosion, are lowered, as is shown by diagrams, almost instantaneously by contact with the metal; and it is for this reason, probably, that the stroke is made so short in proportion to the diameter. The quantity of water consumed for cooling the cylinder amounted to 4¼ cubic feet per horse-power per hour, the temperature being raised 140° F.

M. Tresca has estimated the distribution of the heat generated in the cylinder as follows:—

Heat carried off by the water and the products of combustion,	69	per cent.
Heat converted into work at the brake,.............................	4	„
Losses, not estimated,...	27	„
	100	

The net efficiency at the brake is thus taken as 4 per cent.

Otto and Langen's Atmospheric Gas-Engine—Single-acting.

In this engine, the cylinder is vertical, open to the atmosphere at the upper end; it has a "free piston," its principal feature, which is impelled upwards

[1] *Annales du Conservatoire des Arts et Métiers*, vol. i., 1861, page 894.

against the atmosphere, by the explosion of gas below it. The stress of the explosion of gas is intense, but momentary, and the free piston mounts instantly and quickly against the atmospheric resistance, whilst yet the explosive force continues. Thus the explosive force is utilized efficiently, and the power is derived from the pressure of the atmosphere, by which the piston is driven downwards against the partial vacuum formed under the piston by the collapse of the gaseous products. To cool and contract the gases, the lower half and the bottom of the cylinder are jacketed with cold water. The piston-rod is formed as a rack, and gears into a pinion loose on the fly-wheel shaft. The pinion turns loose with the rack during the ascent, but, during the descent it engages with and turns the shaft by means of a friction-clutch, making two revolutions during one descent. The routine of the engine is as follows:—1. The piston is lifted through $^1/_{11}$th of the stroke to receive the charge of gas and air. 2. The mixture is fired by a gas-light. 3. The piston makes the up-stroke. 4. The plenum under the piston becomes a vacuum of 22 inches of mercury, at the beginning of the down-stroke. 5. The down-stroke is made under an effective pressure of 11 lbs. per square inch, and the force is transmitted to the shaft. 6. When the piston arrives near to the bottom, the vacuum becomes a plenum, by compression of the gases; and, by the weight of the piston and rack, the gaseous products are expelled from the cylinder. The intermittent motion is worked by a tappet on the rack to raise the piston for the next charge.

According to Mr. Crossley,[1] for a ½ horse-power engine, the cylinder is 6 inches in diameter, with a stroke of 40 inches; and the explosions are made at various rates up to that of 30 per minute. The mixture consists of 6½ volumes of air to 1 volume of coal-gas. He takes the heating power of 1 lb. of coal-gas, of density .40, at 24,000 units of heat; and, for a consumption of 1.05 cubic feet of gas per minute, the heat supplied to the engine is equivalent to 584,000 foot-pounds, of which 70,000 foot-pounds, or 12 per cent., is yielded at the brake. The power at the brake is $(70{,}000 \div 33{,}000 =)$ 2.12 horse-power, and the consumption of gas is at the rate of $(1.05 \times 60 \div 2.12 =)$ 30 cubic feet per horse-power at the brake per hour. From indicator-diagrams, it appears that, in the down-stroke, the effective pressure varies from 11 lbs. per square inch to zero at four-fifths of the stroke, averaging 9 lbs. for four-fifths, or about 7 lbs. for the whole of the stroke.

M. Tresca[2] tested a 6-inch single-acting gas-engine, in which the power at the brake, making 81 turns per minute, was .456 horse-power. The gas consumed per hour was—

	Per Minute.	Per Brake Horse-power per Minute.
For work in cylinder,......	20.09 cubic feet.	44.06 cubic feet.
For inflaming, ...	2.08 „	4.57 „
	22.17 „	48.63 „

The water-jacket absorbed only 800 English units of heat per hour. M. Tresca allows a heating power of only 6000 French units per cubic

[1] See a paper by Mr. F. W. Crossley, on "Otto and Langen's Gas-Engine," in the *Proceedings of the Institution of Mechanical Engineers*, 1875, page 191.

[2] *Annales du Conservatoire des Arts et Métiers*, vol. vii., page 628.

metre of coal-gas, equivalent to $(6000 \times 4 \times \frac{30}{35} =)$ 21,000 English units per pound of 30 cubic feet, in round numbers. The quantity of heat generated, according to this allowance, was, then, $(21,000 \times \frac{20}{30} =)$ 14,000 units per hour. The work at the brake was $(.456 \times 33,000 \times 60 \div 772 =)$ 1170 units per hour, which represents an efficiency of $(1170 \times 100 \div 14,000 =)$ 8.4 per cent.

The Otto Gas-Engine.

Whilst the Otto and Langen atmospheric gas-engine superseded the Lenoir gas-engine, it was in its turn superseded by the "Otto Silent Gas-engine," the invention of Dr. Otto, of Deutz: called silent in comparison with other gas-engines, but now known as the Otto Gas-engine. A novel and important feature was the compression of the explosive mixture before being fired: effecting economy of gas by increase of efficiency, and facilitating the use of engines of greater power than before. Constructed by Crossley Brothers, Manchester, the Otto engine is horizontal in its disposition, resembling generally a steam-engine. It is single-acting, having the cylinder open at one end, with a trunk-piston. The cycle of operations is fourfold: in the first out-stroke a charge of gas and air in mixture, in the ratio of 1 to 16, including the burnt gases, is drawn in; and in the first in-stroke, following, the charge is compressed until it reaches a pressure of 35 lbs. per square inch; at the beginning of the second out-stroke the compressed charge is ignited and exploded, and acts on the piston for propulsion, during this, the working stroke; by the second in-stroke the gaseous products of combustion are expelled from the cylinder. Thus there are one charge and one explosion for every four single-strokes, or two double-strokes or revolutions. Strictly, therefore, the engine is half single-acting; and a heavy fly-wheel is necessary, to work the piston through the negative part of the cycle. The cylinder serves alternately as a compression-pump and as a motor-cylinder. It is jacketed with cold water to prevent overheating, although it is estimated that a loss of 42 per cent is thus incurred.

The ignition of the charge has been effected by means of a slide-valve, carrying a gas-jet, kept constantly burning. This form has recently been superseded by a system of ignition within a tube opening to the cylinder, charged with a strong igniting mixture. Space is provided for the compressed charge by a prolongation of the closed end of the cylinder. The initial pressure in the cylinder varies from 120 lbs. to 190 lbs. per square inch above the atmosphere.

The Otto engine is constructed with a single cylinder, of various nominal power, of from ½ H.P. to 20 H.P., indicating from 2 H.P. to 50 H.P.; and with double cylinders, indicating from 16 H.P. to 100 H.P. The 12 N.H.P. engine has been proved to develop 28 I.H.P., and 23 H.P. at the break, or 82 per cent of the indicator power; with a consumption of 20 cubic feet of gas per indicator horse-power per hour, or 24.3 cubic feet per brake horse-power. The total consumption of gas was at the rate of 560 cubic feet per hour; when running without load, 100 cubic feet per hour. In a 4 H.P. engine, 23.3 cubic feet was consumed per brake horse-power. In the use

of Dowson gas instead of ordinary coal-gas, 1⅓ pounds of anthracite coal is consumed per indicator horse-power per hour. The consumption has occasionally been only 1.1 pounds.

Clerk's Gas-Engine.

The gas-engine of Mr. Dugald Clerk is single-acting. A charge is exploded at every out-stroke, the mixture of gas and air being as 1 to 9, admitted for the first half of the stroke. During the second half-stroke pure air is admitted. In order to effect the explosion at every out-stroke a displacer cylinder is employed, and the charged air is compressed till it attains a pressure of 38 lbs. per square inch, when it is exploded and burns during the out-stroke. The exhaust takes place near the end of the stroke, and as the piston returns, the pure air of the charge is exhausted through the pipe, which is cooled.

GASEOUS FUEL.

The Wilson Gas-producer.

The Wilson gas-producer, introduced by Mr. Bernard Dawson, is an upright cylindrical chamber of firebrick, having a solid hearth, kept nearly full of small bituminous coal. A mixture of air and steam, comprising about 20 parts of air to 1 part of steam by weight, is delivered into the lower part of the chamber, the air being induced by two small jets of steam from a steam boiler. The fuel is resolved into combustible gases,—hydrogen, carbonic oxide, and hydrocarbons,—in the manner of the ordinary gas-furnace; and the gases pass through a number of openings above the level of the fuel, into an annular flue, whence they are conducted by an underground conduit to the place of consumption. The fuel is charged in at the top, which is closed by a pendulous conical valve or plug.

As applicable for supplying heat to steam boilers, the results of a test-trial, in November, 1886, at Apsley Paper Mill, Hemel-Hempstead, conducted by the author, may be noticed. Four Cornish boilers were fitted with two 4-cwt. Wilson gas-producers, for generating steam by gas-firing. The boilers are each 5½ feet in diameter, with a 3-feet furnace-tube, and 21 feet long; having eight Galloway tubes in each. The producers stand side by side in an open yard adjacent to the boiler-house. Each producer is cylindrical, 8 feet in diameter, 9 feet high, of firebrick cased in plate-iron. The internal hearth is 5 feet in diameter, having 20 square feet of area, the fuel space above the hearth is 4¼ feet deep, and the gases pass through openings into the annular flue surrounding the neck or upper part of the furnace, whence they are conducted underground to the boilers. Here the supply of gas to each boiler is regulated by means of a valve. The gas is delivered through the doorway, together with air, into the furnace-tube, and combustion takes place.

The four steam boilers are set in a row. No. 1 boiler was separated from Nos. 2, 3, and 4, and was devoted to the generation of steam for supplying the blast injector attached to each gas-producer. The three other boilers were connected for the supply of steam to the factory. The feedwater was measured separately into No. 1 boiler. The coal used in the producers was cobbles from Wyken Colliery, broken up by hand; charged into each hopper about four times per hour.

The leading results of the test-trial are as follows; and for comparison, the results of a six-days' test of the same boilers, which had previously been made with hand-firing, are prefixed. In this case, the fire-grates were 4 feet 8 inches long, presenting an area of 14 square feet for each boiler.

	Hand-firing.	Gas-firing.
Coal consumed per boiler in full steam per hour,	163 lbs.	258.6 lbs.
Water evaporated do. do. do.	14.93 cub. ft.	24.94 cub. ft.
Water per pound of coal from and at 212° F......	6.79 lbs.	7.16 lbs. (net)

It is shown that the boilers in full steam did two-thirds more evaporative duty by gas-firing than by hand-firing; and, with 5½ per cent. more evaporative efficiency net, after allowance made for steam consumed in blowing the producers.

It is also shown that the weight of steam consumed by the producers was equal to 8.29 per cent. of the total quantity generated in the four boilers.

The total evaporative efficiency of the boilers with gas-firing, if no deduction be made for the demands of the producers, is expressed by 6.56 pounds of water per pound of coal, or an equivalent of 7.81 pounds from and at 212°, which is (7.81 – 6.79 = .98 pound, or 14.4 per cent. more efficiency than was obtained by hand-firing. This is an expression of the absolute difference of efficiency in favour of gas-firing. The practical difference, after making the needful allowance, is, as above stated, 5½ per cent.

At intervals no smoke was visible with gas-firing; and there is no good reason why, with good draught, gas-firing should not be conducted entirely without smoke.

Comparative trials have been made at Plas Power Colliery, by Mr. John H. Darby, in which it was shown by the best result that a greater absolute evaporative efficiency was attained of 9.85 per cent. in favour of gas-firing. The following was the average composition of the gases produced:—

Carbonic acid....................................	6.26
Oxygen	0.00
Hydrogen...	14.68
Carbonic oxide..................................	23.98
Marsh gas...	4.72
Nitrogen	50.36
	100.00

The Dowson Generator Gas.

It is known that highly-heating non-luminous gases can be produced by decomposing steam in the presence of carbon: passing steam and air through a fire of incandescent fuel. Mr. J. Emerson Dowson practises this system; and, in addition, he employs special means of generating and superheating the steam. The steam producer and superheater consists of a long coil of tube, of such a form that nearly all of it is exposed to the action of gas flame. Water is forced, under a pressure of from 20 lbs. to 25 lbs. per square inch, into the coil, in which it is converted into superheated steam. The gas required for heating the coil is drawn from the gas-holder. The retort or generator is of iron, lined with ganister. The fuel, anthracite, rests on a grate, above an inclosed chamber, into

which a jet of superheated steam is directed through a small opening, carrying with it, by induction, a current of air into the furnace, for combustion. The gas produced contains by volume, approximately, 20 per cent. of hydrogen, 30 per cent. of carbonic oxide, 3 per cent. of carbonic acid, and 47 per cent. of nitrogen. The gross quantity of fuel consumed in working Otto Gas-engines averages, as before stated, 1.3 pounds per indicator horse-power. Professor Witz, of Lille, tested a 9 horse-power gas-engine on the Delamare-Deboutеville system; and proved a consumption of 89 cubic feet of the Dowson gas, or 1.33 pounds of coal per brake horse-power per hour.

FANS OR VENTILATORS.

Common Centrifugal Fan.

The ordinary fan consists of a number of blades fixed to arms, revolving on a shaft at high speeds. It appears from the results of Mr. Buckle's experiments,[2] that when the fan is revolved in its case without any air being discharged, the pressure generated at the circumference of the fan varies as the square of the velocity of the fan, and the horse-power required to maintain the speed varies as the cube of the velocity. It further appears that the pressure generated at the circumference is one-ninth greater than that which is due to the actual circumferential velocity of the fan. To express the relation of the pressure and the velocity of an air-current, the height due to the velocity is $h = \frac{v^2}{64}$; h is also equal to the height of a column of air equal in weight to the pressure. The velocity due to the pressure may thence be deduced by means of the ordinary relation $v = 8\sqrt{h}$.

Mr. Buckle recommends the following proportions for fans of from 3 to 6 feet in diameter, and for pressures ranging from 3 to 6 oz. per square inch:—

The width and length of the vanes equal to one-fourth of the diameter.

The diameter of the inlet openings in the sides of the fan-chest equal to half the diameter of the fan.

For higher pressures, of from 6 to 9 oz. and upwards, Mr. Buckle recommends that the vanes should be narrower and longer, and the inlet opening smaller, than are prescribed by the above proportions. He gives the following table of dimensions. The number of blades may be 4 or 6. The case is made of the form of an arithmetical spiral, widening the space between the case and the revolving blades, circumferentially, from the origin to the opening for discharge; and it appears that the upper edge of the opening should be level with the lower side of the sweep of the fan:—

[1] *English Mechanic*, December 29, 1876, page 387.

[2] "Experiments Relative to the Fan-Blast," by Mr. Buckle; *Proceedings of the Institution of Mechanical Engineers*, 1847.

Table No. 314.—Dimensions of Fans.

(Mr. Buckle.)

Pressure, from 3 to 6 oz. per square inch.

Diameter of Fans.		Vanes.				Diameter of Inlet Openings.	
		Width.		Length.			
feet.	inches.	feet.	inches.	feet.	inches.	feet.	inches.
3	0	0	9	0	9	1	6
3	6	0	10½	0	10½	1	9
4	0	1	0	1	0	2	0
4	6	1	1½	1	1½	2	3
5	0	1	3	1	3	2	6
6	0	1	6	1	6	3	0
Pressure, from 6 to 9 oz. per square inch, and upwards.							
3	0	0	7	1	0	1	0
3	6	0	8½	1	1½	1	3
4	0	0	9½	1	3½	1	6
4	6	0	10½	1	4½	1	9
5	0	1	0	1	6	2	0
6	0	1	2	1	10	2	4

Mine Ventilators.[1]—Guibal's Fan.

The blades are, for the most part of their length, straight; but they curve forwards at the outer ends. They are fixed on polygonal centres, and at a considerable backward inclination—usually 45°,—to the radius. The wheel is closely surrounded for about two-thirds of the circumference, by a casing of brickwork; for the remaining third, the casing gradually opens out into the discharge vent, which expands upwards as an inverted cone. By so forming the vent, the velocity of the discharged air is reduced, and converted into outward pressure, by the action of which the velocity through the fan is increased, and the efficiency is raised. A Guibal fan, working at Staveley Colliery, is 30 feet in diameter, and 10 feet wide. It makes 60 revolutions per minute in the day. The following are particulars of its performance:—

Speed, in Turns per Minute.	Draft in Inches of Water.	Volume of Air Discharged per Minute.	Efficiency, in parts of the Gross Indicator Power of the Engine.
		cubic feet.	per cent.
32	.70	43,852	40.38
51	1.70	86,283	43.09
64	2.77	101,773	53.27
68	3.10	110,005	53.85

[1] These particulars of mine-ventilators are derived from papers on "Ventilation of Mines," by Mr. J. S. E. Swindell, and Mr. W. Daniel, in the *Proceedings of the Institution of Mechanical Engineers*, 1869, 1875.

The advantage of surrounding the fan by a casing, and of adjusting, by means of a slide, the size of the opening into the vent, is shown by the following results of trials at a mine in Belgium:—

	Gross Efficiency.
Without casing,	22 per cent.
With casing,	31 "
With casing and expanding vent,	57 "
With casing and expanding vent, and with slide adjusted,	61 "

These are the efficiencies in parts of "the gross power supplied from the boiler." An efficiency of 60 per cent. is generally obtained by this ventilator; equivalent to 80 per cent. of the net power of the engine.

Cook's Ventilator.

This is a positive ventilator, making a given discharge for each revolution. It consists of a revolving eccentric within a circular case, against which a flap-valve is maintained constantly in contact, to separate the entering current from the outgoing current. Two ventilators working at Saltburn have casings of 22 feet in diameter, and 11 feet 6 inches wide; the eccentric has a diameter equal to two-thirds of that of the casing, and the eccentricity is one-fourth of its diameter. The period of inlet and discharge of air is 235°, or about two-thirds of a revolution. Making from 26 to 29 turns per minute, with a draught of from 1 to 3.25 inches, the efficiency was found to be from 58.5 to 64 per cent. of the indicator horse-power.

Blowing Engines.

Blowing engines of recent design are direct-acting, the steam-piston and the air-piston being fixed to one rod, and the steam- and air-cylinders in line. There is a pair of blowing cylinders, each of which is worked by a steam-cylinder; and the two steam-cylinders are either a pair or are arranged as compound cylinders. The clearance in the air-cylinders should be reduced to the smallest practicable limits. At Lackenby Ironworks it is only 3 per cent. at each end; the total area of valve-opening at each end, for the inlet, is $^1/_6$th of the area of the piston, and for the outlet, $\frac{1}{8}$th. These proportions are unusually liberal. The two air-cylinders are 80 inches in diameter, with 54 inches of stroke, having each a capacity of 157 cubic feet. They make 24 double strokes per minute, giving a speed of piston of 216 feet per minute; 190,000 cubic feet of atmospheric air are supplied per ton of iron made, and the supply is sufficient for the production of 800 tons of iron per week. The blast-main is 30 inches in diameter, and has a capacity $12\frac{1}{4}$ times the united volumes of the cylinders. The pressure in the main is $4\frac{1}{2}$ lbs. per square inch above the atmosphere, and it is free from fluctuations. The ratio of compression is $\frac{14.7 + 4.5}{14.7} = 1.3$, and the valves, therefore, open to the main when the air-piston has passed through 20 per cent. of the stroke, approximately, allowing for clearance; whilst the air is driven into the compressor during 80 per cent. of the stroke. The clearance is, proportionally, $3 \times 1.3 = 4$ per cent. of the volume of compressed air; and thus the effective charge of air is $(100 - 4 =)$ 96 per cent.

of the quantity compressed. The steam-cylinders are 32 and 60 inches in diameter, and their indicator horse-power is 290 horse-power; whilst that of the air-cylinders is 258 horse-power, representing an efficiency of 89 per cent.[1]

An instance of very low pressure of blast produced by a blowing engine is given by Mr. Briggs. A pair of 12-inch steam-cylinders drive directly a pair of 48-inch air-cylinders, with a stroke of 24 inches. The steam-valves cut off at ⅝ths, and they have "negative" lead and ample cover to the exhaust, for the purpose of counteracting the expansive force of the compressed air left in the clearance. But the clearance in the air-cylinders is very considerable; it is equivalent to 7½ inches of the stroke, or 31¼ per cent. At 60 double strokes per minute, when the speed of piston was 240 feet per minute; the indicator diagrams showed an average effective pressure of 17.8 lbs. per square inch for steam, and 0.8 lbs. for air, representing an efficiency of 72 per cent.[2]

In French blowing engines, according to M. Claudel, the proportion of the air discharged is only 75 per cent. of the volume described by the piston. The stroke is usually equal to the diameter of the air-cylinder, and the speed of piston varies from 100 to 200 feet per minute. The area of the inlet-valve openings is from $^1/_{15}$ to $^1/_{12}$ of that of the piston for speeds of from 100 to 150 feet per minute, and from $^1/_{10}$ to $^1/_9$ for higher speeds. The outlet openings are from $^1/_{15}$ to $^1/_{20}$ of the piston, in area.

In Belgium, Mr. Cockerell employs Woolf cylinders, with the beam, for driving blowing engines. In one example, the engine is of 160 horse-power; the cylinders are 2.79 and 3.94 feet in diameter, adapted for an expansion-ratio of 10, at regular work, to yield a pressure of 7 inches of mercury, or 3½ lbs. per square inch. The engines usually expand nine times, with steam of 4 atmospheres. The intermediate fall of pressure between the first and second cylinders is uniform throughout the stroke, equal to 1.5 lbs. per square inch. The expansion curves are the same as the "theoretical curve." The efficiency, by the indicator applied to the steam- and the air-cylinders, is 81 per cent. for a blast of 4 lbs., and 83½ per cent. for a blast of 4½ lbs. per square inch.[3]

Root's Rotary Pressure-Blower.

Root's rotary blower is positive in its action, and consists of two revolvers on parallel axles, within a close-fitting case, rectangular in section, with semicircular ends. The revolvers consist each of two arms, formed with a bulbous expansion at each end; and being geared together by a pair of spur-wheels on their shafts, outside the case, they necessarily revolve at the same speed; and they work together in such a manner that the ends of the arms of one revolver enter or gear into the middle of the other revolver. Being very correctly fitted, little air is allowed to escape between the revolvers, or between them and the casing. By their harmonious revolutions, one being horizontal whilst the other is vertical, the spaces below and above the revolvers are alternately contracted and enlarged in such an

[1] "Blowing Engines at Lackenby Ironworks," by A. C. Hill. See *Proceedings of the Institution of Mechanical Engineers*, 1871, 1872.
[2] *Journal of the Franklin Institute*, March, 1876.
[3] *Portefeuille de John Cockerell*, 1876, vol. iii.

order that whilst air is drawn into the case at the lower side, it is expelled at the upper side. Four discharges of air are thus performed for each revolution of the machine, and a steady current is maintained.

For blowing air, these machines are made by Messrs. Thwaites & Carbutt, of from 1/8 to 14 nominal horse-power, to supply from 150 to 10,800 cubic feet of air per minute, from delivery orifices of from 2½ to 19 inches in diameter. According to the results of tests made by a committee of engineers, in the United States, the efficiency of the blowers amounted to from 65 to 80 per cent. of the horse-power expended and applied to the machine.

As mine-ventilators, Root's blowers are constructed with revolvers of from 3 feet 10¾ inches to 25 feet in diameter, and 3 feet 2 inches to 13 feet wide, making from 280 to 40 revolutions per minute, delivering a volume of air of from 45 to 5000 cubic feet per turn, or from 12,500 to 200,000 cubic feet per minute. The effective power expended in delivering these volumes of air, for an exhaustion at a 6-inch water-column, is, by formula (14), page 896, from 15.5 to 189 horse-power, and the dimensions of cylinder for a non-condensing engine to drive the ventilators, vary from 14 inches diameter with 18 inches stroke, to 28 inches diameter with 48 inches stroke.

FLOW OF WATER.

FLOW OF WATER THROUGH ORIFICES.

The fundamental formula for the flow of water by the action of gravity is

$$v = 8 \sqrt{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

v = the velocity in feet per second.
h = the height in feet through which it freely falls.

This is the basic formula (6), for the action of gravity, page 279.

The quantity of water delivered per second, through an orifice in the side of a vessel, supposing that there is no contraction, is expressed by the formula,—

$$Q = 8 a \sqrt{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Q = the quantity in cubic feet per second.
a = the normal sectional area of the orifice or the stream in square feet.

But, in effect, the quantity is less than is here expressed, by reason of the contraction of the outflowing stream, and the equation becomes, for practical use,

$$Q = 8\, m a \sqrt{h}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

in which m is a coefficient, less than 1, the value of which varies with the conditions of the orifice.

When the water flows through an orifice in a thin plate, the average value of m is about .62, irrespective of the form of the orifice, and the formula becomes, $Q = 4.96\, a \sqrt{h}$ or, in round numbers,

$$Q = 5 a \sqrt{h}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

in which h = the height in feet, measured to the centre of the orifice.

With an approaching velocity v', the general formula (3) becomes

$$Q = 8\, m a \sqrt{h + v'^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

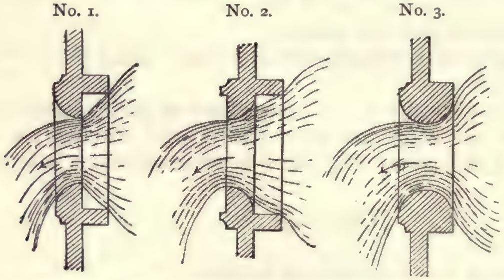
When adjutages or spouts are added to an orifice, the outflow is increased. If an internal tube be added, it is diminished. The average values of the

coefficient *m*, and the product 8 *m*, in formulas (3) and (5), are as follows:—

Form of Adjutage.	Formulas (3) and (5). Value of *m*.	Value of 8 *m*.
Internal tube,	.50	4.0
Thin plate simply,	.62	5.0
Cylinder, at least 2 diameters in length,	.82	6.6
Converging cone, length = 2½ diameters,	.95	7.6
Vena contracta, length = ½ diameter of orifice; smallest diameter = .785 diameter of orifice,	1.00	8.0
Diverging cone, length = 9 diameters,	1.46	11.7

Mr. J. F. Bateman's Experiments at Godby Reservoir, in 1852.[1]

Three rectangular openings, 6 inches deep, and 6 feet long, were made in boards 2½ inches and 5 inches thick, to the sections shown in Figs. 353. The forms of the edges, horizontal and vertical, were quadrantal, to a radius of 2½ inches. In No. 1 the bell-mouth section was



Figs. 353.—Godby Reservoir.—Flow of water through submerged openings in boards. Scale 1/18th.

outwards, in No. 2 inwards, and in No. 3 both outwards and inwards. The openings were entirely submerged on the inside, at depths of from 1 to 4 feet to the centres of the openings, and there was a free discharge. The following are the values of the coefficient 8 *m*, for formula (3), the coefficient for the whole velocity due to the height being expressed by 8.

Coefficients of Velocity of Discharge (8 *m*), in Formula (3).

Deduced from the Results of Experiments at Godby Reservoir.

Head. feet.	Average Coefficients (maximum limit, 8). No. 1 (2½ inches thick).	No. 2 (2½ inches thick).	No. 3 (5 inches thick).
4	5.78	7.04	7.60
3.5	5.66	7.04	7.60
3	5.67	7.04	7.60
2.5	5.60	7.04	7.80
2	5.60	7.04	7.80
1.5	5.50	7.04	7.78
1	5.60	6.89	7.30
0.5	5.30	6.00	6.55
Averages, omitting the last,	5.63	7.02	7.64

[1] See Mr. Bateman's paper on the Manchester Water-works, *Proceedings of the Institution of Mechanical Engineers*, 1866.

Thence the values of m and $8m$ are—

FORM OF OPENING IN BOARDS (Mr. Bateman).	Formulas (3) and (5). Value of m.	Value of $8m$.
No. 1. Quadrantal outwards,	.70	5.6
No. 2. " inwards,	.875	7.0
No. 3. " outwards and inwards,	.94	7.6

It is seen that the values of the coefficients were little affected by the variations of head, except when the head was less than about 1 foot, or double the height of the aperture.

MR. JAMES BROWNLEE'S EXPERIMENTS ON THE FLOW OF WATER THROUGH A SUBMERGED NOZZLE, CONVERGENT AND DIVERGENT.[1]

Mr. Brownlee's experiments were made with a nozzle, the sectional contour of which may be described as a double trumpet-mouth. The entrance was 1⅜ inches in diameter, and 1½ inches long, converging to a diameter of .1982 inch at the throat; whence it diverged to a diameter of 15/16 inch at the other or discharging end, through a length of 5.95 inches, which was equal to thirty times the diameter of the throat. Putting h_1 and h_2 for the heads in feet of water at the entrance to, and the exit from, the nozzle, and v for the velocity of the water passing through the throat, the generating head is $(h_1 - h_2)$, and the relations of this head and the velocity are:—

$$v = 16.02 \sqrt[1.61]{h_1 - h_2} \quad \text{.........................} \quad (6)$$

$$(h_1 - h_2) = \left(\frac{v}{16.02}\right)^{1.61} = \frac{v^{1.61}}{87} \quad \text{.......................} \quad (7)$$

These formulas differ from the normal formula (1), in embodying the 1.61 power of the velocity instead of the 2d power, and they indicate that the velocity of discharge is greater than that normally due to the head. The additional velocity is generated in consequence of a vacuous additional pressure at the throat, the sum of which, and the generating head $(h_1 - h_2)$, is the true head under which the discharge takes place. The table No. 315 contains, for illustration, a selection from the experimental results of Mr. Brownlee.

It appears that, under a double generating head, the water is driven through the compound nozzle, with $(\sqrt[1.61]{2} =)$ 1.538 times the speed for a given generating head; and that a double velocity does not require four times, as by the normal formula, but only $(2^{1.61} =)$ 3.05 times the pressure.

Mr. Brownlee attributes the great augmentation of velocity of flow through the throat of the compound nozzle, to the great length of the divergent outlet, comparatively to the diameter of the throat. The principle of the action upon which the augmented flow is effected, is that the velocity of the outflowing water is, by the necessity of occupying the expanding capacity of

[1] *Transactions of the Institution of Engineers and Shipbuilders in Scotland*, vol. xix., page 81.

the nozzle, rapidly reduced; and that a vacuum or reduction of back-pressure is induced at the throat, which, added to the generating head, makes up the true head to which the velocity is due.

Table No. 315.—Flow of Water through Compound or Double-conical Nozzles (Mr. Brownlee's Experiments).

The Heads are expressed in feet of water.

Generating Head, $h_1 - h_2$.	Vacuum at Throat of Nozzle.	True Head, or Sum of the Generating Head and the Vacuum.	Velocity due to the Generating Head.	Velocity due to the True Head.	Experimental Velocity.	Velocity by Formula (6).
feet.	feet.	feet.	ft. ℔ second.	ft. ℔ second.	ft. ℔ second.	ft. ℔ second.
.25	.52	.77	4.01	7.04	6.66	6.77
.50	1.3	1.8	5.67	10.76	10.23	10.42
.75	2.4	3.15	6.95	14.24	13.6	13.39
1	3.5	4.5	8.02	17.02	16.34	16.02
2	8.2	10.2	11.35	25.63	24.74	24.64
3	14.0	17	13.9	33.09	31.95	31.7
4	19.8	23.8	16.05	38.84	37.9	37.9
5	26.0	31	17.94	44.69	43.45	43.52
6	31.1	37.1	19.66	48.88	48.14	48.74

FLOW OF WATER OVER WASTE-BOARDS, WEIRS, &c.

To find the discharge of water over waste-boards, &c., the general formula is,—

$$Q = \frac{2}{3} ml \sqrt{2g} \left(H \sqrt{H} - h \sqrt{h} \right) \quad \ldots\ldots\ldots\ldots \quad (8)$$

Q = the quantity in cubic feet discharged per second.
m = a coefficient.
l = the width of the notch or overflow, in feet.
H = the height in feet of still-water above the edge of the notch or board.
h = the height in feet of still-water, above the level of the water as it flows over the board.

When the coefficient $m = .62$, the equation becomes, by reduction,

$$Q = 3.31\, l \left(H \sqrt{H} - h \sqrt{h} \right) \quad \ldots\ldots\ldots\ldots \quad (9)$$

FLOW OF WATER IN CHANNELS, PIPES, AND RIVERS.

The friction of fluids upon solid surfaces is independent of the pressure. It is proportional to the area of the surface, or to the area of the sides and bottom directly, and to the volume of moving water inversely; or, in brief, it is as the length of the contour or wetted perimeter of the conduit, c,

divided by the sectional area a, of the current, or to $\frac{c}{a}$. It is proportional to the square of the velocity nearly, or to mv^2. The accelerating force is equal to $\frac{h}{l} \times g$, in which h is the height, l the length of the slope of the channel, and g is gravity, or 32.2. Then, $g \times \frac{h}{l} = \frac{c}{a} \times mv^2$; and,

$$v = \sqrt{mg \times \frac{h}{l} \times \frac{a}{c}} \quad \text{.............................} \quad (10)$$

The quotient $\frac{a}{c}$ is the hydraulic mean depth, or mean radius, and the velocity is proportional to $\sqrt{\frac{a}{c}}$.

Mr. Downing deduces from experiment on the flow of water in pipes, the formula,—

Velocity of water in a channel or pipe.

$$v = \sqrt{\frac{h}{l} \times \frac{a}{c} \times 10{,}000} = 100\sqrt{\frac{h}{l} \times \frac{a}{c}} \quad \text{........} \quad (11)$$

In pipes, $\frac{a}{c} = \frac{d}{4}$; and, when pipes are filled with the flowing water, the formula (11) becomes, by substitution,

Velocity of water in a full pipe.

$$v = 50\sqrt{\frac{h}{l} \times d} \quad \text{..............................} \quad (12)$$

v = the velocity, in feet per second.
h = the head, in feet.
l = the length, in feet.
d = the diameter, in feet.
c = the wetted perimeter, in feet.
a = the sectional area of the current, in square feet.
Q = the quantity of water discharged, in cubic feet per second.
$D = \frac{a}{c}$ = the hydraulic mean depth.
f = the fall, in feet per mile.

The formula (12) is nearly identical with the formula employed by Mr. Hawksley, for the flow of water in a smooth pipe of small and uniform diameter:—

$$v = 48\sqrt{\frac{h}{l} \times d} \quad \text{..............................} \quad (13)$$

Quantity of water discharged by a channel or a pipe.

$$Q = 100\,a\sqrt{\frac{h}{l} \times \frac{a}{c}} = 100\,a\sqrt{\frac{h}{l} \times D} \quad \text{....................} \quad (14)$$

When the pipe runs full under pressure, $Q = .7854\, d^2 \times 50 \sqrt{\frac{hd}{l}}$; from which Mr. Downing deduces the formulas:—

Quantity discharged through a pipe running full under pressure.

(Cubic feet per second) $Q = 39.27 \sqrt{\frac{h}{l} \times d^5}$ .... (15)

(Cubic feet per minute) $Q = 2356 \sqrt{\frac{h}{l} \times d^5}$ .... (16)

(Cubic feet per minute and d in inches......) $Q = 4.72 \sqrt{\frac{d^5 h}{l}}$ .... (17)

(Cubic feet per minute, d in inches, l in yards) $Q = 2.725 \sqrt{\frac{d^5 h}{l}}$ .... (18)

(Gallons per minute, d and l as above....) $Q = 17.03 \sqrt{\frac{d^5 h}{l}}$ .... (19)

(Gallons per hour, d and l as above....) $Q = 1022 \sqrt{\frac{d^5 h}{l}}$ .... (20)

Mr. Downing further deduces from formula (11),

(Feet per second.......) $v = .92 \sqrt{2fD}$ .... (21)
(Feet per minute.......) $v = 55 \sqrt{2fD}$ .... (22)

The average velocity in an open channel is about $4/5$ths of the maximum velocity, which is usually attained at the centre, near the surface.[1]

Limits of Velocity at the Bottom of a Channel.

Mr. Beardmore gives the limits of velocity at the bottom, thus:—

30 feet per minute	does	not disturb clay, with sand and stones.
40 do.	do.	moves coarse sand.
60 do.	do.	moves fine gravel, size of peas.
120 do.	do.	moves 1-inch rounded pebbles.
180 do.	do.	moves angular stones, about 1¾ inches in diameter.

Cast-Iron Water-Pipes.

Water-pipes are made to resist incidental stress, in addition to the normal stress by internal pressure. The proper thickness of cast-iron pipes has been expressed by numerous empirical formulas, widely divergent. The following simple formula is here deduced from Mr. Bateman's practice.

In sixteenths of an inch,... $t = 4 + \frac{Hd}{600}$ .... (23)

In inches and decimals,... $t = .25 + \frac{Hd}{9600}$ .... (24)

Do. do. .... $t = .25 + \frac{pd}{4250}$ .... (25)

[1] See Mr. Downing's work, "Elements of Practical Hydraulics," from which the above formulas have been derived.

t = the thickness of the pipe, in inches and decimals, or in sixteenths of an inch.
H = the head of pressure, in feet of water.
p = the interior pressure, in pounds per square inch.
d = the inside diameter of the pipe, in inches.

Note.—The pressure in lbs. per square inch, is equal to the product of the head in feet of water, by .443.

Mr. Hawksley's formula for the thickness of pipes under pressure, is,—

$$t = \frac{F p d}{2 s} + C \qquad (26)$$

t = thickness, F = factor of strength, p = pressure, d = diameter, s = tensile strength of material, C = a practical constant correction for imperfections of process, method, and workmanship.

For the usual head of 300 feet of water, formula (24) becomes,—

$$\text{in inches} \ldots\ldots t = .25 + \frac{d}{32} \qquad (27)$$

$$\text{,, or } t = .25 + .031\,d \qquad (28)$$

The following are special divisors and multipliers to be employed in formulas (27) and (28) for various heads:—

Head in Feet.	Divisor in (27).	Multiplier in (28).	Equivalent Pressure in lbs. per Square Inch.
100	96	.0104	44.3 lbs.
150	64	.016	66.5 ,,
200	48	.021	88.6 ,,
250	38	.026	110.7 ,,
300	32	.031	132.9 ,,
350	27	.037	155.0 ,,
400	24	.042	177.2 ,,
500	19	.052	221.5 ,,
750	13	.078	332.2 ,,
1000	9.6	.104	443.0 ,,

Socket-end.—For a series of water-pipes cast at Woodside Ironworks, it is calculated, from the sections, that the equivalent length of pipe, of equal weight, for a socket-end, varies from 7.2 inches for 2½-inch pipes, to 8.6 inches for 24-inch pipes. Hence the formula for the equivalent length of pipe for the socket for any diameter:—

$$\text{Equivalent length in inches,} = 7 + \frac{d}{15} \qquad (29)$$

$$\text{,, ,, in feet,} = .6 + \frac{d}{180}; \qquad (30)$$

in which d is the diameter of the pipe in inches.

The table No. 316 gives the thickness and the weight of cast-iron water-pipes of given diameters for a working head of 300 feet of water, or 133 lbs. per square inch. The bursting strengths, taking the ultimate tensile resistance of the metal at 7 tons per square inch, and the factors of safety, are given in the last two columns.

Table No. 316.—Cast-Iron Pipes:—Thickness, Weight, and Strength.

Diameter.	Thickness by Formula (24) or (25).	Nearest Thickness in sixteenths of an Inch.	Net Weight per Foot run, for Thickness in Column 3.	Length of Pipe equal in Weight to the Socket.	Weight of a 9-feet Length of Pipe.	Bursting Pressure per Square Inch, reckoned on Column 3.	Factor of Safety, for Normal Pressure of 300 Feet of Water, or 133 lbs. per Square Inch.
inches.	inches.	whole sixteenths.	pounds.	feet.	cwts.	lbs.	times.
2	.31	5/16	7.09	.60	(6 ft.) .418	4900	36
2½	.33	⅜	10.6	.61	(6 ft.) .625	3920	30
3	.35	⅜	12.4	.62	1.06	3920	30
4	.375	⅜	16.1	.62	1.38	2940	22
5	.41	7/16	23.4	.63	2.01	2744	21
6	.45	7/16	27.7	.63	2.38	2290	17
7	.47	½	36.8	.64	3.17	2240	17
8	.50	½	41.7	.64	3.59	1960	15
9	.53	9/16	52.8	.65	4.55	1960	15
10	.56	9/16	58.3	.66	5.03	1764	13
11	.59	9/16	63.9	.66	5.51	1604	12
12	.62	⅝	77.5	.67	6.69	1633	12
13	.65	⅝	83.6	.67	7.22	1508	11
14	.70	11/16	99.1	.68	8.56	1540	12
15	.71	11/16	105.9	.68	9.15	1440	11
			cwts.				
16	.75	¾	1.10	.69	10.66	1470	11
18	.81	13/16	1.43	.70	13.87	1415	10.6
20	.87	⅞	1.60	.71	15.54	1372	10.3
21	.90	⅞	1.68	.72	16.33	1307	10
24	.99	1	2.19	.73	21.31	1307	10
27	1.09	1 1/16	2.61	.75	25.45	1234	9.3
30	1.18	1 3/16	3.24	.77	31.65	1241	9.3
33	1.27	1 ¼	3.75	.78	36.67	1190	8.9
36	1.36	1 ⅜	4.50	.80	44.10	1198	8.9
39	1.45	1 7/16	5.11	.82	50.18	1156	8.7
42	1.55	1 9/16	5.98	.83	58.78	1167	8.8
45	1.65	1 ⅝	6.67	.85	65.70	1133	8.5
48	1.74	1 ¾	7.63	.87	75.31	1143	8.6

Note to table.—Flanges.—The additional weight for a pair of flanges is reckoned as equivalent to that of a lineal foot of pipe;—equal to 11 per cent. extra for 9-feet lengths.

Gas-pipes.—Mr. Thomas Box gives the following thickness for gas-pipes:[1]—

Table No. 317.—Thickness of Cast-Iron Gas-Pipes.

Diameter.	Thickness.	Diameter.	Thickness.	Diameter.	Thickness.	Diameter.	Thickness.
inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.
1½	.27	5	.37	10	.46	24	.64
2	.29	6	.39	12	.49	30	.69
2½	.3	7	.41	15	.53	36	.75
3	.32	8	.43	18	.57		
4	.35	9	.45	21	.6		

[1] *Practical Hydraulics*, 1867.

WATER-WHEELS.

The work of water-wheels is done by the force of gravity acting on water. The total work in a fall of water is expressed by the product of the weight of water, w, and the height of the fall, h; or by $w\,h$; and, in order that the whole of this work should be realized by the wheel, the water must enter the machine without shock, and leave it without velocity. But there is, unavoidably, a residual velocity v', and the loss of work due to this velocity, is $w\,\frac{v'^2}{2g} = w\,h'$, in which h' is the head due to the residual velocity. The part of the head expended in effective work, and upon internal resistances, is therefore $h - h' = h - w\,\frac{v'^2}{2g}$.

There are two classes of wheels;—first, those which turn on a horizontal axis; second, those which turn on a vertical axis.

WHEELS ON A HORIZONTAL AXIS.

Undershot-Wheels, with Radial Floats or Buckets.

These are constructed from 10 to 25 feet in diameter; the floats are from 14 to 16 inches apart at the circumference, and from 24 to 28 inches deep. Putting v and v' for the initial and final velocities of the water, which flows under the wheel, v' is sensibly equal to the velocity of the middle of the float, and the maximum effect is obtainable when $v' = \frac{v}{2}$, when the efficiency is 50 per cent. Smeaton tried velocities v' of from $.34\,v$ to $.52\,v$, mean $.43\,v$; and obtained an efficiency, with models, of from 29 to 35, mean 33 per cent. By the best modern experiments, the efficiency is usually from 27 to 30 per cent. Experimentally, 40 per cent. is the maximum. Probably, it cannot be exceeded, because the final velocity of the water is not reduced to $\frac{1}{2}\,v$.

Poncelet's Undershot-Wheel.

The floats are curved,—usually a portion of a circle,—and so placed that the hollow of the curve is presented to the entering water, the edge of the float being set at an angle of 30° to the circumference of the wheel. There are 36 floats in wheels of from 10 to 13 feet in diameter, and 48 floats for diameters of from 20 to 23 feet. If the water could enter the wheel without shock, tangentially to the floats, the velocity of the floats being half the velocity of the water, the water would ascend the float, and would then descend by the force of gravity, and drop into the tail-race

with a final forward velocity = 0. The efficiency, under these circumstances, would be 100 per cent. But the conditions of practice do not admit of a tangential entrance, and the efficiency is not more than 65 per cent. for falls of 4 feet and less, 60 per cent. for falls of from 4 feet 3 inches to 5 feet, and from 55 to 50 per cent. for falls of from 6 feet to 6½ feet. These efficiencies are materially greater than that of the undershot-wheels with radial floats; and the experience of the Poncelet-float conspicuously demonstrates the essential importance of providing graduated entrances, and avoiding shocks, concussions, or eddies in the water. The most favourable ratio of the velocity of the floats to that of the water, is 55 per cent. The distance between the inner and outer circumferences that limit the floats should be at least a fourth of the head; Poncelet advises a third of the head.

Paddle-Wheel in an Open Current.

Experience indicates that the most suitable ratio of the velocity of the floats to that of the current is 40 per cent. The depth of the floats should be from ¼th to $^1/_5$th of the radius; it should not be less than 12 or 14 inches. The diameter is usually from 13 to 16½ feet, with 12 floats; but it is thought that there might be an advantage in applying 18 or even 24 floats. The floats should be completely submerged at the lower side, but not more than 2 inches under water.

Breast-Wheel.

This wheel receives the water at a level a little below that of the axis. In practice, the efficiency is 70 per cent. when the height of the fall approaches 8 feet, and 50 per cent. for a fall of 4 feet. For a well constructed wheel, slow-moving, M. Morin found an exceptional efficiency of 93 per cent.

Sir William Fairbairn states that the efficiency of high breast-wheels is 75 per cent., moving at the rate of 5 feet per second at the periphery. The usual velocity adopted by him for high and low falls, was from 4 to 6 feet per second; for a minimum velocity, 3 feet 6 inches per second, for falls of from 40 to 45 feet; for a maximum velocity, 7 feet per second, for falls of 5 or 6 feet.

The water should be delivered to the wheel at a low velocity; or, if the velocity is considerable, the delivery should be at a tangent to the edge of the float. The most suitable velocity of the floats is 4¼ feet per second; the velocity should not exceed the limits of from 3 to 6½ feet per second. The depth of water over the sliding gate should be from 8 to 10 inches, measured from still water. The diameter should be at least 11½ feet; it is seldom more than from 20 to 23 feet. These diameters are suitable for falls of from 3 to 6 or even 8 feet. The distance apart of the floats should be 1⅓ to 1½ times the head over the gate, for slow wheels; for quick wheels, a little more. The depth of the floats should be a little more than 2·3 feet. Normally, the interior capacity between two floats should be nearly double the volume of the water there.

Overshot-Wheel.

The water is delivered nearly at the top of the wheel. The chief causes of loss of head are, first, the relative velocity of the water when it enters the wheel, and the velocity which it possesses at the moment it falls to

the level of the tail-race. Such wheels answer well for heads of from 13 to 20 feet. For heads of less than 10 feet, breast-wheels are preferred. The velocity of the floats should not be less than 3 feet per second; it may be 6½ feet per second for small wheels, and 10 feet for larger wheels, without sensibly affecting the efficiency. The efficiency at a low speed may rise to 80 per cent.; but ordinarily, with velocities of from 3 to 6½ feet, the efficiency varies from 70 to 75 per cent. At higher speeds, and when the buckets are more than two-thirds filled, the efficiency is only 60 per cent. For wheels employed in driving hammers, of from 10 to 13 feet in diameter, having a velocity of from 13 to 16 feet per second, the efficiency occasionally falls to 37 per cent. This low efficiency is due to the impetuosity of the fall of the water into the quick-moving buckets, whence the water is thrown out, partly by reaction, partly by centrifugal force. The capacity of the buckets should be three times the volume of the charge of water; they may be 10 or 11 inches deep, and 12 or 14 inches apart. With a velocity of 4 feet per second, 1 cubic foot of water per foot of breadth of the wheel, may be consumed per second. The stream, as it leaves the gate, is rarely more than 4 or 5 inches deep; it is often less than 2 inches deep.

WHEELS ON A VERTICAL AXIS.

Tub-Wheel.

The old-fashioned spoon-wheel or tub-wheel, consists of a number of paddles fixed on a vertical axis, revolving within a cylindrical well of masonry, with very little clearance. The paddles are slightly concave, and are struck in the hollow side by a horizontal current from a reservoir at a considerable head; the current enters the well tangentially, and, after having expended its force, it falls between the open paddles to the bottom of the well. The maximum efficiency is calculated to be that due to a velocity of the centre of the paddles, when they are struck, equal to one-third of the velocity of the current; and the efficiency is 30 per cent. In practice, the efficiency varies from 15 to 40 per cent.

Whitelaw's Water-Mill.

Mr. James Whitelaw, of Paisley, developed the principle of Barker's mill, and produced an efficient motor. Barker's mill consisted of two hollow radial arms revolving on a central pipe, through which water under pressure passed to the extremities of the arms, and was ejected through an orifice at the end of each arm, in an opposite direction; and so producing rotatory motion. In Whitelaw's mill, the arms taper from the centre towards the circumference, and they are curved in such a manner as to allow the water to pass from the central openings to the orifices, in directions nearly straight and radial, when the machine runs at its proper speed; so that very little centrifugal force is imparted to the water by the revolution of the arms, and that, therefore, a minimum of frictional resistance is opposed to the motion of the water. A model, 15 inches in diameter, measured to the centres of the orifices, with a central opening 6 inches in diameter, and two orifices of discharge, each 2.4 inches by 0.6 inch,—was tested under

a head of 10 feet, making 387 revolutions per minute. The efficiency amounted to 73.6 per cent. At 324 revolutions, the efficiency was 71 per cent. According to the results of tests of another model mill, made by competent engineers, the efficiency amounted to 76 per cent., when the speed of the orifices was equal to that due to the height of the fall.

A water-mill, on Whitelaw's system, 9.55 feet in diameter, having circular orifices 4.944 inches in diameter, with a fall of 25 feet, was erected on the Chard Canal in 1842, for the purpose of hauling boats up an inclined plane. The net work done by the machine represented an efficiency of 67.3 per cent., with the resistance of the gearing in addition. It was estimated that the actual duty of this mill amounted to 75 per cent.[1]

Turbines.

Turbines, like Whitelaw's mill, apply the force of water by impact and reaction combined; but they comprise an additional feature not embodied in Whitelaw's mill, the employment of guide-blades to change the direction of the water descending under a head, so as to cause it to enter tangentially between the blades of the wheel. The blades of the wheel are so curved as to receive the water without shock, and to discharge it horizontally. Turbines are of three kinds;—outward flow, downward flow, and inward flow.

Outward-Flow Turbines.

Fourneyron Turbine.—This turbine acts with an outward flow; that is to say, the water enters from above through a central opening, and is guided by curved blades, to be discharged laterally at the base of a circular chamber, or well, equally at all parts of the circumference, into the buckets or curved blades of the wheel. The wheel is annular, and closely surrounds the circular chamber; thus it receives the whole of the water at its maximum velocity, and it is the function of the curved blades to receive and transmit the force of the water, and to discharge the water at the outer circumference of the wheel, with the least possible residual velocity.

When the supply of water is insufficient for working the turbine at its full power, the exit openings from the well are partially closed by a cylindrical sluice which is lowered upon them to the required extent.

The efficiency is reduced in proportion as the sluice is lowered, for the action of the water on the wheel is less favourably exerted. M. Morin tested a Fourneyron turbine, 2 metres, or 6.56 feet in diameter, and he found that, in this way, the efficiency varied from a maximum of 79 per cent. to 24 per cent., when the supply of water was reduced to a fourth of the full supply. In practice, the radial length of the buckets or floats of the wheel is a fourth of the radius, for falls not exceeding 6½ feet, three-tenths for falls of from 6½ to 19 feet, and two-thirds for higher falls.

Boyden Turbine.—Mr. Boyden, of Massachusetts, designed an outflow-turbine of 75 horse-power, which realized an efficiency of 88 per cent. The peculiar features, as compared with Fourneyron's turbine, are, 1st, and most important, the conducting of the water to the turbine through a vertical truncated cone, concentric with the shaft. The water, as it descends, acquires a gradually increasing velocity, together with a spiral movement in the direction

[1] See *Description of Whitelaw & Stirrat's Patent Water-mill*, 1843.

of the motion of the wheel. The spiral movement is, in fact, a continuation of the motion of the water as it enters the cone. 2d. The guide-plates at the base are inclined, so as to meet tangentially the approaching water. 3d. A "diffuser," or annular chamber surrounding the wheel, into which the water from the wheel is discharged. This chamber expands outwardly, and thus the escaping velocity of the water is eased off and reduced to a fourth when the outside of the diffuser is reached. The effect of the diffuser is to accelerate the velocity of the water through the machine; and the gain of efficiency is 3 per cent. The diffuser must be entirely submerged.

Rules for Outward-Flow Turbines.

Lieutenant F. A. Mahan, U.S., has deduced, from the practice of Mr. Boyden, the following rules for proportioning outward-flow turbines. He has also deduced the formulas which follow from the results of Mr. Francis' experiments on Mr. Boyden's turbines at the Tremont Mills, Lowell.[1]

Rules for Proportioning Outward-flow Turbines.

For falls of from 5 feet to 40 feet, and diameters not less than 2 feet:—1st. The sum of the shortest distances between the buckets should be equal to the diameter of the wheel. 2d. The height of the orifices at the circumference of the wheel should be equal to one-tenth of the diameter of the wheel. 3d. The width of the crowns should be four times the shortest distance between the buckets. 4th. The sum of the shortest distances between the curved guides, taken near the wheel, should be equal to the interior diameter of the wheel.

For falls greater than 40 feet, the 2d rule should be modified: the height of the orifices should be smaller in proportion to the diameter of the wheel.

On the basis of these rules, an efficiency of 75 per cent. may be obtained.

Formulas for the proportions and performance of Outward-flow Turbines.

$$P = .0425\ D^2 h\sqrt{h} \quad (1)$$

$$D = 4.85\sqrt{\frac{P}{h\sqrt{h}}} \quad (2)$$

$$Q = .5\ D^2\sqrt{h} \quad (3)$$

$$v = .56\sqrt{2\,gh} = 4.49\sqrt{h} \quad (4)$$

$$H = .10\ D \quad (5)$$

$$N = 3\ (D + 10) \quad (6)$$

$$w = \frac{D}{N} \quad (7)$$

$$W = \frac{4\ D}{N} \quad (8)$$

$$d = D - \frac{8\ D}{N} \quad (9)$$

$$n = .50\ N \text{ to } .75\ N \quad (10)$$

$$w' = \frac{d}{n} \quad (11)$$

[1] *Water-Wheels and Hydraulic Motors.* New York, 1876.

D = the exterior diameter of the wheel.
d = the interior diameter of the wheel.
H = the height of the orifices of discharge at the outer circumference.
W = the width of the crown of the buckets.
N = the number of buckets.
n = the number of guides.
w = the shortest distance between two adjacent buckets.
w' = the shortest distance between two adjacent guides.
HP = the actual horse-power of the turbine.
h = the height of the fall acting on the wheel.
Q = the quantity of water expended by the turbine, in cubic feet per second.
V = the velocity due to the fall.
V′ = the velocity of the water passing through the narrowest sections of the wheel.
v = the velocity of the interior circumference of the wheel.
C = the coefficient for V′ in terms of V, or $\frac{V'}{V}$.

The dimensions are in feet; and the velocities in feet per second.

These formulas are only to be taken as guides for practice; not as established proportions.

Downward-Flow Turbines.

Fontaine's Turbine.—In turbines constructed with downward flow, the wheel is placed below an annular series of guide-blades, by which the water is conducted to the wheel. The water strikes the curved floats of the wheel, and it falls vertically, or nearly so, into the tail-race. The principle of action is the same as that of the outward-flow turbine; but the centrifugal action of the latter is avoided, and the downward flow is more compact.

The Fontaine turbine yields an efficiency of 70 per cent., when fully charged. When the supply of water is shut off to three-fourths, by the sluice, the efficiency is 57 per cent. The best velocity at the mean circumference of the wheel, is equal to 55 per cent. of that due to the height of fall. It may vary a fourth of this either way, without materially affecting the efficiency. An efficiency of 70 per cent. is guaranteed by manufacturers.

Jonval Turbine.—This turbine is, essentially, the same as Fontaine's; but, for convenience, it is placed at some height above the level of the tail-water, within an air-tight cylinder, or "draft-tube," so that a partial vacuum or reduction of pressure is induced under the wheel, and the effect of the wheel is by so much increased. The resulting efficiency is the same as if the wheel were placed at the level of the tail-race; and thus, whilst the turbine may be placed at any level, advantage is taken of the whole height of the fall.

The efficiency under a full charge is 72 per cent. The best velocity at the exterior of the wheel, is 70 per cent. of that due to the total fall.

Turbine by the North Moor Foundry.—The water is conducted by a pipe into a spiral water-chamber surrounding the wheel, from which it is guided through the guides or water-ports horizontally on to the middle of the circumference of the wheel, so as to enter the curved wings or buckets without any shock. After traversing them, it passes off vertically, or nearly so, on both sides of the wheel, above and below, to the tail-race. The buckets are, for this purpose, constructed in two rings, with right

and left curvatures, meeting at the middle, at an acute angle, where the water enters and is divided equally between them. The flow is partially inwards, combined with vertical flow.

INWARD-FLOW TURBINES.

Vortex Wheel, or Inward-flow Turbine.—The vortex wheel is made with radiating vanes, and is surrounded by an annular case, closed externally, and open internally to the wheel, having its inner circumference fitted with four curved guide-passages. The water is admitted by one or more pipes to the case, and it issues centripetally through the guide-passages upon the circumference of the wheel. The water acting against the curved vanes of the wheel, the wheel is driven round at a velocity dependent on the height of the fall, and the water, having expended its force, passes towards and out at the centre. This wheel has realized an efficiency as high as 77½ per cent. It was originally designed by Professor James Thomson.

Swain Turbine.—In the Swain turbine are combined an inward and a downward discharge. The receiving edges of the floats of the wheel are vertical, opposite the guide-blades, and the lower portions of the edges are bent into the form of a quadrant. Each float thus forms with the surface of the adjoining float an outlet which combines an inward and a downward discharge.[1] A Swain turbine, 72 inches in diameter, was tested at Boott Cotton Mill, U.S., by Mr. J. B. Francis, for several heights of gate, or sluice, from 2 to 13.08 inches, and circumferential velocities of wheel ranging from 60 per cent. to 80 per cent. of the respective velocities due to the heads acting on the wheel. For a velocity of 60 per cent., and for heights of gate varying within the limits already stated, the efficiency ranged from 47½ to 76½ per cent., and for a velocity of 80 per cent., it ranged from 37½ to 83 per cent. The maximum efficiency attained was 84 per cent., with a 12-inch gate and a velocity-ratio of 76 per cent.; but from 9-inch gate to 13-inch gate,—say, from two-thirds gate to full gate,—the maximum efficiency varied within very narrow limits—from 83 to 84 per cent.,—the velocity-ratios being 72 per cent. for the 9-inch gate, and 76½ per cent. for full gate. At half-gate, the maximum efficiency was 78 per cent., when the velocity-ratio was 68 per cent. At quarter-gate, the maximum efficiency was 61 per cent., and the velocity-ratio 66 per cent.

TANGENTIAL WHEELS.

Wheels to which the water is applied at a portion only of the circumference, are called tangential wheels. They are specially suited for very high falls, where a considerable diameter and high tangential velocity may be combined, with a moderate speed of revolution. The Girard turbine belongs to this class. It is employed at Goeschenen station, for the works of the St. Gothard tunnel; and it works under a head of 279 feet. The wheels are 7 feet 10½ inches in diameter; they have 80 buckets, and their regular speed is 160 turns per minute, with a maximum charge of water of 67 gallons per second. It is said that the Girard turbines employed at the Paris water-works have yielded an efficiency as high as 87 per cent.; ordinarily, the efficiency, it is said, is from 75 to 80 per cent.

[1] *Journal of the Franklin Institute*, April, 1875. See also a notice in the *Proceedings of the Institution of Civil Engineers*, vol. xli., page 334.

MACHINES FOR RAISING WATER.

PUMPS.

The effective work done by a pump, in raising water, is equal to the product of the weight of water lifted, by the height through which it is raised. The efficiency of the pump is the ratio of the effective work to the whole work expended in driving the pump.

RECIPROCATING PUMPS.

The efficiency of well-proportioned reciprocating pumps is from 75 to 85 per cent.; ordinarily, it is not more than 75 per cent., and is often less than that. In well-made pumps, in good order, the volume of water passed is 96 or 97 per cent. of the volume described by the piston; but, for ordinary pumps, it is only from 80 to 90 per cent. When the speed of the pump is very rapid, the volume of water may be equal to or greater than the volume described by the piston. In such instances, the column of water continues in motion after the piston has arrived at the end of its stroke. Ordinary well-made pumps may, with certainty, draw water from any depth not exceeding 27 feet. M. Claudel gives the results of experimental tests of various pumps:—

	Efficiency.		Ratio of Volume of Water to that described by the Piston.
Fire-engines:—Height, 10 to 16 feet,	20.7 per cent.	......	91 per cent.
Worked with hose,..	35.8 „	......	91 „
Pumps for drainage, ...	50 to 69 „	......	93 „
Pumps in towns, double-acting,......	70 to 75 „	......	90 to 95 „

Mr. R. Davison[1] gives the duty of 6-inch three-throw pumps employed in raising water at a brewery, with the indicator horse-power consumed, from which the following deductions are made. A barrel of water holds 36 gallons:—

Water per Hour lifted.	Lift.	Gross Power.	Net Power (Duty).	Efficiency.
120 barrels,.....	165 feet,	4.7 H.P.	3.6 H.P.,	or 77 per cent.
160 barrels,......	140 „	6.2 „	4.07 „	or 65.6 „
80 „	54 „	1.0 „	.785 „	or 78.5 „
250 „	48 „	4.87 „	2.18 „	or 45 „
		Average,..................................		66.5 „

[1] *Proceedings of the Institution of Civil Engineers*, vol. ii., year 1843, page 79.

A double-acting pump, constructed by M. Farcot, was tested by M. Tresca. It contains two barrels placed vertically, side by side, and worked from a shaft overhead, with two cranks, driven through a belt pulley. The cranks coincide, so that the buckets of the pumps reciprocate simultaneously. The valves of the first bucket open downwards, and those of the second bucket upwards. They are made as large as possible, and no other valves are required. The water enters at the upper part of the first barrel, and leaves at the upper part of the second barrel, passing from one to the other by a connecting chamber at the bottom. By this arrangement, the first barrel forces the water, and the second barrel draws the water, and a continuous stream is set up. The barrels of the pump were 18 inches in diameter, with a stroke of 6 inches. The following are classified results of performance, in which the efficiency is expressed in parts of the dynametric power expended in driving the pump:—

Speed of the Pumps.	Total Head.	Efficiency.	Ratio of Volume of Water to Capacity of Pump.
turns per minute.	feet.	per cent.	per cent.
33.00	14.10	43.1	96
42.40	14.10	43.1	97.2
55.08	14.10	44.7	92
60.55	16.63	53.7	94.5
Averages,....	14.73	46.1	
23.75	23.22	63.7	—
45.48	24.93	53.0	95.7
60.00	27.32	53.0	95.4
Averages,....	25.16	56.6	
39.62	33.54	66.7	97.6
43.75	33.54	69	98.1
40.50	33.39	61.2	91.4
55.00	35.55	63.2	95.4
28.00	35.55	71.4	91.2
Averages,....	34.31	66.2	
31.00	42.80	73.6	93.9
24.33	45.62	73.7	89.8
52.68	45.62	71.0	95.3
32.50	46.28	66.5	91.7
55.00	46.97	70.4	94.8
50.00	49.33	71.0	95.8
61.98	51.00	68.7	90.5
55.00	75.44	70.4	92.5
Averages,....	50.38	70.7	

From these data, it is evident that the efficiency increases with the height of the lift,—a result which is explained by the less proportion of the constant resistances of the pumps to the work done at higher lifts.

Centrifugal Pumps.

The Appold pump, made with curved receding blades, is the form of centrifugal pump most widely known and accepted. M. Morin tested three kinds of centrifugal or revolving pumps:—1st, on the model of the Appold pump; 2d, having straight receding blades inclined at an angle of 45° with the radius; 3d, having radial blades. They were 12 inches in diameter and 3⅛ inches long, and had 6-inch central openings. Their efficiencies were respectively as follows:—

1. Curved blades, ...	Efficiency,	48 to 68	per cent.
2. Inclined blades,..................	"	40 to 43	"
3. Radial blades, ...	"	24	"

The height to which water ascends in a pipe, by the action of a centrifugal pump, would, if there were no other resistances, be that due to the velocity of the circumference of the revolving wheel, or to $\frac{v^2}{2g}$. The results of experiments made by the author on two pumps, at the International Exhibition of 1862, yielded the following data, showing the height to which the water was raised, without any discharge:—

	Gwynne's Pump (blades partly radial, curved at the ends).	Appold Pump (curved blades).
Diameter of pump-wheel, ...	4 feet,	4 feet 7 inches.
Revolutions per minute, ...	177	95.4
Velocity of circumference, per second,...	37.05 feet,	22.9 feet.
Head due to the velocity, ...	21.45 "	8.194 "
Actual head, ...	18.21 "	5.833 "
Do. in parts of head due to velocity,	85 per cent.	71.2 per cent.

Mr. David Thomson made similar experiments with Appold pumps of from 1.25 to 1.71 feet in diameter, the results of which showed that the actual head was about 90 per cent. of the head due to the velocity.

M. Tresca, in 1861, tested two centrifugal pumps, 18 inches in diameter, with a 9-inch central opening at each side. The blades were six in number, of which three sprung from the centre, where they were ½ inch thick; the alternate three only sprung at a distance equal to the radius of the opening from the centre. They were radial, except at the ends, where they were curved backward, to a radius of about 2¼ inches; and they joined the circumference nearly at a tangent. The width of the blades was taper; the blades were 5¾ inches wide at the nave, and only 2⅝ inches at the ends: so designed that the section of the outflowing water should be nearly constant.

M. Tresca deduced from his experiments, that, in making from 630 to 700 revolutions per minute, the efficiency of the pump, or the actual duty in raising water, through a height of 31.16 feet, amounted to from 34 to 54 per cent. of the work applied to the shaft; or that, in the conditions of the experiment, the pump could raise upwards of 16,200 cubic feet of water per hour, through a height of 33 feet, with about 30 horse-power applied to the shaft, and an efficiency of 45 per cent.

According to Mr. Thomson, the maximum duty of a centrifugal pump worked by a steam-engine, varies from 55 per cent. for smaller pumps to

70 per cent. for larger pumps. They may be most effectively used for low or for moderately high lifts, of from 15 to 20 feet; and, in such conditions, they are as efficient as any pumps that can be made. For lifts of 4 or 5 feet they are even more efficient than others. At the same time, the larger the pump the higher the lift it may work against. Thus, an 18-inch pump works well at a 20-feet lift, and a 3-feet pump at a 30-feet lift. A 21-inch fan at a 40-feet lift has not given good results: high lifts demand very high velocities.[1]

The efficiency is influenced by the form of the casing of the pump. The Hon. R. C. Parsons made experiments with two 14-inch fans on Appold's form and on Rankine's form.[2] In Rankine's fan, the blades are curved backwards, like those of Appold's, for half their length; and curved forwards, reversely, for the outer half of their length. Plotting the results of performance arrived at by Mr. Parsons, the following are the several amounts of work done per pound of water evaporated from the boiler, reduced for a speed of 350 turns per minute:—

	Work done per pound of water evaporated. foot pounds.		Ratio.
Appold fan, in concentric circular casing,...	6,250,	as	1
Do., in spiral casing,....................	9,000,	„	1.44
Rankine fan, in concentric circular casing,..	9,700,	„	1.55
Do., in spiral casing,..................	12,500,	„	2

These data prove two things:—1st, that the spiral casing was better than the concentric casing; 2d, that Rankine's ogee-fan was more efficient by one-half than Appold's fan.

Endless-Chain Pump of Square Boards.

A series of square boards, or paddles, 5 or 6 inches square, strung together; turning on two centres; inclined at an angle of 30° or 40° with the horizon. The lower end is immersed in the water to be raised, and the water is dragged by the paddles up an inclined trough to the higher level. Efficiency, 40 per cent.

When the endless-chain pump works vertically, the paddles pass through a vertical pipe, and this arrangement is suitable for lifts of more than 12 feet. Worked by from 4 to 8 men, the efficiency of the vertical endless-chain pump is 65 per cent., and the volume of water raised is 5/6ths of the volume described by the paddles.

Noria.

The noria is an endless chain of buckets having a capacity of from 1½ to 3 gallons, working vertically. The efficiency increases with the height, because a given excess of elevation above the higher level is required for accommodation for delivering the water. For lifts of from 3 to 12 feet,

[1] "Pumps," page 156, in Mr. Humber's *Water Supply of Cities and Towns.*

[2] See Mr. Parsons' paper on "Centrifugal Pumps," in the *Proceedings of the Institution of Civil Engineers* (1875–76), vol. xlvii., page 267. It is due to the author of this paper to say that he has deduced other conclusions than those drawn in the text, from the comparative results of the experiments with Appold's fan and Rankine's fan.

the efficiency is from 50 to 66 per cent. For higher lifts, an efficiency of from 70 to 80 per cent. may be realized.

Water-Works Pumping Engines.

The indicator-power of the single-acting beam-engines at the East London Water-works, according to Mr. Greaves' experiments, was effective in actual "pumping duty," or efficiency, or quantity of water raised, to the extent of 81 per cent. The difference, 19 per cent., was absorbed in nearly equal proportions by the friction of the engine, including that of the pole, on the one part, and the friction and resistance of the pump on the other part; being each nearly 10 per cent. of the "total load," or indicator-power.[1]

Mr. David Thomson gives data for the "pumping duty" of the double-acting compound-cylinder rotative beam-engines of the Chelsea Water-works, in which the cylinders are placed vertically side by side, under one end of the beam, acting on the Woolf system. It appears that the duty is 80 per cent. of the indicator-power.[2]

A pair of compound rotative beam-engines, designed by Mr. E. D. Leavitt, jun., have been erected at the St. Lawrence Water-works, Mass., U.S.[3] The first and second cylinders of each engine, are connected one to each end of the beam. They are placed together under the main centre, and consequently are directed obliquely each to its proper end of the beam; and, whilst the lower ends are close together, the upper ends lie apart from each other. The connecting-rod to the fly-wheel shaft is connected to the first-cylinder end of the beam; and the rod to the pump, to the second-cylinder end. The cylinders are fitted with gridiron valves, having a large area of opening, and small movement. The pumps are of the bucket-and-plunger type, first introduced by Mr. David Thomson, in England. The effective pressure in the boiler is 90 lbs. per square inch. The cylinders are 18 inches and 38 inches in diameter, with 8 feet of stroke. They are entirely steam-jacketted. The clearance averages, for the first cylinder, 2.43 per cent., and for the second cylinder, 1.68 per cent. of the capacities of the cylinders respectively. The volume of the connecting pipe between their upper ends is 9.92 per cent. of that of the cylinder. The pump-barrel is 26⅛ inches, and the plunger is 18 inches, in diameter, with a stroke of 8 feet. The fly-wheel is 30 feet in diameter, and weighs 16 tons. From the results of the trials, it appears that the steam was cut off at about 30 per cent. of the stroke, with an initial absolute pressure of about 100 lbs. per square inch. The engines made 16¼ turns per minute, and yielded about 195.5 indicator horse-power. The water delivered per stroke amounted to 95 per cent. of the capacity of the pump. The total quantity of coal (Cumberland) consumed was equivalent to 1.69 lbs. per indicator horse-power per hour; and to 2.06 lbs. per horse-power of duty at the pump. The efficiency of the engine was 81.94 per cent. The duty per 100 lbs. of coal amounted to 96,200,000 foot-pounds. The quantity of water evaporated was 8.27 lbs. per pound of coal, and $(8.27 \times 1.69 =)$ 14 lbs. of steam was consumed per indicator horse-power.

[1] Article, "Steam Engine," by the author, in the *Encyclopedia Britannica*, 8th edition.

[2] "On Double-Cylinder Pumping Engines," by Mr. David Thomson, in the *Proceedings of the Institution of Mechanical Engineers*, 1862, page 268.

[3] *Journal of the Franklin Institute*, November, 1876, page 312. Report by Messrs. W. E. Worthen, J. C. Hoadley, and J. P. Davis; with illustrations.

The pumping duties, at the pump, of large pumping engines, whether single-acting or double-acting, are thus seen to average about 81 per cent. of the indicator horse-power.

Hydraulic Rams.

Hydraulic rams are used where there is a considerable flow of water with a moderate fall, to raise a small portion of the flow to a height greater than the fall. The outflow of water falling through a pipe, when the lower end of the pipe is suddenly closed, is suddenly arrested, and the momentum of the current in the pipe is expended in forcing a portion of itself through another pipe,—the delivery-pipe,—into an elevated reservoir. When the momentum is expended, the upward current ceases to flow, the valves in the delivery-pipe are closed against the return of the elevated water; and the valve by which the supply current was arrested, opens as it is relieved of the momentary excessive pressure, and the outflow is resumed. Thus, by a succession of impulses, the water is lifted. Mr. C. L. Hett recommends Daubuisson's formula for the efficiency:—

$$\frac{d_1 h_1}{dh} = 1.42 - .28 \sqrt{\frac{h_1}{h}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

d = the quantity of water used, in gallons per minute.
d_1 = the quantity of water raised, in gallons per minute.
h = the head used, in feet.
h_1 = the lift, in feet.

Mr. Hett[1] gives the following table, calculated by means of Daubuisson's formula, showing the efficiency or percentage of duty due to proportions of lift to fall, of from 4 to 26:—

Lift, when the Fall = 1.	Efficiency.	Lift, when the Fall = 1.	Efficiency.	Lift, when the Fall = 1.	Efficiency.
ratio.	per cent.	ratio.	per cent.	ratio.	per cent.
4	86	12	45	20	17
5	79	13	41	21	14
6	73	14	37	22	11
7	68	15	34	23	8
8	63	16	30	24	5
9	58	17	27	25	2
10	53	18	23	26	0
11	49	19	20		

Mr. Hett recommends, to adopt, allowing for contingencies, only five-sixths of the efficiencies here given. For the diameter of the driving pipe, he gives Eytelwein's formula as sufficient for all practical purposes:—

Diameter of pipe in inches $= .058 \sqrt{\text{quantity of water in gallons}}$. ... (13)

[1] *The Engineer*, January 7, 1876, page 3.

HYDRAULIC MOTORS.

Hydraulic Press.

The action of the hydraulic press depends on the principle that fluids press equally in all directions; and if the pressure applied to the plunger of the force-pump be multiplied in the ratio of the sectional areas or of the squares of the diameters of the plunger and the ram, the product is the pressure applied to the ram.

The ram is packed with a leather collar, and according to the results of experiments made by Mr. John Hick, M.P.,[1] the friction of the collar increases directly with the pressure and with the diameter; and it is independent of the depth of the collar. The friction is equivalent to 1 per cent. of the pressure for a 4-inch ram, ½ per cent. for an 8-inch ram, and ¼ per cent. for a 16-inch ram. The following formula is deduced:—

Leather, new or badly lubricated,... $f = .0471\ dp$, ... (14)
Leather in good condition, ... $f = .0314\ dp$, ... (15)

f = the total frictional resistance of a leather collar.
d = the diameter of the ram, in inches.
p = the pressure, in pounds per square inch.

Armstrong's Hydraulic Machines.

In a paper by Mr. Henry Robinson,[2] the following data are communicated, on the authority of Mr. Percy Westmacott, giving the coefficients of effect obtained in hydraulic machines of ordinary make,—

Direct acting,	93 per cent.
2 to 1,	80 "
4 to 1,	76 "
6 to 1,	72 "
8 to 1,	67 "
10 to 1,	63 "
12 to 1,	59 "
14 to 1,	54 "
16 to 1,	50 "

These coefficients are based on the use of ordinary hemp-packing, and with sheaves and wrought-iron pins, and with no exceptional arrangements for lubrication. But, where special precautions have been taken, with large sheaves and small hard steel pins, the efficiency, multiplying 20 to 1, with a load of 17¼ tons, was as high as 66 per cent. Well made cupped leathers, used instead of hemp-packing, increase the efficiency.

It is considered that the coefficient of effect obtained from a steam-engine pumping into an accumulator, may be taken at 91.7 per cent., the loss by friction amounting to 8.3 per cent. It is found by experiment that the difference of pressure with the accumulator (at 700 lbs.) rising or falling is about 30 lbs., representing .022 of effect. The compounded efficiency will therefore be ascertained by combining the efficiency of the engines with the above varying rates of efficiency.

[1] *Spons' Dictionary of Engineering*, page 1992.
[2] "On the Transmission of Motive-power to Distant Points," in the *Proceedings of the Institution of Civil Engineers*, 1876–77, vol. xlix.

FRICTIONAL RESISTANCES.

INTERNAL RESISTANCE OF STEAM-ENGINES.

It may be assumed that, taken generally, the efficiency of steam-engines, that is to say, the ratio of the work transmitted through the engine to the fly-wheel shaft, to the effective work done by the steam in the cylinder, increases with the power of the engine. When engines are in good working order, and are doing their full duty, the efficiency varies from 80 per cent. for smaller engines, to 90 per cent. for larger engines. In one exceptional instance,—a Corliss engine in France,—the efficiency amounted to 93 per cent.

RESISTANCE OF TOOLS.[1]

The following are results of Dr. Hartig's experiments on the resistance of tools:—

Single-acting Shearing Machines.—The power necessary to drive such tools when empty is expressed by the formula,—

$$P = 0.1 + \frac{n \times t^2}{26.7} \quad \text{.........................} \quad (1)$$

P = horse-power.
t = maximum thickness of plate to be cut.
n = the number of cuts per minute.
a = the area of surface cut or punched per hour, in square inches.
$F = (1166 + 1691\,t)$, a factor expressing the work required to produce a cut or sheared surface of 1 square inch.

The power required to do the work itself, in addition to that required to drive the tool when empty, is

$$P_c = \frac{a\,F}{33{,}000 \times 60} = \frac{a\,F}{1{,}980{,}000} \quad \text{..................} \quad (2)$$

For example, a shearing machine, cutting 4648 square inches of surface per hour, in plates 0.4 inch thick, would absorb 0.68 horse-power empty, and 4.3 horse-power in effective work; total, say, 5 horse-power.

[1] The above particulars of the resistance of tools are abstracted from a notice of the results of Dr. Hartig's experiments, in *Engineering* for October and December, 1874.

Plate-bending Machines.—The net work required to bend a plate or a bar, is expressed by the formula,—

$$F = \frac{85{,}000\, b\, t^2\, l}{r}, \text{ for cold wrought-iron plates} \ldots\ldots \quad (3)$$

$$F = \frac{11{,}300\, b\, t^2\, l}{r}, \text{ for red-hot iron plates} \ldots\ldots\ldots\ldots \quad (4)$$

b, t, and l = the breadth, thickness, and length of the plate, in inches.
r = the radius of curvature, in inches.
F = the net work of bending the plate.

The power required to drive large plate-bending rolls, when empty, is between 0.5 and 0.6 horse-power.

Circular Saws.—The horse-power required to drive circular saws running empty, is

$$P = \frac{n\, d}{32{,}000} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

d = the diameter of the saw, in inches.
n = the number of revolutions per minute.

The net power required to cut with a circular saw, is proportional to the cubic contents of the material removed. For a saw for cutting hot iron, moving at a circumferential speed of 7875 feet per minute, and making a cut 0.14 inch wide, the power is expressed by the formulas—

$$P_c = 0.702\ A, \text{ for red-hot iron} \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$P_c = 1.013\ A, \text{ for red-hot steel} \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

A = the sectional area of surface cut through, in square feet.

Work of Ordinary Cutting Tools, in Metal.—Materials of a brittle nature, as cast-iron, are reduced most economically in power consumed, by heavy cuts; whilst materials which yield tough curling shavings are more economically reduced by thinner cuttings. The following formulas apply to light cutting work:—

The power required to plane away cast-iron is,—

$$\textit{Planing}\text{ cast-iron,} \ldots\ldots\ P = W\left(.0155 + \frac{1}{11{,}000\, s}\right) \ldots\ldots \quad (8)$$

W = the weight of cast-iron removed per hour, in pounds.
s = the average sectional area of the shavings, in square inches.

For planing steel, wrought-iron, and gun-metal, with cuts of an average character,—

Planing steel,.............. P = 0.112 W,......... (9)
,, wrought-iron,..... P = .052 W,......... (10)
,, gun-metal, ... P = .0127 W,....... (11)

For turning off metals, the power required is less than for planing

them off, and it was found that the power was greater for smaller diameters than for larger diameters.

Turning cast-iron,......... $P = .0314\ W$,......... (12)
,, wrought-iron,.... $P = .0327\ W$,......... (13)
,, steel,.............. $P = .047\ W$,........... (14)

For drilling metals, the power required to remove a given weight of material is greater than in planing. Volume is taken instead of weight in the formulas: applicable to holes of from 0.4 to 2 inches in diameter:—

Drilling cast-iron, dry,.............. $P = q\,(.0168 + \frac{.00067}{d})$ ... (15)

,, wrought-iron, with oil,... $P = q\,(.0168 + \frac{.0269}{d})$ .... (16)

q = the volume removed, in cubic inches per hour.
d = the diameter of the hole.

In the use of shaping machines, the resistance is greater for notch-cuts than for cuts on flat surfaces, and for skin-cuts than for under-cuts:—

Shaping cast-iron skin-cuts..................... $P = .1087\ W$,... (17)
,, ,, under-cuts,.................. $P = .0604\ W$,... (18)
,, ,, wheel-teeth (notch-cuts),. $P = .118\ W$,..... (19)

Power required to drive Ordinary Cutting Tools when empty.—For lathes, the power varies with the number of shafts between the driving shaft and the main spindle:—

Number of Intermediate Shafts.	Light Lathes, Empty.	Heavy Lathes, Empty.	
0,......	$P = .05 + .0005\,n.$	$P = 0.25 + .0031\,n$....	(20)
1 or 2,......	$P = .05 + .0012\,n.$	$P = 0.25 + .053\,n$	(21)
3 or 4,......	$P = .05 + .05\,n.$	$P = 0.25 + 0.18\,n$	(22)

n = the number of turns of the lathe-spindle per minute.

For drilling machines, when empty, the power varies according to the construction of the machines, with or without intermediate gearing:—

a. Drill, without gearing, ... $P = .0006\,n_1 + .0005\,n_2$.. (23)
b. ,, with gearing for the spindle, $P = .0006\,n_1 + .001\,n_2$... (24)
c. ,, radial drills, without gearing, $P = .0006\,n_1 + .004\,n_2$... (25)
d. ,, ,, with gearing,..... $P = .04 + .0006\,n_1 + .004\,n_2$ .. (26)

n_1 = the number of turns per minute of the gearing shaft.
n_2 = the number of turns of the drill.

For a slotting machine, having a stroke of 8 inches, and a tool-holder and slide weighing 93½ lbs., running empty,—

$$P = .045 + \frac{n\,s}{4000} \ \cdots\cdots \ (27)$$

n = the number of strokes per minute.
s = the stroke in inches.

For shaping machines, the movements of which are slow, the power required for moving the machine, when empty, is only from 10 to 15 per cent. of the whole power required to work the machine.

Screw-cutting Machines.—The power required by one of Sellers' tools for cutting screws on shafts or on bolts, and for tapping nuts, is expressed by the formulas:—

$$\text{Screwing (net work)}, \ldots\ldots\ldots\ldots \quad P = \frac{5\,l\,d^3}{64} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

$$\text{Tapping} \quad ,, \quad \ldots\ldots\ldots\ldots \quad P = \frac{l\,d^3}{29} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

d = the diameter in inches.
l = the length in feet cut per hour.

The additional power required for driving a screwing machine of medium size, is about one-fifth of a horse-power.

Wood-cutting Machines.—The power required for driving circular-saws, when empty, has been given by formula (5), page 952. The net, or additional, power required to do the work, is proportional to the cubic contents of the wood reduced to sawdust, at the rate of 1 horse-power for 1 cubic foot per hour of soft wood, or for half a cubic foot of hard wood.

$$\text{Circular-saw, hard wood}, \ldots\ldots\ldots \quad P_c = \frac{A\,c}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (30)$$

$$,, \quad \text{soft wood}, \ldots\ldots\ldots \quad P_c = \frac{A\,c}{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

A = the sectional area of surface in square feet cut through per hour.
c = the width of the cut, in inches.

For saw-frames, cutting dry pine timber, the net power required to do the work, exclusive of the work to drive the machine when empty, is as follows:—

$$\text{Saw-frame, pine}, \ldots\ldots \quad P_c = .00428 + .0065\frac{S\,c}{f} \quad \ldots\ldots\ldots\ldots \quad (32)$$

S = the stroke of the saws, in feet.
c = the width of the cuts, in inches.
f = the feed per cut, in inches.
P_c = the horse-power required per square foot cut through per hour.

For band-saws, the power required for the work itself is:—

$$\text{Band-saw, pine}, \ldots\ldots \quad P_c = .0034 + \frac{0.758\,c\,v}{10{,}000\,f} \quad \ldots\ldots\ldots\ldots \quad (33)$$

$$,, \quad \text{oak}, \ldots\ldots \quad P_c = .00483 + \frac{0.957\,c\,v}{10{,}000\,f} \quad \ldots\ldots\ldots\ldots \quad (34)$$

$$,, \quad \text{beech}, \ldots\ldots \quad P_c = .00576 + \frac{1.127\,c\,v}{10{,}000\,f} \quad \ldots\ldots\ldots\ldots \quad (35)$$

v = the velocity of the band-saw, in feet per minute.
f = the rate of feed, in feet per minute.
c = the width of the cut, in inches.

In planing and moulding machines, which are driven at high speeds, a large proportion of the total power is absorbed in driving the machine itself. The formula is,—

$$\text{Empty,} \ldots\ldots\ldots \quad P = \frac{N}{2000} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (36)$$

N = the sum of the turns per minute made by the several shafts.

The net power required for moulding and shaping wood, is given by the formulas:—

$$\text{Pine,} \ldots\ldots\ldots \quad P = .0566 + \frac{.02268}{h} \ldots\ldots \quad (37)$$

$$\text{Red beech, using cutters,} \ldots\ldots \quad P = .08895 + \frac{.00731}{h} \ldots\ldots \quad (38)$$

$$\text{„ using cutting discs,..} \quad P = .0895 + 9.138\,s \ldots\ldots \quad (39)$$

P = horse-power required to produce 1 cubic foot of shavings per hour.
h = the height of wood cut down to form the moulding, in inches.
s = the average sectional area of the shavings, in square inches.

In drilling timber with holes of from 0.4 to 4 inches in diameter, for depths up to 6 inches:—

$$\text{Drilling pine,} \ldots\ldots\ldots \quad P = q\left(.000125 + \frac{.000656}{d}\right) \ldots \quad (40)$$

$$\text{„ alder,} \ldots\ldots\ldots \quad P = q\left(.000472 + \frac{.001423}{d}\right) \ldots \quad (41)$$

$$\text{„ white beech,} \ldots \quad P = q\left(.003442 + \frac{.001495}{d}\right) \ldots \quad (42)$$

Grindstones.—To drive grindstones empty, the power is expressed by the formulas:—

$$\text{Large grindstones empty,} \ldots \quad P = .0000409\,dv \ldots\ldots\ldots \quad (43)$$

$$\text{or} \quad P = .000128\,d^2n \ldots\ldots\ldots \quad (44)$$

$$\text{Small fine „ „} \ldots \quad P = 0.16 + .0000895\,dv \ldots\ldots\ldots \quad (45)$$

$$\text{or} \quad P = 0.16 + .00028\,d^2n \ldots\ldots\ldots \quad (46)$$

The coefficients of friction between grindstones and metals are as follows:—

	Coarse Grindstones, at high speeds.		Fine Grindstones, at low speeds.
For cast iron,.............	coefficient .22	....	coefficient 0.72
For wrought iron,.........	„ .44	....	„ 1.00
For steel,..................	„ .29	....	„ 0.94

$$\text{Grindstones, net work,} \ldots\ldots \quad P = \frac{p\,K\,v}{33{,}000} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (47)$$

p = the pressure between the material and the stone.
v = the circumferential velocity of the stone, in feet per minute.
K = the coefficient of friction.

RESISTANCE OF COLLIERY WINDING ENGINES.

For working shafts from 246 to 580 yards deep, in the county of Durham, the following are particulars of dimensions and performance of engines and winding gear:[1]—

DIMENSIONS.

No.	DIRECT-ACTING ENGINE.	Cylinder.	Speed of Piston.	Steam in Boiler.	Drums.	Mean Diameter.
		in. in.	ft. per minute.	lbs.		feet.
1	1 cyl., vertical, condensing	65 × 84	176	19	flat	26
2	1 „ „ „	68 × 84	224	20	„	23½
3	2 „ horizontal, non-condensing	40 × 72	253	—	conical	21
4	2 „ „ „	34 × 72	291	40	„	17½
5	1 „ „ „	48 × 72	232	—	flat	21½

PERFORMANCE.

No.	Gross Engine Power per Minute.	Duty, Coal Raised, per Minute.	Efficiency.	Ropes.	Average Speed of Ropes.
	foot-pounds.	foot-pounds.	per cent.	in. in.	feet per minute.
1	10,342,350	7,729,344	75	flat, iron, 6½ × ⅞	1020
2	15,403,000	9,744,000	63	„ „ 6 × ⅞	1180
3	7,470,000	4,077,000	55	round, steel, 1⅝	1689
4	4,375,596	2,700,000	62	round, iron, 1⅜	1302
5	5,281,107	3,956,178	75	—	1300

RESISTANCE OF WAGGONS IN COAL PITS.

The average resistance of waggons used in the Midland coal pits, having four 15-inch wheels at 21-inch centres, is $^1/_{50}$th of the weight, or 45 lbs. per ton. The bodies of the waggons are 4 feet 3 inches by 3 feet wide, and 20 inches deep. On a roadway having an average fall of 1 in 30, worked by an endless chain, passed over a 7½-feet grooved pulley at the end of the course, the gravitation of full waggons descending, supplies sufficient hauling power to overcome all the wheel-friction and take up the empty waggons. The greatest traverse is from 5000 to 6000 feet; the speed is 3 miles per hour, and the tubs or waggons are attached to the rope at intervals of from 20 to 25 yards.[2]

[1] See paper by Mr. G. H. Daglish, on Winding Engines, in the *Proceedings of the Institution of Mechanical Engineers*, 1875, page 217.

[2] Mr. G. Fowler, *Proceedings of the Institution of Mechanical Engineers*, 1870.

RESISTANCE OF MACHINERY OF FLAX MILLS.

M. E. Cornut, in 1871–72, tested, by the indicator, the resistance of the steam-engines, shafting, and machinery of the flax mill at Hamégicourt.[1] There were two Woolf engines, with vertical cylinders and beam, condensing, which made, at regular speed, 25 turns per minute.

First cylinder of each engine,......	12.9 inches diameter,	44.3 inches stroke.
Second „ „	22.0 „ „	59.8 „ „

The power required to drive the machinery was very variable. It varied 15 or 20 per cent., according to the lubricant employed, and the mode of lubrication:—

	Atmospheres.
With vegetable oil and hand oiling, steam pressure required,	5 to 5¼.
With mineral oil and continuous oiling, „ „	4¼ maximum.

Making a difference in pressure of at least 15 per cent. In accordance with this result, it was found, by direct test, that, in lubricating with vegetable oil (*huile grasse*), 2.90 horse-power per 100 spindles (wet system), was consumed, and only 2.44 horse-power with mineral oil, making a difference of 16 per cent. But the modes of lubrication are not distinguished. Finally, with vegetable oil, the belts, if tight, were broken at starting on Monday mornings, after a day's stoppage; but, with mineral oil, there were no breakages.

The quantity of mineral oil consumed per day, in lubricating the same three pedestals, was,—

By hand oiling,..	29.0 grains.
By continuous oiling,................................	16.2 „

Showing a reduction of 44 per cent. by continuous oiling.

To test the increase of resistance by want of lubrication, the engines, shafting, and belts, were, one day in March, 1871, started empty (*à vide*), at 4 A.M. At 6 A.M., 30.08 horse-power was indicated. All the lubricators were then removed, except two next the engines, and the oil that remained was cleared off the journals. The engines and shafting continued in motion, and observations on the power expended were made at intervals.

Observation.	Time Elapsed after Removing the Lubricators.	Indicator Horse-Power.	Augmentation of Resistance.
	hours.		per cent.
1st	0	30.08	—
2d	2	31.60	5½
3d	3	33.47	10.9
4th	4	35.34	17.45
5th	6	37.33	30.78

[1] *Essais Dynamometriques*, Lille, 1873. These experiments were carefully and intelligently conducted, and M. Cornut's conclusions appear to be worthy of confidence.

At this time, the journals began to heat, and the experiment was ended. Taking the first three observations to represent ordinary practice in hand-oiling, a variation is manifested, of from 5 to 10 per cent. of resistance.

From comparative tests made at 6 A.M. and at 11 A.M., respectively ½ hour and 5½ hours after starting in the morning, the total resistance of the engines and machinery in ordinary work, for the second test, was from 8 to 9 per cent. less than for the first test. The temperature in the workshops had risen 10° or 11° F. in the interval.

The resistance of the engines and machines, separately, was tested during the period from August, 1871, to February, 1872:—

The engines, shafting, and belts only, making 25 turns of the engine per minute, expended an average of 30.41 indicator horse-power. The maximum power was 32.10 H.P., in December, 1871.

Four cards consumed from 9.10 to 7.40;—average 8.42 H.P.

14 drawing-frames of 29 heads, or 156 slivers, consumed from 6.82 to 8.04;—average 7.19 H.P.

6 roving-frames, of 330 spindles, 9.04 to 8.25;—average 8.67 H.P., or 2.627 H.P. per 100 spindles.

4 combing-machines, 2.228 H.P.

20 spinning-frames, dry, 1480 spindles, 47.50 H.P.

20 spinning-frames, wet, 2080 spindles; spinning Nos. 25 to 30, 46.59 H.P., or 2.24 H.P. per 100 spindles; Nos. 40 to 70, 35.82 H.P., or 1.72 H.P. per 100 spindles. For several particular numbers, the power was determined to be as follows:—

No. 16,..........................	3.200	H.P. per 100 spindles.
„ 20,..........................	2.760	„ „
„ 25,..........................	2.262	„ „
„ 28,..........................	2.190	„ „
„ 30,..........................	2.140	„ „
„ 40,..........................	1.917	„ „

M. Cornut deduced from those data, that the horse-power per 100 spindles varied inversely as the square root of the number.

The average indicator-power that would have been required for driving the whole of the machinery in full work, was 148.42 horse-power. The actual total average power required was only 131.23 horse-power, or 88 per cent. of the power for full work; being 17.19 horse-power less than for the whole of the machines. This reduction represents the power for the proportion of machines at rest.

The power required to drive the machines when empty was also measured.

The table No. 318 contains particulars of the horse-power required for each machine, at work, and empty. The indicator-power may be divided thus:—

Steam-engine, shafting, and belts,..............	30 I.H.P.	or 20 per cent.
Preparing and spinning machinery, &c.,......	120 „	or 80 „
	150 „	100 „

Table No. 318.—Flax Mill at Hamégicourt—Horse-Power required to Drive the Engines, Shafting, and Machinery.

(From M. Cornut's data.)

Description.	Indicator Horse-Power.			Efficiency of the Machines.
	Total at Work.	For One Machine at Work.	For One Machine Empty.	
	H.-P.	H.-P.	H.-P.	per cent.
Engines, shafting, and belts,..	30.41	—	—	—
4 Cards, ...	8.42	2.105	1.423	32
14 Drawing frames (29 heads or 156 slivers), ...	7.19	.0934 per sliver	.0794	15
4 Combing machines,...........	2.22	.555	.151	75
6 Roving frames (330 spindles)	7.78	2.627 p. 100 spindles	2.434	7.3
20 Spinning frames, dry (1480 spindles),.............	47.50	3.21 „	2.515	21.6
20 Spinning frames, wet (2080 spindles),.............	46.59	2.24 „	1.613	19
Total horse-power, all at work, calculated from separate experiments,.....	151.00			
Total hor.-power that would have been actually expended, all at work, ...	148.42			

Estimation of Horse-power required for a Flax Spinning-mill.—Let the power be expressed in terms of the number of spindles driven by 1 indicator horse-power. M. Cornut gives the following data:—

M. Feray d'Essone,......	55 spindles per H.P. for No. 40.
	20 „ „ for No. 6.
English estimate (2000 to 12,000 spindles),	25 spindles per H.P., for No. 35 long staple, No. 18 tow.
Dr. Hartig, ...	40 spindles per H.P., for Nos. 35 to 51 long staple, Nos. 16 to 29 tow.
Do.	26 spindles per H.P., for Nos. 25 to 30 long staple, Nos. 14 to 23 tow.

M. Cornut:—

2080 spindles, wet,..................	34.4	spindles per H.P.,	long fibre.
640 „ dry,..................	20.1	„ „	„
840 „ „	14.5	„ „	tow.
3560 „	23.7	„ „	

RESISTANCE OF MACHINERY OF WOOLLEN MILLS.

Dr. Hartig, of Dresden, tested the machinery of a woollen mill, by means of a dynamometer, which was applied directly to each machine. The principal results, showing the actual horse-power expended in driving each machine, when empty and when at work, are given in table No. 319.

Table No. 319.—Woollen Machinery—Horse-power Required to Drive the Machines.

(From Dr. Hartig's data.)

Description.	Ordinary Speed in Turns per Minute.	Indicator Horse-power.			
		Empty.	At Work.	Efficiency.	At Work, Resistance of Shafting included.
	turns.	H.P.	H.P.	℔ cent.	H.P.
Washing machine, two cylinders, ...	35	—	.223	—	.25
Centrifugal pump for this machine, ...	300	—	.77	—	.80
Hydro-extractor, ...	1000 to 1200	.75	1.20	39	1.00
Burring machine, ...	350 to 500	1.47	1.77	17	1.75
Scutcher, continuous feed, ...	300	.47	.66	29	.70
Opening machine, ...	350 to 450	.42	.95	55	1.00
Scribbler card, 40 inches wide, ...		.34	.58	40	
Intermediate card, ,, ...	110	.27	.43	36	1 75
Finishing card, ,, ...		—	—	—	
1 spindle, spinning, ...	2500	—	.0027	—	.003
1 spindle, doubling, ...	1100	.005	.006	13	.007
Sizing and warping machine (without ventilator), ...	17	—	.071	—	.075
Loom, 7.54 feet wide, ...	40 to 45	—	.113	—	.13
Scouring machine, for 2 pieces, ...	40	.13	.50	74	.55
Fulling mill, 1 cylinder, for *nouveautés*,	100	.19	2.54	93	2.25
Do. 3 cylinders, for cloth, ...	45	.17	1.59	89	1.50
	110	.71	2.74	73	2.75
Double fulling mill, for *nouveautés*, ...	100	.30	3.40	91	3.25
Do. do. for cloth, ...	45	.16	3.26	95	2.75
	blows.				
Single fulling stock, two stamps (Dobb's), ...	125	.53	1.64	78	1.70
Double fulling stock, two stamps (Spranger's), ...	116	.43	1.99	74	.75
	turns.				
Gig (*Laineur*), single, without spreader,	90	.19	.73	74	.75
Do. double, ...	100	.20	1.38	86	2.75
Machine for dressing the reverse side, with cylinders, ...	100	.11	2.03	94	.45
Machine for dressing the reverse side, with eccentrics, ...	100	.17	2.45	93	.90
Wringing machine, lengthwise, ...	650	.52	.61	14	.60
Do. do. across, ...	1000	.25	.31	22	.25
Brushing machine, one cylinder, ...	250	—	—	—	.40
Do. do. two cylinders, ...	250	.37	1.03	64	.90

RESISTANCE OF MACHINERY FOR CONVEYANCE OF GRAIN.

Conveyance of Grain horizontally by Screws and by Bands.—A 12-inch screw, having 4 inches pitch, turning in a trough, with a clearance of ¼ inch, revolving with the speed of maximum effect, 60 turns per minute, discharged 6¾ tons of grain per hour, expending .04 horse-power per foot run. The sectional area of the body of grain moved was 49 per cent. of that of the screw. At speeds above 60 turns per minute, the grain did not advance, but revolved with the screw.

A 12-inch screw, having a 12-inch pitch, delivered 34 tons per minute, at 70 turns per minute, expending .125 horse-power per lineal foot, or 37 per cent. less power for equal weights of grain. The sectional area of the grain was 72 per cent. of that of the screw.

An endless band, 18 inches wide, travelling at about 9 feet per second, delivered 70 tons of grain per hour; power expended, .014 horse-power per foot run.

To convey 50 tons per hour through 100 feet, the power expended by the screw was 18.38 H.P.; by the endless band, 1.02 H.P.

Grain has been raised by a dredger 30 feet long; efficiency, 50 per cent.[1]

Mr. Davison states that 95 feet of horizontal Archimedian screw, 15 inches in diameter, with an elevator lifting 65 feet, convey 40 quarters of malt per hour, for an expenditure of 3.13 indicator horse-power.

RESISTANCE TO TRACTION ON COMMON ROADS.

From the results of recent and carefully conducted experiments made by M. Dupuit, he made the following deductions as to the resistance to traction on macadamized roads and on uniform surfaces generally:—

1. The resistance is directly proportional to the pressure.
2. It is independent of the width of tyre.
3. It is inversely as the square root of the diameter.
4. It is independent of the speed.

M. Dupuit admits that, on paved roads, which give rise to constant concussion, the resistance increases with the speed; whilst it is diminished by an enlargement of the tyre up to a certain limit.

M. Debauve[2] has deduced from experiment, that the advantage of a pavement over a metalled road is considerable for waggons, is less for stage-coaches, and is nearly nothing for *voitures de luxe*, or private carriages. He summarizes the resistances as follows:—

Vehicle.	Resistance to Traction. On Metalled Roads.		Resistance to Traction. On Paved Roads.
Waggon, ...	67 lbs. per ton.	......	38 lbs. per ton.
Stage-coach, ...	67 „	......	45 „
Cabriolet, ...	81 „	......	76 to 83 „

M. Tresca tested the resistance of a tramway omnibus on Loubat's system, adapted with wheels for running on a common road. The experiments were made on an inclined street in Paris, in good condition, having ascending gradients of 1 in 55, one part of which was paved, and another part macadamized. The frictional resistance, after the gravitation on the incline was eliminated, was as follows:—

Surface.	Gross Weight. tons.		Speed. miles per hour.		Frictional Resistance. lbs. per ton.
Macadam,.........	5.67	......	10.7	......	83
Pavement,.........	5.67	......	10.1	......	66

[1] The above data are derived from Mr. Westmacott's paper on "Corn-Warehousing Machinery," *Proceedings of the Institution of Mechanical Engineers*, 1869, page 208.

[2] *Manuel de l'Ingénieur des Ponts et Chaussées*, 1873; 9me fascicule, page 32.

RESISTANCE OF CARTS AND WAGGONS ON COMMON ROADS AND ON FIELDS.

The resistance to traction of agricultural carts and waggons was tested at Bedford in July, 1874, by means of a horse-dynamometer designed by Messrs. Eastons & Anderson.[1] The first course was a piece of hard road 200 yards in length, rising 1 in 430; it was dry and in fair condition, largely made of gravel. The surface was in many places somewhat loose. The second course was along an arable field, growing oats, on a rising gradient of 1 in 1000; it was very dry, and was harder than in average condition.

The fore-wheels of the waggons averaged 3 feet 3 inches, and the hind-wheels 4 feet 9 inches in diameter; the width of tyres was from 2½ to 4 inches. The weight, empty, averaged about a ton, and it was nearly equally divided between the front and hind wheels. The cart wheels were, say, 4 feet 6 inches high, with tyres 3½ and 4 inches wide. The weight of the empty carts averaged 10 cwt.

The following results arise out of the published data:—

On Road.	Pair-horse Waggon, without Springs, Loaded with Roots.	Waggon without Springs, Loaded with Maize.	Waggon with Springs, Loaded with Roots.	Cart without Springs, Loaded with Roots.
Load,	44 cwt.	80 cwt.	44 cwt.	20 cwt.
Gross weight drawn, about	64 "	100 "	64 "	30 "
Average speed per hour,...	2½ miles	2.60 miles	2.47 miles	2.65 miles
Maximum draft,	320 lbs.	400 lbs.	300 lbs.	180 lbs.
Average draft,	159 "	251 "	133 "	49.4 "
Horse-power developed, at 33,000 foot-pounds per minute,	1.06 H.P.	1.74 H.P.	.88 H.P.	.35 H.P.
Draft per ton gross, on level,	43.5 lbs., or 1/52	44.5 lbs., or 1/50	34.7 lbs., or 1/65	28 lbs., or 1/80
On Field.				
Load,	44 cwt.	80 cwt.	44 cwt.	20 cwt.
Gross weight drawn, about	64 "	100 "	64 "	30 "
Average speed per hour,...	2.35 miles	2.52 miles	2.35 miles	2.61 miles
Maximum draft,	1000 lbs.	1200 lbs.	1000 lbs.	400 lbs.
Average draft	700 "	997 "	710 "	212 "
Horse-power developed, at 33,000 foot-pounds per minute,	4.36 H.P.	6.70 H.P.	4.45 H.P.	1.48 H.P.
Draft per ton gross, on level,	210 lbs., or 1/11	194 lbs., or 1/12	210 lbs., or 1/11	140 lbs., or 1/16

From these data it appears that, on the hard road, the resistance is only from ¼th to 1/6th of the resistance on the field. The lowest resistance is that of the cart on the road—28 lbs. per ton; due, no doubt, as observed in *Engineering*, to the absence of small wheels like those of the waggons.

[1] See a report of the trials in *Engineering*, July 10, 1874, page 23.

The highest resistance is 210 lbs. per ton—on the field. The addition of springs reduced the resistance 20 per cent. on the road; but, on the field, the resistance was not reduced by the addition of springs.

Also, that the horse-power, of 33,000 foot-pounds per minute, developed, varied from ⅓ H.P. to 7 H.P. Allowing a pair of horses for the first and third columns above, two pairs for the second column, and one horse for the last column, the following are the total works done per horse, in technical horse-power:—

	Horse-power per Horse. On Road.	Horse-power per Horse. On Field.
Pair-horse waggon, without springs,	.53 H.P.	2.18 H.P.
Two pair-horse waggon, without springs,	.44 „	1.68 „
Pair-horse waggon, with springs,	.44 „	2.22 „
One-horse cart, without springs,	.35 „	1.48 „
Total averages,	.44 „	1.89 „
Averages, without springs,	.44 „	1.78 „

Taking the average power exerted without springs, .44 horse-power, on the road, as the average for a day's work, it represents $.44 \times 33{,}000 = 14{,}520$, say 15,000, foot-pounds per minute, for the power of a horse on a hard road.

The resistance of a smooth well-made granite tramway, like the tramways in the City of London and Commercial Road, made with stones 5 or 6 feet in length, is from 12½ lbs. to 13 lbs. per ton of weight.

Experiments on the tractional resistance for a loaded omnibus, on various kinds of roads, were made by a committee of the Society of Arts:[1]—

	lbs.
Weight of omnibus,	2480
Load, 22 sacks of oats, at 149 lbs.,	3278
Total weight, 2.57 tons, or	5758

The loaded omnibus was drawn to and fro over each trial surface, and the mean result was taken as the resistance for an exact level:—

	Average Speed. miles per hour.	Resistance. Total. lbs.	Resistance. Per Ton. lbs.
Granite pavement, sets 3 to 4 inches wide,	2.87	44.75	17.41
Asphalte roadway,	3.56	69.75	27.14
Wood pavement,	3.34	106.88	41.60
Good gravelly Macadam road,	3.45	114.32	44.48
Granite Macadam, newly laid,	3.51	259.80	101.09

There is a want of consistency here, in the excessive resistance on an asphalte pavement, compared with that on a granite pavement. There can be no doubt that asphalte pavement, properly made, is, of all pavements, the least resistant; and that its resistance cannot be greater than the resistance of a granite tramway.

[1] Report of the Committee, *Journal of the Society of Arts*, June 25, 1875.

Sir John Macneil gives the tractive force necessary to move a waggon weighing 21 cwt., at 2½ miles per hour, on roads of the following descriptions:—

	Total Resistance. lbs.		Resistance per Ton. lbs.
Well-made pavement,	33	...	31.2
Road made with 6 inches of broken stone of great hardness, on a foundation of large stones set in the form of a pavement, or upon a bottoming of concrete,	46	...	44
Old flint road, or a road made with a thick coating of broken stone, laid on earth,	65	...	62
Made with a thick coating of gravel, laid on earth,	147	...	140

Sir John Macneil made a series of experiments on the tractive resistance of a stage-coach, on a section of the Holyhead Road. The weight of the coach, empty, was 18 cwt., and the weight of seven passengers in addition, allowing 1½ cwt. for each passenger, was 10½ cwt.; total weight 28½ cwt. The experimental gradients ranged from 1 in 20 to 1 in 600, and the speeds were 6, 8, and 10 miles per hour. It was found that, by some unexplained cause, the net frictional resistance at equal speeds varied considerably according to the gradient. The resistances were a maximum for the steepest gradient, and a minimum for gradients of 1 in 30 to 1 in 40; for these they are less than for 1 in 600. The mode of action of the horses on the carriage may have been an influential element. The averages show,—

For a Stage-coach.	On a Metalled Road.
At 6 miles per hour,	62 lbs. per ton, frictional resistance.
At 8 " "	73 " "
At 10 " "	79 " "

With these may be associated the resistance, by Sir John Macneil's experiments, of a waggon on a good road, namely, 44 lbs. per ton, at 2½ miles per hour. Plotting the resistances for the above four speeds, the following approximate formula is deduced:—

Frictional Resistance to Traction of a Stage-coach on a Metalled Road in good condition.

$$R = 30 + 4v + \sqrt{10v} \quad \text{.......................} \quad (1)$$

R = the frictional resistance to traction per ton.
v = the speed in miles per hour.

Note.—The formula is applicable to waggons at low speeds. It is simpler than the formulas deduced by Sir John Macneil.[1]

M. Charié-Marsaines made observations of a general character, on the performances of Flemish horses drawing loads upon the paved and the macadamized roads in the north of France, where the country is flat, and the loads are considerable.

[1] Sir John Macneil's formulas are given in *Sir Henry Parnell on Roads*, page 464.

Table No. 320.—PERFORMANCE OF HORSES ON ROADS IN FRANCE.

(M. Charié-Marsaines.)

Season of the Year.	Description of Road.	Weight per Horse.	Speed in Miles per Hour.	Work Done per Hour in Tons Drawn One Mile.	Ratio of Paved Road to Macadamized Road.
		tons.	miles.	ton-miles.	
Winter,........	Pavement	1.306	2.05	2.677	1.644 to 1
	Macadam	.851	1.91	1.625	
Summer,......	Pavement	1.395	2.17	3.027	1.229 to 1
	Macadam	1.141	2.16	2.464	

The average daily work of a Flemish horse in the north of France is, on the same authority, as follows:—

Winter,........................	21.82	ton-miles per day in winter.
Summer,........................	27.82	„ „ in summer.
Mean for the year, say,...	25.00	„ „

It has already been stated, page 720, that a good horse can draw a weight of 1 ton at 2½ miles per hour, for from 10 to 12 hours a day—equivalent to ($1 \times 2\frac{1}{2} \times 10 =$) 25 tons drawn one mile per day. This is the same amount of performance as is above given from M. Charié-Marsaines.

Conclusion.—With the exception of Messrs. Eastons and Anderson at Bedford, the authorities on the tractional resistance to vehicles on common roads, ignore, with remarkable unanimity, the influence of sizes of the wheels and other essential particulars. It is better, therefore, to refrain from attempting to draw general conclusions, and to leave the figures "to speak for themselves."

RESISTANCE ON RAILWAYS.

The Author[1] deduced from experimental data, the following formulas for the resistance of locomotives and trains, under these conditions:—the permanent way in good order; the engine, tender, and train in good order; a straight line of rails; fair weather, and dry and clean rails; an average side wind, of average strength, varying (in the experiments) from *slight* to VERY *strong*:—

Resistance of Engine, Tender, and Train.	Resistance of Train alone.	
$R = 8 + \dfrac{v^2}{171}$;	$R' = 6 + \dfrac{v^2}{240}$;	(2), (3)

R = total resistance of engine, tender, and train, in lbs. per ton gross; R′ = resistance of train alone, in lbs. per ton; v = speed, in miles per hour.

[1] *Railway Machinery*, 1855, pages 297, 298.

For ordinary practice, to meet the unfavourable conditions, which may occur in combination, of frequent quick curves under one mile radius, and strong side and head winds, the Author estimated from his own observations that the resistance, as calculated by means of the foregoing rules, should be increased 50 per cent., or one-half more. On this basis, for speeds of

5, 10, 15, 20, 30, 40, 50, 60 miles per hour,

the frictional resistances per ton of engine, tender, and train, are,

12.2, 13, 14, 15.5, 20, 26, 34, 43.5 lbs.,

and the frictional resistances per ton of the train alone are,

9.15, 9.6, 10.5, 11.4, 14.6, 19.0, 24, 31.5 lbs.

RESISTANCE ON STREET TRAMWAYS.

The rails of street tramways are rolled with a groove for the guidance of the wheels by the flanges. The wheels of cars, therefore, do not run so freely as those of carriages or waggons on railways. The average frictional resistance of vehicles on tramways is 30 lbs. per ton, although an occasional maximum of 60 lbs. per ton may be reached, and, on the contrary, a minimum of, say, 15 lbs. per ton, when the rails are wet and clean, straight and new. The resistance due to clogging of the grooves of rails was brought into evidence by Mr. J. Arthur Wright, who found that, on a dusty day, on the steam lines of the Rouen tramways, when, despite every effort to keep the rails clear, the grooves became filled with dust and dirt, the engines consumed about 2¼ lbs. of coke per mile more than they did under more favourable circumstances, when the consumption averaged about 12 lbs. per mile. The excess is nearly 20 per cent. over.[1]

[1] These data are derived from *Tramways, their Construction and Working*, 1882, by D. Kinnear Clark, p. 180.

APPENDIX.

DR. SIEMENS' WATER-PYROMETER.

[*Appended to "Pyrometers," page* 326.]

This apparatus belongs to the second class of pyrometers described at page 327. In it, Dr. Siemens has reduced the water-pyrometer to a form complete and exact for purposes of scientific observation. The water is contained in a copper vessel capable of holding rather more than a pint. The sides and bottom of the vessel are fitted with an outer casing or jacket, the interspace of which is filled with felt, by which any radiation of heat from the vessel is prevented. A mercurial thermometer is fixed in the vessel, and immersed in the water. It is fitted with a small sliding scale in addition to the ordinary scale, which is graduated and figured with 50 degrees to 1 degree of the ordinary scale. Six solid copper cylinders are supplied with the pyrometer, each of which is accurately adjusted so that its capacity for absorbing heat shall be one-fiftieth of that of a pint of water. In using the pyrometer, a pint of water is measured into the copper vessel, and the sliding scale is set with its zero at the temperature of the water as indicated by the ordinary scale of the thermometer. A copper cylinder is put into the furnace or the hot-blast current, of which the temperature is to be measured, where it is left for a space of time of from 2 to 10 minutes, according to the intensity of heat to be measured. Having been thus raised to the temperature of the furnace or current, the cylinder is withdrawn and quickly dropped into the water; the temperature of the water is raised at the rate of 1° for each 50° of the temperature of the copper cylinder. The rise of the temperature may then be read off direct on the pyrometer scale. Add to the temperature thus noted, the observed initial temperature of the water, and the sum is the exact temperature required. For very high temperatures, cylinders of platinum may be employed.

ATMOSPHERIC HAMMERS.

[*Appended to "Air Machinery," page* 915.]

In the hammers designed by M. Chenot Ainé,[1] atmospheric air is employed as a spring, for the purpose of accumulating and of applying the motive power to the hammer. The hammer is a cylinder turned from end to end, and bored out to two different diameters. It is divided into two

[1] *Revue Industrielle*, December, 1876, page 521

chambers by a diaphragm at the middle. The lower end is thus completely inclosed, whilst the upper end is open. Two pistons fixed to one rod, passing through the diaphragm, play in the upper and lower chambers; and they receive a reciprocating motion from a crank overhead, driven by a band passed over a pulley fixed on the crank-shaft. The cylinder-hammer is thus floated on the pistons by means of air-cushions, of which there is one above the diaphragm, one below it, and a third below the lower piston; and it is impressed with a reciprocating movement following the reciprocations of the pistons, by the agency of these cushions of air. The height of the fall and the force of the blow, are regulated by the speed at which the machine is driven. There is no sensible heating or cooling of the working parts, and M. Chenot estimates that the efficiency of the machine amounts to 75 per cent. of the power communicated to the driving pulley.

The chief feature of interest in this machine is the employment of air compressed and expanded to two or three times its normal volume, without any inconvenience by either heat or cold. It is obvious that during the momentary actions and reactions, time is not afforded for the heating and cooling effects of changes of temperature in the air to take place. Hence the high efficiency.

BERNAYS' CENTRIFUGAL PUMP.

[*Appended to "Pumps," page* 944.]

Mr. Joseph Bernays constructs the discs of his pump with a double joint, the inner one being the joint universally employed around the suction-openings, by which the water is admitted from the suction-passages into the disc. The second, or outer, joint is at the extreme diameter of the disc, and it prevents the pressure of the water in the delivery-pipe from reacting on the outer faces of the revolving disc. A saving of power is thus effected, in reducing the loss by friction on the disc.

The form of the vanes of Mr. Bernays' encased pump, may be roughly described as semi-elliptical, or the half of a flat ellipse divided at its longest diameter; the concave surface being presented to the water in the direction of the motion. By the adoption of such a form, it is designed that the blade should scoop up the water arriving at the centre of the pump by its inner edge, and should project the water forward in a direction as nearly tangential as possible by its outer edge, at the circumference. When the pump is not encased, and the water is delivered into an open well or reservoir, the outer end of the vane is curved backwards for the purpose of facilitating the discharge radially. In this case, the blades acquire an ogee form, like Rankine's fan.

Mr. Bernays has supplied the following note on centrifugal pumps:—

"The only parts of a centrifugal pump which, when at work, absorb more power by friction than is due to the mere velocity of the water passing through the pump, are the outer faces of the revolving disc. These outer faces are surrounded by water quite or nearly stationary, and as they themselves revolve at a speed proportionate to the height to which the water is to be lifted, more or less independent of the quantity that passes through the pump, they tend to carry the surrounding water round with them.

According to the greater or less pressure under which the pump works, the friction produced will be greater or less, just as there is greater skin resistance in a vessel of greater draft than in one of light draft, both having an equal extent of surface. The saving of power effected by the removal of the pressure of the water in the delivery pipe of Mr. Bernays' pump, as above explained, is all the more necessary, since, with a centrifugal pump, the power required for driving it increases rapidly, and in a greater ratio than the heights of delivery to which such pump may have to be applied. This is a point to which the attention of engineers and users of centrifugal pumps has never been called, or, if it has ever been mentioned, it has been, and is, constantly lost sight of. Nevertheless the fact is clear, and the explanation very simple indeed. The only working parts of a centrifugal pump which, irrespective of the friction previously mentioned, actually propel the water and absorb the power applied to it, are the arms or vanes radiating from the centre to the outside of the disc. The shape of these arms may for a moment be left out of consideration, as their more or less perfect form accounts for a mere percentage only of the whole power used for driving a centrifugal pump. The main power is used in driving or pushing the arms against the water at a speed calculated to produce a pressure equal to or rather in proportion to the height to which the water is raised. Now, the speed at the outside diameter of the pump disc is approximately equal to that of a body falling from the height to which the water is lifted, or it is directly proportionate to the square root of that height. And as the direct resistance which the arms meet with in their rotation, is simply proportional to the height to which the water is to be lifted (the same as in common reciprocating pumps), it follows that the amount of power necessary for working the pump, is a function of the height multiplied by its square root, or $h\sqrt{h} = \sqrt{h^3}$.

Thus, a pump requiring 20 H.P. to raise water 10 feet high, will, if the height be increased to 40 feet, not merely require 4 times the power for the same quantity delivered, but 4 multiplied by $\sqrt{4}$, or 8 times, that is, 160 H.P. And it evolves from this, that although centrifugal pumps are an exceedingly useful and simple mechanical appliance for raising large quantities of liquids to moderate height,—and it may here be added, *variable* quantities to *variable* heights,—they should not be made use of for great heads of delivery, where the cost of the power employed to work them is any consideration."

STEAM-VACUUM PUMP.

[*Appended to "Pumps," page* 944.]

The steam-vacuum pump belongs to the class of pumps on Savary's old system, in which steam from the boiler is admitted into direct contact with the surface of the water to be forced. Experiments were conducted by Mr. J. F. Flagg, at the Cincinnati Exposition of 1875, on such a pump.[1] The water was drawn directly from a canal, through a 3-inch pipe, 155 feet

[1] *Journal of the American Society of Civil Engineers*, December, 1876, page 381.

long, with a lift of 10.83 feet. The head of pressure was regulated by means of a cock applied to the discharge pipe. The proportion of primed water in mixture with the steam was ascertained, and allowed for:—

	1st trial.	2d trial.
Effective pressure in boilers, lbs. per sq. inch,	72 lbs.	57 lbs.
Temperature of water in canal, Fahrenheit,	60°	61°.5
Do. effluent water, „	86°.9	73°.1
Pressure at the gauge, lbs. per sq. inch,	35.3 lbs.	16.3 lbs.
Do. feet of head,	81.8 ft.	37.7 ft.
Steam consumed per horse-power, per hour,	477.5 lbs.	390.7 lbs.
Coal do. do. (allowing 9 lbs. water per pound of coal),	53.1 lbs.	43.4 lbs.
Duty per 100 lbs. of coal, foot-pounds.	3,732,260	4,561,200

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THE END.

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